Programming Languages - Assignment-2 (6/2/24)

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SM11326

1) $(\lambda n.\lambda y.yn)(5+2)\lambda n.n+1$

Call by Name

$\Rightarrow (\lambda n.(\lambda y.(yn)))(5+2)(\lambda t.(t+1))$ α-substitution

$\Rightarrow (y.(yn))\left[(5+2)/n\right](\lambda t.(t+1))$ β-reduction

$\Rightarrow (y \cdot (y\,(5+2)))\ (\lambda t \cdot (t+1))$

$\Rightarrow (y\,(5+2))\left[(\lambda t \cdot (t+1))/y\right]$ β-reduction

$\Rightarrow (t+1)\left[(5+2)/t\right]$

$\Rightarrow (5+2)+1$

$\Rightarrow 7+1$

$\boxed{\Rightarrow 8}$

Call by Value

$\Rightarrow (\lambda n \cdot (\lambda y \cdot (y\,n)))\ (5+2)\ (\lambda t \cdot (t+1))$ α-substitution $(t \to n)$

$\Rightarrow (\lambda n \cdot (\lambda y \cdot (y\,n)))\ (7)\ (\lambda t \cdot (t+1))$

$\Rightarrow (\lambda y.(y\,n))\,[7/n]\,(\lambda t.(t+1))$ β-reduction

$\Rightarrow (\lambda y\,(y\,7))\,(\lambda t.(t+1))$

$\Rightarrow (y\,7)\,\left[(\lambda t.(t+1))/y\right]$ β-reduction

$\Rightarrow (\lambda t.(t+1))\,7$

$\Rightarrow (t+1)\,[7/t]$ β-reduction

$\Rightarrow 7+1$

$\Rightarrow \boxed{8}$

2> $(\lambda f.f\,7)\,((\lambda x.x\,x)\,\lambda y.y)$

Call by Name

$\Rightarrow (f\,7)\,\left[((\lambda x.(x\,x))(\lambda y.y))/\right]$ β-reduction

$\Rightarrow ((\lambda n.(n\,n))(\lambda y.y))\,7$

$\Rightarrow (n\,n)\left[(\lambda y.y)/n\right]7$ β-reduction

$\Rightarrow ((\lambda y.y)(\lambda y.y))\,7$

$\Rightarrow ((\lambda y.y)(\lambda t.t))\,7$ α-substitution $(t \to n)$

$\Rightarrow (y)\left[(\lambda t.t)/y\right]7$ β-reduction

$\Rightarrow (\lambda t.t)\,7$

$\Rightarrow (t)\left[7/t\right]$ β-reduction

$\Rightarrow \boxed{7}$

Call by Value

$\Rightarrow (\lambda f.(f\,7))\,(x\,x)\left[{}^{(\lambda y.y)}/_{x}\right]$ β-reduction

$\Rightarrow (\lambda f.(f\,7))\,((\lambda y.y)\,(\lambda y.y))$

$\Rightarrow (\lambda f.(f\,7))\,((\lambda y.y)\,(\lambda t.t))$ α-substitution

$\Rightarrow (\lambda f.(f\,7))\,(y\left[{}^{(\lambda t.t)}/_{y}\right])$ β-reduction

$\Rightarrow (\lambda f.(f\,7))\,(\lambda t.t)$

$\Rightarrow (f\,7)\left[{}^{(\lambda t.t)}/_{f}\right]$ β-reduction

$\Rightarrow (\lambda t.t)\,7$

$\Rightarrow (t)\left[7/t\right]$ β-reduction

$\Rightarrow \boxed{7}$

3> $(\lambda y.(\lambda x.x)y)((\lambda u.u)(\lambda v.v))$

Call by Name

$\Rightarrow ((\lambda x.x)\,y)\left[((\lambda u.u)(\lambda v.v))/y\right]$ β-reduction

$\Rightarrow (\lambda x.x)((\lambda u.u)(\lambda v.v))$

$\Rightarrow (x)\left[((\lambda u.u)(\lambda v.v))/x\right]$ β-reduction

$\Rightarrow (\lambda u.u)(\lambda v.v)$

$\Rightarrow (u)\left[(\lambda v.v)/\right]$ β-reduction

$\Rightarrow \boxed{(\lambda v.v)}$

Call by Value

$\Rightarrow (\lambda y.(\lambda x.x)y)(u[(\lambda v.v)/u])$ β-reduction

$\Rightarrow (\lambda y.(\lambda x.x)y)(\lambda v.v)$

$\Rightarrow ((\lambda x.x)y)[(\lambda v.v)/y]$

$\Rightarrow (\lambda x.x)(\lambda v.v)$

$\Rightarrow (x)[(\lambda v.v)/x]$

$\Rightarrow \boxed{(\lambda v.v)}$

4) $(\lambda n \cdot n)\ 5 + 1$

Call by Name

$\Rightarrow (n)[5/n] + 1$ — β-reduction

$\Rightarrow 5 + 1$

$\Rightarrow \boxed{6}$

Call by Value

$\Rightarrow (n)[5/n] + 1$ — β-reduction

$\Rightarrow 5 + 1$

$\Rightarrow \boxed{6}$

Here "+1" is not directly bound to 5, hence even

→ Here if ... in "Call by Value" we will pass 5 first.

5) $(\lambda x \cdot x)(5+1)$

Call by Name

$(x)\left[(5+1)/x\right]$ β-reduction

$\Rightarrow (5+1)$

$\Rightarrow \boxed{6}$

Call by Value

$\Rightarrow (\lambda x \cdot x)(6)$

$\Rightarrow (x)\left[6/x\right]$ β-reduction

$\lfloor n/n \rfloor$

$\Rightarrow$ $\boxed{b}$

Thank you

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