

DIC Practice Questions for Minor – II

1. Evaluate $\iint_R dx dy$ where R is the parallelogram in the xy-plane with vertices (1,0), (3,1), (2,2) and (0,1) by using transformation $u = x + y$ and $v = x - 2y$.
2. Find the volume of the surface in first octant and bounded by $2x + 3y + 4z = 12$.
3. Evaluate $\iiint (x^2 + y^2 + z^2) dx dy dz$ taken over the volume enclosed by the sphere $x^2 + y^2 + z^2 = 1$ by changing into spherical polar coordinates.
4. Evaluate (a) $\int_0^1 \int_1^2 \int_2^3 (x + y + z) dx dy dz$. (b) $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dx dy dz$
5. a. Find constants a, b, c so that the vector $\vec{F} = (x + 2y + az) \vec{i} + (bx - 3y - z) \vec{j} + (4x + cy + 2z) \vec{k}$ is irrotational.
b. Prove that $r^n \vec{r}$ is Solenoidal if $n = -3$.
6. Prove that $\vec{F} = (2xyz^2) \vec{i} + (x^2z^2 + z \cos yz) \vec{j} + (2x^2yz + y \cos yz) \vec{k}$ is conservative. Find its scalar potential.
7. Find directional derivative of the function $f = 4xy^2 + 2x^2yz$ at the point P (1, 2, 3) in the direction of PQ where Q (5, 0, 4).
8. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2).
9. If $\vec{f} = (x^2 + y^2) \vec{i} - 2xy \vec{j}$, evaluate $\int_C \vec{f} \cdot d\vec{r}$ where C is the rectangle in xy-plane bounded by $x=0$, $x=a$, $y=0$, $y=b$.
10. Evaluate $\int_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = z \vec{i} + x \vec{j} - 3y^2z \vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z=0$ and $z=5$.
11. Verify Green's theorem for $\int_C (xy + y^2) dx + x^2 dy$ Where C is bounded by $y = x$ and $y = x^2$.
12. Verify Green's theorem for $\int_C (2xy - x^2) dx + (x^2 + y^2) dy$ Where C is bounded by $x = y^2$ and $y = x^2$.
13. Verify Gauss theorem for $\vec{f} = 4xz \vec{i} - y^2 \vec{j} + yz \vec{k}$ taken over the surface of the cube bounded by $x=0$, $x=a$, $y=0$, $y=a$, $z=0$, $z=a$.
14. Verify Gauss Divergence theorem for the function $\vec{F} = 4x \vec{i} - 2y^2 \vec{j} - z^2 \vec{k}$ taken over the surface bounded by the cylinder $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.
15. Verify Stoke's theorem for $\vec{F} = (2x - y) \vec{i} - yz^2 \vec{j} - y^2z \vec{k}$ over upper half of the surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by the projection in xy-plane.
16. Evaluate $\int_S \text{curl } \vec{f} \cdot \hat{n} ds$ where $\vec{f} = (x^2 + y - 4) \vec{i} + 3xy \vec{j} + (2xz + z^2) \vec{k}$, where S is the surface of the semi sphere $x^2 + y^2 + z^2 = 16$ above the xy-plane.