Lecture 2-3 Stock Positions and Portfolio Return

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Problem A

Return on KLV stock in 6/2010

The return on KLV stock is given by:

$$R_{KLV}(6/10) = \frac{P_1 - P_0}{P_0} = \frac{57.50 - 50}{50} = 15\%$$

Problem B

1. Value of Brokerage Account at the start of 6/2010

The value of Jane's brokerage account at the start of 6/2010:

$$\text{Value of Brokerage Account (start 6/10)} = \$12000$$

Jane borrows \$3000 and invests \$12000, thus:

$$15000 - 3000 = 12000$$

2. Fraction of Portfolio in KLV Stock (start 6/2010)

Let $w_{KLV,p}(\text{start } 6/10)$ denote the fraction of the portfolio invested in KLV stock:

$$w_{KLV,p}(\text{start } 6/10) = \frac{15000}{12000} = 1.25$$

3. Fraction of Portfolio in Riskless Asset (start 6/2010)

Let $w_{f,p}(\text{start } 6/10)$ denote the fraction of the portfolio invested in the riskless asset:

$$w_{f,p}(\text{start } 6/10) = \frac{-3000}{12000} = -0.25$$

Since:

$$w_{f,p}(\text{start } 6/10) = 1 - w_{KLV,p}(\text{start } 6/10) = -0.25$$

4. Percent Margin (start 6/2010)

The percent margin at the start of 6/2010 is:

$$\text{Percent Margin(start 6/10)} = \frac{\text{Net Worth (start 6/10)}}{\text{Value of Stock (start 6/10)}} = \frac{12000}{15000} = 0.80$$

5. Return on Jane's Portfolio in 6/2010

The return on Jane's portfolio is given by:

$$R_p(6/10) = w_{KLV,p}(\text{start } 6/10)R_{KLV}(6/10) + w_{f,p}(\text{start } 6/10)R_f(6/10)$$

$$R_p(6/10) = 1.25 \times 15\% + (-0.25) \times 1\% = 18.5\%$$

6. Value of Brokerage Account at the end of 6/2010

The value of Jane's brokerage account at the end of 6/2010 is:

$$\text{Net Worth (end 6/10)} = \text{Net Worth (end 5/10)} \times [1 + R_p(6/10)]$$

$$\text{Net Worth (end 6/10)} = 12000 \times (1 + 0.185) = 12000 \times 1.185 = \$14220$$

Problem C

1. Value of Brokerage Account at the start of 6/2010 (Short Sell)

The value of the brokerage account at the start of 6/2010 is:

$$\text{Value of Brokerage Account (start 6/10)} = \$12000$$

Since Jane short-sells $9000 worth of KLV stock, we get:

$$-9000 + 21000 = 12000$$

2. Fraction of Portfolio in KLV Stock (start 6/2010)

The fraction of the portfolio invested in KLV stock is:

$$w_{KLV,p}(\text{start 6/10}) = \frac{-9000}{12000} = -0.75$$

3. Fraction of Portfolio in Riskless Asset (start 6/2010)

The fraction of the portfolio invested in the riskless asset is:

$$w_{f,p}(\text{start 6/10}) = \frac{21000}{12000} = 1.75$$

Since:

$$w_{f,p}(\text{start 6/10}) = 1 - w_{KLV,p}(\text{start 6/10}) = 1.75$$

4. Percent Margin (start 6/2010)

The percent margin at the start of 6/2010 is:

$$\text{Percent Margin(start 6/10)} = \frac{\text{Net Worth (start 6/10)}}{\text{Value of Stock (start 6/10)}} = \frac{12000}{9000} = 1.3333$$

5. Return on Jane's Portfolio in 6/2010

The return on Jane's portfolio in 6/2010 is:

$$R_p(6/10) = w_{KLV,p}(\text{start 6/10})R_{KLV}(6/10) + w_{f,p}(\text{start 6/10})R_f(6/10)$$

$$R_p(6/10) = -0.75 \times 15\% + 1.75 \times 1\% = -9.5\%$$

6. Value of Brokerage Account at the end of 6/2010

The value of Jane's brokerage account at the end of 6/2010 is:

$$\text{Net Worth (end 6/10)} = \text{Net Worth (end 5/10)} \times [1 + R_p(6/10)]$$

$$\text{Net Worth (end 6/10)} = 12000 \times (1 - 0.095) = 12000 \times 0.905 = \$10860$$

Lecture 3 3-4 CQ

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I. State Expansion, Recovery, Crisis

Suppose there are three possible states at the end of the period (expansion, recovery, and crisis) and Jim has three assets available (DFQ, BML, and the riskless asset). Jim only cares about the expected return on his portfolio and the standard deviation of return on his portfolio. The following data is available concerning the return on DFQ (R_{DFQ}), the return on BML (R_{BML}), and the probability of each state:

State	**Probability**	R_{DFQ}	R_{BML}
Expansion (Ex)	0.2	33%	20%
Recovery (Re)	0.7	8%	10%
Crisis (Cr)	0.1	-12%	-10%

A. Expected Return

1. Expected return for DFQ:

$$E[R_{DFQ}] = 0.2 \times 33\% + 0.7 \times 8\% + 0.1 \times -12\% = 11\%$$

2. Expected return for BML:

$$E[R_{BML}] = 0.2 \times 20\% + 0.7 \times 10\% + 0.1 \times -10\% = 10\%$$

3. Expected return for the riskless asset:

$$E[R_f] = R_f = 6\%$$

What does expected return measure?
Expected return measures the average or mean return of an asset.

B. Standard Deviation of Return

1. Variance of DFQ:

$$\sigma^2[R_{DFQ}] = 0.2 \times (33 - 11)^2 + 0.7 \times (8 - 11)^2 + 0.1 \times (-12 - 11)^2 = 156$$

Standard deviation:

$$\sigma[R_{DFQ}] = \sqrt{156} = 12.49\%$$

2. Variance of BML:

$$\sigma^2[R_{BML}] = 0.2 \times (20 - 10)^2 + 0.7 \times (10 - 10)^2 + 0.1 \times (-10 - 10)^2 = 60$$

Standard deviation:

$$\sigma[R_{BML}] = \sqrt{60} = 7.75\%$$

3. Standard deviation of the riskless asset:

$$\sigma^2[R_f] = 0.2 \times (6 - 6)^2 + 0.7 \times (6 - 6)^2 + 0.1 \times (6 - 6)^2 = 0$$

Standard deviation:

$$\sigma[R_f] = 0\%$$

What does the standard deviation of return measure?

The standard deviation measures the dispersion or volatility of an asset's return.

C. Covariance and Correlation between DFQ and BML

1. Covariance between DFQ and BML:

$$\sigma[R_{DFQ}, R_{BML}] = 0.2\times(33-11)(20-10)+0.7\times(8-11)(10-10)+0.1\times(-12-11)(-10-10) = 90$$

Correlation between DFQ and BML:

$$\rho[R_{DFQ}, R_{BML}] = \frac{90}{12.49 \times 7.75} = 0.93$$

2. Covariance between DFQ and the riskless asset:

$$\sigma[R_{DFQ}, R_f] = 0.2\times(33-11)(6-6)+0.7\times(8-11)(6-6)+0.1\times(-12-11)(6-6) = 0$$

Correlation:

$$\rho[R_{DFQ}, R_f] = \text{undefined}$$

What do covariance and correlation measure?

Covariance and correlation measure the extent to which two assets' returns move together. Correlation is a standardized measure of covariance and must lie between -1 and 1.

D. Regression of DFQ on BML

Consider the regression:

$$R_{DFQ} = a + bR_{BML} + e$$

where b is the regression slope coefficient, a is the regression intercept, and e is the residual term.

E. Portfolio Allocation Decision

Jim only cares about the expected return and standard deviation of return. He needs:

$$E[R_{DFQ}], E[R_{BML}], \sigma[R_{DFQ}], \sigma[R_{BML}], \sigma[R_{DFQ}, R_{BML}], R_f$$

II. Portfolio Management with DFQ and the Riskless Asset

A. A Portfolio with 40% DFQ and 60% in the Riskless Asset

1. Expected return:

$$E[R_p] = 0.4 \times 11\% + 0.6 \times 6\% = 8\%$$

2. Standard deviation:

$$\sigma[R_p] = 0.4 \times 12.49\% = 4.996\%$$

B. A Portfolio with 140% DFQ and -40% in the Riskless Asset

1. Expected return:

$$E[R_p] = 1.4 \times 11\% - 0.4 \times 6\% = 13\%$$

2. Standard deviation:

$$\sigma[R_p] = 1.4 \times 12.49\% = 17.49\%$$

C. A Portfolio with -40% DFQ and 140% in the Riskless Asset

1. Expected return:

$$E[R_p] = -0.4 \times 11\% + 1.4 \times 6\% = 4\%$$

2. Standard deviation:

$$\sigma[R_p] = |-0.4| \times 12.49\% = 4.996\%$$

D. Short-Selling and Buying on Margin

1. Will Jim short-sell DFQ?

Jim will not short-sell DFQ because $E[R_{DFQ}] > R_f$. Short-selling DFQ results in a portfolio on the negative-sloped portion of the arrowhead in $\{E[R], \sigma[R]\}$ space, which is suboptimal.

2. Will Jim buy DFQ on margin?

Jim might buy DFQ on margin if his risk aversion is low, as it leads to a portfolio on the positive-sloped portion of the arrowhead.

E. Choice Between DFQ and BML

Jim will choose to invest in the asset with the steepest Capital Allocation Line (CAL). Since:

$$\text{slope}[CAL_{DFQ}] = \frac{11-6}{12.49} = 0.40, \quad \text{slope}[CAL_{BML}] = \frac{10-6}{7.75} = 0.516$$

Jim will prefer to hold BML with the riskless asset.