Lecture Notes: Multifactor Asset Pricing Models and Empirical Evidence

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1 Multifactor Asset Pricing Models and Empirical Evidence

1.1 I. Investors and Covariance

Let R_p be portfolio return and E_I be a macroeconomic indicator that summarizes the state of the economy. Briefly discuss why investors might care about $\text{cov}(R_p, E_I)$ in addition to $E[R_p]$ and $\text{cov}(R_p)$.

Answer: Multi-period individuals who receive labor income each period dislike receiving negative shocks to their human capital value over a period due to an increased probability of a low bonus or, worse, job loss. Negative shocks to human capital value are more likely to occur if the economy is unexpectedly weaker at the end of the period. E_I summarizes the state of the economy at the end of the period and so investors dislike unexpectedly low values of E_I at the end of the period. Consequently, each individual, to a degree that depends on her risk aversion and the cyclicality of her labor income, would like her portfolio return to have a covariance with E_I that is as low as possible. In other words, individuals dislike $\text{cov}(R_p, E_I)$.

1.2 II. Expected Return on an Asset in the Economy

Let R_i be the return on asset i and E_I be a macroeconomic indicator. Suppose every individual in the economy only cares about $\{E[R_p], \sigma[R_p], \sigma[R_p, E_I]\}$ when forming his/her portfolio p. What determines the expected return on an arbitrary asset i, $E[R_i]$, in the economy?

Answer: The expected return on asset i, $E[R_i]$, can be expressed as follows:

$$E[r_i] = \beta_{i,M}\lambda_M + \beta_{i,E_I}\lambda_{E_I}$$

where λ_M and λ_{E_I} are the risk premia associated with market and E_I risks, respectively. $\beta_{i,M}$ and β_{i,E_I} are regression coefficients from a time-series regression of r_i on r_M and r_{E_I}, where $r_i = R_i - R_f$, $r_M = R_M - R_f$, and $r_{E_I} = R_{E_I} - R_f$.

1.3 III. Growth in GDP and Expected Return on Total Tech

Let ΔG be the growth in GDP over the period and R_i be the return on asset i over the period. Suppose each individual cares about $\{E[R_p], \sigma[R_p], \sigma[R_p, \Delta G]\}$ when forming his/her portfolio p. Let $R_{\Delta G}$ be the return on the portfolio most highly correlated with ΔG. The following additional information is available:

Asset $\beta_{i,\Delta G}$	Expected Return($E[R_i]$)	$\beta_{i,M}$
All Steel 0.4	2.36%	1.2
Total Tech -0.5	?	1.2

where $\beta_{i,M}$ and $\beta_{i,\Delta G}$ are regression coefficients from a multiple regression (time-series) of r_i on r_M and $r_{\Delta G}$.

1.3.1 A. Risk Premium for Bearing $\beta_{i,\Delta G}$ Risk

We know a two-factor model holds with ΔG as the state variable. So all assets lie on:

$$E[r_i] = \beta_{i,M} E[r_M] + \beta_{i,\Delta G} E[r_{\Delta G}]$$

Using this formula for All Steel:

$$2.36\% - 0.6\% = 1.2 \times (1.9\% - 0.6\%) + 0.4 \times E[r_{\Delta G}]$$

Solving for $E[r_{\Delta G}]$:

$$E[r_{\Delta G}] = \frac{1.76\% - 1.2 \times 1.3\%}{0.4} = 0.5\%$$

Thus, the risk premium for bearing $\beta_{i,\Delta G}$ risk is 0.5%.

1.3.2 B. Expected Return on the Hedging Portfolio Highly Correlated with ΔG

The expected return on the hedging portfolio highly correlated with ΔG, $E[R_{\Delta G}]$, is:

$$E[R_{\Delta G}] = E[r_{\Delta G}] + R_f = 0.6\% + 0.5\% = 1.1\%$$

1.3.3 C. Expected Return on Total Tech

The expected return on Total Tech is:

$$E[r_{\text{TT}}] = \beta_{\text{TT},M} E[r_M] + \beta_{\text{TT},\Delta G} E[r_{\Delta G}] = 1.2 \times 1.3\% + (-0.5) \times 0.5\% = 1.31\%$$

Thus, the expected return on Total Tech is:

$$E[R_{\text{TT}}] = E[r_{\text{TT}}] + R_f = 0.6\% + 1.31\% = 1.91\%$$

1.4 IV. Fama-French 3-Factor Model

Explain how the Fama-French 3-factor model is able to deliver the positive CAPM alphas documented for small firms and value firms in the U.S. economy.

Answer: The Fama-French model is a multifactor pricing model that has two state variables that investors care about in addition to the expected return and standard deviation of return on their portfolios. Each of these two state variables has a portfolio with which it is maximally correlated. The SMB zero-investment portfolio, which is the excess return on one of these portfolios, is long small and short big stocks, while being book-to-market neutral. The HML zero-investment portfolio, which is the excess return on the other, is long high and short low book-to-market stocks, while being size neutral.

The Fama-French model implies that all assets satisfy:

$$E[r_i] = \beta_{i,M} E[r_M] + \beta_{i,SMB} E[r_{SMB}] + \beta_{i,HML} E[r_{HML}]$$

Value stocks are highly correlated with r_{HML} (since $\beta_{i,HML}$ for value stocks is high), so value stocks require a high expected return (all else equal) to induce investors to hold them and will have positive CAPM alphas. Small stocks are highly correlated with r_{SMB} (since $\beta_{i,SMB}$ for small stocks is high), so small stocks require a high expected return (all else equal) to induce investors to hold them and have positive CAPM alphas.

Lecture 7-8: Fixed Income Markets

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I. Treasury Bill Data

The following price data is available today from the WSJ website for a given Treasury bill:

Days to Maturity	75
Ask Yield	3.50%

A. Hold-to-Maturity Return (R)

The bond equivalent yield (Y) can be thought of as the hold-to-maturity return expressed as an annual percentage rate (APR) with n-day compounding. The hold-to-maturity return, R, can be calculated from the ask yield using the following formula:

$$R = Y \times \frac{360}{n}$$

Substituting the values:

$$R = 3.50\% \times \frac{360}{75} = 16.8\%$$

Thus, the hold-to-maturity return for the T-bill is 16.8%.

B. Price of the T-Bill

The price of a Treasury bill can be calculated using the following formula, where P is the price, F is the face value (assumed to be 100), Y is the ask yield, and n is the number of days to maturity:

$$P = \frac{F}{(1 + Y \times \frac{n}{360})}$$

Substituting the values:

$$P = \frac{100}{(1 + 0.0350 \times \frac{75}{360})} = 99.27$$

Thus, the price of the T-bill today is $99.27.

II. Federal Reserve's Treasury Bill Sale

T-bills are initially sold at an auction. The two types of bids are:

- Competitive bids specify an amount and a price.
- Non-competitive bids do not specify a price but can be entered for an amount up to $1 million.

The Federal Reserve arranges the competitive bids in order of descending price. It then works its way down the list until the total amount bid (including non-competitive interest) is equal to the amount it wishes to sell. All successful bids, both competitive and non-competitive, are filled at the lowest competitive bid price that is filled.

III. Treasury Note Price Calculation

The following price information is available for 2/15/05:

Rate	6%	Aug 10 note
Ask Yield	4.00%	

The asked price for the 6% Aug 10 note on 2/15/05 can be calculated using the following formula:

$$V_0 = C \times \text{PVAF}_{YTM/2,N} + 100 \times \text{PVIF}_{YTM/2,N}$$

Where:

- C is the coupon payment,
- N is the number of coupon payments to maturity,
- V_0 is the invoice price today.

The price is calculated as follows:

$$V_0 = 29.361 + 80.426 = 109.787$$

Thus, the price of the 6% Aug 10 note is 109.79.

IV. Bond Price and Yield Relationship

A. If the Bond Equivalent Yield is Greater than the Coupon Rate

If the bond equivalent yield (YTM) is greater than the coupon rate, then the coupon rate is less than the YTM, which means that the bond is selling at a discount relative to par value.

$$\text{Coupon Rate} < YTM \implies \text{Price} < 100$$

B. If the Bond Equivalent Yield is Less than the Coupon Rate

If the bond equivalent yield (YTM) is less than the coupon rate, then the coupon rate is greater than the YTM, which means that the bond is selling at a premium relative to par value.

$$\text{Coupon Rate} > YTM \implies \text{Price} > 100$$

V. Treasury Strip Calculation

Today is 2/15/10, and you decide to strip 1000 of the 10% Aug 11 Treasury notes, each with a face value of \$100. The following Treasury strips will be received back from the U.S. Treasury:

2/15/10	8/15/10	8/15/11
	0	3

The Treasury strips you will receive back are:

- 5000 = 50 x Aug 10 coupon interest strips,
- 5000 = 50 x Feb 11 coupon interest strips,
- 5000 = 50 x Aug 11 coupon interest strips,
- 100000 = 1000 x Aug 11 note principal strips.

Lecture 8: Bond Pricing and Forward Rates

1 I. U.S. Treasury Notes and Strips Prices on 8/15/10

A. Price of the Aug 11 Strip

We are given the yield for the Aug 11 strip on 8/15/10 expressed as an APR with semi-annual compounding. To calculate the price of the Aug 11 strip, we use the following formula:

$$\text{Price} = \frac{100}{(1 + \frac{y}{2})^{2 \cdot t}}$$

where y is the yield expressed as an APR, and t is the time to maturity in years. Given the yield for the Aug 11 strip (5.1957%) and the time to maturity of 1 year, the price can be calculated as:

$$\text{Price} = \frac{100}{(1 + \frac{5.1957}{200})^2} = 95.00$$

Thus, the price of the Aug 11 strip is 95.00.

B. Yield of the Feb 12 Strip

We are asked to calculate the yield of the Feb 12 strip on 8/15/10. The yield of the 1.5-year discount bond can be calculated as follows:

$$y_{1.5} = 2 \times \left(\left(\frac{100}{88} \right)^{\frac{1}{3}} - 1 \right)$$

$$y_{1.5} = 8.7064\%$$

Thus, the yield of the Feb 12 strip on 8/15/10 is 8.7064%.

C. Price of the 8% Aug 11 Note

The price of the 8% Aug 11 note can be calculated using the no-arbitrage formula. We are given the prices of the Feb 11 strip (98) and the Aug 11 strip (95). Using the formula:

$$P_{8\%\text{ Aug }11}(0) = d_{\frac{1}{2}}(0) \times \left(\frac{8}{2}\right) + d_1(0) \times \left(100 + \frac{8}{2}\right)$$

where the discount factors are calculated as:

$$d_{\frac{1}{2}}(0) = \frac{98}{100} = 0.98$$
$$d_1(0) = \frac{95}{100} = 0.95$$

Thus, the price is:

$$P_{8\%\text{ Aug }11}(0) = 0.98 \times 4 + 0.95 \times 104 = 102.72$$

Therefore, the price of the 8% Aug 11 note is 102.72.

D. Arbitrage Opportunity

If the price of the 8% Aug 11 note is 102.50, $itislowerthanitsno-arbitragepriceof$ 102.72. To take advantage of this opportunity, we can create an arbitrage position by buying the 8% Aug 11 note and selling a "synthetic" 8% Aug 11 note created from the Feb 11 and Aug 11 strips. Let a be the number of Aug 11 strips and b be the number of Feb 11 strips bought.

The position and cash flows are as follows:

$$\begin{array}{cc} \text{Buy 1 8\% Aug 11 note:} & -102.50 \quad \text{(today)} \\ \text{Sell 1.04 Aug 11 strips:} & +1.04 \times 95 = 98.80 \\ \text{Sell 0.04 Feb 11 strips:} & +0.04 \times 98 = 3.92 \end{array}$$

Thus, the arbitrage profit is:

$$\text{Arbitrage Profit} = -102.50 + 98.80 + 3.92 = 0.22$$

E. Yield Difference between Feb 12 Strip and 10% Feb 12 Note

The yield on the Feb 12 strip on 8/15/10 is 8.7064%, which is higher than the yield on the 10% Feb 12 note (8.5153%). The reason for this difference is:

1. The strip only pays its par value in 18 months, while the note pays coupons in 6 months and in 1 year, in addition to coupon and principal in 18 months.
2. The yields on the Feb 11 and Aug 11 strips are lower than the yield on the Feb 12 strip, affecting the weighted average yield of the 10% Feb 12 note.

F. Synthetic Aug 11 Strip Creation

To create a synthetic Aug 11 strip using the 2% Feb 11 and 8% Aug 11 notes, let a and b represent the number of 8% Aug 11 and 2% Feb 11 notes to be bought, respectively.

The cash flows are:

$$\begin{aligned} \text{Buy a 8\% Aug 11 notes:} \quad & a \times 4 \quad \text{at } 2/15/11 \\ \text{Buy b 2\% Feb 11 notes:} \quad & b \times 101 \quad \text{at } 2/15/11 \end{aligned}$$

To solve for a and b, we find:

$$\begin{aligned} a \times 104 = 100 \quad & \Rightarrow \quad a = 0.96154 \\ a \times 4 + b \times 101 = 0 \quad & \Rightarrow \quad b = -0.03808 \end{aligned}$$

Thus, the synthetic Aug 11 strip is created by buying 0.96154 of the 8% Aug 11 notes and selling 0.03808 of the 2% Feb 11 notes.

G. Holding Period Return and Yield Comparison for Feb 12 Note

1. Holding Period Return

To calculate the holding period return on the 10% Feb 12 note over the period 8/15/10 to 2/15/11:

$$\text{Holding Period Return} = \frac{P_{\text{final}} - P_{\text{initial}} + \text{Coupons}}{P_{\text{initial}}}$$

Given the price on 8/15/10 and 2/15/11, the holding period return is:

$$\text{Holding Period Return} = 7.7413\%$$

2. Comparison of YTM on 8/15/10 and 2/15/11

Since the holding period return on the 10% Feb 12 note is lower than the YTM on 8/15/10, the YTM on 2/15/11 must be higher than on 8/15/10.

H. FRA Contract for Tom

Tom has $5000toinveston8/15/11for6months.Tolockintherate, heshouldenterintoa12x18FRAon8/1$
is calculated as:

$$f_{1,1.5}(0) = 15.9091\%$$

Thus, Tom will be able to lock in a forward rate of 15.9091% for the period from 8/15/11 to 2/15/12.