FINDING SOLUTION OF NONLINEAR CONSTRAINED OPTIMIZATION USING SPIRAL OPTIMIZATION ALGORITHM

Michael Sipayung and Kuntjoro Adji Sidarto

Jl. Jamin Ginting 578, Medan 20155
Jl. Ganesha 10, Bandung 40132
20920306@mahasiswa.itb.ac.id, sidarto@math.itb.ac.id

Abstract. A Nonlinear Constrained optimization is one of the interesting problems in optimization which arises frequently in a wide range of operational applications, scientific and engineering. The goal of this problem is solving an optimization problem where some of the constraints or the objective function are nonlinear. Some algorithms fail to find global solutions and focus only on local solutions. The augmented Lagrangian, sequential minimization method based on the use of penalty functions and interior-point methods, which are widely used, have advantage on their speed of convergence, but several important new challenges arise such as nonconvexity, the presence of nonlinearities and the need to ensure progress toward the solution. These methods are also assumed that the objective function is twice differentiability, for simple objective function it is easy to find the gradient and the hessian of the objective function but for complicated expression which is hard to find the derivative of the objective function analytically. In general, to satisfy optimality condition for nonlinearly constrained optimization, the differential form of the current objective function is required. In this paper, the proposed method is to find the global minimizer of general nonlinearly constrained optimization problems that its differential form is not needed.

Keywords: *Nonlinear constrained optimization, spiral optimization algorithm, penalty function*

1 Introduction

Optimization is an important tool in decision science, many problems in the real world can be modeled or transform as optimization problems. The goals are to find values of the variables that optimize the objective function. In the real world, often there are exist some constrains or restricted to the variables. The existing of the constrains always make the problem more complicated and we cannot use the previous or common algorithm for solving unconstrained optimization to solve constrained optimization. There are two kinds of constrains in optimization problems, they are called linear and nonlinear constrained optimization that may consist of inequality or equality. Figures 1

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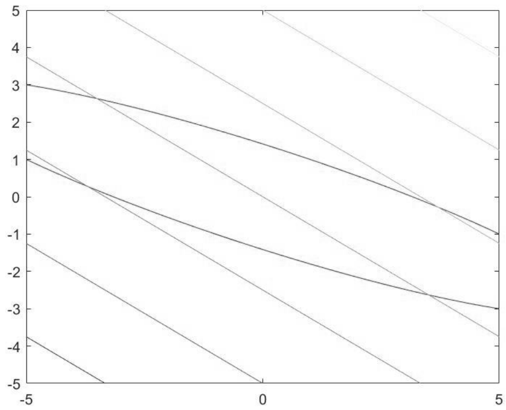

Figure 1. the graph of objective function with additional equality constrains

and figures 2 show the graph of the objective function with an additional equality and inequality constrains respectively. In this paper we are focuses on nonlinear constrained optimization. The general structure of constrained optimization problem is essentially contained in the following

$$\min_{x \in \mathcal{R}^n} f(x) \quad \text{subject to} \quad c_i(x) = 0, i = \mathcal{E} \qquad (1)$$
$$c_i(x) \geq 0, i = I$$

where f(x) is the objective function and $c_i(x)$ are additional constrains functions.

$\mathcal{E}$ is the index set of equality constraints in the problem, I is the set of inequality constrains and both these sets are finite. If both the objective function and constraint functions may be nonlinear, this problem refers to nonlinear programming and if all constraints and the objective functions may be linear the problem refers to linear programming. There are exist numbers of method to solve linear programming and nonlinear programming.

Most of algorithms in optimization based on derivative information and they also use local search methods. The phase to find the optimum point usually starts with the initial guess, and then tries to improve the quality of search through iterative process to find the optimum or the best possible solution. The newton and quasi newton methods are widely known because they have high speed of convergence once the initial guess is close to the optimum point. Even if these methods have high speed of convergence, they always fail to find the solutions when the given objective function is nonconvex or concave function, and the given initial guess is not really close to the optimum point. These methods also require the derivative information from the objective function, and many of them are hard to find analytically. To overcome this weakness, the researchers are recently proposed metaheuristic methods. Metaheuristic is a heuristic approximation framework for continuous or discontinuous global optimization problems [7]. The term of global optimization is referred to an attempt to find the global minima or global maxima from an objective function. The metaheuristic methods are not required complete information from the given problems such as the existence of the derivative or the convexity of the objective functions, but the methods may provide a good solution to the optimization problems. There are exist numbers of metaheuristics method such as ant colony optimization, spiral optimization algorithm, particle swarm optimization, genetic algorithm, tabu search and simulated annealing, and they are all based on the natural systems.

Methods for minimizing a function subject to nonlinear constraints can be divided broadly into two classes, those which set up an equivalent unconstrained minimization problem by adding a penalty term to either the objective function or the Lagrangian function, and those which seek to generate a sequence of feasible-descent steps [6]. Indeed, there is no general agreement on the best approach and much research still be done, the earliest developments were sequential minimizations methods based on the use of penalty and barrier functions, these methods suffer from some computational disadvantages and are not entirely efficient. The problem of finding the solution of nonlinear constrained optimization based on metaheuristic or non-metaheuristic optimization have been proposed on some optimization articles. In general, for metaheuristic method, the first step for solving the constrains optimization begin by transform the constrains optimization problem to unconstrained optimization problem. Liu [1] made the genetic algorithm for solving nonlinear constrained optimization problems. Zahara et al. [4] proposed a new hybrid optimization method for solving nonlinear constrained optimization problem. Hua et al. [3] made the election-survey algorithm for solving nonlinear constrained optimization problems based on dynamic penalty function method presented by Joiness. Wang et al. [2] made the homotopy method for solving a nonlinear programming problem. For each approach, the homotopy method must ensure that the function is smooth or convex, in [2] the procedure for solving nonlinear constrained optimization by using genetic algorithm must be considered, caused the genetic algorithm may be difficult to implement and this algorithm effectively in the context of spatial multiobjective problem. If we are using this approach we may find the difficulties, one approach must require that the objective function is convex, and the other approach is hard to implement in the large-scale optimization problems.

Having transformed the constrained optimization problem into unconstrained optimization problem, the approach in this paper focuses for finding the solution of nonlinear constrained optimization using spiral dynamic optimization algorithm with either equality or inequality constrains. Since the number of solutions to each nonlinear constrained optimization problem is finite, the proposed method focuses only for finding single solution to each problem. In the next section, the problem for solving nonlinear constrained

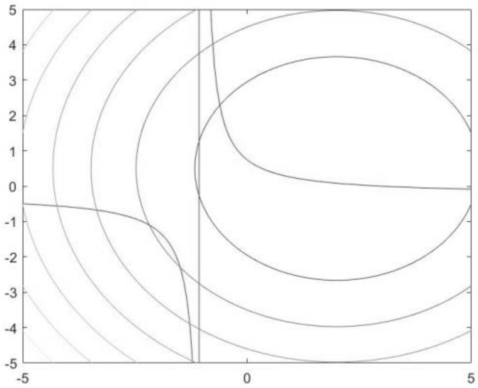

Figure 2. the graph of objective function with additional inequality constrains

optimization problem is reformulated as unconstrained optimization problems. Spiral Dynamic optimization algorithm is review in section 3 and section 4 is focuses on a problem sets which can be the benchmark of the last developments methods and the best solution from the implementation of spiral dynamic optimization algorithm is presented. In the last section, some conclusion from the proposed method is given.

2 Formulations

This section reviews the nonlinear constrained optimization problem, penalty method and briefly introduce powerful method called Langrange Multipliers. Suppose we have the nonlinear constrained optimization problem which has formulated as:

$$\underset{x \in \mathcal{R}^n}{\text{minimize}}\ f(x) \quad \text{subject to} \tag{2}$$

$$\Phi_i(x) = 0, i = \mathcal{E}$$
$$\Psi_i(x) \geq 0, i = I$$

The equation (2) has some behaviors with equation (1) above, in this paper we will follow this notation to show the difference between equality and inequality constrains. For simple optimization problem which is consist of simple objective function, equality constrains, or inequality constrains, the problem can handle by using penalty method. The penalty method for solving nonlinear constrained optimization problems formulated as:

$$\prod(x, \mu_i, \upsilon_i) = f(x) + \sum_{i=1}^{M} \mu_i \Phi_i(x)^2 + \ + \sum_{j=1}^{N} \upsilon_j \Psi_j(x)^2 \tag{3}$$

The equation (3) called penalty method because the constrained optimization problems solve by define a new penalty function, in other words the constrained optimization problem transform into unconstrained optimization problem by using a new penalty function, and in this case, there are exist 2 penalty parameters. Another classic but powerful methods for solving constrained optimization problems especially nonlinear constrained optimization problems called Langrange Multipliers method, and formulated as:

$$\Pi = f(x) + \lambda g(x) \tag{4}$$

Where λ is the Langrange Multiplier. In this case for *n* equality constrains, the Langrange Multipliers require *n* parameter for solving nonlinear constrains optimization with which is only handle equality constrains. The idea of penalty method and Langrange Multipliers for solving constrained optimization problem is very important. In this paper, we use the proposed approach by Yang [9], the equations (5) show us the new approach for solving the nonlinear constrained optimization problem.

$$\mathrm{L(x)} = f(x) + \sum_{i=1}^{M} \mu_i |\Phi_i(\mathrm{x})| \;\; + + \sum_{j=1}^{N} \upsilon_j \max(0, \Psi_j(\mathrm{x}))^2 \tag{5}$$

In equation (5) we have 2 constants parameter, and it should sufficiently large value. The new function has transformed nonlinear constrained optimization problem into unconstrained optimization problem, and the next section we're briefly describe the Spiral Dynamics Optimization Algorithm for solving nonlinear constrained optimization problem.

3 Spiral Model

In this section, we are briefly reviewing 2-dimensional spiral optimization. For 2-dimensioanl rotation, the standard rotation matrix has the following form:

$$\mathrm{R}^2 = \begin{matrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{matrix} \tag{6}$$

This rotates all column vectors by mean the following matrix multiplication:

$$\begin{matrix} x' \\ y' \end{matrix} = \begin{matrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{matrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{7}$$

After counterclockwise rotation, the new coordinates are

$$x' = x\cos(\theta) - y\sin(\theta)$$
$$y' = x\sin(\theta) - y\cos(\theta) \quad (8)$$

Tamura [7] using this rotation matrix to formulate a new discrete logarithmic spiral model, which generates a point converging at the origin from arbitrary initial point on *(x,y)* plane.

$$\begin{bmatrix} x_1^{k+1} \\ x_2^{k+1} \end{bmatrix} = rR_{1,2}^{2}(\theta)\begin{bmatrix} x_1^{k} \\ x_2^{k} \end{bmatrix} = S_2(r,\theta)x(k) \quad (9)$$

For *n*-dimensional spiral rotation, Tamura [7] and Sidarto [8] provide a comprehensive discussion. Based on the model above, algorithm of 2-dimensional Spiral Optimization Algorithm for solving optimization problem is developed below.

1. **Input:** objective function f(x), k_{max} maximum iteration, k =0, initial points, r ($0 < r < 1$), θ ($0 \le \theta \le 2\pi$).
2. **Initialization:** set initial point $x_i(0) \in \mathscr{R}^2$, I =1,2, …………, m in the feasible region and center $x^* = x_{ig}(0)$, $i_g = \arg\min f(x_i(0))$, i=1,2, …, m.
3. **Update:** $x_i(k+1) = S_2(r,\theta)\, x_i(k) - S_2(r,\theta) - I_2)\, x^*$, i=1,2,3, ……., m, $x^* = x_{ig}(0)$, $i_g = \arg\min f(x_i(k+1))$.
4. **Termination Condition:** if $k = k_{max}$ then stop, otherwise set k = k+1 and return to phase 3.
5. **Output:** x^* as a minimum point of objective function f(x).

Tamura [7] has confirmed the effectiveness of this spiral optimization algorithm, and the model was naturally realizing the strategy from diversification to intensification. The Spiral Optimization Algorithm for n-dimensional, and the algorithm for solving optimization problem is written below.

1. **Input:** objective function f(x), k_{max} maximum iteration, k =0, initial points, r ($0 < r < 1$), θ ($0 \le \theta \le 2\pi$).
2. **Initialization:** set initial point $x_i(0) \in \mathscr{R}^2$, I =1,2, …………, m in the feasible region and center $x^* = x_{ig}(0)$, $i_g = \arg\min f(x_i(0))$, i=1,2, …, m.
3. **Update:** $x_i(k+1) = S_n(r,\theta)\, x_i(k) - S_n(r,\theta) - I_n)\, x^*$, i=1,2,3, ……., m, $x^* = x_{ig}(0)$, $i_g = \arg\min f(x_i(k+1))$.
4. **Termination Condition:** if $k = k_{max}$ then stop, otherwise set k = k+1 and return to phase 3.
5. **Output:** x^* as a minimum point of objective function f(x).

4 Numerical Experiments

To evaluate the effectiveness and the efficiency of the proposed method, a set of nonlinear constrained optimization problems from various benchmarks problems have been tested. In this study, all numerical experiments were implemented on a personal computer with 4 GB ram with processor AMD-A4 and running Windows 10. The code was written in C/ C++ with Cmake for build in build automation, testing, packaging, installation of software by using a compiler-independent method and Eigen3 for performing matrix and vector operations. In table 1, all the results of the numerical experiments are given.

4.1 Test Problems

Problem 1:

From Wang et al. [2], the system of nonlinear constrained and our chosen inequalities constrains are as follows:

$$\min \quad f(x) = (x_1 - 3)^2 + (x_2 - 1)^2 \tag{10}$$

subject to

$$g_1(x) = (x_2 - 4)^2 \le 0$$
$$g_2(x) = 1 - x_1^2 - (x_2 - 2)^2 \le 0$$
$$g_3(x) = -x^2 + \frac{1}{4}x_1^2 \le 0$$

In [2] the total number of the best solutions is unspecified.

Problem 2:

From Liu [1], the system of nonlinear constrained and our chosen inequalities constrains are as follows:

$$\min \quad f(x) = (x_1 - 10)^2 + (x_2 - 20)^2 \tag{11}$$

subject to

$$g_1(x) = (x_1 - 5)^2 - (x_2 - 5)^2 - 100 \ge 0$$
$$g_2(x) = (x_1 - 6)^2 - (x_2 - 5)^2 - 82.81 \ge 0$$
$$13 \le x_1 \le 100, 0 \le x_2 \le 100$$

The best solution for this problem is specified in the literature.

Problem 3:

From Wang [2], the system of nonlinear constrained and our chosen inequalities constrains are as follows:

$$\min \quad f(x) = (x_1 - 3)^2 + (x_2 - 1)^2 \tag{12}$$

subject to

$$g_1(x) = x_1^2 + x_2^2 - 16 \le 0$$
$$g_2(x) = 4 - x_1^2 - x_2^2 \le 0$$

Problem 4:

From Nocedal [5], the system of nonlinear constrained and our chosen inequalities constrains are as follows:

$$\min \quad f(x) = 1/2(x_1 - 2)^2 + 1/2(x_2 - 1/2)^2 \tag{13}$$

subject to

$$g_1(x) = (x_1 + 1)^{-1} - x_2 - 1/4 \geq 0$$
$$0 \leq x_1 \leq 1$$
$$0 \leq x_2 \leq 1$$

Problem 5:

From Hua et al. [3], the system of nonlinear constrained and our chosen constrains are as follows:

$$\min \quad f(x) = x_1^2 + (x_2 - 1)^2 \tag{14}$$

subject to

$$0 \leq x_1 \leq 1$$
$$0 \leq x_2 \leq 1$$
$$g_1(x) = x_2 - x_1^2 = 0$$

The best solution for this problem is specified in the literature.

Problem 6:

From Nocedal [5], the system of nonlinear constrained and our chosen constrains are as follows:

$$\max \quad f(x) = 2 * (x_1^2 + x_2^2 - 1) - x_1 \tag{14}$$

subject to

$$g_1(x) = x_1^2 + x_2^2 - 1 = 0$$

Problem 7:

From Griva et al. [5], the system of nonlinear constrained and our chosen constrains are as follows:

$$\min \quad f(x) = e^{3x_1} + e^{-4x_2} \tag{15}$$

subject to

$$g_1(x) = x_1^2 + x_2^2 - 1 = 0$$

The best solution for this problem is unspecified in the literature.

4.2 Numerical Results

All problems have tested by using the following parameter values: M= 250, r=0.98, α, β = 10^{20}, k_{max}=500, θ= π/4.0. Experimental results on the problem are shown in **table 1**.

Table 1 Results for the test problems

Test Problem	X*	Best	F*
1	2.42677, 1.47231	2.42680, 1.47240	0.551669
2	14.0953, 9.15638	14.0953, 9.15638	480.227
3	3.00000, 1.00000	3.00000, 1.00000	0
4	1.95280, 0.088662	1.95300, 0.08900	0.0857134
5	0.001167, 0.50000	0,001167, 0.50000	0.250001
6	0.999562, 0.029582	1.0, 0.0	-0.999563
7	-0.879053, -0.47672	-0.879053, -0.47672	6.90442

5 Conclusions

In this paper, Spiral Optimization Algorithm is proposed for solving the nonlinear constrained optimization problem, our proposed method can handle either equality constrains, or inequality constrains or both. The numerical experiment results clearly illustrate the attractiveness of the Spiral Optimization Algorithm for solving nonlinear constrained optimization problem. The proposed techniques also easy to implement in large scale optimization problem, especially for nonlinear constrained optimization problems. In the future, we will apply spiral optimization algorithm to various optimization problem in the real world, and the techniques to find all feasible solution from optimization problem which has appear in recent articles such as in Sidarto [8].

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