Solution Set: Multifactor Asset Pricing Models, Fixed Income Valuation, and Bond Portfolio Management

I. Multifactor Models

A. Expected January Return for Yellow

Using the equation:

$$E[R_i(Jan)] = R_f(Jan) + \beta_i^*, M\lambda_M^* + \beta_i^*, GDP\lambda_{GDP}^*,$$

we calculate λ_{GDP}^* using the information for Red:

$$18\% = 8\% + 1.4 \times (14\% - 8\%) + 0.4 \times \lambda_{GDP}^*,$$

$$\lambda_{GDP}^* = \frac{1.6\%}{0.4} = 4\%.$$

Then, the expected return for Yellow:

$$E[R_{Yellow}(Jan)] = 8\% + 1.1 \times 6\% + 1.2 \times 4\% = 19.4\%.$$

B. Risk Premium for Bearing GDP Risk

As calculated above:

$$\lambda_{GDP}^* = 4\%.$$

C. Covariance of Portfolio Return with GDP

An individual may care about the covariance of their portfolio return with GDP(Jan) because GDP(Jan) may be correlated with their human capital value at the end of January.

II. Implied Yield Curve, Forward Rates, and No-Arbitrage

A. Implied Yield Curve (APRs with Semiannual Compounding)

1. **Yield on 6-Month Discount Bond:**

$$d_{\frac{1}{2}}(Feb96) = \frac{98.34375}{100 + 2.25} = 0.96180,$$

$$y_{\frac{1}{2}}(Feb96) = \left(\frac{1}{0.96180} - 1\right) \times 2 = 7.9440\%.$$

2. **Yield on 1-Year Discount Bond:**

$$d_1(Feb96) = \frac{99.03125 - 1.125 \times 0.96180}{102.625} = 0.94038,$$

$$y_1(Feb96) = \left(\frac{1}{0.94038}^{0.5} - 1 \right) \times 2 = 6.2425\%.$$

3. **Yield on 1.5-Year Discount Bond:**

$$d_{1.5}(Feb96) = \frac{98.71875 - 1.4375 \times (0.96180 + 0.94038)}{103.4375} = 0.90644,$$

$$y_{1.5}(Feb96) = \left(\frac{1}{0.90644}^{\frac{1}{3}} - 1 \right) \times 2 = 6.6571\%.$$

4. **Yield on 2-Year Discount Bond:**

$$d_2(Feb96) = \frac{98.46875 - 3 \times (0.96180 + 0.94038 + 0.90644)}{103} = 0.87420,$$

$$y_2(Feb96) = \left(\frac{1}{0.87420}^{0.25} - 1 \right) \times 2 = 6.8364\%.$$

B. Implied Forward Rates (APRs with Semiannual Compounding)

Using $d_{t,t+\tau}(0) = \frac{d_{t+\tau}(0)}{d_t(0)}$, we calculate:

$$d_{\frac{1}{2},1}(Feb96) = \frac{d_1(Feb96)}{d_{\frac{1}{2}}(Feb96)} = \frac{0.94038}{0.96180} = 0.97773,$$

$$f_{\frac{1}{2},1}(Feb96) = 2 \left(\frac{1}{0.97773} - 1 \right) = 4.5554\%.$$

C. Price of Aug 97 U.S. Treasury Strip

$$P_{Aug97\,strip}(Feb96) = d_{1.5}(Feb96) \times 100 = 0.90644 \times 100 = 90.644.$$

III. Duration and Interest Rate Sensitivity

A. Price of Feb 96 Note (Aug 94)

$$P_{4.75\%}(Feb96) = \frac{2.375}{1.0275} + \frac{2.375}{1.0275^2} + \frac{102.375}{1.0275^3} = 98.9341.$$

B. Macaulay Duration

$$D_{4.75\%}(Feb96) = 0.5 \times \frac{2.3114}{98.9341} + 1 \times \frac{2.2496}{98.9341} + 1.5 \times \frac{94.3731}{98.9341} = 1.46527.$$

Modified Duration:

$$D_{modified} = \frac{D_{Macaulay}}{1 + \frac{y}{2}} = \frac{1.46527}{1.0275} = 1.42605.$$

IV. Immunization

A. Liability Present Value

$$L(Aug94) = \sum_{k=1}^{5} \frac{5M}{(1.03)^{2k}} = 4.7130M{+}4.4424M{+}4.1874M{+}3.9470M{+}3.7205M = 21.0103M.$$

B. Immunization Strategy

$$DA = DL = 2.8820.$$

$$DA = (1 - \omega_{note}) \times 10 + \omega_{note} \times 1.4527,$$

$$\omega_{note} = \frac{10 - 2.8820}{10 - 1.4527} = 0.8328.$$

Investment:

$$\text{In Note: } 0.8328 \times 21.0103M = 17.4969M,$$

$$\text{In Strip: } 3.5134M.$$