(a - 15 points) Explain diagonalization principle and use it show that the set of Real Numbers in the closed interval [0,1] is Uncountable.

Diagonaliation - Is a technique that you can use to show that ther complememnt of the diagonal is not part of it.

[0,1]

$$\begin{array}{l|lll} & a_{11} & a_{12} & a_{13} \\ \hline & a_{21} & a_{22} & a_{23} \\ & a_{31} & a_{32} & a_{33} \end{array}$$

(b - 12 points) Write out the *set*, $L$, of letters in your first name (write all letters in lower case)? What is the size of $2^L$ and why (give a one sentence justification)? Show three elements of $2^L$.

My name = Mike $L = \{M, i, k, e\}$

$2^L \to P(L) \to$ Power Set $\to$ Set of all subsets

Size of $2^L = 2^{|L|} = 2^4 = 16 \to$ Size of power set $= 2^n$ $n =$ size of set

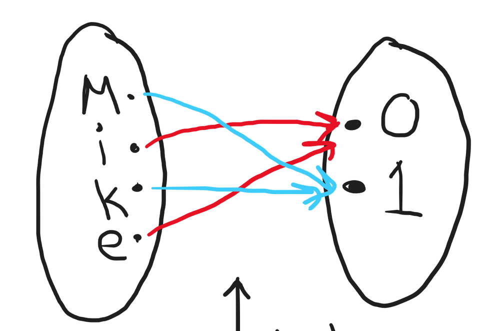
3 elements: $\{\emptyset\}$ $\{M, i\}$ $\{k, e\}$

↑ the null set is always a subset ☆

(c - 8 points) Define a function from $L$ (the same set of part b) to set $B = \{0, 1\}$ as follows: All vowels in $L$ are mapped to 0 and all consonants to 1. Diagram this function as in the book. How many functions are there from $L$ to the set $B$? Why (give a one sentence justification)?

$L = \{M, i, k, e\}$ $B = \{0, 1\}$

If a set A has m elements and set B has n elements, then the number of functions possible from A to B is $n^m$.




→ Consonant
→ VoweL

↑ don't draw bubbles on exam, use chart instead ↗

| | f(n) |
|---|---|
| M | 1 |
| i | 0 |
| k | 1 |
| e | 0 |

$\underbrace{|L| = 4}_{m}$ $\underbrace{|B| = 2}_{n}$ $\to$ $2^4 = 16$

(d - 4 points) What is the Busy Beaver problem?

The busy beaver problem aims at finding a terminating program of a given size that produces the most output possible.

I'll come back to this one ‼

(e - 2+6+3 points) Let $L$ from part b be considered an alphabet. Define a specific infinite language over $L$ in set-theoretic notation and sketch a proof (main ideas) that this language is countably infinite. Is the set of all *finite* languages over $L$ countable or uncountable (no proof needed)?

$L = \{M, i, k, e\}$

$P = \{M^*\}$

↑ my infinite language

$M^*$ examples:

$M, MM, MMM, \ldots \infty$

Proof ideas:

We have a way to count things by being able to list out the different types of elements

Since the language is the reption of M, we can map it to N (set of Natural Numbers) where the amount of M's is equivalent of q where q is an element of N. N is by definition countable, therefore our correspondance shows that the infinite language P is countable