

CSE 218
Quiz (Open-book Exam)
January 2021 Term
Date: 10 July 2021

Full marks: 35

Time: 35 mins + 10 mins for upload

Show all the necessary calculations wherever applicable. Each question carries 5 marks. Answer all the questions.

Answer of each question has to be written on a separate page. No page should contain the answers of two separate questions.

1. The distance covered by a rocket in meters from $t = 8\text{ s}$ to $t = 30\text{ s}$ is given by:

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

Consider the given integral $\int_8^{30} f(t)dt$ as the sum of two integrals $\int_8^{20} f(t)dt$ and $\int_{20}^{30} f(t)dt$. Evaluate the whole integral as follows:

- Evaluate the first integral $\int_8^{20} f(t)dt$ using Simpson's 1/3 rule. For this case, divide the interval into four equally spaced sub-intervals.
 - Evaluate the second integral $\int_{20}^{30} f(t)dt$ using trapezoidal rule. For this case, divide the interval into two sub-intervals.
 - Add the two results obtained in (a) and (b) to determine the value of the whole integral.
2. Use Newton's divided differences formula to find the polynomial interpolating $f(x) = \sin(x)$ through the equispaced points $0; \pi/4; \pi/2$.
3. From the clinical trial of an experimental growth factor drug, we got the following data, where D is the administered dosage unit and ΔH is the total unit of neutrophil increased. From this data, can we predict Δn for $D = 10$? If yes, what should be the most likely predicted value (you need to show all the calculations)? If not, why (you need to show all the reasonings)?

D	1	2	3	4	5	6	7
Δn	0.5	2.5	2.0	4.0	3.5	6.0	5.5

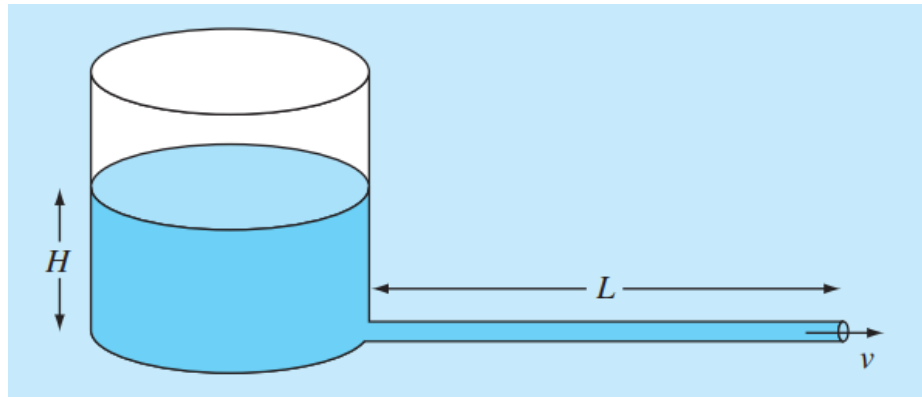
4.

x	1	2	3	5	7	8
$f(x)$	7	12	19	33	51	99

Calculate $f(4)$ using Lagrange polynomials of order 3. Choose the sequence of the points for your estimates to attain the best possible accuracy. Your solution should show the intermediate steps of calculation.

5. Explain why Gauss-Jordan technique requires more operations than Gauss elimination one in solving a system of linear equations.

6.



As depicted in the figure, the velocity of water, v (m/s), discharged from a cylindrical tank through a long pipe can be computed as

$$v = \sqrt{2gH} \tanh\left(\frac{\sqrt{2gH}}{2L}t\right)$$

where $g = 9.8 \text{ m/s}^2$, H = initial head (m), L = pipe length (m), and t = elapsed time.

Determine the head needed to achieve $v = 6.5 \text{ m/s}$ in 2s for a 4.5m long pipe by bisection method with a stopping criterion of $e_s = 1\%$. The height of the cylindrical tank is 3m.

For each iteration, show the lower bound, upper bound and estimated value of H and approximate relative error (e_s).

7. The variables x and y are related with each other by the following equation.

$$y = axe^{bx}$$

Here, a and b are constants.

Some values of y for the corresponding x are given in the following table.

x	y
0.1	0.75
0.2	1.25
0.4	1.45

Show how you can compute the values of a and b by least-square regression. **You don't have to perform detailed computation.** Mentioning the procedure will suffice.