Programming Languages – Assignment I

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1> $(\lambda x.x)(\lambda x.x)$

$\Rightarrow (\lambda x.x)(\lambda t.t)$ α-substitution $(x \to t)$

$\Rightarrow (x)\left[(\lambda t.t)/x\right]$ β-reduction

$\Rightarrow \boxed{\lambda t.t \equiv I}$ I is Identity Function

2> $(\lambda x.x\ x)(\lambda x.\lambda y.x\ x)$

$\Rightarrow (\lambda x.(x\,x))(\lambda t.(\lambda y.(t\,t)))$ α-substitution $(x \to u)$

$\Rightarrow (x\,x)\left[(\lambda t.(\lambda y.(t\,t)))/x\right]$ β-reduction

$\Rightarrow (\lambda t.\lambda y.tt)\ (\lambda t.\lambda y.tt)$

$\Rightarrow (\lambda p.(\lambda y.(pp)))(\lambda t.(\lambda y.(tt)))$ α-substitution $(t \to p)$

$\Rightarrow \lambda y.(pp)\left[(\lambda t.(\lambda y.(tt)))/p\right]$ β-redu.

$\boxed{\Rightarrow (\lambda y.((\lambda t.(\lambda y.(tt)))(\lambda t.(\lambda y.(tt)))))}$

⇒ Here we cannot proceed further because it is already in its most simplest form.

3) $((\lambda x.(xy))(\lambda z.z))$

$\rightarrow (xy)[(\lambda z.z)/x]$ — β-reduction replace x with $\lambda z.z$

$\rightarrow (\lambda z.z)\, y$

$\rightarrow z[y/z]$ — β-reduction, replace z with y

$\rightarrow \boxed{y}$

4) $(\lambda z.z)(\lambda y.y\,y)(\lambda x.x\,a)$

$\rightarrow (z)[(\lambda y.(y\,y))/z](\lambda x.x\,a)$ — β-reduction

$\rightarrow (\lambda y.(y\,y))(\lambda x.(x\,a))$

$\rightarrow (yy)[(\lambda x.(xa))/y]$ — β-reduction, replace y with $(\lambda x.(xa))$

$\rightarrow (\lambda x.(xa))(\lambda x.(xa))$

$\rightarrow (\lambda t.(ta))(\lambda x.(xa))$ — α-Substitution

$\rightarrow (ta)\left[(\lambda n.(ne_1))/t\right]$ B-reduction,
$t \rightarrow (\lambda n.(nq))$

$\rightarrow (\lambda n.(na))\ a$

$\rightarrow (na)\left[a/n\right]$ B-reduction
$(n \rightarrow a)$

$\Rightarrow \boxed{a\ a}$

5) $(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$

$\rightarrow (\lambda z.z)(\lambda t.(tt))(\lambda z.(zy))$ α-substitution
$(z \rightarrow t)$

$\rightarrow (\lambda z.z)(\lambda t.(tt))(\lambda p.(py))$ α-substitution
$(p \rightarrow z)$

$\rightarrow (z)\left[(\lambda t.(tt))/z\right](\lambda p.(py))$ B-reduction

$\rightarrow (\lambda t.(tt))(\lambda p.(py))$

$\rightarrow (tt)\left[(\lambda p.(py))/t\right]$ B-reduction

$\rightarrow (\lambda p.(py))(\lambda p.(py))$

$\rightarrow (\lambda p.(py))(\lambda u.(uy))$ α-substitution
$(p \rightarrow u)$

$\rightarrow (py)\left[(\lambda u.(uy))/p\right]$ B-reduction

$\rightarrow (\lambda u.(uy))\ y$

$\rightarrow (uy)\left[\,y/u\,\right]$ β-reduction

$\rightarrow \boxed{y\ y}$

6) $(\lambda x.\lambda y.x\ y\ y)\ (\lambda a.a)\ b$

$\rightarrow (\lambda y.(x\ y\ y))\left[\,\lambda a.a/x\,\right] b$ → β-reduction

$\rightarrow (\lambda y.((\lambda a.a)\ y\ y))\ b$

$\rightarrow ((\lambda a.a)\ y\ y)\left[\,b/y\,\right]$ β-reduction

$\rightarrow (\lambda a.a)\ b\ b$

$\rightarrow a\left[\,b/a\,\right] b$ β-reduction

$\rightarrow \boxed{b\ b}$

7) $(\lambda x.x\ x)\ (\lambda y.y\ x)\ z$

$\rightarrow (\lambda x.(x\ x))\ (\lambda y.(y\ x))\ z$

$\rightarrow (x\ x)\left[\,(\lambda y.(y\ x))/x\,\right] z$ β-reduction

$\rightarrow (\lambda y.(yx))\ (\lambda y.(yx))\ z$

$\rightarrow (\lambda y.(yx))\ (\lambda t.(tx))\ z$ α-substitution $(y \rightarrow t)$

$\rightarrow (yx)\left[(\lambda t.(tx))/y\right] z$ β-reduction

$\rightarrow (\lambda t.(tx))\ x\ z$

$\rightarrow (tx)\left[x/t\right] z$ β-reduction

$\rightarrow (xx)\ z$

$\rightarrow \boxed{x\ x\ z}$

8. $(\lambda x.(\lambda y.(xy))\ y)\ z$

$\rightarrow (\lambda x.(\lambda y.(xy))\ y)\ z$

$\rightarrow (\lambda x.(\lambda t.(xt))\ y)\ z$ α-substitution $(y \rightarrow t)$

$\rightarrow ((\lambda t.(xt))\ y)\left[z/x\right]$ β-reduction $(z \rightarrow x)$

$\rightarrow ((\lambda t.(zt))\ y)$

$\rightarrow (zt)\left[y/t\right]$ β-reduction $(y \rightarrow t)$

$\rightarrow \boxed{zy}$

q. $((\lambda x.xx)(\lambda y.y))(\lambda y.y)$

$\rightarrow ((\lambda x.xx)(\lambda u.u))(\lambda y.y)$ α-substitution $(y \to u)$

$\rightarrow (xx)[\lambda u.u/x])(\lambda y.y)$ β-reduction

$\rightarrow ((\lambda u.u)(\lambda u.u))(\lambda y.y)$

$\rightarrow ((\lambda t.t)(\lambda u.u))(\lambda y.y)$ α-substitution $(t \to u)$

$\rightarrow (t[\lambda u.u/t])(\lambda y.y)$ β-reduction

$\rightarrow (\lambda u.u)(\lambda y.y)$

$\rightarrow u[\lambda y.y/u]$ β-reduction

$\rightarrow \boxed{\lambda y.y}$

10. $(((\lambda n.\lambda y.(n\,y))\,(\lambda y.y))\,w)$

$\rightarrow (((\lambda n.\lambda t.(n\,t))\,(\lambda y.y))\,w)$

α-substitution $(y \to t)$

$\rightarrow (((\lambda t.(n\,t))\,[(\lambda y.y)/n])\,w)$

β-reduction

$\rightarrow (((\lambda t.((\lambda y.y)\,t))\,w)$

$\rightarrow ((\lambda y.y)\,t)\,[w/t]$ β-reduction

$\rightarrow (\lambda y.y)\,w$

$\rightarrow y[w/y]$ β-reduction

$\rightarrow$ $\boxed{w}$

The End