Lecture 9: Bond Portfolio Management

1 Reinvestment Risk and Liquidation Risk

1.1 A. Reinvestment Risk

Reinvestment risk arises when an investor holds an instrument with a fixed cash flow to be received before a specified time horizon, T. The risk is associated with the uncertainty regarding the yields prevailing at the time of receiving the cash flow. In other words, the investor faces reinvestment risk because the future yield at the time of reinvestment is uncertain.

1.2 B. Liquidation Risk

Liquidation risk refers to the uncertainty about the price of an instrument at the time of liquidation. If an investor holds an instrument generating cash flows to be received after the time horizon T, there is uncertainty regarding the price of the instrument at the time of liquidation. This uncertainty is known as liquidation risk.

2 Problem Setup (2/15/10)

2.1 A. Price of the 6% February 11 Note

Given that the yield to maturity (YTM) is 4% per annum with semi-annual compounding, the price of a 6% February 11 note can be computed as follows:

$$P = \frac{3}{\left(1+\frac{0.04}{2}\right)} + \frac{103}{\left(1+\frac{0.04}{2}\right)^2}$$

$$P = 2.9412 + 99.0004 = 101.9416$$

2.2 B. Macaulay Duration of the Note

The Macaulay duration is calculated using the following formula, where k represents the weight of the cash flow at each time period:

$$k\left(\frac{1}{2}\right) = \frac{2.9412}{101.9416} = 0.028852$$

$$k(1) = \frac{99.0004}{101.9416} = 0.971148$$

The duration is:

$$D = \frac{1}{2} \times k\left(\frac{1}{2}\right) + 1 \times k(1) = \frac{1}{2} \times 0.028852 + 1 \times 0.971148 = 0.9856 \text{ years}$$

3 For Coupon Bonds and Strips

3.1 III. Price Behavior of Coupon Bonds and Strips

For coupon bonds and strips, the price is always a decreasing convex function of yield. This implies that as the yield increases, the price decreases in a convex manner.

4 Firm QV's Liability Management

Firm QV needs to make a \$5M payment in 2 years and a \$15M payment in 4 years. The yield curve is flat at 8% APR with semi-annual compounding.

4.1 A. Portfolio with Treasury Strips and Bonds

1. Composition of the Bond and Strip Portfolio:

Firm QV must construct a portfolio with a composition of strips such that it can still fund the payments regardless of any future shift in the yield curve. The solution is to buy 2-year strips with a face value of \$5M and 4-year strips with a face value of \$15M. This strategy will ensure that the firm can meet its future obligations.

2. Rebalancing the Portfolio:

Since this is a dedication strategy, firm QV will not need to rebalance the portfolio, regardless of future shifts in the yield curve.

4.2 B. Portfolio with 3-Year and 5-Year U.S. Treasury Strips

1. Composition of the Strip Portfolio:

Suppose firm QV only has access to 3-year and 5-year U.S. Treasury strips to fund the payments. The goal is to construct a portfolio that matches both the value and Macaulay duration of the liability stream.

The value of the liability stream today is computed as:

$$L = \frac{5M}{\left(1+\frac{0.08}{2}\right)^4} + \frac{15M}{\left(1+\frac{0.08}{2}\right)^8}$$

$$L = \frac{5M}{1.169859} + \frac{15M}{1.372520} = 4.2740M + 10.9604M = 15.2344M$$

The duration of the liabilities today is:

$$D_L = 2 \times \left(\frac{4.2740}{15.2344}\right) + 4 \times \left(\frac{10.9604}{15.2344}\right) = 3.4389 \text{ years}$$

Let A_3 be the dollar amount invested today in the 3-year strips, and A_5 be the dollar amount invested in the 5-year strips. The total portfolio value must equal the value of the liabilities:

$$A_3 + A_5 = 15.2344M$$

The duration of the portfolio is given by:

$$D_P = \omega_{3,P} \times D_3 + (1 - \omega_{3,P}) \times D_5$$

where $D_3 = 3$ years and $D_5 = 5$ years. Setting the portfolio duration equal to the liability duration:

$$3.4389 = \omega_{3,P} \times 3 + (1 - \omega_{3,P}) \times 5$$

Solving for $\omega_{3,P}$:

$$\omega_{3,P} = 0.78055$$

Thus, the amount invested in the 3-year strips is:

$$A_3 = 0.78055 \times 15.2344M = 11.8912M$$

and the amount invested in the 5-year strips is:

$$A_5 = 15.2344M - 11.8912M = 3.3432M$$

2. Rebalancing the Portfolio:

Unlike the previous case where Treasury strips of all maturities were available, firm QV will need to rebalance the portfolio over time to remain able to fund the payments if the yield curve shifts.

Lecture 9-10: Derivatives - Definitions and Payoffs

1 I. Difference Between American and European Options

1.1 A. Difference Between American and European Options

A European option can only be exercised at the option's expiration date, while an American option can be exercised at any time up to and including the expiration date.

2 II. European Call Option on a Stock

Consider a European call option on a stock with an exercise price of \$50, that expires in T periods time. Let $S(t)$ be the stock price at time t, where today is time 0.

2.1 A. Circumstances for Exercising the Option

The holder will exercise the call option at expiration if $S(T) > 50$. Otherwise, the holder will not exercise the option.

2.2 B. Payoff to the Holder of the Option at Expiration

The payoff to the holder of the European call option at expiration as a function of the stock price at expiration, $S(T)$, is:

$$\text{Payoff}_{\text{holder}} = \max(S(T) - 50, 0)$$

2.3 C. Payoff to the Writer of the Option at Expiration

The payoff to the writer (seller) of the European call option at expiration as a function of the stock price at expiration, $S(T)$, is:

$$\text{Payoff}_{\text{writer}} = -\max(S(T) - 50, 0)$$

2.4 D. Relation Between the Payoff to the Holder and the Writer

The payoff to the writer is always the negative of the payoff to the holder.

2.5 E. Cost to Enter Long and Short Positions in the Option

Since the payoff to the holder of the call option at expiration is non-negative, the price of the call option must be non-negative, otherwise, there would be an arbitrage opportunity. Therefore, the holder must pay a non-negative price to enter a long position, and the writer receives the non-negative price to enter a short position.

3 III. European Put Option on a Stock

Consider a European put option on a stock with an exercise price of \$50, that expires in T periods time. Let $S(t)$ be the stock price at time t, where today is time 0.

3.1 A. Circumstances for Exercising the Option

The holder will exercise the put option at expiration if $S(T) < 50$. Otherwise, the holder will not exercise the option.

3.2 B. Payoff to the Holder of the Option at Expiration

The payoff to the holder of the European put option at expiration as a function of the stock price at expiration, $S(T)$, is:

$$\text{Payoff}_{\text{holder}} = \max(50 - S(T), 0)$$

3.3 C. Payoff to the Writer of the Option at Expiration

The payoff to the writer (seller) of the European put option at expiration as a function of the stock price at expiration, $S(T)$, is:

$$\text{Payoff}_{\text{writer}} = -\max(50 - S(T), 0)$$

3.4 D. Relation Between the Payoff to the Holder and the Writer

The payoff to the writer is always the negative of the payoff to the holder.

3.5 E. Cost to Enter Long and Short Positions in the Option

Since the payoff to the holder of the put option at expiration is non-negative, the price of the put option must be non-negative, otherwise, there would be an arbitrage opportunity. Therefore, the holder must pay a non-negative price to enter a long position, and the writer receives the non-negative price to enter a short position.

4 IV. Forward Contracts

Consider a forward contract entered into today for the delivery of an asset in T periods at a forward price of \$50. Let $S(t)$ be the asset's price at time t, where today is time 0.

4.1 A. Payoff to the Buyer of the Forward Contract at the Settlement Date

The payoff to the buyer of the forward contract at the settlement date as a function of the asset's price at expiration, $S(T)$, is:

$$\text{Payoff}_{\text{buyer}} = S(T) - 50$$

4.2 B. Payoff to the Seller of the Forward Contract at the Settlement Date

The payoff to the seller of the forward contract at the settlement date as a function of the asset's price at expiration, $S(T)$, is:

$$\text{Payoff}_{\text{seller}} = 50 - S(T)$$

4.3 C. Timing of Cash Flows in a Forward Contract

Money only changes hands at the settlement date. The buyer's cash flow at the settlement date is $[S(T) - 50]$, while the seller's cash flow is $[50 - S(T)]$.

4.4 D. Relation Between the Payoff to the Buyer and Seller of the Forward Contract

The payoff to the seller is always the negative of the payoff to the buyer.

4.5 E. Cost to Buy or Sell a Forward Contract

No money changes hands between the buyer and the seller of the forward contract when the contract is entered into today. The forward price is set such that the contract is entered into with no initial payment.

5 V. Futures Contracts vs Forward Contracts

5.1 A. Relationship Between Futures and Forward Contracts

Forward and futures contracts with the same settlement date have similar exposures to the underlying asset, except futures contracts have daily resettlement, whereas forward contracts do not. With a forward contract, no money changes hands until the settlement date, when the value of the contract at the settlement date changes hands.

In contrast, with a futures contract, no money is paid or received on the day the contract is bought or sold. However, each day after the contract is bought or sold, the value of the contract at the end of the day changes hands. The sum of the daily changes in the value of the futures contract roughly equals the value of the forward contract on the settlement date. The timing difference in cash flows explains why futures and forward prices are typically slightly different.

5.2 B. Key Differences Between Futures and Forward Contracts

- Forward contracts are tailored to the needs of the parties, while futures contracts traded on exchanges like the CME have standardized unit sizes and standardized maturity dates.
- Forward contracts are usually traded in dealer markets, while futures contracts are traded only on exchanges by law.
- In a forward market, you must be concerned with the creditworthiness of your counterparty. In a futures market, exchange members are responsible for their customers' transactions, and the clearing house interposes itself between the two parties by buying from the seller and selling to the buyer.
- With a forward contract, no money changes hands until settlement, while with a futures transaction, the buyer and seller are required to post funds with their brokers and all exchange members are required to post funds with the clearing house.

Lecture 10: Options - Valuation

1 I. Option Valuation with Dell Data

Use the data for May 05 Dell calls and puts at the end of Concept Questions 10 to answer the following questions. The data is from Bloomberg screens on 4/13/05 for Dell options. The May 05 options expire 5/21/05, and Dell was not paying any dividend between 4/13/05 and 5/21/05. Note that the "Fin Rate" of 2.76% per annum is the continuously compounded risk-free rate, and that the Dell stock price on 4/13/05 is \$36.91. Days to expiration are 38, so $T = 38$.

The "Fin Rate" is the risk-free rate and is 2.76%. Since this is a continuously compounded annual rate, it implies a discount factor on a 38-day discount bond of:

$$d_T(0) = e^{-0.0276\times\frac{38}{365}} = 0.9971$$

The Dell stock price on 4/13/05 is \$36.91, so $S(0) = 36.91$.

1.1 A. May 05 Call Option with Exercise Price \$32.5

Exercise price: $X = 32.5$
Last trade price: $C_{X,T}(0) = 4.70$

$$\text{Intrinsic Value} = \max\{S(0) - X \times d_T(0), 0\} = \max\{36.91 - 32.5 \times 0.9971, 0\} = \max\{4.50, 0\} = 4.50$$

$$\text{Time Value} = C_{X,T}(0) - \text{Intrinsic Value} = 4.70 - 4.50 = 0.20$$

1.2 B. May 05 Call Option with Exercise Price \$37.5

Exercise price: $X = 37.5$
Last trade price: $C_{X,T}(0) = 0.75$

$$\text{Intrinsic Value} = \max\{S(0) - X \times d_T(0), 0\} = \max\{36.91 - 37.5 \times 0.9971, 0\} = \max\{-0.48, 0\} = 0$$

$$\text{Time Value} = C_{X,T}(0) - \text{Intrinsic Value} = 0.75 - 0 = 0.75$$

1.3 C. May 05 Put Option with Exercise Price \$32.5

Exercise price: $X = 32.5$
Last trade price: $P_{X,T}(0) = 0.10$

$$\text{Intrinsic Value} = \max\{X \times d_T(0) - S(0), 0\} = \max\{32.5 \times 0.9971 - 36.91, 0\} = \max\{-4.50, 0\} = 0$$

$$\text{Time Value} = P_{X,T}(0) - \text{Intrinsic Value} = 0.10 - 0 = 0.10$$

1.4 D. May 05 Put Option with Exercise Price \$37.5

Exercise price: $X = 37.5$
Last trade price: $P_{X,T}(0) = 1.25$

$$\text{Intrinsic Value} = \max\{X \times d_T(0) - S(0), 0\} = \max\{37.5 \times 0.9971 - 36.91, 0\} = \max\{0.48, 0\} = 0$$

$$\text{Time Value} = P_{X,T}(0) - \text{Intrinsic Value} = 1.25 - 0 = 0.77$$

2 II. Exercising an American Call Option on a Non-Dividend Paying Stock

Would you ever exercise an American call option on a non-dividend paying stock early? Explain why or why not. What would you do if your only exposure to a non-dividend stock is American call options on that stock, and you receive some reliable information that tells you that the stock's price is going to drop dramatically in the next few days?

The holder of an American call option on a non-dividend paying stock should never exercise early. Prior to expiration, the value of the option to someone who is willing to hold it to expiration is always more than what the holder gets if she exercises immediately. This is because the intrinsic value of the option is always more than what the holder gets if she exercises immediately.

The reason is that if the holder exercises now, she has to pay the exercise price immediately, whereas the party who waits until expiration to decide whether to exercise delays paying the exercise price until the expiration date.

Consequently, if your only exposure to a non-dividend stock is American call options on that stock, and you receive some reliable information that tells you that the stock's price is going to drop dramatically in the next few days, you should sell the options, not exercise them.

3 III. Price of a European Call Option Using Put-Call Parity

Dell's price today is 36.91. The price of a discount bond (face value of 100) maturing in 38 days is 99.71. A European put expiring in 38 days with an exercise price of 37.5 has a price of \$1.25 today. Dell will not pay a dividend in the next 38 days. What is the price today of a European call expiring in 38 days with an exercise price of 37.5?

Using put-call parity:

$$S(0) = C_{37.5,38\text{day}}(0) - P_{37.5,38\text{day}}(0) + 37.5 \times d_{38\text{day}}(0)$$

$$C_{37.5,38\text{day}}(0) = P_{37.5,38\text{day}}(0) + S(0) - 37.5 \times d_{38\text{day}}(0)$$

Substituting the values:

$$C_{37.5,38\text{day}}(0) = 1.25 + 36.91 + 37.5 \times 0.9971 = 0.77$$

4 IV. Price Comparison Between American and European Options

4.1 A. American vs. European Call Option

An American call option always has a price at least as high as the price of an otherwise identical European call option because the American option gives you the option to exercise at expiration like the European call, as well as the additional option to exercise early, which must have a non-negative value.

When the underlying asset does not pay dividends, the holder of an American call option never exercises early, and so the option to do so must be worth 0. Consequently, if the underlying asset does not pay dividends, then the American and otherwise identical European call options always have the same price.

4.2 B. American vs. European Put Option

An American put option always has a price at least as high as the price of an otherwise identical European put option because the American option gives you the option to exercise at expiration like the European put, as well as the additional option to exercise early, which must have a non-negative value.

When the underlying asset does not pay dividends, the holder of an American put option will exercise early if the price of the underlying is sufficiently low, and so the option to do so must be worth at least 0. Consequently, if the underlying asset does not pay dividends, then an American put option always has a price at least as high as the price of an otherwise identical European put.

5 V. Black-Scholes Option Pricing Model

5.1 A. Five Inputs into the Black-Scholes Model

The five inputs are:

- $S(0)$: the current price of the underlying asset.
- X: the exercise price.
- σ: the volatility of the continuously compounded annual return on the underlying asset.
- r': the continuously compounded annual risk-free rate (i.e., $1investedtodayattherisklessrateisworth1 \cdot e^{r' \cdot t}$ in t years).
- T: the time to expiration.

5.2 B. How Each Input Affects the Price of a European Call Option on a Non-Dividend Paying Stock

- $S(0)$: $C_{X,T}(0)$ is monotonically increasing in $S(0)$.
- X: $C_{X,T}(0)$ is monotonically decreasing in X.
- σ: $C_{X,T}(0)$ is monotonically increasing in σ.
- r': $C_{X,T}(0)$ is monotonically increasing in r'.
- T: $C_{X,T}(0)$ is monotonically increasing in T.

5.3 C. How Each Input Affects the Price of a European Put Option on a Non-Dividend Paying Stock

- $S(0)$: $P_{X,T}(0)$ is monotonically decreasing in $S(0)$.
- X: $P_{X,T}(0)$ is monotonically increasing in X.
- σ: $P_{X,T}(0)$ is monotonically increasing in σ.
- r': $P_{X,T}(0)$ is monotonically decreasing in r'.
- T: The effect of T on $P_{X,T}(0)$ is ambiguous.