MALLA REDDY UNIVERSITY
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1. Reduce the matrix to Echelon form and find its rank. $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$.

2. Find the rank of $\begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$ using Echelon form.

3. If the matrix rank of $\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ K & 2 & 2 & 2 \\ 9 & 9 & K & 3 \end{bmatrix}$ is 3 then find the value of K.

4. Find the rank of $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing into the normal form

5. Find the rank of $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ by reducing it to normal form

6. Discuss for what values of $\lambda\, and\, \mu$ the simultaneous system of equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.

7. Find whether the following system of equations is consistent. If so solve them. $x+2y-z=3, 3x-y+2z=-1, 2x-2y+3z=2, x-y+z=-1$.

8. Find the values a & b for which the equations $x+y+z=3, x+2y+2z=6$ $x+ay+3z=b$ have (i) No Solution (ii) a unique solution and (iii) infinite no. of solutions.

9. Show that the only real number λ for which the system $x+2y+3z=\lambda x, 3x+y+2z=\lambda y, 2x+3y+z=\lambda z$ has non-zero solution is 6 and solve them when $\lambda = 6$.

10. Determine whether the following equations will have a non-trivial solution if so solve them. $4x+2y+z+3w=0, 6x+3y+4z+7w=0, 2x+y+w=0$.

11. a) Solve the system of equations $x+y+z=1,\ 4x+3y-z=6,\ 3x+5y+3z=4$ by LU decomposition method

    b) Solve the system of equations $x+2y+3z=9,\ 4x+5y+6z=24,\ 3x+y-2z=4$ by LU decomposition method

12. Reduce the following Quadratic forms to canonical form by orthogonal transformation and hence discuss the nature of it

    (a) $2x^2+y^2+z^2-2xz-4yz+2xy$

    (b) $x^2+2y^2+3z^2-2xy$

    (c) $3x^2+5y^2+3z^2-2yz+2zx-2xy$

13. Find the characteristic roots of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

14. Find the Eigen values and the corresponding Eigen vectors of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

15. Find the Eigen values and the corresponding Eigen vectors of the matrix

    $$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

16. Verify Cayley-Hamilton Theorem and find A^{-1} for $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$

17. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ verify Cayley-Hamilton theorem. Find A^{-1} and A^4.

18. Find the characteristic equation of the matrix, $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$ and hence

computeA^{-1}. Also find the matrix represented by

$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$

19. Diagonalize the matrix $A=\begin{bmatrix}1&1&1\\0&2&1\\-4&4&3\end{bmatrix}$ find a) A^8 b)A^4

20. Find a matrix P which transforms the matrix $A=\begin{bmatrix}1&0&-1\\1&2&1\\2&2&3\end{bmatrix}$ to diagonal form. Also

find a) A^{-1} b)A^4

21. Determine the diagonal matrix orthogonally similar to the following symmetric matrix

$A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$.

22. Diagonalize the matrix by an orthogonal transformation $\begin{bmatrix}2&0&1\\0&6&0\\4&0&2\end{bmatrix}$

23. Determine the modal matrix P for $A=\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$ and hence diagonalize A.

24. Determine the model matrix P of $A=\begin{bmatrix}-2&2&-3\\2&1&-6\\-1&-2&0\end{bmatrix}$ verify that $P^{-1}AP$ is a diagonal

matrix.

25. Show that the given Differential Equations are Exact and Solve

(a) $2xydx+x^2dy=0$

(b) $(2xy+y-\tan y)dx+(x^2-x\tan^2 y+\sec^2 y)dy=0$

(c) $(y^2e^{xy^2}+4x^3)dx+(2xye^{xy^2}-3y^2)dy=0$

(d) $(y(1+\frac{1}{x})+\cos y)dx+(x+\log x-x\sin y)dy=0$

(e) Solve $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$

(f) Solve $\left(\cos x - x\cos y\right)dy - \left(\sin y + y\sin x\right)dx = 0$

(g) Solve $\left(\sec x \tan x \tan y - e^{x}\right)dx + \sec x \sec^2 y dy = 0$

(h) Solve $\dfrac{dy}{dx} + \dfrac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$

26. Solve by finding Integrating Factors

(a) $xdy - ydx = xy^2 dx$

(b) $(1+xy)xdy + (1-yx)ydx = 0$

(c) $xdx + ydy + \dfrac{xdy - ydx}{x^2 + y^2} = 0$

(d) $ydx - xdy + 3x^2 y^2 e^{x^3} dx = 0$

(e) $x^2 ydx - (x^3 + y^3)dy = 0$

(f) $(y^2 + x^2)\dfrac{dy}{dx} = xy$

(g) $y^2 dx + (x^2 - xy - y^2)dy = 0$

(h) $\left(x^4 + y^4\right)dx - xy^3 dy = 0$

(i) $\dfrac{dy}{dx} = \dfrac{y}{x} + \dfrac{x^2 + y^2}{x^2}$

(j) $(xy^2 + 2x^2 y^3)dx + (x^2 y - x^3 y^2)dy = 0$

(k) $(1+xy)xdy + (1-xy)ydx = 0$

(l) $(2xy^2 + y)dx + (x + 2x^2 y - x^4 y^3)dy = 0$

(m) $(xy^2 \sin xy + y\cos xy)dx + (x^2 y \sin xy - x\cos xy)dy = 0$

(n) $(y + \dfrac{y^3}{3} + \dfrac{x^2}{2})dx + \dfrac{1}{4}(x + xy^2)dy = 0$

(o) $(xy^2 - e^{\frac{1}{x^3}})dx - x^2 ydy = 0$

(p) $(3xy - 2ay^2)dx + (x \quad - 2axy)dy = 0$

(q) $(xy^3 + y)dx + 2(x^2 y^2 + x + y^4)dy = 0$

(r) $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

(s) $(2xy^4 e^y + 2xy^3 + y)dx + (x^2 y^4 e^y - x^2 y^2 - 3x)dy = 0$

27. Solve the Linear differential Equations

(a) $\dfrac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$

(b) $\dfrac{dy}{dx} + \dfrac{3x^2}{1+x^3} y = \dfrac{\sin^2 x}{1+x^3}$

© $x\dfrac{dy}{dx} + y = x^2 + 3x + 2$

(d) $\dfrac{dy}{dx} + y\tan x = x^m \cos x$

(e) $x\log x\frac{dy}{dx}+y=2\log x$

(f) $(1+x+xy^2)\frac{dy}{dx}+(y+y^3)=0$

(g) $(1+y^2)dx=(\tan^{-1}y-x)dy$

28. Solve the Bernoulli's Equations

(a) $\frac{dy}{dx}+\frac{y}{x}=y^2x\sin x, x>0$

(b) $x\frac{dy}{dx}+y=y^2\log x$

© $xy-\frac{dy}{dx}=y^3e^{-x^2}$

(d) $\frac{dy}{dx}(x^2y^3+xy)=1$

(e) $\frac{dy}{dx}=\frac{x^2+y^2+1}{2xy}$

(f) $\frac{dy}{dx}=2y\tan x+y^2\tan^2 x$

29. (a)Find the Orthogonal Trajectories of the family of semi cubical parabolas $ay^2=x^3$

(b)Find the Orthogonal Trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$ where λ is a parameter.

(c) Find the Orthogonal trajectories of the family of curves $r^n\sin n\theta=a^n$

(d) Find the Orthogonal trajectories of the family of curves $r=4a\sec\theta\tan\theta$

(e) Find the Orthogonal trajectories of the family of curves $(a+x)y^2=x^2(3a-x)$

(f) Find the Orthogonal trajectories of the family of curves $r=a(1+\sin^2\theta)$

(g) Show that the family of con-focal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self-orthogonal

30.(a)Suppose that an object is heated to 300F and allowed to cool is a room whose air temperature 20F, it after 10 min, the temperature of the object is 250F, what will be its temperature after 20 min?

(b) body cools from 60^0C to 50^0C in 10 minutes when kept in air at 30^0C in the next 10 minutes what is the temperature of the body

31. (a)The number of bacteria cultures grows at the rate proportional to N, the value of N was initially 100 and it increases to 332 in one hr. What would be the value of N after $1\frac{1}{2}hr$

(b)If radioactive carbon-14 has a half-life of 5750 years, what will remain of 1 gram after 3000 years?

(c) If 30% of radioactive substance disappear in 10 days. How long will it take for 90% of it to disappear.

32. a) Solve $\left(D^2-2D+1\right)y = xe^x \sin x$

b) Solve $\left(D^2-2D\right)y = e^x \sin x$ by the method of variation of parameters.

c) Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=-2\cosh x$ *Also find when* $y=0, \frac{dy}{dx}=1\, at\; x=0$

d) Solve $\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+\frac{y}{x^2}=\frac{2\log x}{x^2}$.

e) Solve $\left(D^2-4D+4\right)y = 8x^2e^{2x}\sin 2x$

f) Solve $y''-6y'+9y=\frac{e^{3x}}{x^2}$ by the method of variation of parameter.

g) Solve $\frac{d^2x}{dt^2}+n^2x = k\cos(nt+\alpha)$

h) Solve $\frac{d^4x}{dt^4}+2\frac{d^2x}{dt^2}+x=t^2\cos t$

i) Solve $\left(D^2+1\right)y = x^2e^{3x}$

j) Solve by the method of variation of parameters $\frac{d^2y}{dx^2}-y=\frac{2}{\left(1+e^x\right)}$.

k) Solve $\left(D^2-2D+2\right)y = e^x\tan x$.

l) *Solve* $\left(D^2-3D+2\right)y=\sin\left(e^{-x}\right)$

m) Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$

n) Solve $\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+\frac{y}{x^2}=\frac{2\log x}{x^2}$

o) Solve $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-20y=(x+1)^2$

p) Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x\sin(\log x)$

33. Find the Laplace transform of $(i) e^{2t}+4t^3-2\sin 3t+3\cos 3t$ $(ii)\sin(\omega t+\alpha)$
(iii) $\sin^3 2t$ (iv) $\sin 2t.\cos 3t$

34. Find the Laplace transform of
$(i)\ e^{-3t}(2\cos 5t-3\sin 5t)$ $(ii)\cosh at\cos at$ $(iii) e^{3t}\sin^2 t$ (iv $L\left\{e^t\left(\cos 2t+\frac{1}{2}\sinh 2t\right)\right\}$

35. Find the Laplace transforms of $(i) t^2\cos at$ $(ii)\ te^{-t}\sin 3t$ $(iii) L\left\{t^2\sin 2t\right\}$

36. Find $(i)\ L\left\{\int_0^t \frac{e^{-t}\sin t}{t}dt\right\}$ $(ii)\ L\left\{\int_0^t \frac{1-e^{-t}}{t}dt\right\}$ $(iii)\ L\left\{\frac{1-\cos t}{t}\right\}$ (iv) $\frac{\sin t\sin 5t}{t}$

37. Evaluate $(i)\int_0^\infty te^{-2t}\cos 3t\,dt$ $(ii)\int_0^\infty t^2e^{-4t}\sin 2t\,dt$

38. Find inverse Laplace transform of (i) $\frac{s+1}{s^2+6s+25}$ (ii) $\frac{s+2}{s^2(s+3)}$ (iii) $\tan^{-1}\left(\frac{a}{s}\right)$
(iv) $\frac{1}{s\left(s^2+a^2\right)}$

39. Apply convolution theorem to evaluate

$(i) L^{-1}\left\{\frac{s}{(s^2-a)^2}\right\}$ $(ii)\ L^{-1}\left\{\frac{s^2}{(s^2+a)^2(s^2+b)^2}\right\}$ $(iii)\ L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$ $(iv)\ L^{-1}\left\{\frac{1}{(s+2)^2(s-2)}\right\}$ $(v) L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

40. Use Laplace transform to solve
(i) $y''-3y'+2y=4t+e^{3t}$, $y(0)=1, y'(0)=1$
(ii) $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$, $y=\frac{dy}{dt}=0$ when $t=0$
(iii) $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=3te^{-t}$ given $x=4$ and $\frac{dx}{dt}=0$ at $t=0$
(iv) $\left(D^2+5D-6\right)y=x^2e^{-x}, y(0)=a, y'(0)=b$
(v) $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t$, $y(0)=0, y'(0)=1$