



Figure 1: Position (x,y,z) by geometry

Position and Rotation matrixes are found using geometry instead of DH parameters in this approach. Let the world frame be the frame zero and position (x,y,z) is (0,0,0).

$$P_0^1 = (0, 0, 10)$$

P_0^2 can be determined by projecting $d2 * \sin(45)$ on the y-axis of the world frame. See the picture 1 for clarity.

$$P_0^2 = (-d2 * \cos(\pi/4) * \sin(\theta_1), d2 * \sin(\pi/4) * \cos(\theta_1), 10 - d2 * \cos(\pi/4))$$

$$P_0^3 = (-d2 * \cos(\pi/4) * \sin(\theta_1) - 5 * \cos(\theta_3) * \sin(\theta_1), d2 * \sin(\pi/4) * \cos(\theta_1) + 5 * \cos(\theta_3) * \cos(\theta_1), 10 - d2 * \cos(\pi/4) + 5 * \sin(\theta_3))$$

$$R_0^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-3\pi/4) & -\sin(-3\pi/4) \\ 0 & \sin(-3\pi/4) & \cos(-3\pi/4) \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} \cos(\theta_1) & \frac{\sin(\theta_1)}{\sqrt{2}} & \frac{-\sin(\theta_1)}{\sqrt{2}} & 0 \\ \sin(\theta_1) & \frac{-\cos(\theta_1)}{\sqrt{2}} & \frac{\cos(\theta_1)}{\sqrt{2}} & 0 \\ 0 & \frac{\sqrt{2}}{-1} & \frac{-1}{\sqrt{2}} & 10 \end{bmatrix}$$

R_0^1 is rotated by 135 degrees CW. By multiplying $R_0^1 = R_0^0 * R_0^1$ we get rotation from frame 0 to 1.

$$R_1^{inter1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\pi/2) & -\sin(-\pi/2) \\ 0 & \sin(-\pi/2) & \cos(-\pi/2) \end{bmatrix}$$

$$R_1^{inter2} = \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) \\ 0 & 1 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) \end{bmatrix}$$

$$R_0^2 = R_0^1 * R_1^{inter1} * R_1^{inter2};$$

$$T_0^2 = \begin{bmatrix} \frac{-\sin(\theta_1)}{\sqrt{2}} & \frac{\sin(\theta_1)}{\sqrt{2}} & \cos(\theta_1) & -d_2 * \cos(\pi/4) * \sin(\theta_1) \\ \frac{\cos(\theta_1)}{\sqrt{2}} & \frac{-\cos(\theta_1)}{\sqrt{2}} & \sin(\theta_1) & d_2 * \sin(\pi/4) * \cos(\theta_1) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 10 - d_2 * \cos(\pi/4) \end{bmatrix}$$

For R_2^3 rotation:

$$R_2^{inter0} = \begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) & 0 \\ \sin(\pi/4) & \cos(\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^{inter1} = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^{inter2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi/2) & -\sin(\pi/2) \\ 0 & \sin(\pi/2) & \cos(\pi/2) \end{bmatrix}$$

$$R_2^{inter3} = \begin{bmatrix} \cos(\pi/2) & -\sin(\pi/2) & 0 \\ \sin(\pi/2) & \cos(\pi/2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^3 = R_0^2 * R_2^{inter0} * R_2^{inter1} * R_1^{inter2} * R_2^{inter3}$$

$$T_0^3 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1)\cos(\theta_3) & \sin(\theta_1)\sin(\theta_3) & -d_2 * \cos(\pi/4) * \sin(\theta_1) - 5 * \cos(\theta_3) * \sin(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1)\cos(\theta_3) & -\cos(\theta_1)\sin(\theta_3) & d_2 * \sin(\pi/4) * \cos(\theta_1) + 5 * \cos(\theta_3) * \cos(\theta_1) \\ 0 & -\sin(\theta_3) & -\cos(\theta_3) & 10 - d_2 * \cos(\pi/4) + 5 * \sin(\theta_3) \end{bmatrix}$$

Inverse Kinematics

$$R_0^3 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1)\cos(\theta_3) & \sin(\theta_1)\sin(\theta_3) \\ \sin(\theta_1) & \cos(\theta_1)\cos(\theta_3) & -\cos(\theta_1)\sin(\theta_3) \\ 0 & -\sin(\theta_3) & -\cos(\theta_3) \end{bmatrix}$$