Baye's Theorem

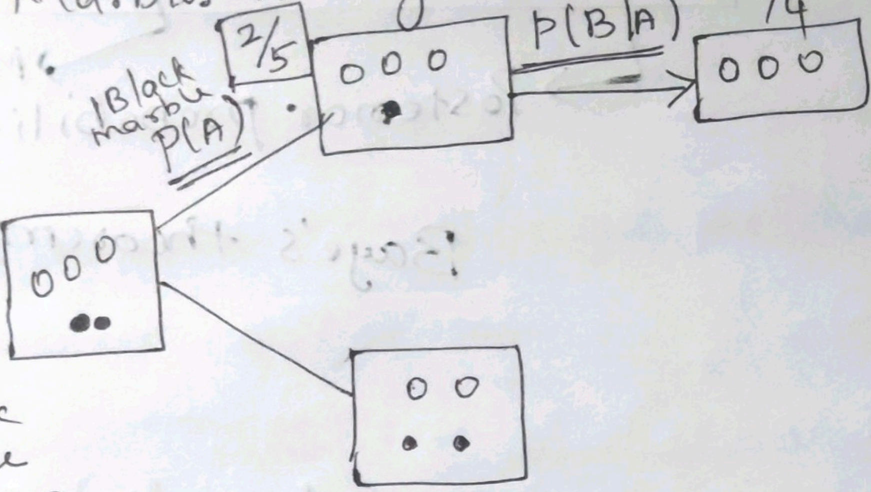
① Conditional Probability

② Independent Events

③ Dependent Events

Dependent Event.

Eg: Marbles in Bag (Red, Black) = 5



1 black marble

$P(B) = \frac{2}{5}$

$P(B|A) = \frac{1}{4} \longrightarrow$ choose 1 black marble.

$P(A \cap B) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$

$P(B|A) \Longrightarrow$ Meaning
⇓
probability of an event B and given A (already done)

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

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$$P(A\cap B) = P(A|B) * P(B)$$ $\Big\}$ $P(A\cap B) = P(B$

$$P(B\cap A) = P(B|A) * P(A)$$

$$P(A/B) * P(B) = P(B/A) * P(A)$$

$$\boxed{P(A/B) = \frac{\overbrace{P(B/A)}^{\text{Likelihood}} * \overbrace{P(A)}^{\text{Prior}}}{\underbrace{P(B)}_{\text{Marginal}}}}$$

$P(A/B)$ → Posterior probability.

Baye's theorem.

<u>Note:</u>

If A and B are independent events,

then $P(A|B) = P(A)$

$$\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}}$$

Conditional Probability

↳ possibility of an event or outcome happening based on the existence of the previous event or outcome.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability of A given that B has already occurred

Similarly,

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \qquad P(A) \neq 0$$

~~4 boys and 2 girls are randomly~~

~~seated in a row.~~

~~[illegible]~~

Independent Events

$$P(A \cap B) = P(A) \cdot P(B)$$

proof:

$$P(A/B) = P(A)$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = P(A)$$

$$\boxed{\Rightarrow P(A \cap B) = P(A) \cdot P(B)}$$

A bag contains 5 red balls and 4 green balls

Event $A = \{$first ball is red$\}$

Event $B = \{$second ball is red$\}$

Explain the concepts of dependent and independent events by finding P(A and B)

with replacement

$P(A) = \frac{5}{9}$

$P(B) = \frac{5}{9}$

$P(A \text{ and } B) = \frac{5}{9} \cdot \frac{5}{9}$

Independent Event

$P(A \cap B) = P(A) \cdot P(B)$.

Without replacement

$P(A) = \frac{5}{9}$

If the first ball is red, w/o replacement

o o o o red

• • • • green

$P(B/A) = \frac{4}{8} = \frac{1}{2}$

prob. of second red ball when 1st is known to be red

$P(A \text{ and } B) = P(A) \cdot P(B)$

$= \frac{5}{9} \cdot \frac{1}{2} = \frac{5}{18}$

$P(A/B) \Rightarrow$ 1) probability of A with respect to B

2) prob. of A taking B as sample space.

3) prob. of A when B has already occurred.

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \qquad P(B) \neq 0$$

Question Mother, father and daughter line up at random in a queue. Find $P(A/B)$ for A = Daughter on one end, B = father in middle.

$$S = \{MFD, FMD, MDF, DMF, DFM, FDM\}$$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{2}{2} = 1$$

$\boxed{Ans = 1}$

Qn: Dice is thrown twice and sum of numbers is observed to be 4. What is probability that number 2 has appears atleast once?

Soln:

Let A = sum of numbers is 4

B = No '2' has appeared atleast once.

$A = \{ \underline{(2,2)} (1,3) (3,1) \}$ $n(A) = 3$

$B = \{ (2,1) \underline{(2,2)} (2,3) (2,4) (2,5)$
$(2,6) (1,2) (3,2) (4,2), (5,2)$
$(6,2) \}$

$n(B) = 11$

find conditional probability.

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1}{3}$$

$$= \frac{1}{3}.$$

Find $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$

$$2P(A) = \frac{5}{13}$$

$$\boxed{P(A) = \frac{5}{26}}$$

$$P(B) = \frac{5}{13}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{5}$$

$$\frac{P(A\cap B)}{P(B)} = \frac{2}{5}$$

$$\frac{2}{\cancel{8}} = \frac{P(A\cap B)}{\cancel{8}} \times 13$$

$$\boxed{P(A\cap B) = \frac{2}{13}}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

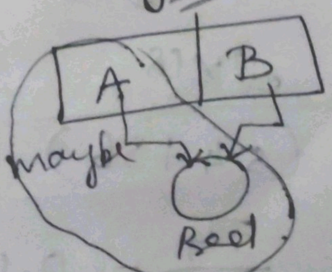
$$P(A\cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$$

$$= \frac{5}{26} + \frac{3}{13}$$

$$\boxed{P(A\cup B = \frac{11}{26}}$$

Baye's Theorem (problems)

Bag A contains 3 red and 4 green balls. Bag B contains 4 red and 5 green balls. One ball is drawn at random from one of the bags and found to be red. What is the probability that it was drawn from Bag A?



Bag A	Bag B	
• • •	• • • •	red
o o o o	o o o o o	green

probability of selecting bag A $P(A) = \frac{1}{2}$

probability of selecting bag B

$$P(B) = \frac{1}{2}$$

prob. of drawing red ball from A

$$P(R|A) = \frac{3}{7}$$

$$P(R|B) = \frac{4}{9}$$

$$P(A|R) = \frac{P(A).P(R/A)}{P(A).P(R/A)+P(B).P(R/B)}$$

$$= \frac{\frac{1}{2}\times\frac{3}{7}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{4}{9}}$$

$$\begin{array}{r|l} 2 & 14, 18 \\ \hline 3 & 7, 9 \\ \hline & 7, 3 \end{array}$$

$$= \frac{\frac{3}{14}}{\frac{3}{14}+\frac{4}{18}}$$

$$\left[\frac{3\times18+4\times14}{14\times18}\right.$$

$$= \frac{3}{\cancel{14}}\times\frac{\cancel{14}\times18}{(3\times18+4\times14)}$$

$$= \frac{3\times18}{(54+56)} = \frac{54}{54+56}$$

$$= \frac{54}{110} = 0.49 //$$

Amy has two bags. Bag I has 7 red and 2 blue balls and bag II has 5 red and 9 blue balls. Amy draws a ball at random and it turns out to be red. Determine the probability that the ball was from the bag I using the Bayes theorem.

Soln:-

Let X and Y be the events that the ball is from the bag I and bag II.

Assume A to be the event of drawing a red ball.

probability of choosing a bag for drawing a ball is ½ that is,

$p(X) = p(Y) = \frac{1}{2}$

Since there are 7 red balls out of a total of 11 balls in the bag I,

∴ P(drawing a red ball from the bag I) = $P(A|X) = \frac{7}{11}$

P(drawing a red ball from bag II)

$P(A|Y) = \frac{5}{14}$

~~P(X)~~

To determine the value of

P(the ball drawn is from the bag I given that it is a red ball)

ie) $P(X|A)$

$$P(X|A) = \frac{P(A|X)\,P(X)}{P(A|X)P(X) + P(A|Y)P(Y)}$$

$$= \left[\left((7/11)(1/2) \right) / \left((7/11)(1/2) + (5/14)(1/2) \right) \right]$$

$$= \underline{0.64}$$

Ans: Hence, the probability that the ball is drawn is from bag I is <u>0.64</u>.