Minor - 2

① The gain in weight of 2 random samples of patients fed on 2 different diets. A and B are given below. Examine whether the difference in mean increase in weight is significant?

Diet - A	13	14	10	11	2	16	10	8	
Diet - B	7	10	12	8	10	11	9	10	11

X	Y	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
13	7	2.5	6.25	-2.78	7.7284
14	10	3.5	12.25	0.22	0.0484
10	12	-0.5	0.25	2.22	4.9284
11	8	0.5	0.25	-1.78	3.1634
2	10	-8.5	72.25	0.22	0.0484
16	11	5.5	30.25	1.22	1.4884
10	9	-0.5	0.25	-0.78	0.6084
8	10	-2.25	5.0625	0.22	0.0484
				1.22	1.4884
	11				
84	88		126.8125		19.5556

$$\bar{x} = \frac{\Sigma x_i}{n_1} = \frac{84}{8} = 10.5, \quad \bar{y} = \frac{\Sigma y_i}{n_2} = \frac{88}{9} = 9.78$$

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left[\Sigma(x_i - \bar{x})^2 + \Sigma(y_i - \bar{y})^2\right]$$

$$= \frac{1}{8+9-2}\left[126.81 + 19.56\right] = \frac{146.37}{15}$$

$s^2 = 9.758$, $S = 3.12$

step① $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 < \mu_2$ (one tail)

step② Level of significance $\alpha = 5\%$

step③

$$t = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{10.5 - 9.78}{3.12\sqrt{\frac{1}{8} + \frac{1}{9}}}$$

$$t = \frac{0.72}{3.12(0.49)} = 0.47$$

step④ degree of freedom $\nu = n_1 + n_2 - 2 = 15$

$t_\alpha = 1.753$

step⑤ $t_{cal} < t_\alpha$

Hence; Accept H_0, Reject H_1

② To Determine where there really is a relationship b/w an employee's performance in company's training program and his or her ultimate success in the job, the company takes a sample of 400 cases from its very extensive files and obtained the results shown in the following table.

		Performance in Training program		
		Below Average	Average	Above average
Success in Job (Employers Rating)	Poor	23	60	29
	Average	28	79	60
	Very Good	9	49	63

		Performance in training Prog			
		Below Avg	Avg.	above avg	Row total
Success in Job (employers Rating)	Poor	23	60	29	112
	Average	28	79	60	167
	very Good	9	49	63	121
	Column total	60	188	152	400

The expected frequency for each

$$E_i = \frac{\text{Row total} \times \text{column total}}{\text{Grand Total}}$$

$(E_{23}) = \frac{112 \times 60}{400} = 16.8$

$(E_{60}) = \frac{112 \times 188}{400} = 52.6$

$(E_{29}) = \frac{112 \times 152}{400} = 42.5$

$E_{(28)} = \frac{167 \times 60}{400} = 25.0$

$E(79) = \frac{167 \times 188}{400} = 78.4$

$E(60) = \frac{167 \times 152}{400} = 63.4$

$E(9) = \frac{121 \times 60}{400} = 18.15$

$E_{(49)} = \frac{121 \times 188}{400} = 56.8$

$E(63) = \frac{121 \times 152}{400} = 45.9$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
23	16.8	6.2	38.44	2.28
60	52.6	7.4	54.76	1.02
29	42.5	-13.5	182.25	4.32
28	25.0	3	9	0.34
79	78.4	0.6	0.30	0.003
60	63.4	-3.4	11.56	0.18
9	18.15	-9.15	83.12	4.61
49	56.8	-7.8	60.84	1.08
63	45.9	17.1	292.41	6.30
				$\Sigma = 20.133$

Step 1

H_0: no sig diff

H_1: sig diff

Step 2 L.O.S $\alpha = 5\%$

(3) Test stats $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 20.133$

(4) Degree of freedom $= (m-1)(n-1) = (3-1)(3-1)$
$= 2 \times 2 = 4$

(5) Critical value (χ^2_α) at 5% LOS & dof

$\therefore \chi^2_\alpha = 9.48$

(5) $\chi^2 > \chi_\alpha^2 \Rightarrow$ reject H_0
Accept H_1

3) Calculate correlation coeff. b/w x and y

x	1	3	4	5	7	8	10
y	2	6	8	10	14	16	20

x	y	x^2	y^2	xy
1	2	1	4	2
3	6	9	36	18
4	8	16	64	32
5	10	25	100	50
7	14	49	296	98
8	16	64	256	128
10	20	100	400	200
38	264	76	1056	528

$n=7$

$$r = \frac{n\sum x_iy_i - (\sum x_i)(\sum y_i)}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\sqrt{n\sum y_i^2 - (\sum y_i)^2}}$$

$$= \frac{7(528) - 38(76)}{\sqrt{7(264) - 1444}\sqrt{7(1056) - 5776}} = \frac{3696 - 2888}{20.099 \times 040.19}$$

$r = 1.000049877$

4) Given following Aptitude and I.Q scores for a group of students, compute the rank correlation coefficient b/w them

Aptitude score	57	58	59	59	60	61	60	64
I.Q score	95	108	95	106	120	126	106	110

59 repeated 2 times $m_1 = 2$

60 also repeated 2 times $m_2 = 2$

X	Y	Rx	Ry	d=Rx−Ry	d^2
57	97	8	7	1	1
58	108	7	5	2	4
59	95	5.5	8	−2.5	6.25
59	106	5.5	6	−0.5	0.25
60	120	3.5	2	1.5	2.25
61	126	2	1	1	1
60	113	3.5	3	0.5	0.25
64	110	1	4	−3	9
					24

$$\rho = 1 - \left[\frac{6\left[\Sigma d^2 + \frac{1}{12} m_1(m_1^2 - 1) + \frac{1}{2} m_2(m_2^2 - 1)\right]}{n(n^2-1)}\right]$$

$$\rho = 1 - \left[\frac{6\left[24 + \frac{1}{12}(6) + \frac{1}{12}(6)\right]}{504}\right]$$

$$\rho = 1 - \left[\frac{150}{504}\right] = \boxed{\rho = 0.702}$$

5) for the following bivariate data obtain the two lines of regression. Determine of value of y when X=3.5.

X	1	2	3	4	5	6.
Y	14	33	40	63	76	85

3
76
× 5
380

X	Y	X^2	Y^2	XY
1	14	1	196	14
2	33	4	1089	66
3	40	9	~~1089~~ 1600	120
4	63	16	3969	252
5	76	25	5776	380
6	85	36	7225	510
21	311	91	19855	1342

Regression of y on X

$y-\bar{y} = byx\,(x-\bar{x})$

$$byx = \frac{n\Sigma xy-(\Sigma x)(\Sigma y)}{n\Sigma x^2-(\Sigma x^2)} = \frac{6(1342)-(21)(311)}{6(91)-441}$$

$$= \frac{8052-653)}{546-441} = \frac{1521}{105}$$

$byx = 14.486$

$\bar{x} = \frac{\Sigma x}{n} = \frac{21}{6} = 3.5$

$\bar{y} = \frac{\Sigma y}{n} = \frac{311}{6} = 51.833$

$y-51.83 = 14.486\,(x-3.5)$

$y = 14.486x+51.833-50.701$

$\boxed{y = 14.486x+1.132}$

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if $x = 3.5$

$y = 14.486(3.5) + 1.132$

$y = 50.701 + 1.132$

$\boxed{y = 51.833}$

Regression of x on y

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$$b_{xy} = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{6(1342) - (21)(311)}{6(19855) - 96721} = \frac{8052 - 6531}{119130 - 96721}$$

$$= \frac{1521}{22409}$$

$b_{xy} = 0.068$

$\bar{x} = 3.5 \quad , \quad \bar{y} = 51.833$

$(x - 3.5) = 0.067(y - 51.83)$

$x = 0.067y + 3.5 - 3.\cancel{4}472$

$\boxed{x = 0.067y - 0.02739}$

if $x = 3.5 \quad y = ?$

$y = 14.485x + 1.133$

$y = 14.485(3.5) + 1.133 \Rightarrow y = 50.6975 + 1.133$

$y = 51.830$

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Fit

6) Second degree parabola $y=a+bx+cx^2$ for the following data and use to determine value of y corresponding to value of $x=6.2$ and value of x when $y=14.5$

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

req eq

$$y=a+bx+cx^2 \Rightarrow \Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

x	y	x^2	x^3	x^4	xy	x^2y
1	9	1	1	1	9	9
2	8	4	8	16	16	32
3	10	9	27	81	30	90
4	12	16	64	256	48	192
5	11	25	125	625	55	275
6	13	36	216	1296	78	468
7	14	49	343	2401	98	686
8	16	64	512	4096	128	1024
9	15	81	729	6561	135	1215
45	108	285	2025	15333	597	3991

$n=9$

$$108 = 9a + b(45) + c(285)$$

$$597 = a(45) + b(285) + c(2025)$$

$$3991 = a(285) + b(2025) + c(15333)$$

$a = \frac{307}{42} = 7.309$, $b = 0.917$, $C = 3.246$

$$\boxed{y = 7.309 + 0.917x + 3.246x^2}$$

Value of y when $x = 6.2$

$y = 7.309 + 0.917(6.2) + 3.246(6.2)^2$

$= 137.77$

Value of x when $y = 14.5$

$14.5 = 7.309 + 0.917x + 3.246(x)^2$

$3.246x^2 + 0.917x = 7.191$

$x_1 = 1.353$

$x_2 = -1.636$

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8. Consider a single server queuing system with Poisson input, exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour and the maximum permissible calling units in the system is two. Derive the steady-state probability distribution of the of no. of calling units in the system, and then calculating the expected number in the system.

λ = arrival rate = 3 units/hr

Expected service time is 0.25 hr

service time $\mu = 4$ units/hr

Traffic intensity $\rho = \frac{\lambda}{\mu} = \frac{3}{4}$

no. of units allowed in the system $N = 2$

let P_n be the prob for n units to be in the sys

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-\frac{3}{4}}{1-\left(\frac{3}{4}\right)^3} = \frac{(1/4)}{\left(\frac{64-27}{64}\right)} = 1/4 \times \frac{64}{37}$$

$$= \frac{16}{37}$$

$P_n = \rho^n P_0$

$P_1 = \rho . P_0 = \frac{3}{4} \times \frac{16}{37} = \frac{12}{37}$

$P_2 = \rho^2 . P_0 = \frac{9}{16} \times \frac{16}{37} = 9/37$

$\therefore P_0 + P_1 + P_2 = 1$

The expected no. in the system

$$E(n) = \sum_{n=0}^{2} nP_n = \cancel{\ }$$

$$= 1P_1 + 2P_2 = \frac{12}{37} + \frac{9}{37}(2)$$

$$= \frac{12}{37} + \frac{18}{37} = \frac{30}{37}$$

$\therefore \frac{30}{37}$ units on avg in the sys

~~#~~ infinite model

$\lambda = 10/hr$, $\mu = 12/hr$

$\rho = \frac{\lambda}{\mu} = \frac{10}{12} = \frac{5}{6}$

(i) $P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-5/6}{1-\cancel{8}(\frac{5}{6})^2} = \frac{1-0.83}{1-(0.83)^2} = \frac{1-0.83}{1-0.688}$

$= \frac{0.17}{0.321}$

$= 0.529$

(i) $P_0 = 1 - \frac{5}{6} = \frac{1}{6}$

(ii) ~~A~~vg no. of customers in the que

$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{(10)^2}{12(12-10)} = \frac{100}{12(2)}$

$= \frac{\cancel{100}^{50}}{\cancel{24}_{12}}$

$= \frac{50}{12}$

$= \frac{25}{6}$

$= 4.166$

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