12. (a) $(D^2-2D+1)y = xe^x \sin x$

$A.E \Rightarrow m^2-2m+1=0$

$\boxed{m=1,1}$

$\boxed{y_c = (C_1+C_2x)e^x}$

$$y_p = \frac{xe^x \sin x}{D^2-2D+1}$$

$$= \frac{xe^x \text{ Im part of } e^{ix}}{(D+1)^2-2(D+1)+1}$$

$$= \frac{xe^x \text{ Im part of } e^{ix}}{D^2+\cancel{2D}+\cancel{1}-\cancel{2D}-\cancel{2}+1}$$

$$= \frac{xe^x \text{ Im part of } e^{ix}}{(D+i)^2}$$

$$= \frac{xe^x \text{ Im part of } e^{ix}}{D^2+2Di-1}$$

$$= \frac{-xe^x \text{ Im part of } e^{ix}}{(1-(D^2+2Di))}$$

$$= -xe^x \text{ Im part of } e^{ix}\,(1-(D^2+2Di))^{-1}$$ $D(x)=1$

$D^2(x)=0$

$$= -xe^x \text{ Im part of } e^{ix}\left(1+D^2+2Di+(D^2+2Di)^2 - \cdots\right)$$

$$= -e^x \text{ Im part of } e^{ix}(x+2i)$$

$$= -e^x \text{ Im part } (\cos x + i\sin x)(x+2i)$$

$$= -e^x \text{ Im part } [x\cos x + 2i\cos x + xi\sin x - 2\sin x]$$

$\boxed{y_p = -e^x(2\cos x + x\sin x)}$

$y = y_c + y_p$

$\boxed{y = (C_1+C_2x)e^x - e^x(2\cos x + x\sin x)}$

(b) $(D^2-2D)y = e^x \sin x$

$$A.F \Rightarrow \cancel{D^2}\ m^2 - 2m = 0$$

$$m(m-2) = 0$$

$$\boxed{m = 0, 2}$$

$$y_c = C_1 e^0 + C_2 e^{2x}$$

$$\boxed{y_c = C_1 + C_2 e^{2x}}$$

$$u = 1,\quad v = e^{2x}$$

$$W(u,v) = u\frac{dv}{dx} - v\frac{du}{dx}$$

$$= 2e^{2x}$$

$$y_p = Au + Bv$$

$$A = -\int \frac{vR\,dx}{W(u,v)}$$

$$= -\int \frac{\cancel{e^{2x}}\, e^x \sin x}{2\cancel{e^{2x}}}\,dx$$

$$= -\frac{1}{2}\int e^x \sin x\, dx$$

$$= -\frac{1}{2}\left[\frac{e^x}{2}\left[\sin x - \cos x\right]\right]$$

$$= -\frac{e^x}{4}(\sin x - \cos x)$$

$$B = \int \frac{uR\,dx}{W(u,v)}$$

$$= \int \frac{\cancel{e^x}\sin x}{2e^{\cancel{2x}}}\,dx$$

$$= \frac{1}{2}\int e^{-x}\sin x\, dx$$

$$= \frac{1}{2}\left[\frac{e^{-x}}{2}\left(-\sin x - \cos x\right)\right]$$

$$= -\frac{e^{-x}}{4}(\sin x + \cos x)$$

$$\cancel{A} = y_p = Au + Bv$$

$$= \frac{-e^x}{4}\sin x + \frac{e^x}{4}\cos x \cancel{+} \frac{e^{-x}}{4}(\sin x + \cos x)\, e^{2x}$$

$$= \frac{-e^x}{4}\sin x + \cancel{\frac{e^x}{4}\cos x} - \frac{e^x}{4}\sin x - \cancel{\frac{e^x}{4}\cos x}$$

$$\boxed{y_p = \frac{-e^x}{2}\sin x}$$

$$y = y_c + y_p$$

$$\therefore \boxed{y = C_1 + C_2 e^{2x} - \frac{e^x}{2}\sin x}$$

(c) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x$

Sol^n; S.F $\Rightarrow$ $(D^2+4D+5)y = -2\cosh x$.

A.F $\Rightarrow$ $m^2 + 4m + 5 = 0$.

$\boxed{\cancel{m=2\pm i}}$ $\boxed{m = -2 \pm i}$

$$\boxed{y_c = e^{-2x}(C_1 \cos x + C_2 \sin x)}$$

$$y_p = \frac{-2\cosh x}{(D^2+4D+5)}$$

$$= \frac{-e^x}{D^2+4D+5} - \frac{e^{-x}}{D^2+4D+5}$$

$$= \frac{-e^x}{1+4+5} - \frac{e^{-x}}{1-4+5}$$

$$= \frac{-e^x}{10} - \frac{e^{-x}}{2}$$

$$\boxed{y_p = -\left[\frac{e^x}{10} + \frac{e^{-x}}{2}\right]}$$

$y = y_c + y_p$

$$\boxed{y = e^{-2x}(C_1 \cos x + C_2 \sin x) - \frac{e^x}{10} - \frac{e^{-x}}{2}}$$

(d) $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + \frac{y}{x^2} = \frac{2\log x}{x^2}$

$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2\log x$

S.F $\Rightarrow (x^2D^2 - xD + 1)y = 2\log x$

To reduce it to Cauchy euler's

$x = e^z ; z = \log x$

$x^2D^2 = \theta(\theta-1)$

$xD = \theta$

$(\theta(\theta-1) - \theta + 1).y = 2z$

$(\theta^2 - \theta - \theta + 1)y = 2z$

$(\theta^2 - 2\theta + 1)y = 2z$

A.E $\Rightarrow m^2 - 2m + 1 = 0$

$\boxed{m = 1, 1}$

$\boxed{\therefore y_c = (c_1 + c_2 z)e^z}$

$y_p = \frac{2z}{\theta^2 - 2\theta + 1}$

$= 2z\{1 + (\theta^2 - 2\theta)\}^{-1}$

$= 2z(1 - (\theta^2 - 2\theta) + (\theta^2 - 2\theta)^2 + \dots)$

$= 2(z + 2)$

$\boxed{y_p = 2z + 4}$

$y = y_c + y_p$

$= (c_1 + c_2 z)e^z + 2z + 4$

$= (c_1 + c_2 \log x)x + 2\log x + 4$

$\therefore \boxed{y = (c_1 + c_2\log x)x + \log x^2 + 4}$

(c) $(D^2-4D+4)\,y = 8x^2e^{2x}\sin 2x$

$A.F \Rightarrow m^2-4m+4=0$

$\boxed{m=2,2}$

$\boxed{y_c=(c_1+c_2x)e^{2x}}$

$$y_p = \frac{8x^2e^{2x}\sin 2x}{D^2-4D+4}$$

$$= \frac{8x^2e^{2x}\sin 2x}{(D+2)^2-4(D+2)+4}$$

$$= \frac{8x^2e^{2x}\sin 2x}{D^2+\cancel{4D}+4-\cancel{4D}-8+4}$$

$$= \frac{8x^2e^{2x}\sin 2x}{D^2}$$

$$= \frac{8x^2e^{2x}\,\text{Im part of } e^{2ix}}{(D+2i)^2}$$

$D(x^2)=2x$

$D^2(x^2)=2$

$D^3(x^2)=0$

$$= \frac{\overset{4}{\cancel{8}}x^2e^{2x}\,\text{Im part of } e^{2ix}}{(\cancel{2}i)^2\left(1+\frac{D}{2i}\right)^2}$$

$$= \frac{\overset{-2}{\cancel{8}}x^2e^{2x}}{\cancel{4i^2}}\,\text{Im part of } e^{2ix}\left(1+\frac{D}{2i}\right)^{-2}$$

$$= \frac{-\cancel{2}x^2e^{2x}}{\cancel{8}}\,\text{Im part of } e^{2ix}\left(1-2\left(\frac{D}{2i}\right)+3\left(\frac{D}{2i}\right)^2-\cdots\right)$$

$$= \frac{-2e^{2x}}{\cancel{8}}\,\text{Im part of } e^{2ix}\left(x^2-\frac{2x}{i}+\frac{6}{4}+\cdots\right)$$

$$= -2e^{2x}\,\text{Im part of } (\cos 2x+i\sin 2x)\left(x^2-\frac{2x}{i}-\frac{3}{2}\right)$$

$$= -2e^{2x}\,\text{Im part of } \left(x^2\cos 2x-\frac{2x}{i}\cos 2x-\frac{3}{2}\cos 2x+x^2i\sin 2x-2x\sin 2x-\frac{3}{2}i\sin 2x\right)$$

$$= -2e^{2x}\,\text{Im part of } \left(x^2\cos 2x+2xi\cos 2x-\frac{3}{2}\cos 2x+x^2i\sin 2x-2x\sin 2x-\frac{3}{2}i\sin 2x\right)$$

$$\boxed{y_p = -2e^{2x}\left(2x\cos 2x+x^2\sin 2x-\frac{3}{2}\sin 2x\right)}$$

$y=y_c+y_p$

$$\boxed{y=(c_1+c_2x)e^{2x}-2e^{2x}\left(2x\cos 2x+x^2\sin 2x-\frac{3}{2}\sin 2x\right)}$$

(f) $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$

Soln: S.F $\Rightarrow (D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$

A.F $\Rightarrow m^2 - 6m + 9 = 0$

$\boxed{m = 3, 3}$

$\boxed{y_c = c_1 e^{3x} + c_2 x e^{3x}}$

$w(u,v) = u\frac{dv}{dx} - v\frac{du}{dx}$

$= e^{3x}(3xe^{3x} + e^{3x}) - 3xe^{3x}e^{3x}$

$= \cancel{3xe^{6x}} + e^{6x} \cancel{- 3xe^{6x}}$

$= e^{6x}$

$y_p = Au + Bv$

$A = -\int \frac{vR\,dx}{w(u,v)}$

$= -\int \frac{\cancel{x}e^{3x}e^{3x}}{x^{\cancel{2}}(\cancel{e^{6x}})}\,dx$

$= -\int \frac{1}{x}\,dx$

$= -\log x$

$B = \int \frac{uR\,dx}{w(u,v)}$

$= \int \frac{e^{3x}\,\cancel{e^{3x}}}{x^2\,\cancel{e^{6x}}}\,dx$

$= \int \frac{1}{x^2}\,dx$

$= -\frac{1}{x}$

$\frac{1}{x^2} =$

$y_p = Au + Bv$

$= -\log x(e^{3x}) - \frac{1}{\cancel{x}}\cancel{x}e^{3x}$

$\boxed{y_p = -e^{3x}(\log x + 1)}$

$y = y_c + y_p$

$\therefore \boxed{y = (c_1 + c_2 x)e^{3x} - e^{3x}(\log x + 1)}$ //

(h) $\frac{d^4x}{dt^4} + 2\frac{d^2x}{dt^2} + x = t^2\cos t$.

Sol^n: S.F $\Rightarrow (D^4+2D^2+1)x = t^2\cos t$.

AF $\Rightarrow m^4+2m^2+1 = 0$

$(m^2+1)^2 = 0$

$(m^2+1)(m^2+1) = 0$

$\boxed{m = \pm i, \pm i}$

$\boxed{y_c = (C_1 + C_2 t)\cos t + (C_3 + C_4 t)\sin t}$

$y_p = \frac{t^2\cos t}{D^4+2D^2+1}$

$= \frac{t^2 \text{ Rp part of } e^{it}}{(D^2+1)^2}$

$= \frac{t^2 \text{ Rp part of } e^{it}}{((D+i)^2+1)^2}$

$= \frac{t^2 \text{ Rp part of } e^{it}}{(D^2+2Di-\cancel{1}+\cancel{1})^2}$

$= \frac{t^2 \text{ Rp of } e^{it}}{(2Di)^2 \cdot \left(1+\frac{D^2}{2Di}\right)^2}$

$= \frac{-t^2 \text{ Rp part of } e^{it}}{4D^2} \left(1+\frac{D}{2i}\right)^{-2}$

$= \frac{-t^2}{4D^2} \text{ Rp part of } e^{it}\left(1 - 2\left(\frac{D}{2i}\right) + 3\left(\frac{D}{2i}\right)^2 + \dots\right)$

$= \frac{-1}{4D^2} \text{ Rp of } e^{it}\left(t^2 - \frac{2t}{i} + \frac{3}{2}\right)$

$= \frac{-1}{4} \text{ Rp of } e^{it} \iint t^2 - \frac{2t}{i} + \frac{3}{2}\, dt\, dt$

$= \frac{-1}{4} \text{ Rp of } e^{it} \int \frac{t^3}{3} - \frac{2t^2}{2i} + \frac{3}{2}t\, dt$

$= \frac{-1}{4} \text{ Rp of } e^{it}\left(\frac{t^4}{12} - \frac{t^3}{3i} + \frac{3t^2}{4}\right)$

$= \frac{-1}{48} \text{ Rp of } (\cos x + i\sin x)\left(t^4 - \frac{4t^3}{i} + 9t^2\right)$

$= \frac{-1}{48} \text{ Rp of } (t^4\cos x + 4t^3 i\cos x + 9t^2\cos x + t^4 i\sin x - 4t^3\sin x + 9t^2 i\sin x)$

$$\begin{array}{r|l} 3 & 12, 3, 4 \\ 4 & 4, 1, 4 \\ & 1, 1, 1 \end{array}$$

$y = y_c + y_p$

$$\boxed{y = (c_1 + c_2 t)\cos t + (c_3 + c_4 t)\sin t - \frac{1}{48}\left(t^4\cos x + 9t^2\cos x - 4t^3\sin x\right)}$$

(ii) .. $(D^2+1)y = x^2 e^{3x}$

Sol^n.

A.F $\Rightarrow$ $m^2+1=0$

$$\boxed{m = \pm i}$$

$$\boxed{y_c = c_1\cos x + c_2\sin x}$$

$y_p = \dfrac{x^2 e^{3x}}{D^2+1}$

$\cancel{= \dfrac{x^2 e^{3x}}{(D+i)(D-i)}}$

$\cancel{= \dfrac{1}{2i}\left[\dfrac{x^2 e^{3x}}{D-i} - \dfrac{x^2 e^{3x}}{D+i}\right]}$

$\cancel{1 = A(D+i) + B(D-i)}$

$\cancel{\boxed{D=i}}$ | $\cancel{\boxed{D=-i}}$

$\cancel{\boxed{A = \frac{1}{2i}}}$ | $\cancel{\boxed{B = \frac{-1}{2i}}}$

$\cancel{y_{1,} = \dfrac{x^2 e^{3x}}{D-i}}$

$\cancel{= e^{ix}\int x^2 e^{3x} e^{-ix}\,dx}$

$\cancel{= e^{ix}\int x^2 e^{3x}(\cos x - i\sin x)\,dx}$

$\cancel{= e^{ix}\int x^2 e^{3x}\cos x -}$

$= x^2 e^{3x}\dfrac{1}{(D+3)^2+1}$

$= x^2 e^{3x}\dfrac{1}{D^2+9+6D+1}$

$= x^2 e^{3x}\dfrac{1}{D^2+6D+10}$

$D\,x^2 = 2x$

$D^2\,x^2 = 2$

$= \dfrac{x^2 e^{3x}}{10}\dfrac{1}{\left(1 + \frac{D^2+6D}{10}\right)}$

$= \dfrac{x^2 e^{3x}}{10}\left(1 + \dfrac{D^2+6D}{10}\right)^{-1}$

$= \dfrac{x^2 e^{3x}}{10}\left(1 - \dfrac{(D^2+6D)}{10} + \left(\dfrac{D^2+6D}{10}\right)^2 + \cdots\right)$

$\frac{72}{100} - \frac{2}{10} \Rightarrow \frac{52}{100}$

$= \dfrac{e^{3x}}{10}\left(x^2 - \dfrac{(2+12x)}{10} + \dfrac{72}{100}\right)$

$$\boxed{y_p = \frac{e^{3x}}{10}\left(x^2 - \frac{6x}{5} + \frac{52}{100}\right)}$$

$y = y_c + y_p$

$$\boxed{y = c_1\cos x + c_2\sin x + \frac{e^{3x}}{10}\left(x^2 - \frac{6x}{5} + \frac{52}{100}\right)}$$

9) $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$

Sol^n: $A.F \Rightarrow (D^2-1)y = \frac{2}{1+e^x}$

$A.F \Rightarrow m^2 - 1 = 0$

$\boxed{m = \pm 1}$

$\boxed{y_c = c_1 e^{-x} + c_2 e^{x}}$

$\cdot\, w(u,v) = u\frac{dv}{dx} - v\frac{du}{dx}$

$= e^{-x}e^{x} + e^{x}e^{-x}$

$\boxed{w(u,v) = 2}$

$y_p = Au + Bv$

$A = -\int \frac{vR\,dx}{w(u,v)}$

$= -\int \frac{e^x \cancel{2}}{\cancel{2}(1+e^x)}dx$

$\boxed{A = -\log(1+e^x)}$

$B = \int \frac{uR\,dx}{w(u,v)}$

$= \int \frac{e^{-x}\cancel{2}}{\cancel{2}(1+e^x)}dx$

$= \int \frac{1}{e^{x}(1+e^{x})}$

$= \int \frac{1}{v^2(1+v)}dv$

$= \int \frac{-1}{v} + \frac{1}{v^2} + \frac{1}{1+v}dv$

$= -\log v - \frac{1}{v} + \log(1+v)$

$= -\frac{1}{v} + \log\left|\frac{1+v}{\;}\right| - \log v$

$\boxed{B = -e^{-x} + \log(1+e^x) - x}$

Put $e^x = v$

$e^x dx = dv$

$dx = \frac{dv}{v}$

$1 = A(v)(1+v) + B(1+v) + C(v^2)$

$\boxed{v = -1}$, $\boxed{C = 1}$

$1 = A(v^2+v) + B(1+v) + C(v^2)$

$1 = v^2(A+C) + v(A+B) + B$

$A + C = 0$

$A + B = 0$

$\boxed{B = 1}$

$\boxed{A = -1}$

$\boxed{C = 1}$

$y_p = Au + Bv$

$y_p = -e^{-x}\log(1+e^x) - e^{-x}e^{x} + e^{x}\log(1+e^x) - xe^{x}$

$\boxed{y_p = \log(1+e^x)(e^x - e^{-x}) - 1 - xe^{x}}$

$y = y_c + y_p$

$\therefore \boxed{y = c_1 e^{-x} + c_2 e^{x} - 1 - xe^{x} + \log(1+e^x)(e^x - e^{-x})}$

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(k) $(D^2-2D+2)y = e^x \tan x$

$\frac{2\pm\sqrt{4-8}}{2} = 1\pm i$

Soln:

A.E $\Rightarrow m^2-2m+2=0$

$\boxed{m = 1\pm i}$

$\boxed{y_c = (c_1 \cos x + c_2 \sin x)e^x}$

$w(u,v) = u\frac{dv}{dx} - v\frac{du}{dx}$

$= e^x\cos x(\cos x e^x + \sin x e^x) - e^x\sin x(\cos x e^x - \sin x e^x)$

$= e^{2x}\cos^2 x + \cancel{e^{2x}\sin x\cos x} - \cancel{e^{2x}\sin x\cos x} + e^{2x}\sin^2 x$

$= e^{2x}(\cos^2 x + \sin^2 x)$

$= e^{2x}$

$y_p = Au + Bv$

$A = -\int \frac{vR\,dx}{w(u,v)}$

$= -\int \frac{e^x \sin x\, e^x \tan x}{e^{2x}}\,dx$

$= -\int \frac{\sin^2 x}{\cos x}\,dx$

$= -\int \frac{1-\cos^2 x}{\cos x}\,dx$

$= -\int \sec x\,dx + \int \cos x\,dx$

$\boxed{A = -\log(\sec x + \tan x) + \sin x}$

$B = \int \frac{uR\,dx}{w(u,v)}$

$= \int \frac{\cancel{e^x}\cos x\, \cancel{e^x}\tan x}{\cancel{e^{2x}}}\,dx$

$= \int \sin x\,dx$

$\boxed{B = -\cos x}$

$y_p = Au + Bv$

$= -e^x\cos x(\log(\sec x + \tan x)) + e^x\cos x\sin x - e^x\sin x\cos x$

$\boxed{y_p = -e^x\cos x(\log|\sec x + \tan x|)}$

$y = y_c + y_p$

$\boxed{y = (c_1\cos x + c_2\sin x)e^x - e^x\cos x\log|\sec x + \tan x|}$

(1) $(D^2-3D+2)y = \sin(e^{-x})$

Sol^n: A.F $\Rightarrow (m^2-3m+2)=0$

$\boxed{m=2,1}$

$\boxed{y_c = c_1e^x + c_2e^{2x}}$

$y_p = \dfrac{\sin(e^{-x})}{D^2-3D+2}$

$= \dfrac{\sin e^{-x}}{(D-1)(D-2)}$

$= -\dfrac{\sin(e^{-x})}{(D-1)} + \dfrac{\sin(e^{-x})}{D-2}$

$\frac{1}{}$

$1 = A(D-2)+B(D-1)$

$\boxed{D=1}$ $\boxed{A=-1}$ | $\boxed{D=2}$ $\boxed{B=1}$

Put $e^{-x}=v$

$-e^{-x}dx = dv$

$e^{-x}=v$

$-e^{-x}dx = dv$

$\boxed{y_{p_1}=} -e^x\int \sin(e^{-x})e^{-x}\,dx$

$= e^x\int \sin v\,dv$

$\boxed{y_{p_1} = -e^x\cos(e^{-x})}$

$y_{p_2} = e^{2x}\int \sin(e^{-x})e^{-2x}\,dx$

$= -e^{2x}\int v\sin v\,dv$

$= -e^{2x}\left[v(-\cos v) - \int 1(-\cos v)\,dv\right]$

$= -e^{2x}\left[-v\cos v + \sin v\right]$

$= -e^{2x}\left[-e^{-x}\cos(e^{-x}) + \sin e^{-x}\right]$

$\boxed{y_{p_2} = e^x\cos(e^{-x}) - e^{2x}\sin(e^{-x})}$

$y_p = y_{p_1} + y_{p_2}$

$= \cancel{-e^x\cos(e^{-x})} + \cancel{e^x\cos(e^{-x})} - e^{2x}\sin(e^{-x})$

$\boxed{y_p = -e^{2x}\sin(e^{-x})}$

$y_G = y_c + y_p$

$\boxed{y = c_1e^x + c_2e^{2x} - e^{2x}\sin(e^{-x})}$

(a) $(x^2D^2+2xD-20)y = (x+1)^2$

Soln, G.F $\Rightarrow (x^2D^2+2xD-20)y = (x+1)^2$

To reduce it to cauchy euler's

i.e $x = e^z$; $z = \log x$

$x^2D^2 = \theta(\theta-1)$

$xD = \theta$

$(\theta(\theta-1)+2\theta-20)y = (e^z+1)^2$

$(\theta^2+\theta-20)y = (e^z+1)^2$

1
2
4
5

$\cancel{\frac{3\pm\sqrt{9-80}}{2} = \frac{3\pm i\sqrt{7}}{2}}$

A.F $\Rightarrow$ $m^2+m-20=0$

$\boxed{m = 4, -5}$

$\boxed{y_c = c_1 e^{-5z} + c_2 e^{4z}}$

$y_p = \frac{(e^z+1)^2}{\theta^2+\theta-20}$

$= \frac{e^{2z}+2e^z+1}{(\theta+5)(\theta-4)}$

$= \frac{e^{2z}}{(\theta+5)(\theta-4)} + \frac{2e^z}{(\theta+5)(\theta-4)} + \frac{1}{(\theta+5)(\theta-4)}$

$\boxed{y_p = -\frac{e^{2z}}{14} - \frac{2e^z}{18} - \frac{1}{20}}$

$y_G = y_c + y_p$

$y = c_1e^{-5z}+c_2e^{4z} - \frac{e^{2z}}{14} - \frac{e^z}{9} - \frac{1}{20}$

$\therefore \boxed{y = c_1x^{-5}+c_2x^4 - \frac{x^2}{14} - \frac{x}{9} - \frac{1}{20}}$

(P) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y = \log x \cdot \sin(\log x)$

Soln: S.F $\Rightarrow (x^2D^2+xD+1)y = \log x \sin(\log x)$

To reduce it to cauchy euler's

$x = e^z;\ z = \log x$

$x^2D^2 = \theta(\theta-1)$

$xD = \theta$

$(\theta(\theta-1)+\theta+1)y = z\sin z$

$(\theta^2+1)y = z\sin z$

A.F $\Rightarrow$ $m^2+1=0$

$\boxed{m=\pm i}$

$\boxed{y_c = c_1\cos z + c_2\sin z}$

$y_p = \dfrac{z\sin z}{\theta^2+1}$

$= \dfrac{z \text{ Im part of } e^{iz}}{(\theta+i)^2+1}$

$= \dfrac{z \text{ Im part of } e^{iz}}{D^2+2Di}$

$= \dfrac{z \text{ Im part of } e^{iz}}{2Di\left(1+\frac{D}{2i}\right)}$

$= \dfrac{z}{2Di} \text{ Im part of } e^{iz}\left(1+\frac{D}{2i}\right)^{-1}$

$= \dfrac{z}{2Di} \text{ Im part of } e^{iz}\left(1-\frac{D}{2i}+\left(\frac{D}{2i}\right)^2\right)$

$= \dfrac{1}{2Di} \text{ Im part of } e^{iz}\left(z-\frac{1}{2i}\right)$

$= \text{Im part of } \dfrac{e^{iz}}{2i}\int\left(z-\frac{1}{2i}\right)dz$

$= \dfrac{1}{2i} \text{ Im part of } (\cos z + i\sin z)\left(\frac{z^2}{2}-\frac{z}{2i}\right)$

$= \dfrac{1}{4i}\left(\frac{z^2}{2}\cos z + zi\cos z + z^2 i\sin z - z\sin z\right)$

$\boxed{\cancel{y_p = \frac{1}{4}(z\cos x + z^2\sin x)}}$ $\quad$ $\cancel{y = y_c + y_p}$

$y = c_1$

$\boxed{y_p = \frac{1}{4}(z^2\cos z + z\sin z)}$ //

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$$y = y_c + y_p$$

$$= C_1 \cos z + C_2 \sin z + \frac{1}{4}\left(z^2 \cos z - z \sin z\right)$$

$$\boxed{y = C_1 \cos(\log x) + C_2 \sin(\log x) + \frac{1}{4}\left((\log x)^2 \cos(\log x) - \log(x) \sin(\log x)\right)}$$

(c) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x$

Soln S.F $\Rightarrow (D^2 + 4D + 5)y = -2\cosh x$

(m) $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x+\frac{1}{x}\right)$

Soln: S.F $\Rightarrow (x^3D^3+2x^2D^2+2)y = 10\left(x+\frac{1}{x}\right)$

$\Rightarrow (x^3D^3+2x^2D^2+2)y = 10(x+x^{-1})$

To reduce it to cauchy euler's

$x = e^z$; $z = \log x$

$x^3D^3 = \theta(\theta-1)(\theta-2)$

$x^2D^2 = \theta(\theta-1)$

$xD = \theta$

$(\theta(\theta-1)(\theta-2) + 2\theta(\theta-1)+2)y = 10(e^z+e^{-z})$

$((\theta^2-\theta)(\theta-2) + 2\theta^2-2\theta+2)y = 10(e^z+e^{-z})$

$(\theta^3-\cancel{2\theta^2}-\theta^2+\cancel{2\theta}+\cancel{2\theta^2}-\cancel{2\theta}+2)y = 10(e^z+e^{-z})$

$(\theta^3-\theta^2+2)y = 10(e^z+e^{-z})$

A.F $\Rightarrow$ $m^3-m^2+2 = 0$

$(m+1)(m^2-2m+2) = 0$

$\boxed{m = -1, 1\pm i}$

$\boxed{y_c = c_1e^{-z} + e^z(c_2\cos z + c_3\sin z)}$

$y_p = \frac{10(e^z+e^{-z})}{D^3-D^2+2}$

1+2+2

$= 10\left[\frac{e^z}{D^3-D^2+2} + \frac{e^{-z}}{D^3-D^2+2}\right]$

$= 10\left[\frac{e^z}{1-1+2} + \frac{e^{-z}}{\cancel{-1-1+2}}\right]$

$= 10\left[\frac{e^z}{2} + \frac{ze^{-z}}{1!(D^2-2D+2)}\right]$

$\boxed{y_p = 10\left[\frac{e^z}{2} + \frac{ze^{-z}}{5}\right]}$

$y = y_c + y_p$

$= c_1e^{-z} + e^z(c_2\cos z + c_3\sin z) + 10\left[\frac{e^z}{2} + \frac{ze^{z}}{5}\right]$

$\boxed{y = c_1x^{-1} + x(c_2\cos(\log x) + c_3(\log x)) + 10\left[\frac{x}{2} + \frac{x\log x}{5}\right]}$

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