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Homework 1

Problem 1

Answer to the problem goes here.

1. Assume $a_1 + b_1\sqrt{2}$ and $a_2 + b_2\sqrt{2} \in Q[\sqrt{2}]$ where a, b are $\in Q$ So, we can say that $a_1+b_1\sqrt{2}+a_2+b_2\sqrt{2} = a_1a_2+2b_1b_2+(a_1b_2+b_1a_2)\sqrt{2}$ which is $\in Q[\sqrt{2}]$ So $(Q[\sqrt{2}], +, x)$ is a field. We know that addition and multiplication is associative , distributive and commutative under real numbers then it is also closed under $Q[\sqrt{2}]$ $(1 + \sqrt{2})$ is the identity because if this is added or multiplied it gives the same output as the input.

   Additive Inverse of $(a + b\sqrt{2})$ is $(-a + -b\sqrt{2})$ because $(a + b\sqrt{2}) + (-a + -b\sqrt{2}) = 0 + 0\sqrt{2} = 0$

   Multiplicative Inverse of $(a + b\sqrt{2})$ is $\frac{a-b\sqrt{2}}{a^2-2b^2}$ because $(a + b\sqrt{2}) * \frac{a-b\sqrt{2}}{a^2-2b^2} = 1 + 0\sqrt{2} = 1$
   Due to all these above, we know that it is a field (F,+, x)

2. Using the notation that complex number $z = x + i * yor(x, y)$ here $i = \sqrt{-1}$ for example $(x_1, y_1) * (x_2, y_2) * (x_3, y_3)$ Complex numbers are closed under addition because $z = x_1 + iy_1$ $w = x_2 + iy_2$ then $z + w = (x_1 + x_2) + i(y_1 + y_2)$ we know that real numbers are closed under addition and since x_1, x_2, y_1, y_2 are in Real Numbers then $(x_1 + x_2)$ and $(y_1 + y_2)$ is closed under addition.

   Complex numbers are associative under addition $z_1 = x_1 + iy_1 or(x_1, y_1)$, $z_2 = x_2 + iy_2$ or (x_2, y_2) , $z_3 = x_3 + iy_3$ or (x_3, y_3) Then

   $$z_1 + z_2 + z_3 = (x_1, y_1) + ((x_2 + y_2) + (x_3, y_3))$$

   $$z_1 + z_2 + z_3 = (x_1, y_1) + ((x_2 + x_3, y_2, y_3))$$

   $$z_1 + z_2 + z_3 = (x_1 + (x_2 + x_3), y_1 + (y_2 + y_3))$$

   $$z_1 + z_2 + z_3 = ((x_1 + x_2) + x_3, (y_1 + y_2) + y_3)$$

   $$z_1 + z_2 + z_3 = ((x_1 + x_2, y_1 + y_2)) + (x_3, y_3)$$

   $$z_1 + z_2 + z_3 = ((x_1, y_1) + (x_2, y_2)) + (x_3, y_3)$$

   $$z_1 + z_2 + z_3 = (z_1 + z_2) + z_3$$

   Which shows that complex numbers are associative using the addition associative of real numbers

   Complex numbers are commutative under addition $z_1 = x_1 + iy_1 or(x_1, y_1)$, $z_2 = x_2 + iy_2$ or (x_2, y_2) then

   $$z_1 + z_2 = (x_1, y_1) + (x_2, y_2)$$

   $$z_1 + z_2 = (x_1 + x_2, y_1 + y_2)$$

   $$z_1 + z_2 = (x_2 + x_1, y_2 + y_1)$$

$$z_1 + z_2 = (x_2, y_2) + (x_1, y_1)$$

$$z_1 + z_2 = z_2 + z_1$$

Complex numbers have addition identity z_1 is a complex number. Then $0 + 0i$ is the identity. $z_1 + (0 + 0i) = (x_1 + iy_1) + (0 + 0i) = (x_1 + 0) + i(y_1 + 0) = x_1 + iy_1$ and $(0+0i)+z_1 = (0+0i)+(x_1+iy_1) = (0+x_1)+i(0+y_1) = x_1+iy_1$ Complex numbers have addition inverse z_1 is a complex number.Then negative of z_1 is the identity. $z_1 - z_1) = (x_1 + iy_1) - (x_1 + iy_1) = (x_1 - x_1) + i(y_1 - y_1) = 0 + 0i$ Since, $0 + 0i$ is the addition identity we know that the inverse for $x_1 + iy_1$ is $-x_1 - iy_1$

Complex numbers are closed under multiplication because $z_1 = x_1 + iy_1$ $z_2 = x_2 + iy_2$ then $z_1 * z_2 = (x_1x_2 - y_1y_2) + (x_1y_2 - x_2y_1)$ then we know that $(x_1x_2 - y_1y_2)$ and $(x_1y_2 + x_2y_1) \in$ R because they are closed under multiplication and addition

Complex numbers are associative under multiplication $z_1 = x_1 + iy_1 or (x_1, y_1)$, $z_2 = x_2 + iy_2$ or (x_2, y_2) , $z_3 = x_3 + iy_3$ or (x_3, y_3) Then

$$z_1(z_2z_3) = (x_1, y_1)((x_2 + y_2)(x_3, y_3))$$

$$z_1(z_2z_3) = (x_1, y_1)(x_2x_3 - y_2y_3, x_2y_3 + y_2x_3)$$

$$z_1(z_2z_3) = (x_1(x_2x_3 - y_2y_3) - y_1(x_2y_3 + y_2x_3), y_1(x_2x_3 - y_2y_3) + x_1(x_2y_3 + y_2x_3))$$

$$z_1(z_2z_3) = (x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)(x_3, y_3)$$

$$z_1(z_2z_3) = ((x_1, y_1)(x_2, y_2))(x_3, y_3)$$

$$z_1(z_2z_3) = (z_1z_2)z_3$$

Complex numbers are commutative under multiplication $z_1 = x_1 + iy_1 or (x_1, y_1)$, $z_2 = x_2 + iy_2$ or (x_2, y_2) then

$$z_1z_2 = (x_1, y_1)(x_2, y_2)$$

$$z_1z_2 = (x_1x_2 - y_1y_2, x_1y_2 + x_2y_1)$$

$$z_1z_2 = (x_2x_1 - y_2y_1, x_1y_2 + x_2y_1)$$

$$z_1z_2 = (x_2x_1 - y_2y_1, x_2y_1 + x_1y_2)$$

$$z_1z_2 = (x_2, y_2)(x_1, y_1)$$

$$z_1z_2 = z_2z_1$$

Complex numbers have multiplication identity is $(x_1 + iy_1)(1 + 0i) = (x_1 * 1 - y_1 * 0) + i(x_1 * 0 + y_1 * 1) = (x_1 + iy_1)$

So the identity is $(1 + 0i)$

Complex numbers have multiplication inverse $1/z$ given z is a complex number z_1 is the conjugate of z is $1/z = \frac{x-iy}{x^2+y^2} = \frac{z_1}{zz_1}$ Complex multiplication is distributive over addition

$$z_1(z_2 + z_3) = (x_1, y_1)((x_2, y_2) + (x_3, y_3))$$

$$z_1(z_2 + z_3) = (x_1, y_1)(x_2, y_2) + (x_1, y_1)(x_3, y_3)$$

$$z_1(z_2 + z_3) = z_1z_2 + z_1zz_3$$

These proprieties show that complex number form a field under addition and multiplication $(C, +, *)$

Problem 2

Matrix multiplication is not commutative because if we have two matrices A and B then AB $\neq$ BA

Problem 3

+	[0]	[1]	[2]
[0]	[0]	[1]	[2]
[1]	[1]	[2]	[0]
[2]	[2]	[0]	[1]

×	[0]	[1]	[2]
[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]
[2]	[0]	[2]	[1]

Problem 4

Please refer to my Q1 part 2.

Problem 5

B, D, are in reduced row-echelon form.

Problem 6

A + B = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $A^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Problem 7

This matrix is non-invertable for t=2.

Problem 8

Matrix a is inconsistent for h=2 Matrix b is inconsistent for all h

Problem 9a

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Problem 9b

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & * & * \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & * \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & * \\ 0 & 1 & * \end{bmatrix} \begin{bmatrix} 1 & * & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 9c

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

Problem 10

a can be any real number, b = 0, c = 1, d = 0, e =0

Problem 11

Problem 12 Q1

RREF of the augmented matrix = $\begin{bmatrix}1 & 0 & 0 & 4\\0 & 1 & 0 & 3\\0 & 0 & 1 & 0\end{bmatrix}$ used extended euclidean algorithm to modify fractions from python rref

Problem 12 Q2

$$x = \begin{bmatrix}4\\3\\0\end{bmatrix}$$

Problem 13

```
In [71]: M = Matrix([[3,11,19,-2],[7,23,39,10],[-4,-3,-2,6]])

In [72]: M.rref()

Out[72]: (Matrix([
          [1, 0, -1, 0],
          [0, 1,  2, 0],
          [0, 0,  0, 1]]),
          (0, 1, 3))
```

No solution

Problem 14

```
In [73]: M = Matrix([[3,6,9,5,25,53],[7,14,21,9,53,105],[-4,-8,-

In [74]: M.rref()

Out[74]: (Matrix([
          [1, 2, 3, 0, 5, 6],
          [0, 0, 0, 1, 2, 7],
          [0, 0, 0, 0, 0, 0]]),
          (0, 3))
```

$$\begin{bmatrix}1\\0\\0\end{bmatrix} \text{a} + \begin{bmatrix}2\\0\\0\end{bmatrix} \text{b} + \begin{bmatrix}3\\0\\0\end{bmatrix} \text{c} + \begin{bmatrix}0\\1\\0\end{bmatrix} \text{d} + \begin{bmatrix}5\\2\\0\end{bmatrix} \text{e} = \begin{bmatrix}6\\7\\0\end{bmatrix}$$

Problem 15

$$\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}\text{a} + \begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}\text{b} + \begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix}\text{c} + \begin{bmatrix}0\\0\\0\\1\\0\end{bmatrix}\text{d} + \begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix} = \begin{bmatrix}\frac{-8221}{4340}\\\frac{8591}{8680}\\\frac{4695}{434}\\\frac{-459}{434}\\\frac{699}{434}\end{bmatrix}$$

Problem 16

Problem 17

$$\begin{bmatrix}2&1\\3&2\end{bmatrix} * \begin{bmatrix}1&3\\4&1\end{bmatrix} = \begin{bmatrix}6&7\\11&11\end{bmatrix}$$
$$\begin{bmatrix}6&7\\11&11\end{bmatrix} * \begin{bmatrix}6&7\\11&11\end{bmatrix} = \begin{bmatrix}85&119\\187&198\end{bmatrix}$$
$$\begin{bmatrix}2&1\\3&2\end{bmatrix}^2 * \begin{bmatrix}1&3\\4&1\end{bmatrix}^2 = \begin{bmatrix}123&94\\250&241\end{bmatrix} \text{ So } (AB)^2 \neq A^2B^2$$

Problem 18

$$A = \begin{bmatrix}0&1\\1&0\end{bmatrix}$$
$$A^T = \begin{bmatrix}0&1\\1&0\end{bmatrix}$$
$$A * A^T = \begin{bmatrix}1&0\\0&1\end{bmatrix} \text{ which is the identity of A}$$

Problem 19 Q1

Examples of Symmetric

$$\begin{bmatrix}1&2\\2&3\end{bmatrix}$$

$$\begin{bmatrix}5&6&7\\6&3&2\\7&2&1\end{bmatrix}$$

$\begin{bmatrix} 9 & 13 & 3 & 6 \\ 13 & 11 & 7 & 6 \\ 3 & 7 & 4 & 7 \\ 6 & 6 & 7 & 10 \end{bmatrix}$

Examples of Skew-Symmetric

$\begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & 2 & 3 \\ -1 & 0 & 4 & 5 \\ -2 & -4 & 0 & 6 \\ -3 & -5 & -6 & 0 \end{bmatrix}$

Problem 19 Q2

The main diagonal only contains 0.

Problem 19 Q3

Null Matrix

Problem 19 Q4

$$(A + A^T)^T = A^T + (A^T)^T$$
$$(A + A^T)^T = A^T + A$$

because we know that $A^T = A$ for symmetric matrices. Therefore $A + A^T$ is symmetric

$$(AA^T)^T = A^{T^T}A^T$$
$$(AA^T)^T = AA^T$$

because we know that $A^T = A$ for symmetric matrices. Therefore AA^T is symmetric

$$(A^TA)^T = A^T(A^T)^T$$
$$(A^TA)^T = A^TA$$

Therefore $A^T A$ is symmetric

$$(A - A^T)^T = A^T - (A^T)^T = -(A - A^T)$$

so we know that $A - A^T$ is skew symmetric

Problem 19 Q5

Assume A is an nxn matrix then we can write $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$ The first part $\frac{1}{2}(A + A^T)$ is a symmetric matrix because we know from above that $(A + A^T)$ is a symmetric matrix. The second part $\frac{1}{2}(A - A^T)$ is a skew-symmetric matrix because we know from above that $(A - A^T)$ is a skew-symmetric matrix.

Problem 20a

Not injective. $F(1) = F(-1)$ but $x \neq y$ so it is a contradiction and hence it is not injective.

Problem 20b

Assume it is injective then f(a) = f(b) which means $\frac{a^3}{a^2+1} = \frac{b^3}{b^2+1}$ which is only possible if $a = b$. Therefore it is injective

Problem 20c

Not injective $F(0) = F(2) = F(3)$ but $x \neq y$ so it is a contradiction and hence it is not injective.

Problem 20d

$e^x + 2$ is injective because it has always has positive derivative or strictly increasing.

Problem 21

$$\text{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.25 & 1 & 0 & 0 \\ 0 & 0.267 & 1 & 0 \\ 0 & 0 & 0.268 & 1 \end{bmatrix} \text{U} = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 0 & 3.75 & 1 & 0 \\ 0 & 0 & 3.73 & 1 \\ 0 & 0 & 0 & 3.73 \end{bmatrix}$$

Problem 22

$$q_1 = d_1$$
$$l_i = \frac{a_k}{d_{k-1}}$$
$$d_k = q_k - l_k r_k$$

Problem 23

Problem 24 Q1

$H^T = (I - 2uu^T)^T = I^T - 2(uu^T)^T = I - 2(u^T)^T u^T = I - 2uu^T = H$ so it is symmetric because it equals its transpose

Problem 24 Q2

We know that $H = H^T$ and $H^T H = 1$ so H is orthogonal matrix.

Problem 24 Q3

$H^2 = (I - 2uu^T)^2 = I^2 - 4uu^T + 4uuu^T uu^T$ and because we know that $u^T u = 1$ then we know $H^2 = I$

Problem 24 Q4

$Hu = (I - 2uu^T)u = u - 2uu^T u = u - 2u = -u$

Problem 24 Q5

$$H_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{-2}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{-2}{3} & \frac{-2}{3} & \frac{1}{3} \end{bmatrix}$$

$$H_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} \end{bmatrix}$$