Math 7243

Homework 1. Matrix calculus:

Using the denominator layout notation conventions.
(One point each question except 4, 7,8 with 2 points each.)
Problems 4,7,8 have longer calculations.

Problem 1. Assume $\vec{x} \in \mathbb{R}^n$, $A \in \mathbb{R}^{m\times n}$ and $\vec{b} \in \mathbb{R}^m$. Let $f(\vec{x}) = \vec{b}^T A\vec{x}$. Find ∇f.

Problem 2. Assume $\vec{x} \in \mathbb{R}^n$. Find $\dfrac{\partial \vec{x}^T \vec{x}}{\partial \vec{x}}$.

Problem 3. Assume $\vec{x}$ and $\vec{a} \in \mathbb{R}^n$. Find $\dfrac{\partial (\vec{x}^T \vec{a})^2}{\partial \vec{x}}$

Problem 4. Suppose $\vec{x} : \mathbb{R}^n \to \mathbb{R}^m$ is a map sending $\vec{z} \in \mathbb{R}^n$ to $\vec{x}(\vec{z}) \in \mathbb{R}^m$. Similarly, suppose $\vec{y} : \mathbb{R}^n \to \mathbb{R}^m$ and A is an $m \times m$ constant matrix. Prove that $\dfrac{\partial(\vec{y}^T A\vec{x})}{\partial \vec{z}} = \dfrac{\partial \vec{y}}{\partial \vec{z}} A\vec{x} + \dfrac{\partial \vec{x}}{\partial \vec{z}} A^T \vec{y}$

Problem 5. Suppose $A(x) : \mathbb{R} \to \mathbb{R}^{n\times n}$ is a map from $\mathbb{R}$ to $\mathbb{R}^{n\times n}$.

Show that if $A(x)$ is invertible, then $\dfrac{dA^{-1}}{dx} = -A^{-1}\dfrac{dA}{dx}A^{-1}$

Problem 6. Let $\vec{x}$ and $\beta \in \mathbb{R}^p$. Prove that $\dfrac{\partial \vec{x}^T \beta}{\partial \vec{x}} = \beta$

Problem 7. Chain Rule. Assume that Y is an n vector but assume that Y depends on X and X depends on some $Z \in \mathbb{R}^q$. Show that

$$\frac{\partial Y}{\partial Z} = \frac{\partial X}{\partial Z}\frac{\partial Y}{\partial X}$$

Does the order matter?

Hint: This means that $X : \mathbb{R}^q \to \mathbb{R}^p$ and $Y : \mathbb{R}^p \to \mathbb{R}^n$.

Problem 8. Let $z : \mathbb{R}^p \to \mathbb{R}$ be a function that depends on $\vec{x} \in \mathbb{R}^p$ and let Y be a n-vector that depends on $\vec{x} \in \mathbb{R}^p$. Prove that

$$\frac{\partial}{\partial \vec{x}}(zY) = z\frac{\partial Y}{\partial \vec{x}} + \frac{\partial z}{\partial \vec{x}} Y^T$$