Math 4570 Homework 1 Jack Thompson

1

a) Let $F = \{f = a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$ $F \subset \mathbb{R}$ since $a, b, \sqrt{2} \in \mathbb{R}$

- F is a monoid since $\forall f \in F\ \exists e \in F$ s.t. $f * e = f$
  - In this case $e = 1$, which is in F: $1 = 1 + 0\sqrt{2}$.
  - F is also associative in multiplication because it is a subset of $\mathbb{R}$
  - F is commutative in multiplication because it is a subset of $\mathbb{R}$
- To prove F is a group, we must show that $\forall a, b \in \mathbb{Q}$ $\exists x, y \in \mathbb{Q} \mid (a + b\sqrt{2})(x + y\sqrt{2}) = 1$, or $(ax + 2by) + (ay + bx)\sqrt{2} = 1$

  So, we want $ax + 2by = 1$ and $ay + bx = 0$

  $\begin{bmatrix} a & 2b & 1 \\ b & a & 0 \end{bmatrix}$ taking this matrix to rref gives us $x = \frac{a}{a^2 - 2b^2}$ and $y = \frac{-b}{a^2 - 2b^2}$.

  These x and y satisfy $ax + 2by = 1$, and since $a, b \in \mathbb{Q}$, x, y must be in $\mathbb{Q}$, and therefore $x + y\sqrt{2} \in F$
- F is an abelian group because addition and multiplication are commutative in $\mathbb{R}$
- $\forall f \in F\ \forall e' \mid f + e' = 0$; $e' = 0$ or $e' = 0 + 0\sqrt{2}$ $\therefore (F, +, 0)$ is an additive abelian group
- F is distributive under addition and multiplication
  - $(a + b\sqrt{2}) + (c + d\sqrt{2}) = (a + c) + (b + d)\sqrt{2}$
  - $(a + b\sqrt{2}) \cdot (c + d\sqrt{2}) = ac + 2bd + ad\sqrt{2} + bc\sqrt{2}$

Therefore F is a field

b) This field is $\mathbb{C}$

2 Let us assume that the set of all $n \times n$ matrices with $n > 2$ and matrix multiplication and addition is a field: $(\mathbb{R}^{n \times n}, +, \cdot)$

- Therefore, $\mathbb{R}^{n \times n}$ must be commutative in multiplication
  - For example, $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} w & x \\ y & z \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = N$
  - However, we find that $M = \begin{bmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{bmatrix}$
  - While $N = \begin{bmatrix} wa + xc & wb + xd \\ ya + zc & yb + zd \end{bmatrix}$ $M \neq N$, this is a contradiction

3

+	[0]	[1]	[2]
[0]	[0]	[1]	[2]
[1]	[1]	[2]	[0]
[2]	[2]	[0]	[1]

x	[0]	[1]	[2]
[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]
[2]	[0]	[2]	[1]

4 We see that

- $(a+bi)+(x+yi)=((a+x)+(b+y)i) \in \mathbb{C}$
- $(a+bi)\cdot(x+yi) = ax+ayi+xbi+byi^2 = ((ax-by)+(ay+xb)i) \in \mathbb{C}$ $i^2=-1$
- $0 \in \mathbb{C}$ when $a,b=0$, this is the additive identity
- $1 \in \mathbb{C}$ when $a=1, b=0$, this is the multiplicative identity
- $(-a-bi) \in C$ since $-a,-b \in \mathbb{R}$, this is the additive inverse
- To find the multiplicative inverse, we want to find $c,d \in \mathbb{R}$ s.t. $(a+bi)(c+di)=1$ or $(ac-bd)+(ad+bc)i=1$

$\begin{bmatrix} a & -b & 1 \\ b & a & 0 \end{bmatrix} \xrightarrow{rref}$ $c=\frac{a}{a^2+b^2}$ $d=\frac{-b}{a^2+b^2}$ $c,d \in \mathbb{R}$ since $a,b \in \mathbb{R}$ as long as $a^2+b^2 \neq 0$

Therefore $\mathbb{C}$ is a field

5 B and D are in reduced row echelon form

6 $\mathbb{Z}_2 = \{[0],[1]\}$

$A+B=\begin{bmatrix} [1] & [0] & [0] \\ [1] & [0] & [0] \\ [1] & [0] & [0] \end{bmatrix}$ $A^2=\begin{bmatrix} [1] & [1] & [0] \\ [0] & [0] & [1] \\ [0] & [1] & [1] \end{bmatrix}$ $A^*B=\begin{bmatrix} [0] & [1] & [0] \\ [0] & [0] & [1] \\ [1] & [1] & [1] \end{bmatrix}$

7 Matrix A not invertible when $\det(A)=0$

$$\left\|\begin{matrix} 6 & -1 & 1 \\ t & 0 & 1 \\ 0 & 1 & t \end{matrix}\right\| = 6(0-1)-(-1)(t^2-0)+1(t-0)$$
$$= -6+t^2+t=0=(t+3)(t-2)$$
$$t=-3,2$$

8 $\left[\begin{array}{cc|c} 1 & h & 4 \\ 3 & 6 & 8 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & \frac{8h-24}{3(h-2)} \\ 0 & 1 & \frac{4}{3(h-2)} \end{array}\right]$ h can be any number but 2

b $\left[\begin{array}{cc|c} -4 & 12 & h \\ 2 & -6 & -3 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & 0 & 1 \end{array}\right]$ $0 \neq 1$, there is no solution

☆

9

a $\begin{bmatrix} 1 & \bullet \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & \bullet \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$ c) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

b $\begin{bmatrix} 1 & \bullet & \bullet \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & \bullet \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & \bullet & \bullet \\ 0 & 1 & \bullet \end{bmatrix}, \begin{bmatrix} 1 & \bullet & \bullet \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & \bullet \\ 0 & 0 & 1 \end{bmatrix}$

10 a, b, d can be any number, c must be 1, e must be Ø

11

a $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -3 & -2 \\ 2 & 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -3 & -2 \\ 0 & -4 & -5 & -6 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & -4 & -5 & -6 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 7 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & \frac{2}{7} \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & \frac{8}{7} \\ 0 & 0 & 1 & \frac{2}{7} \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & \frac{22}{7} \\ 0 & 1 & 0 & \frac{8}{7} \\ 0 & 0 & 1 & \frac{2}{7} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & \frac{6}{7} \\ 0 & 1 & 0 & \frac{8}{7} \\ 0 & 0 & 1 & \frac{2}{7} \end{bmatrix}$ $x_1 = \frac{-6}{7} x_4$ $x_4 = x_1$

$x_2 = \frac{-8}{7} x_4$

$x_3 = \frac{-2}{7} x_4$

$x = \begin{bmatrix} -\frac{6}{7} \\ -\frac{8}{7} \\ -\frac{2}{7} \\ 1 \end{bmatrix} x_4$

b ☆ $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 2 \end{bmatrix} \xrightarrow[\mathbb{Z}_7]{\text{rref}} \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) after #24

c) In python

d No

12 $\text{rref}(A|\vec{b}) = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

13 System has no solutions

14 $x_1 = 6 - 2x_2 - 3x_3 - 5x_5$, $x_4 = 7 - 2x_5$, $x_2 = x_2$, $x_3 = x_3$, $x_5 = x_5$

$$x = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -5 \\ 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} x_5 + \begin{bmatrix} 6 \\ 0 \\ 0 \\ 7 \\ 0 \end{bmatrix}$$

15 $x_1 = \frac{-1525}{434}$, $x_2 = \frac{781}{434}$, $x_3 = \frac{4695}{434}$, $x_4 = \frac{-457}{434}$, $x_5 = \frac{697}{434}$

16

a If A, B, C are $n \times n$ matrices and $ABC = I_n$, we have $A(BC) = I$ or $AA^{-1} = I$. Therefore A is invertible, its inverse is BC. C's inverse is AB. B's inverse is AC

b In order for AB to be invertible, $\det(AB)$ must not equal zero. $\det(AB) = \det(A)\det(B)$. The only way for $\det(AB)$ to be zero is for either $\det(A)$ or $\det(B)$ or both to be zero. Since AB is invertible, neither $\det(A)$ nor $\det(B)$ can be zero, and therefore both A and B are invertible.

17 $A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 3 \\ 7 & 9 \end{bmatrix}$ is a counterexample

18 $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

19

a symmetric: $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 3 & 8 \\ 3 & 19 & 12 \\ 8 & 12 & 7 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 5 & 8 & 13 \\ 2 & 8 & 21 & 34 \\ 3 & 13 & 34 & 55 \end{bmatrix}$

skew symmetric: $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 3 & -8 \\ -3 & 0 & 12 \\ 8 & -12 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & -1 & 2 & -3 \\ 1 & 0 & -8 & 13 \\ -2 & 8 & 0 & -34 \\ 3 & -13 & 34 & 0 \end{bmatrix}$

b The main diagonal of a skew symmetric matrix consists of zeros

c The definition of a symmetric matrix A is $A^T = A$. The definition of a skew symmetric matrix A is $A^T = -A$. For a matrix to be both symmetric and skew symmetric,
$A^T = A = -A$ This is not possible.

d - Assume the matrix $B = A + A^T$ is not symmetric. Therefore $B^T \neq A + A^T$. However, we see that $B^T = (A+A^T)^T = A^T + A = A + A^T$. This is a contradiction.
- AA^T is symmetric because $(AA^T)^T = A^TA = AA^T$
- Similarly, A^TA is symmetric because $(A^TA)^T = AA^T = A^TA$
- $A - A^T$ is skew symmetric because $(A-A^T) = A^T - A = -(A-A^T)$
- If A is an $n \times n$ matrix, then $A + A^T$ is symmetric and $A - A^T$ is skew symmetric. $(A+A^T) + (A-A^T) = 2A$, therefore
$\frac{1}{2}((A+A^T) + (A-A^T)) = A$

20 a) none b) bijective c) surjective d) injective

21 In python

22 $q_i = d_i + l_{i-1}u_{i-1}$, $p_i = d_i * l_i$, $r_i = u_i$

23 $u_t = 1$ $\quad l_t = \frac{1}{d_t}$

$d_1 = 4$ $\quad d_t = 4 - \frac{1}{l_{t-1}}$

$p_t = d_t l_t$

$p_t = 1$

24

a Yes, H_n is symmetric. $H_n^T = (I_n - 2\vec{u}\vec{u}^T)^T = I_n^T - 2\vec{u}^T\vec{u} = I - 2\vec{u}\vec{u}^T$

b Yes. $H_n^T \cdot H_n = (I - 2\vec{u}\vec{u}^T)(I - 2\vec{u}\vec{u}^T) = I^2 - 4\vec{u}\vec{u}^T I + 4(\vec{u}^T u)^2 I = I^2 = I$

c Because H_n is symmetric, $H_n^T = H_n$, therefore $H_n^2 = H_n H_n = H_n^T H_n = I$

d $H_n \vec{u} = -\vec{u}$

e In python

11

b $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 2 \end{bmatrix} \xrightarrow[R_3-2R_1]{R_2-R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -3 & -2 \\ 0 & -4 & -5 & -6 \end{bmatrix} \xrightarrow{R_3-4R_2} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 6 & 4 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix} 6R_2$

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix} \xrightarrow[R_2-R_3]{R_1-2R_2} \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \xrightarrow{4R_3} \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

In [1]:
```
from sympy import *
import numpy as np
import numba as nb
import galois
import sympy as sym
```

Question 6

In [2]:
```
A = Matrix([[1,1,1],[0,1,1],[0,1,0]])
A
```

Out[2]:
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

In [3]:
```
B = Matrix([[0,1,1],[1,1,1],[1,1,0]])
B
```

Out[3]:
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

In [4]:
```
A+B
```

Out[4]:
$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

In [5]:
```
A**2
```

Out[5]:
$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

In [6]:
```
A*B
```

Out[6]:
$$\begin{bmatrix} 2 & 3 & 2 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Question 8

Part a

In [7]:
```
h = sym.Symbol('h')
```

In [8]:
```
A = Matrix([[1,h,4],[3,6,8]])
A
```

Out[8]:

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

In [9]:
```
B = A.rref()
B
```

Out[9]:
```
(Matrix([
 [1, 0, (24 - 8*h)/(6 - 3*h)],
 [0, 1,          -4/(6 - 3*h)]]), (0, 1))
```

Part b

In [10]:
```
A = Matrix([[-4,12,h],[2,-6,-3]])
A
```

Out[10]:

$$\begin{bmatrix} -4 & 12 & h \\ 2 & -6 & -3 \end{bmatrix}$$

In [11]:
```
B = A.rref()
B
```

Out[11]:
```
(Matrix([
 [1, (144 - 24*h)/(8*h - 48), 0],
 [0,                       0, 1]]), (0, 2))
```

Question 11

Part b

In [12]:
```
GF7 = galois.GF(7)
```

In [13]:
```
A = GF7([[1,2,3,4],[1,1,0,2],[2,0,1,2]])
A
```

Out[13]:
```
GF([[1, 2, 3, 4],
    [1, 1, 0, 2],
    [2, 0, 1, 2]], order=7)
```

In [14]:
```
GF7.row_reduce(A)
```

Out[14]:
```
GF([[1, 0, 4, 0],
    [0, 1, 3, 0],
    [0, 0, 0, 1]], order=7)
```

Part c

In [15]:
```
GF2 = galois.GF(2)
GF3 = galois.GF(3)
```

In [16]:
```
A2 = GF2([[1,0,1,0],[1,1,0,0],[0,0,1,0]])
A2
```

Out[16]:
```
GF([[1, 0, 1, 0],
    [1, 1, 0, 0],
    [0, 0, 1, 0]], order=2)
```

In [17]:
```
GF2.row_reduce(A2)
```

Out[17]:
```
GF([[1, 0, 0, 0],
    [0, 1, 0, 0],
    [0, 0, 1, 0]], order=2)
```

In [18]:
```
A3 = GF3([[1,2,0,1],[1,1,0,2],[2,0,1,2]])
A3
```

Out[18]:
```
GF([[1, 2, 0, 1],
    [1, 1, 0, 2],
    [2, 0, 1, 2]], order=3)
```

In [19]:
```
GF3.row_reduce(A3)
```

Out[19]:
```
GF([[1, 0, 0, 0],
    [0, 1, 0, 2],
    [0, 0, 1, 2]], order=3)
```

Question 12

Part a

In [20]:
```
A = GF7([[3,1,4],[5,2,6],[0,5,2]])
b = GF7([[1],[5],[1]])
Ab = GF7([[3,1,4,1],[5,2,6,5],[0,5,2,1]])
Ab
```

Out[20]:
```
GF([[3, 1, 4, 1],
    [5, 2, 6, 5],
    [0, 5, 2, 1]], order=7)
```

In [21]:
```
GF7.row_reduce(Ab)
```

Out[21]:
```
GF([[1, 0, 0, 4],
    [0, 1, 0, 3],
    [0, 0, 1, 0]], order=7)
```

Part b

In [22]:
```
np.linalg.solve(A, b)
```

Out[22]:
```
GF([[4],
    [3],
    [0]], order=7)
```

Question 13

In [23]:
```
M = Matrix([[3,11,19,-2],[7,23,39,10],[-4,-3,-2,6]])
M
```

Out[23]:

$$\begin{bmatrix} 3 & 11 & 19 & -2 \\ 7 & 23 & 39 & 10 \\ -4 & -3 & -2 & 6 \end{bmatrix}$$

In [24]:
```
M.rref()
```

Out[24]:
```
(Matrix([
 [1, 0, -1, 0],
 [0, 1,  2, 0],
 [0, 0,  0, 1]]), (0, 1, 3))
```

In [25]:
```
#We have x1=x3,x2=-2x3,0=1. 0 does not equal 1.
#Therefore there is no solution.
```

Question 14

In [26]:
```
M = Matrix([[3,6,9,5,25,53],[7,14,21,9,53,105],[-4,-8,-12,5,-10,11]])
M
```

Out[26]:

$$\begin{bmatrix} 3 & 6 & 9 & 5 & 25 & 53 \\ 7 & 14 & 21 & 9 & 53 & 105 \\ -4 & -8 & -12 & 5 & -10 & 11 \end{bmatrix}$$

In [27]:
```
M.rref()
```

Out[27]:
```
(Matrix([
 [1, 2, 3, 0, 5, 6],
 [0, 0, 0, 1, 2, 7],
 [0, 0, 0, 0, 0, 0]]), (0, 3))
```

Question 15

In [28]:
```
M = Matrix([[2,4,3,5,6,37],
            [4,8,7,5,2,74],
            [-2,-4,3,4,-5,20],
            [1,2,2,-1,2,26],
            [5,-1,4,6,4,24]])
M
```

Out[28]:

$$\begin{bmatrix} 2 & 4 & 3 & 5 & 6 & 37 \\ 4 & 8 & 7 & 5 & 2 & 74 \\ -2 & -4 & 3 & 4 & -5 & 20 \\ 1 & 2 & 2 & -1 & 2 & 26 \\ 5 & -1 & 4 & 6 & 4 & 24 \end{bmatrix}$$

In [29]:
```
M.rref()
```

Out[29]:
```
(Matrix([
 [1, 0, 0, 0, 0, -1525/434],
 [0, 1, 0, 0, 0,   781/434],
 [0, 0, 1, 0, 0,  4695/434],
 [0, 0, 0, 1, 0,  -459/434],
 [0, 0, 0, 0, 1,   699/434]]), (0, 1, 2, 3, 4))
```

Question 17

In [30]:
```
A = Matrix([[2,1],[4,3]])
A
```

Out[30]:

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

In [31]:
```
B = Matrix([[5,3],[7,9]])
B
```

Out[31]:

$$\begin{bmatrix} 5 & 3 \\ 7 & 9 \end{bmatrix}$$

In [32]:
```
(A*B)**2 == (A**2)*(B**2)
```

Out[32]:
```
False
```

Question 21

In [33]:
```
import pprint
import scipy
import scipy.linalg
```

In [34]:
```
A = scipy.array([[4,1,0,0],
                 [1,4,1,0],
                 [0,1,4,1],
                 [0,0,1,4]])
A
```

Out[34]:
```
array([[4, 1, 0, 0],
       [1, 4, 1, 0],
       [0, 1, 4, 1],
       [0, 0, 1, 4]])
```

In [35]:
```
P, L, U = scipy.linalg.lu(A)
```

In [36]:
```
P
```

Out[36]:
```
array([[1., 0., 0., 0.],
       [0., 1., 0., 0.],
       [0., 0., 1., 0.],
       [0., 0., 0., 1.]])
```

In [37]:
```
L
```

Out[37]:
```
array([[1.        , 0.        , 0.        , 0.        ],
       [0.25      , 1.        , 0.        , 0.        ],
       [0.        , 0.26666667, 1.        , 0.        ],
       [0.        , 0.        , 0.26785714, 1.        ]])
```

In [38]:
```
U
```

Out[38]:
```
array([[4.        , 1.        , 0.        , 0.        ],
       [0.        , 3.75      , 1.        , 0.        ],
       [0.        , 0.        , 3.73333333, 1.        ],
       [0.        , 0.        , 0.        , 3.73214286]])
```

Question 23

In [39]:
```
d1 = sym.Symbol('d1')
d2 = sym.Symbol('d2')
d3 = sym.Symbol('d3')
d4 = sym.Symbol('d4')
u1 = sym.Symbol('u1')
u2 = sym.Symbol('u2')
u3 = sym.Symbol('u3')
l1 = sym.Symbol('l1')
l2 = sym.Symbol('l2')
l3 = sym.Symbol('l3')
```

In [40]:
```
L = Matrix([[1,0,0,0],
            [l1,1,0,0],
            [0,l2,1,0],
            [0,0,l3,1]])
L
```

Out[40]:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ l_1 & 1 & 0 & 0 \\ 0 & l_2 & 1 & 0 \\ 0 & 0 & l_3 & 1 \end{bmatrix}$$

In [41]:
```
U = Matrix([[d1,u1,0,0],
            [0,d2,u2,0],
            [0,0,d3,u3],
            [0,0,0,d4]])
U
```

Out[41]:

$$\begin{bmatrix} d_1 & u_1 & 0 & 0 \\ 0 & d_2 & u_2 & 0 \\ 0 & 0 & d_3 & u_3 \\ 0 & 0 & 0 & d_4 \end{bmatrix}$$

In [42]:
```
A = L*U
A
```

Out[42]:

$$\begin{bmatrix} d_1 & u_1 & 0 & 0 \\ d_1 l_1 & d_2 + l_1 u_1 & u_2 & 0 \\ 0 & d_2 l_2 & d_3 + l_2 u_2 & u_3 \\ 0 & 0 & d_3 l_3 & d_4 + l_3 u_3 \end{bmatrix}$$

Question 24

In [43]:
```
u3 = (1/(3**.5))*Matrix([[1],
                          [1],
                          [1]])
u3
```

Out[43]:

$$\begin{bmatrix} 0.577350269189626 \\ 0.577350269189626 \\ 0.577350269189626 \end{bmatrix}$$

In [44]:
```
u3_tp = np.transpose(u3)
u3_tp
```

Out[44]:
```
array([[0.577350269189626, 0.577350269189626, 0.577350269189626]],
      dtype=object)
```

In [45]:
```
H3 = np.identity(3) - 2*u3*u3_tp
H3
```

Out[45]:

$$\begin{bmatrix} 0.333333333333333 & -0.666666666666667 & -0.666666666666667 \\ -0.666666666666667 & 0.333333333333333 & -0.666666666666667 \\ -0.666666666666667 & -0.666666666666667 & 0.333333333333333 \end{bmatrix}$$

In [46]:
```
H3_tp = np.transpose(H3)
H3*H3_tp
```

Out[46]:

$$\begin{bmatrix} 1.0 & 3.88578058618805 \cdot 10^{-16} & 3.88578058618805 \cdot 10 \\ 3.88578058618805 \cdot 10^{-16} & 1.0 & 3.88578058618805 \cdot 10 \\ 3.88578058618805 \cdot 10^{-16} & 3.88578058618805 \cdot 10^{-16} & 1.0 \end{bmatrix}$$

In [47]:
```
H3*H3
```

Out[47]:

$$\begin{bmatrix} 1.0 & 3.88578058618805 \cdot 10^{-16} & 3.88578058618805 \cdot 10 \\ 3.88578058618805 \cdot 10^{-16} & 1.0 & 3.88578058618805 \cdot 10 \\ 3.88578058618805 \cdot 10^{-16} & 3.88578058618805 \cdot 10^{-16} & 1.0 \end{bmatrix}$$

In [48]:
```
H3*u3
```

Out[48]:

$$\begin{bmatrix} -0.577350269189626 \\ -0.577350269189626 \\ -0.577350269189626 \end{bmatrix}$$

In [49]:
```
u4 = (1/(4**.5))*Matrix([[1],
                         [1],
                         [1],
                         [1]])
u4
```

Out[49]:

$$\begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

In [50]:
```
u4_tp = np.transpose(u4)
u4_tp
```

Out[50]:
```
array([[0.500000000000000, 0.500000000000000, 0.500000000000000,
        0.500000000000000]], dtype=object)
```

In [51]:
```
H4 = np.identity(4) - 2*u4*u4_tp
H4
```

Out[51]:

$$\begin{bmatrix} 0.5 & -0.5 & -0.5 & -0.5 \\ -0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & -0.5 \\ -0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix}$$