

SMDM Week 2 Quiz 2 - Solution

1. If there are 12 horses in a race, numbered 1 to 12, and all the horses have an equal probability of winning, what is the probability that the horses bearing number 3, 5 or 8 will win the race?

Probability of A or B or C = Probability of A + Probability of B + Probability of C

Probability of 3 or 5 or 8 = Probability of 3 + Probability of 5 + Probability of 8

$$= 1/12 + 1/12 + 1/12$$

$$= \frac{1}{4}$$

2. From a full deck of 52 cards, 1 card is drawn randomly. What is the probability that the card is either a spade or a king?

$$(13+4-1)/52 = 16/52$$

13 spades

4 kings

1 is subtracted because a king could be of a spade.

3. A card is drawn at random from a standard deck of cards. Let A be the event 'card is a spade' and let B be the event 'card is a king.'

Note: We keep the card back into the deck after drawing.

(a) Are A and B mutually exclusive?

(b) Are A and B independent?

Please select the right set of answers from below:

Two events are independent if the following are true:

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

$$P(A \text{ AND } B) = P(A) * P(B)$$

Here $P(A|B) = 13/52$ - because the case is with replacement

$P(A) = 13/52$ - As there are 13 spades in a deck of cards

$P(B|A) = 4/52$ - because the case is with replacement

$P(B) = 4/52$ - As there are 4 kings in a deck of cards

So we can say that they are independent.

A and B are mutually exclusive events if they cannot occur at the same time. This means that A and B do not share any outcomes and $P(A \text{ AND } B) = 0$.

As we can see,

$$P(A \text{ AND } B) = P(A) * P(B) = 13/52 * 4/52 \neq 0$$

So we can't say that they are mutually exclusive.

4. An upscale sari shop stocks only silk and cotton handloom saris. Customers are found to buy silk sari with a probability of 45% and cotton sari with a probability of 75%. If 5% of the customers do not buy anything, then what proportion buys both cotton and silk sari?

A: Buys silk $P(A) = 0.45$

B: Buys cotton $P(B) = 0.75$

$$P(A \cup B)^c = P(\text{Does not buy any sari}) = 0.05 = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$P(A \cup B) = 1 - 0.05 = 0.95$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.45 + 0.75 - 0.95 = 0.25 \text{ or } 25\%$$

5. A consumer research survey sampled 200 men and 200 women to find out whether they prefer to drink plain water or soft drink when they are really thirsty. 280 reported they prefer to drink plain water. Of the group preferring a soft drink, 80 were men and 40 were women. What is the probability that a randomly chosen man will prefer a soft drink?

Number of men = 200

Number of men preferring to drink soft drink: 80

The probability that a randomly chosen man will prefer a soft drink: $80/200 = 0.4 = 40\%$

6. A survey of 300 families was conducted to study the income level versus brand preference.

Income\ Brand Preferred	Brand1	Brand2	Brand3	Total
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High	55	45	20	120
Medium	45	25	25	95
Low	25	35	25	85
Total	125	105	70	300

If a family is selected at random, what is the probability that it belongs to High Income?

Probability of a randomly selected family belonging to high income:

Number of families belonging to high income / Total number of families = $120/300 = 0.4$

7.

	B'	B'
A	10	20
A'	20	40

The conditional probability of A|B is

$P(A \text{ AND } B)/P(B) = (10/90)/(30/90) = 10/30$

8. A company has 2 machines that produce widgets. An older machine produces 23% defective widgets, while the new machine produces only 8% defective widgets. In addition, the new machine produces 3 times as many widgets as the older machine does. Given that a randomly chosen widget was tested and was found to be defective, what is the probability that it was produced by the new machine?

Prob of new = 0.75

Prob of old = 0.25

Prob of old defect = 0.23

Prob of new defect = 0.08

$$(0.75 \cdot 0.08) / ((0.75 \cdot 0.08) + (0.25 \cdot 0.23)) = 0.511$$

9. A student is taking a multiple-choice exam which has 7 test questions and each has 4 options. Assume that the student has no knowledge of the subject material but guesses the correct answer with a probability of 0.75. What is the probability that she will answer at most 3 questions correctly?

Here, we have to calculate the $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

We can use the binomial probability dist formula for this given as follows:

$$P(x) = n! / [(n-x)! \cdot x!] \cdot \text{prob_of_success}^x \cdot \text{prob_of_faliure}^{(n-x)}$$

So,

$$P(0) = 7! / [(7-0)! \cdot 0!] \cdot 0.75^0 \cdot (1-0.75)^7 = 0.00006103$$

Similarly,

$$P(1) = 0.0012817$$

$$P(2) = 0.0115356$$

$$P(3) = 0.057678$$

$$\text{So, } P(X \leq 3) = P(0) + P(1) + P(2) + P(3) = 0.0705$$

That is equal to 7 percent.

10. In Normal Distribution mean, median & mode are all equal to one another.

Yes, in a perfectly normal distribution, all three will produce the same value.

