Subject: Probability and Statistics **Subject code: MR22-1BS0108**			
Qno	**Question**	**Marks**	**Section**
1	(a) A Statistics class for engineers consists of 25 Industrial, 10 Mechanical, 10 Electrical and 8 Civil Engineering students. If a student is randomly selected by the instructor to answer a question, find the probability that the student chosen is (i) an industrial engineering (ii) a civil or an electrical engineering. (b) There are 3 economists, 4 engineers, 2 statisticians and 1 doctor. A committee of 4 from among them is to be formed. Find the probability that the committee: (i) consist of one of each kind (ii) has at least one economist (iii) has the doctor as a member and three others.	8	Section-I
2	A fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective?	8	Section-I
3	In a certain assembly plant, three machines B_1, B_2 and B_3 make 30%, 45% and 25% respectively of the products. It is known from the past experience that 2%, 3% and 2% of the products made by each machine, respectively are defective. What is the probability that the product is defective? If the product was chosen randomly and found to be defective, what is the probability that it was made by machine: (i)B_1 (ii)B_2	8	Section-I
4	In a bolt factory, machines A, B and C manufactures respectively 20%, 30% and 50% of the total output. Of their outputs 5, 4 and 2 percent are known to be defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by (i) Machine A (ii) Machine B?	8	Section-I
5	Verify whether the following functions can serve as probability distributions or not. If yes, write down the cumulative probability distributions in each case: $(a)\ \ f(x) = \frac{x-2}{2},\ for\ x = 1,2,3,4.$ $(b)\ f(x) = \frac{x^2}{25},\ for\ x = 0,1,2,3,4$	8	Section-I
6	The error in reaction temperature, in ^0C, for a controlled laboratory experiment is a continuous random variable X having the probability density function: $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & elsewhere \end{cases}$, (a) Verify that the above is probability density function or not (b) Find $P(0 < X < 1), P(0 < X \leq 1.5)$ (c) Compute the mean, variance and standard deviation of X	8	Section-I
7	Assuming that half of the population is vegetarian so that the chance of an individual being a vegetarian is 0.5 and assuming that 100 investigators can take a sample of 10 individuals to see whether they are vegetarians, how many investigators would you expect to report that three people or less were vegetarians?	8	Section-I

8	In a certain town 20% of the population is literate, and assume that 200 investigators take a sample of 10 individuals, each to see whether they are literate. How many investigators would you expect to report that three people or less are literates in the sample?	8	Section-I
9	The number of breakdowns of a computer is a random variable having Poisson distribution with a mean of 1.8 per month. Find the probability that the computer will function for a month a) without any breakdowns b) with only one breakdown c) with at least 2 breakdowns.	8	Section-I
10	A manufacturer of cotter pins knows that 2% of his product is defective. If he sells cotter pins in boxes of 200 and guarantees that not more than 5 pins will be defective. What is the probability that a box will fail to meet the guaranteed quality by using (i) formula for binomial distribution (ii) Poisson approximation to the binomial distribution?	8	Section-I
11	Given a standard normal distribution, find the area under the curve that lies: a) to the left of z = -1.39; b) to the right of z = -0.89 c) between z = -0.48 and z = 1.74	8	Section-II
12	If a random variable has the standard normal distribution, find the probability that it will take on a value: a) less than 1.65; b) greater than -1.95; c) lies in between -1.75 and -1.04	8	Section-II
13	Given a standard normal distribution, find the value of k such that a) $P(Z > k) = 0.2946$; b) $P(Z < k) = 0.0427$; c) $P(k < Z < -0.18) = 0.4197$. d) $P(-0.93 < Z < k) = 0.7235$.	8	Section-II
14	A research scientist reports that mice will live an average of 40 months when their diets are sharply restricted and then enriched with vitamins and proteins. Assuming that the lifetimes of such mice are normally distributed with a standard deviation of 6.3 months, find the probability that a given mouse will live a) more than 32 months; b) less than 28 months; c) between 37 and 49 months.	8	Section-II
15	The time required to assemble a piece of machinery is a random variable having approximately a normal distribution with mean (μ) is 12.9 minutes and standard deviation (σ) is 2.0 minutes. What is the probability that the assembly of a piece of machinery of this kind will take: (i) at least 11.5 minutes (ii) anywhere from 11.00 to 14.80 minutes?	8	Section-II
16	The hourly wages of 1000 workmen are normally distributed with a mean of Rs. 70 and a standard deviation of Rs.5. Estimate the number of workers whose hourly wages will be	8	Section-II

	(i) between Rs.69 and Rs.72 (ii) more than Rs.75 (iii) less thanRs.63.		
17	A population consists of 4 observations 10, 20, 30, 40. Determine the mean and variance of the population. Write all the possible samples of size 2 (with replacement and without replacement). Construct the sampling distribution about mean. Show that the mean of sample means is equal to the population mean.	8	Section-II
18	A population consists of observations 2, 3, 6, 8 and 11. Determine the mean and variance of the population. Write all the possible samples of size 2 (with replacement and without replacement). Construct the sampling distribution about mean. Show that the mean of sample means is equal to the population mean.	8	Section-II
19	An industrial engineer collected data on the labor time required to produce an order of automobile mufflers using a heavy stamping machine. The data on times (hours) for 52orders of different parts has a mean of 1.865 hours with a standard deviation of 1.250 hours. What can one assert with 99% confidence about the maximum error, if sample mean is used as a point estimate the true population means labor time required to run the heavy stamping machine?	8	Section-II
20	The management of a manufacturing firm wishes to determine the average time required to complete a certain manual operation. There should be 95% confidence that the error in the estimate will not exceed 2 minutes. How large a sample size is required if the standard deviation of the time needed to complete the manual operation is estimated by a time and motion study expert as 10minutes?	8	Section-II
21	(a) An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a 0.05 level of significance. (b) The manufacturer of television tubes knows from the past experience that the average life of a tube is 2000 hours with a standard deviation of 200 hours. A sample of 100 tubes has an average life of 1950 hours. Test at 0.05 level of significance if this sample came from a normal population of mean 2000hours.	8	Section-III
22	An investigation of two kinds of photocopying equipment showed that 71 failures of the first kind of equipment took on the average 83.2 minutes to repair with a standard deviation of 19.3 minutes, while 75 failures of the second kind of equipment took on the average 90.8 minutes to repair with a standard deviation of 21.4 minutes. Test the null hypothesis $\mu_1 - \mu_2 = 0$ (the hypothesis that on the average it takes an equal amount of time to repair either kind of equipment) against the alternative hypothesis $\mu_1 - \mu_2 \neq 0$ at the 0.05 level of significance.	8	Section-III
23	A manufacturer claims that the average tensile strength of Thread-A exceeds the average tensile strength of Thread-B by at least 12 kilograms. To test this claim, 50 pieces of each type of thread were tested under similar conditions. Type A thread had an average strength of 86.7 kilograms with a standard deviation of 6.28 kilograms, while Type B thread had an average tensile strength of 77.8 kilograms with a standard deviation of 5.61kilograms. Test themanufacturer's claim using a 0.05 level of significance.	8	Section-III

<table>
<tr><td>24</td><td>Ten individuals are chosen at random from a normal population and their heights are found to be: 63, 63, 66, 67, 68, 69, 70, 70, 71, 71 in inches. Test if the sample belongs to the population whose mean height is 66 inches?</td><td>8</td><td>Section-III</td></tr>
<tr><td>25</td><td>A manufacturer of gunpowder had developed a new powder which is designed to produce a muzzle velocity equal to 3000ft/sec. Seven shells are loaded with the charge and the muzzle velocities are measured. The resulting velocities are: 3005, 2935, 2965, 2995, 3905, 2935 and 2905. Do these data present sufficient evidence to indicate that the average velocity differs from 3000ft/sec?</td><td>8</td><td>Section-III</td></tr>
<tr><td>26</td><td>(a) The gain in weight of two random samples of patients fed on two different Diets: A and B are given below. Examine whether the difference in mean increase in weight is significant?
<table>
<tr><td>Diet-A</td><td>13</td><td>14</td><td>10</td><td>11</td><td>2</td><td>16</td><td>10</td><td>8</td><td></td></tr>
<tr><td>Diet-B</td><td>7</td><td>10</td><td>12</td><td>8</td><td>10</td><td>11</td><td>9</td><td>10</td><td>11</td></tr>
</table>
(b) The following data represent the running times (in minutes) of films produced by two motion-picture companies:
<table>
<tr><td>Company-1</td><td>102</td><td>86</td><td>98</td><td>109</td><td>92</td><td></td><td></td></tr>
<tr><td>Company-2</td><td>81</td><td>165</td><td>97</td><td>134</td><td>92</td><td>87</td><td>114</td></tr>
</table>
Test the hypothesis that the average running time of films produced by company-2 exceeds the average running time of film produced by the company- 1 by 10 minutes. Use a 0.01 level of significance.</td><td>8</td><td>Section-III</td></tr>
<tr><td>27</td><td>Two independent samples of 8 and 7 items respectively had the following values of the variable:
<table>
<tr><td>Sample – I</td><td>9</td><td>11</td><td>13</td><td>11</td><td>15</td><td>9</td><td>12</td><td>14</td></tr>
<tr><td>Sample – II</td><td>10</td><td>12</td><td>10</td><td>14</td><td>9</td><td>8</td><td>10</td><td></td></tr>
</table>
Do the estimates of population variances differ significantly?</td><td>8</td><td>Section-III</td></tr>
<tr><td>28</td><td>200 digits were chosen at random from a set of tables. The frequency of the digits were:
<table>
<tr><td>Digits</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>18</td><td>19</td><td>23</td><td>21</td><td>16</td><td>25</td><td>22</td><td>20</td><td>21</td><td>15</td></tr>
</table>
Use Chi-Square test, to assess the correctness of hypothesis that the digits were distributed in equal number in the table at the level of significance0.05.</td><td>8</td><td>Section-III</td></tr>
<tr><td>29</td><td>A die is thrown 60times with the following results.
<table>
<tr><td>Face</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>Frequency</td><td>8</td><td>7</td><td>12</td><td>8</td><td>14</td><td>11</td></tr>
</table>
Test at 5% level of significance if the die is unbiased, at 0.05 level of significance.</td><td>8</td><td>Section-III</td></tr>
<tr><td>30</td><td>To determine whether there really is a relationship between an employee’s performances in the company’s training programme and his or her ultimate success in the job, the company takes a sample of 400 cases from its very extensive files and obtained the results shown in the following table:
<table>
<tr><td></td><td>Performance in Training</td></tr>
</table></td><td>8</td><td>Section-III</td></tr>
</table>

			Programme			
			Below Average	Average	Above Average	
	Success in Job (Employers Rating)	Poor	23	60	29	
		Average	28	79	60	
		Very Good	9	49	63	
31	a) Calculate the correlation coefficient between X and Y from the following data: X: 65, 66, 67, 67, 69, 68, 70, 72 Y: 67, 68, 65, 68, 72, 72, 69, 71 b) Calculate the correlation coefficient between X and Y from the following data: X: 1, 3, 4, 5, 7, 8, 10 Y: 2, 6, 8, 10, 14, 16, 20 c) Calculate the correlation coefficient between X and Y from the following data: X: -3, -2, -1, 1, 2, 3 Y: 9, 4, 1, 1, 4, 9			8	Section-IV
32	The marks obtained by 10 students in Mathematics (X) and in Statistics (Y) are given below. Compute the correlation coefficient between X and Y. Roll No.: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 X: 75, 30, 60, 80, 53, 35, 15, 40, 38, 48 Y: 85, 45, 54, 91, 58, 63, 35, 43, 45, 44			8	Section-IV
33	The marks secured by recruits in Selection Test (X) and in the Proficiency Test (Y) are given below. Use rank correlation method to determine the relationship between X and Y. X: 10, 15, 12, 17, 13, 16, 24, 14, 22, 20 Y: 30, 42, 45, 46, 33, 34, 40, 35, 39, 38			8	Section-IV
34	Given the following Aptitude and I.Q. Scores for a group of students. Compute the rank correlation coefficient between them. Aptitude Score: 57, 58, 59, 59, 60, 61, 60, 64 I.Q Score: 97, 108, 95, 106, 120, 126, 113, 110			8	Section-IV
35	A Chemical company wishing to study the effect of extraction time(X) on the efficient of extraction operation(Y), obtained the following data: X: 27, 45, 41, 19, 35, 39, 19, 49, 15, 31 Y: 57, 64, 80, 46, 62, 72, 52, 77, 57, 68 Obtain the two regression lines. Also determine the extraction efficiency one can expect when the extraction time is 35 minutes.			8	Section-IV
36	The following data gives the experience of the machine operators and their performance ratings as given by the number of good parts turned out per 100 pieces. Experience(X): 16, 12, 18, 4, 3, 10, 5, 12 Performance Ratings (Y): 88, 87, 89, 68, 78, 80, 75, 83 Obtain the regression line of performance ratings on experience and estimate the probable performance if the operator has 7 years of experience.			8	Section-IV
37	For the following bivariate data obtain the two lines of regression.			8	Section-IV

	Determine the value of Y when X=3.5 X 1 2 3 4 5 6 Y 14 33 40 63 76 85		
38	Fit a Straight line of the form: Y = a + bX for the following data For 8 randomly selected observations, the following data were recorded: X 1 2 3 4 5 6 7 8 Y 1.0 1.2 1.8 2.5 3.6 4.7 6.6 9.1	8	Section-IV
39	Fit a Second-degree parabola of the form: $Y = a + bX + cX^2$ for the following data For 10 randomly selected observations, the following data were recorded: Over time (X) 1 1 2 2 3 3 4 5 6 7 Additional Units (Y) 2 7 7 10 8 12 10 14 11 14	8	Section-IV
40	Fit a Second-degree parabola of the form: $Y = a + bX + cX^2$ for the following data and use to determine the value of Y corresponding to the value of X=6.2 and the value of X when Y=14.5 X 1 2 3 4 5 6 7 8 9 Y 9 8 10 12 11 13 14 16 15	8	Section-IV
41	A television repairman finds that the time spent on his jobs has an exponential distribution with mean of 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets follows a Poisson distribution approximately with an average rate of 10 per 8-hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?	8	Section-V
42	Students arrive at the head office according to a Poisson input process with a mean rate of 40 per hour. The time required to serve a student has an exponential distribution with a mean of 50 per hour. Assume that the students are served by a single individual, find the average waiting time of a student.	8	Section-V
43	New Delhi Railway Station has a single ticket counter. During the rush hours, customers arrive at the rate of 10 per hour. The average number of customers that can be served is 12 per hour. Find out the following: (i) Probability that the ticket counter is free. (ii) Average number of customers in the queue.	8	Section-V
44	There is congestion on the platform of a railway station. The trains arrive at a rate of 30/days. The service time for any train is ED with an average of 36mins. Calculate: (a) Mean queue size (b) Probability that there are more than 10 trains in the system.	8	Section-V
45	At a one-man barber shop customers arrive according to P.D with a mean arrival rate of 5/hr. The hair cutting time is ED with a haircut taking 10 min on an average assuming that the customers are always willing to wait find: a) Average number of customers in the shop b) Average waiting time of a customer c) The percent of time an arrival Can walk right without having to wait d) The probability of a customer waiting more than 5mins.	8	Section-V
46	Consider a single server queuing system with Poisson input, exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour and the maximum permissible calling units in the system is two. Derive the steady-state probability distribution of the number of calling units in	8	Section-V

	the system, and then calculate the expected number in the system.		
47	At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady-state probabilities for the various number of trains in the system. Also find the average waiting time of a new train coming into the yard.	8	Section-V
48	If for a period of 2 hours in the day (8 to 10 a.m.) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period a) the probability that the yard is empty, and b) the average number of trains in the system, on the assumption that the line capacity of the yard is limited to 4 trains only.	8	Section-V
49	A one – person barbershop has six chairs to accommodate people waiting for haircut. Assume customers who arrive when all six chairs are full, leave without entering the barbershop. Customers arrive at the average rate of 3 per hour and spend on average of 15 minutes in the barbershop. Then find the a) the probability a customer can get directly into the barber chair upon arrival. b) Expected number of customers waiting for haircut. c) Effective arrival rate. d) The time a customer can expect to spend in the barbershop.	8	Section-V
50	Trains arrive at the yard every 15 minutes and the service rate is 33 minutes. If the line capacity at the yard is limited to 4 trains, find a) The probability that the yard is empty. b) The average number of trains in the system.	8	Section-V