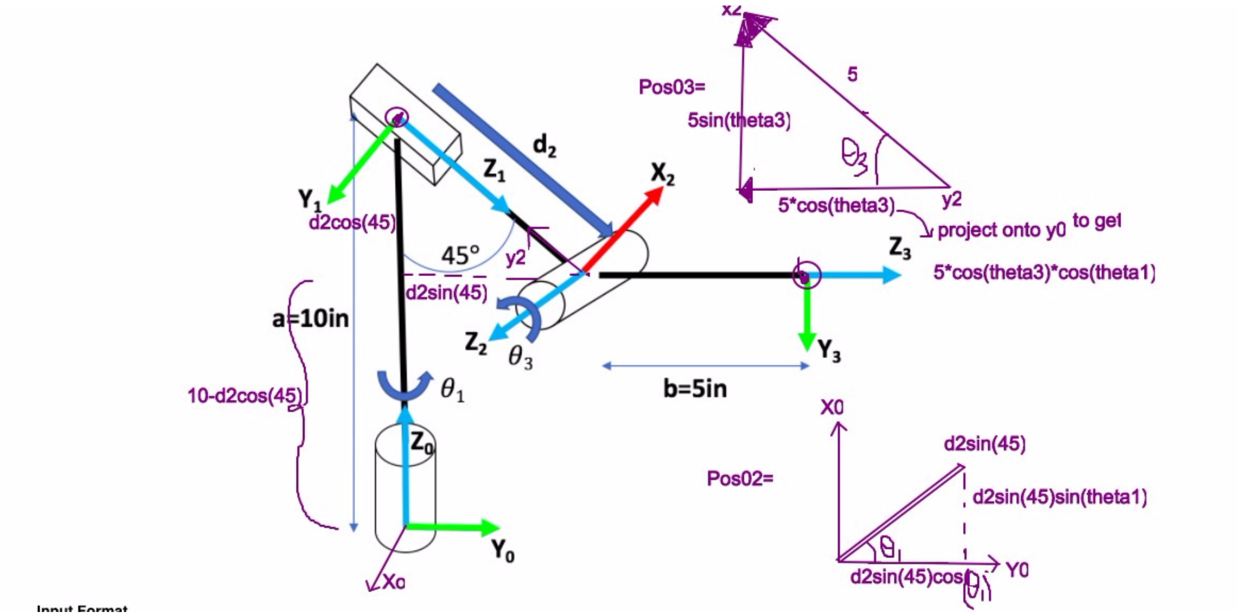

Figure 1: Postion (x,y,z) by geomentry

Position and Rotation matrixes are found using geomentry instead of DH parameters in this approach. Let the world frame be the frame zero and position (x,y,z) is (0,0,0).

$P_0^1 = (0, 0, 10)$

P_0^2 can be determined by projecting $d2 * sin(45)$ on the y-axis of the world frame. See the picture 1 for clarity.

$P_0^2 = (-d2 * cos(\pi/4) * sin(\theta_1), d2 * sin(\pi/4) * cos(\theta_1), 10 - d2 * cos(\pi/4))$

$P_0^3 = (-d2 * cos(\pi/4) * sin(\theta_1) - 5 * cos(\theta_3) * sin(\theta_1), d2 * sin(\pi/4) * cos(\theta_1) + 5 * cos(\theta_3) * cos(\theta_1), 10 - d2 * cos(\pi/4) + 5 * sin(\theta_3))$

$$R_0^0 = \begin{bmatrix} cos(\theta_1) & -sin(\theta_1) & 0 \\ sin(\theta_1) & cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(-3pi/4) & -sin(-3pi/4) \\ 0 & sin(-3pi/4) & cos(-3pi/4) \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} cos(\theta_1) & \frac{sin(\theta_1)}{\sqrt{2}} & \frac{-sin(\theta_1)}{\sqrt{2}} & 0 \\ sin(\theta_1) & \frac{-cos(\theta_1)}{\sqrt{2}} & \frac{cos(\theta_1)}{\sqrt{2}} & 0 \\ 0 & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 10 \end{bmatrix}$$

R_0^1 is rotated by 135 degress CW. By multiplying $R0^1 = R0^0 * R0^1$ we get rotation from frame 0 to 1.

$$R_1^2 inter1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(-pi/2) & -sin(-pi/2) \\ 0 & sin(-pi/2) & cos(-pi/2) \end{bmatrix}$$

$$R_1^2 inter2 = \begin{bmatrix} cos(pi/2) & 0 & sin(pi/2) \\ 0 & 1 & 0 \\ -sin(pi/2) & 0 & cos(pi/2) \end{bmatrix}$$

$R02 = R01 * R12_inter1 * R12_inter2;$

$$T_0^2 = \begin{bmatrix} \frac{-sin(\theta_1)}{\sqrt{2}} & \frac{sin(\theta_1)}{\sqrt{2}} & cos(\theta_1) & -d2 * cos(\pi/4) * sin(\theta_1) \\ \frac{cos(\theta_1)}{\sqrt{2}} & \frac{-cos(\theta_1)}{\sqrt{2}} & sin(\theta_1) & d2 * sin(\pi/4) * cos(\theta_1) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 10 - d2 * cos(\pi/4) \end{bmatrix}$$

For R_2^3 rotation:

$$R_2^3 inter0 = \begin{bmatrix} cos(\pi/4) & -sin(\pi/4) & 0 \\ sin(\pi/4) & cos(\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^3 inter1 = \begin{bmatrix} cos(\theta_3) & -sin(\theta_3) & 0 \\ sin(\theta_3) & cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 inter2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(pi/2) & -sin(pi/2) \\ 0 & sin(pi/2) & cos(pi/2) \end{bmatrix}$$

$$R_2^3 inter3 = \begin{bmatrix} cos(\pi/2) & -sin(\pi/2) & 0 \\ sin(\pi/2) & cos(\pi/2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_0^3 = R_0^2 * R_2^3 inter0 * R_2^3 inter1 * R_2^3 inter2 * R_2^3 inter3$

$$T_0^3 = \begin{bmatrix} cos(\theta_1) & \frac{sin(\theta_1)(cos(\theta_3)-sin(\theta_3)}{2} - \frac{sin(\theta_1)(cos(\theta_3)+sin(\theta_3)}{2} & \frac{-sin(\theta_1)(cos(\theta_3)+sin(\theta_3)}{2} - \frac{sin(\theta_1)(cos(\theta_3)-sin(\theta_3)}{2} & -d2 * cos(\pi/4) \\ sin(\theta_1) & \frac{cos(\theta_1)(cos(\theta_3)+sin(\theta_3)}{2} - \frac{cos(\theta_1)(cos(\theta_3)-sin(\theta_3)}{2} & \frac{cos(\theta_1)(cos(\theta_3)+sin(\theta_3)}{2} + \frac{cos(\theta_1)(cos(\theta_3)-sin(\theta_3)}{2} & d2 * sin(\pi/4) * \\ 0 & -cos(\theta_3) & sin(\theta_3) & 10 - d2 \end{bmatrix}$$

Inverse Kinematics

$$R_0^3 = \begin{bmatrix} cos(\theta_1) & \frac{sin(\theta_1)(cos(\theta_3)-sin(\theta_3)}{2} - \frac{sin(\theta_1)(cos(\theta_3)+sin(\theta_3)}{2} & \frac{-sin(\theta_1)(cos(\theta_3)+sin(\theta_3)}{2} - \frac{sin(\theta_1)(cos(\theta_3)-sin(\theta_3)}{2} \\ sin(\theta_1) & \frac{cos(\theta_1)(cos(\theta_3)+sin(\theta_3)}{2} - \frac{cos(\theta_1)(cos(\theta_3)-sin(\theta_3)}{2} & \frac{cos(\theta_1)(cos(\theta_3)+sin(\theta_3)}{2} + \frac{cos(\theta_1)(cos(\theta_3)-sin(\theta_3)}{2} \\ 0 & -cos(\theta_3) & sin(\theta_3) \end{bmatrix}$$

From the Rotation matrix, we can find θ_1 and θ_3. d_2 can be found using the z component of Pos_0^3.

$\theta_1 = atan2(R(2,1), R(1,1));$

$\theta_3 = atan2(R(3,3), -R(3,2));$

$d2 = -(z - 5 * sin(\theta_3) - 10)/cos(\pi/4);$