Predicting extragalactic distance errors using Bayesian inference in multi-measurement catalogs

Germán Chaparro-Molano,[1][*] Juan Carlos Cuervo,[2] Oscar Alberto Restrepo Gaitán[1,3] Sergio Torres Arzayús[4]

[1] *Vicerrectoría de Investigación, Universidad ECCI, 111311 Bogotá, Colombia*
[2] *Department, Institution, Street Address, City Postal Code, Country*
[3] *Radio Astronomy Instrumentation Group, Universidad de Chile, Santiago de Chile, Chile*
[4] *Centro Internacional de Física, Bogotá, Colombia*

ABSTRACT

This is a simple template for authors to write new MNRAS papers. The abstract should briefly describe the aims, methods, and main results of the paper. It should be a single paragraph not more than 250 words (200 words for Letters). No references should appear in the abstract.

Key words: Galaxies: distances – keyword2 – keyword3

1 INTRODUCTION

Understanding the uncertainties in redshift-independent extragalactic distance measurements is absolutely necessary before reporting statistically sound conclusions regarding the structure of the local universe (Nasonova & Karachentsev 2011; Courtois et al. 2012; Ma et al. 2013; Springob et al. 2014; Sorce et al. 2014; Said et al. 2016; Kourkchi & Tully 2017), large scale structure (McClure & Dyer 2007; Roman & Trujillo 2017; Javanmardi & Kroupa 2017; Torres & Cuervo 2018), and events like transient gravitational wave detections (White et al. 2011). Hubble constant estimations have been using increasingly sophisticated statistical tools for primary distance determination methods, such as SNIa (Barris & Tonry 2004; Rubin et al. 2015; Dhawan et al. 2018), Cepheids Humphreys et al. (2013) or both (Riess et al. 2016). Although most estimates of the Hubble constant use Cepheid calibration for calibrating secondary methods (Tully & Pierce 2000; Freedman et al. 2001; Freedman & Madore 2010), Mould & Sakai (2008) have explored changes in Hubble constant estimation using the Tully-Fisher relation (TF) relation without Cepheid calibration. Secondary methods for extragalactic distance determination like the TF relation, or the Fundamental Plane (FP) have recently become more precise thanks to increasing volumes of data from surveys like 6dF (Springob et al. 2014) and 2MASS (Jarrett et al. 2000; Springob et al. 2007) together with Spitzer data (Sorce et al. 2013), along with improved statistical methods (Obreschkow & Meyer 2013).

As of 2018, three multi-measurement catalogs including a substantial amount of redshift-independent distance measurements have been released: HyperLEDA (Makarov et al. 2014), NED-D (Mazzarella & Team 2007; Steer et al. 2017), and Cosmicflows-3 (Tully et al. 2016). HyperLEDA includes a homogenized catalog for extragalactic distances in the nearby universe, with 12866 distance measurements for 518 galaxies to date. NED-D is the NASA/IPAC Extragalactic Distance catalog of Redshift-Independent Distances, which compiles XXX distance measurements for XXX galaxies, for which ~ 1800 galaxies ($\sim 1\%$) have more than 12 distance measurements, and 180 galaxies ($\sim 0.1\%$) have distance measurements using more than 6 different methods. Cosmicflows-3 is the most up-to-date catalog, which reports distance measurements for 10616 galaxies for up to four distance determination methods, and calibrated with supernova luminosities. However, unlike HyperLEDA or NED-D, Cosmicflows-3 only reports the latest distance measurement for each method. In HyperLEDA, NED-D and Cosmicflows-3 errors are reported as one standard deviation from the reported distance modulus. Treatment of errors for combining distance moduli across methods or across measurements is suggested by Mazzarella & Team (2007) and Tully et al. (2016) to be based on weighted estimates such as the uncertainty of the weighted mean, albeit with caution due to the hererogeneous origin of the compiled data. In the case of NED-D, this is complicated by the fact that many errors are not reported or are reported as zero. In fact, the TF relation method has the largest number of galaxies with non-reported distance modulus errors (818 to date). Even though extragalactic distances measured using the TF relation were originally reported to have a relative error

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[*] E-mail: gchaparrom@ecci.edu.co

in distance modulus of $10 - 20\%$ (Tully & Fisher 1977), we consider that this conservative estimate can be improved upon by using a predictive model based on the distance error of galaxies that use the same distance determination method. This requires a robust estimation of the variance of extragalactic distances based on the available data.

For many galaxies in all three catalogs, the random error for each distance modulus measurement ϵ_i (for $i = 1, ..., N$, for N distance measurements per galaxy) is not representative of the scatter across measurements, even when considering the same method for determining distances. In addition, distance modulus distributions for each measurement (which are assumed to be Gaussian) are transformed to log-normal distributions in metric distance space. We improve upon previous methods by robustly estimating the underlying variance across measurements and distance determination methods for the three catalogs by bootstrap sampling the posterior distribution of each extragalactic distance (Chaparro Molano et al. 2018), and comparing our results to more commonly used frequentist methods, such as the weighted estimates mentioned above. Furthermore, we build a predictive Bayesian model for the 818 galaxies in the NED-D catalog whose distances were measured using the TF relation but have non-reported errors.

Here go the sections.

2   POSTERIOR DISTRIBUTION FOR EXTRAGALACTIC DISTANCES

As mentioned in the Introduction, the best approach to consider the effects of random and scattering errors in catalog-wide, multi-method distance analyses is to perform a robust estimation of the variance of the posterior distribution of each extragalactic distance. The posterior distribution of the distance to a given galaxy can be obtained by drawing distance modulus samples from $P(\mu)$, which is the unweighted mixture of normal distributions corresponding to each distance modulus measurement μ_i,

$$\mu \sim \sum_i^N \mathcal{N}(\mu_i, \epsilon_i^2) \ ,$$

and then converting to metric distance,

$$D = 10^{\frac{\mu}{5}+1} \ .$$

Therefore,

$$D_G \sim \sum_i^N \text{lognormal}(M_i, \sigma_{M_i}^2) \ .$$

Here $M_i = \ln D_i$ and $\sigma_{M_i} = \epsilon_i \cdot \ln 10$.

However, this method is not very efficient for a standardized treatment of errors. It is more convenient to treat each extragalactic metric distance D_G as a normal random variable with a single-valued σ_D as a measure of the uncertainty in the estimation of an extragalactic distance,

$$D_G \sim \mathcal{N}(D, \sigma_D)$$

For this reason we compare four methods for estimating the D, σ_D pair. Two of these methods (H, M) use robust measures of the posterior distribution of each extragalactic distance, and the other two (P, Q) use measures based on propagation of errors.

2.1   Estimating the variance of $P(D_G)$

Method H takes D as the median of the posterior and σ_D as the half-distance (H) between the 84th and 16th percentiles of the posterior. Method M takes D as the median of the posterior and σ_D as the median absolute deviation (MAD) of the posterior. Method P consists on calculating D from the weighted mean distance modulus $\bar{\mu}^*$ with weights $w_i = \epsilon_i^{-2}$. σ_D is calculated by propagation (P) of measurement errors i.e. from the uncertainty of the weighted mean (Tully et al. 2016),

$$\sigma_D^P = 0.461 \, \bar{D}^* \left(\sum_i^N w_i \right)^{-1/2} \ , \tag{1}$$

Method P does not take into account the scatter in distance measurements for single galaxies, which is why method Q calculates D same as method P, but σ_D is calculated as the sum in quadrature (Q) of the propagated uncertainty of the weighted mean and the propagated unbiased weighted sample variance σ_D^*:

$$\sigma_D^Q = \left[\left(\sigma_D^P \right)^2 + \left(\sigma_D^* \right)^2 \right]^{1/2} \ . \tag{2}$$

Here σ_D^* is calculated as (Brugger 1969),

$$\sigma^* = 0.461 \, \bar{D}^* \sqrt{\frac{N}{N - 1.5} \frac{\sum_i^N w_i(\mu_i - \bar{\mu}^*)^2}{\sum_i^N w_i}} \ . \tag{3}$$

2.2   Comparison of variance estimation methods

Even though our analysis for error estimation can be used across different distance determination methods for single galaxies, we think that it is more informational to separate the analysis by method. Without loss of generality, we will focus on galaxies whose distances have been measured using the Tully-Fisher method in the NED-D catalog because it is the method with the most non-reported errors in the database. From here on, when we mention distance measurements in the NED-D catalog, we will be excluding from our analysis measurements that require the target redshift to calculate the distance, as indicated in the `redshift (z)` field.

Fig. 1 shows that the center and variance of the posterior distribution of each extragalactic distance is best explained using the H method, whereas the less robust P and Q methods under-predict the variance for galaxies in the whole distance range. The M method also under-predicts the variance, but being a robust method, it is not as sensitive to outliers as the mehods P and Q, as seen in the case of NGC 1558 in Fig. 1. For the more symmetrical posterior distribution of UGC 12792, the M and Q methods predict the same center and variance.

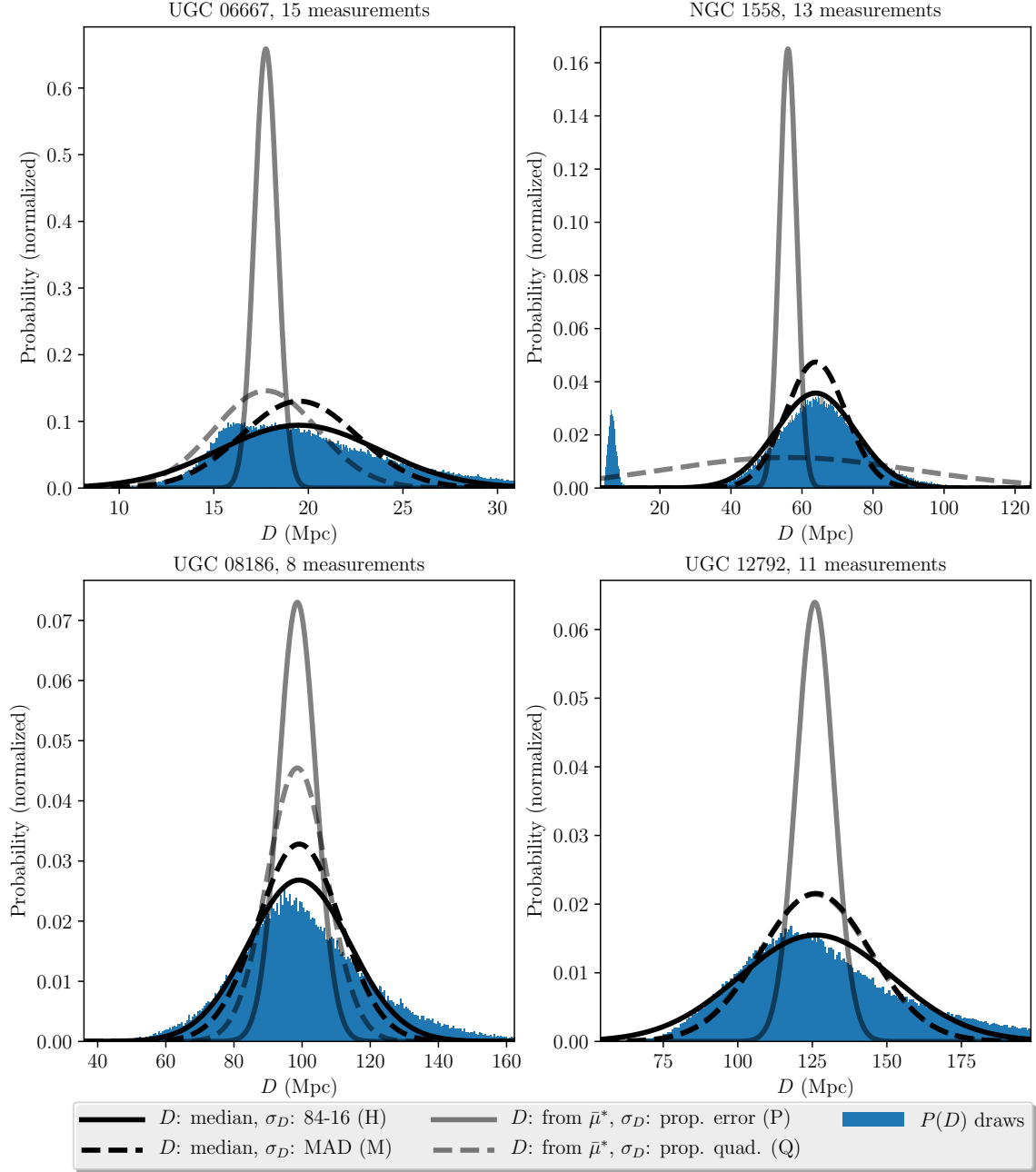Distance errors grow linearly with distance, as seen

Figure 1. Comparison of extragalactic distance posterior distribution draws and modeled distributions for UGC 06667, NGC 1558, UGC 08186, and UGC 12792 using the Tully-Fisher Method for distance determination in NED-D. The four methods used for approximating the posterior distribution (H, M, P, and Q) are described in the text.

in Fig. 2. This means that there is a strong systematic component in the variance of $P(D_G)$. Furthermore, the quadrature (Q) and propagation (P) methods underpredict distance errors for most galaxies in the sample. Fig. 4 shows that method Q underpredicts distance errors with respect

to the median absolute deviation method (M), which also shows a tighter linear correlation due to its robustness.

Given that σ_D calculated using the H method is obtained from many realizations from the posterior distribu-
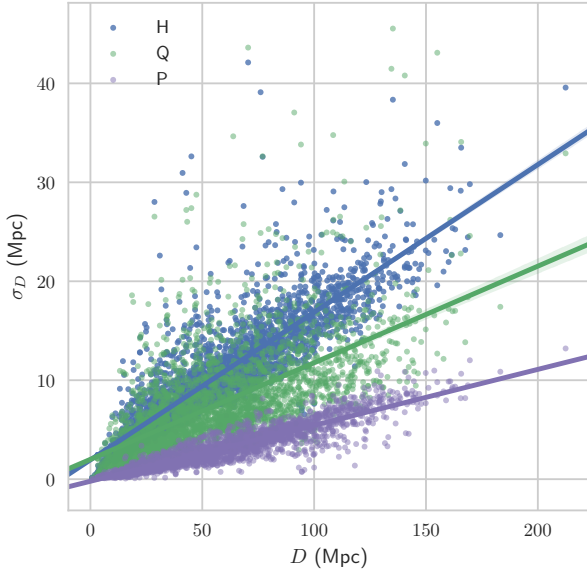
Figure 2. Median extragalactic distance vs. predicted extragalactic distance errors for galaxies with more than 5 TF distance measurements in NED-D according to the H, Q, P error models, showing a linear regression and confidence intervals computed using the `seaborn.regplot` Python function.
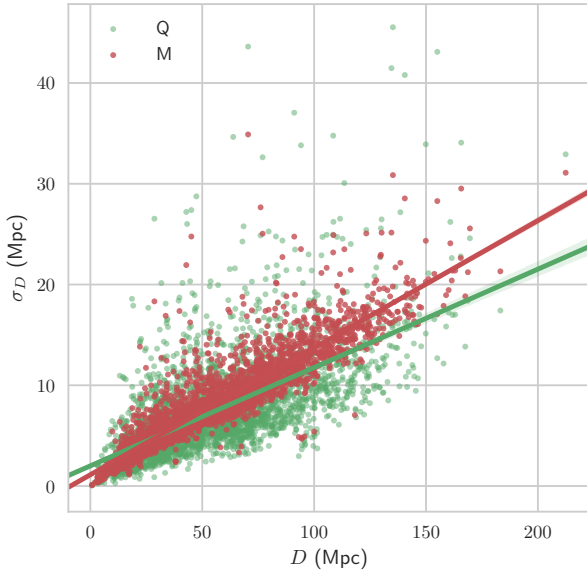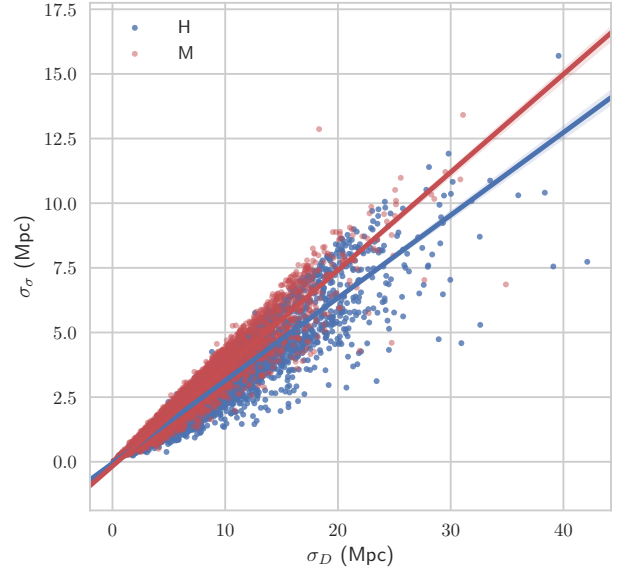

Figure 4. Predicted extragalactic distance errors vs. variance of the error as determined by the H and M methods, showing a linear regression and confidence intervals computed using the `seaborn.regplot` Python function.

tion of extragalactic distances, it is also possible to calculate its variance as the half-distance between the 84th and 16th percentile of σ_D realizations. Fig. **??** shows that the variance of the estimated error is proportional to the error for the H and M methods. This will be relevant in Section 3 when we construct a predictive model for non-reported errors.

3   PREDICTIVE BAYESIAN MODEL

As seen in Figs. 2 and 4, TF distance errors estimated using the robust methods H and M grow linearly with distance. This means that we should be able to predict missing distance errors for galaxies in NED-D. For this reason we try out several Bayesian models using the `emcee` affine invariant Markov Chain Monte Carlo (MCMC) ensemble sampler (Foreman-Mackey et al. 2013), which has been widely used due to its usability and efficiency. Recently, Zhang & Shields (2018) used `emcee` for model assessment using Bayesian and Akaike Information Criteria along with Bayes factors, focusing on small datasets, where it is not relevant to reproduce the original variance of the data. `emcee` has also been proved to be useful in recovering probabilistic models for photometric redshifts Speagle & Eisenstein (2017a,b).

Several models in order to reach a predictive model.
. samplers such as Foreman-Mackey et al. (2013) have been widely used for fitting data to models. Gelman et al. (1996) Brooks et al. (2000). linmix Kelly (2007) is a widely used Bayesian linear regressor which uses a Gaussian Mixture Model to approximate the distibution of unobserved "true" data values and from this information estimate the

Figure 3. Median extragalactic distance vs. predicted extragalactic distance errors for galaxies with more than 5 TF distance measurements in NED-D according to the Q, M error models, showing a linear regression and confidence intervals computed using the `seaborn.regplot` Python function.

Table 1. This is an example table. Captions appear above each table. Remember to define the quantities, symbols and units used.

A	B	C	D
1	2	3	4
2	4	6	8
3	5	7	9

regression coefficients but does not use model checking. Exploration of prior discrepancy modeling in model estimation Ling et al. (2014), maybe not needed? Other discrepancy measures for model selection de la Horra & Teresa Rodriguez-Bernal (2012). Chi2 model selection De la Horra (2008).

3.1 Figures and tables

Figures are referred to as e.g. Fig. **??**, and tables as e.g. Table 1.

4 CONCLUSIONS

The last numbered section should briefly summarise what has been done, and describe the final conclusions which the authors draw from their work.

REFERENCES

Barris B., Tonry J., 2004, ASTROPHYSICAL JOURNAL, 613, L21

Brooks S. P., Catchpole E. A., Morgan B. J. T., 2000, Statistical Science, 15, 357

Brugger R. M., 1969, The American Statistician, 23, 32

Chaparro Molano G., Restrepo Gaitán O. A., Cuervo Marulanda J. C., Torres Arzayus S. A., 2018, in Revista Mexicana de Astronomia y Astrofisica Conference Series. pp 63–63

Courtois H. M., Hoffman Y., Tully R. B., Gottloeber S., 2012, ASTROPHYSICAL JOURNAL, 744

Dabringhausen J., Fellhauer M., 2016, MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY, 460, 4492

De la Horra J., 2008, COMMUNICATIONS IN STATISTICS-THEORY AND METHODS, 37, 1412

Dhawan S., Jha S. W., Leibundgut B., 2018, ASTRONOMY & ASTROPHYSICS, 609

Foreman-Mackey D., Hogg D. W., Lang D., Goodman J., 2013, PUBLICATIONS OF THE ASTRONOMICAL SOCIETY OF THE PACIFIC, 125, 306

Freedman W. L., Madore B. F., 2010, in Blandford R., Faber S., van Dishoeck E., Kormendy J., eds, Annual Review of Astronomy and Astrophysics, Vol. 48, ANNUAL REVIEW OF ASTRONOMY AND ASTROPHYSICS, VOL 48. pp 673–710, doi:10.1146/annurev-astro-082708-101829

Freedman W. L., et al., 2001, The Astrophysical Journal, 553, 47

Gelman A., li Meng X., Stern H., 1996, Statistica Sinica, 6, 733

Hernan-Caballero A., Spoon H. W. W., Lebouteiller V., Rupke D. S. N., Barry D. P., 2016, MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY, 455, 1796

Humphreys E. M. L., Reid M. J., Moran J. M., Greenhill L. J., Argon A. L., 2013, ASTROPHYSICAL JOURNAL, 775

Jarrett T. H., Chester T., Cutri R., Schneider S., Skrutskie M., Huchra J. P., 2000, The Astronomical Journal, 119, 2498

Javanmardi B., Kroupa P., 2017, ASTRONOMY & ASTROPHYSICS, 597

Kelly B. C., 2007, ASTROPHYSICAL JOURNAL, 665, 1489

Kourkchi E., Tully R. B., 2017, ASTROPHYSICAL JOURNAL, 843

Ling Y., Mullins J., Mahadevan S., 2014, JOURNAL OF COMPUTATIONAL PHYSICS, 276, 665

Ma Y.-Z., Taylor J. E., Scott D., 2013, MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY, 436, 2029

Makarov D., Prugniel P., Terekhova N., Courtois H., Vauglin I., 2014, ASTRONOMY & ASTROPHYSICS, 570

Mazzarella J. M., Team N., 2007, in Shaw R., Hill F., Bell D., eds, ASTRONOMICAL SOCIETY OF THE PACIFIC CONFERENCE SERIES Vol. 376, ASTRONOMICAL DATA ANALYSIS SOFTWARE AND SYSTEMS XVI. pp 153–162

McClure M. L., Dyer C. C., 2007, NEW ASTRONOMY, 12, 533

Mould J., Sakai S., 2008, ASTROPHYSICAL JOURNAL LETTERS, 686, L75

Nasonova O. G., Karachentsev I. D., 2011, ASTROPHYSICS, 54, 1

Obreschkow D., Meyer M., 2013, ASTROPHYSICAL JOURNAL, 777

Riess A. G., et al., 2016, The Astrophysical Journal, 826, 56

Roman J., Trujillo I., 2017, MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY, 468, 703

Rubin D., et al., 2015, ASTROPHYSICAL JOURNAL, 813

Said K., Kraan-Korteweg R. C., Staveley-Smith L., Williams W. L., Jarrett T. H., Springob C. M., 2016, MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY, 457, 2366

Sorce J. G., et al., 2013, The Astrophysical Journal, 765, 94

Sorce J. G., Courtois H. M., Gottlober S., Hoffman Y., Tully R. B., 2014, MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY, 437, 3586

Speagle J. S., Eisenstein D. J., 2017a, MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY, 469, 1186

Speagle J. S., Eisenstein D. J., 2017b, MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY, 469, 1205

Springob C. M., Masters K. L., Haynes M. P., Giovanelli R., Marinoni C., 2007, The Astrophysical Journal Supplement Series, 172, 599

Springob C. M., et al., 2014, MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY, 445, 2677

Steer I., et al., 2017, ASTRONOMICAL JOURNAL, 153

Torres S., Cuervo J. C., 2018, Tecciencia, 24, 53

Tully R. B., Fisher J. R., 1977, A&A, 54, 661

Tully R. B., Pierce M. J., 2000, The Astrophysical Journal, 533, 744

Tully R. B., Courtois H. M., Sorce J. G., 2016, ASTRONOMICAL JOURNAL, 152

White D. J., Daw E. J., Dhillon V. S., 2011, CLASSICAL AND QUANTUM GRAVITY, 28

Zhang J., Shields M. D., 2018, MECHANICAL SYSTEMS AND SIGNAL PROCESSING, 98, 465

de la Horra J., Teresa Rodriguez-Bernal M., 2012, SORT-STATISTICS AND OPERATIONS RESEARCH TRANSACTIONS, 36, 69

APPENDIX A: SOME EXTRA MATERIAL

If you want to present additional material which would interrupt the flow of the main paper, it can be placed in an Appendix which appears after the list of references.

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