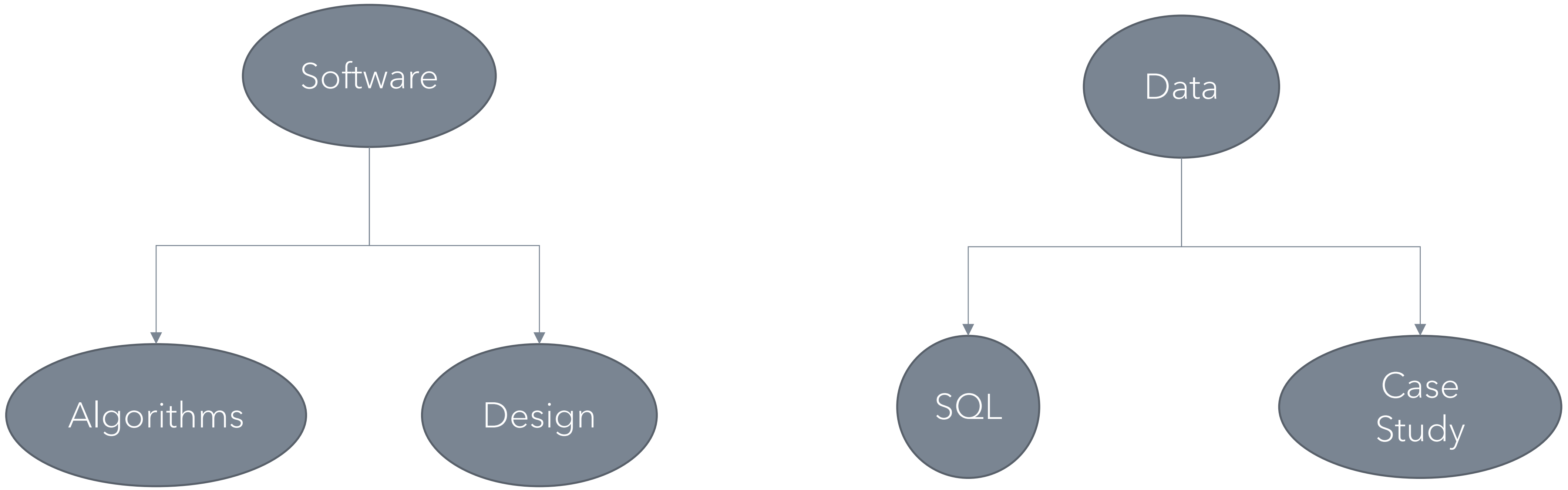
DATA CLUB – MEET I

Topic: Complexity

- Types of interviews



- Types of job roles (for Applied Mathematicians)
  1. Research Engineer (Algorithms, Distributed Systems)
  2. Machine Learning Engineer (Algorithms, OOPS, Distributed Systems, ML System Design)
  3. Quantitative Researcher (Algorithms, Mathematics, Case Studies)
  4. Software Engineers (Algorithms, OOPS, Distributed Systems)
  5. Data Scientist (Algorithms, SQL, Case Studies)
  6. Data Analyst (Algorithms, SQL, Case Studies)

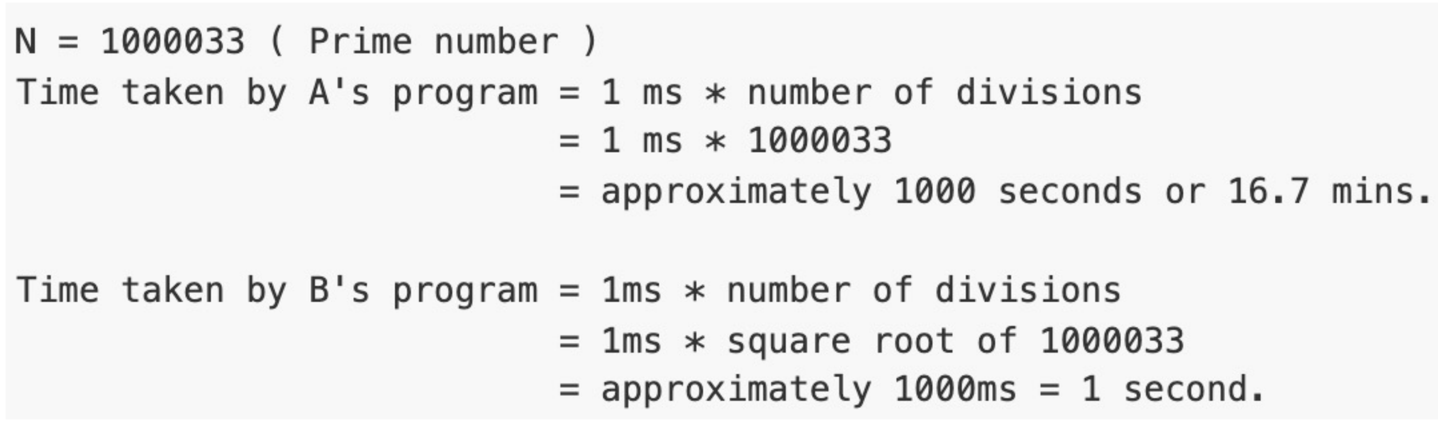
1. Top-down learning
2. Learn by examples
3. More practice
4. Interactive
5. NOT EASY

Let's assume we ask 2 interviewees A and B to write a program to detect if a number N >= 2 is prime.

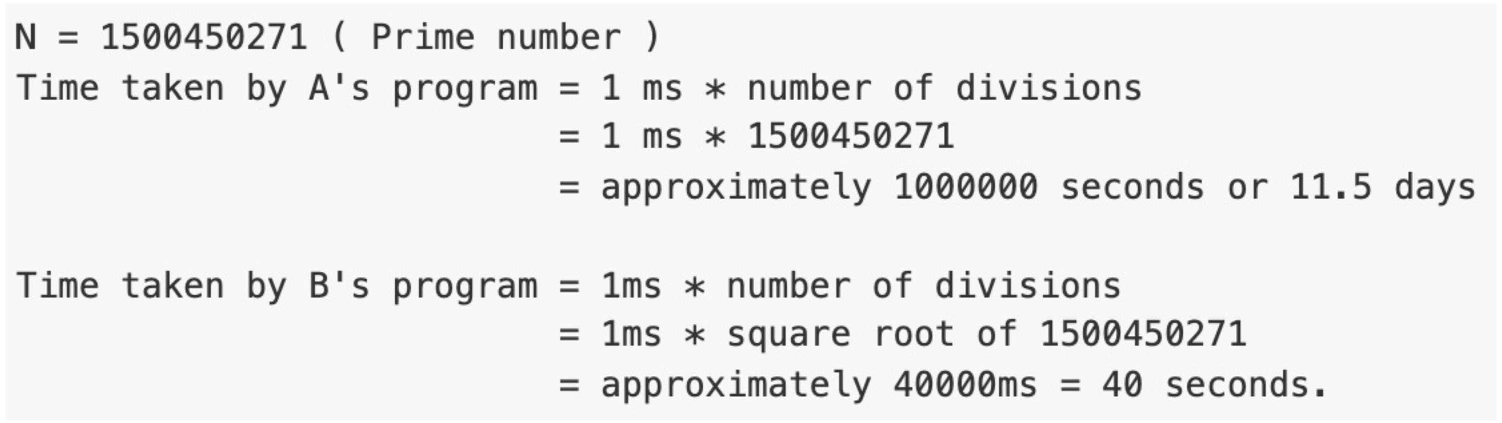
```
i = 2
while i < N
  if N is divisible by i
     N is not prime
  add 1 to i
```

```
i = 2
while i <= square root of N
  if N is divisible by i
    N is not prime
  add 1 to i
```

Let’s assume that the operation N is divisible by **i** takes 1 ms.



```
N = 1000033 ( Prime number )
Time taken by A's program = 1 ms * number of divisions
                          = 1 ms * 1000033
                          = approximately 1000 seconds or 16.7 mins.

Time taken by B's program = 1ms * number of divisions
                          = 1ms * square root of 1000033
                          = approximately 1000ms = 1 second.
```



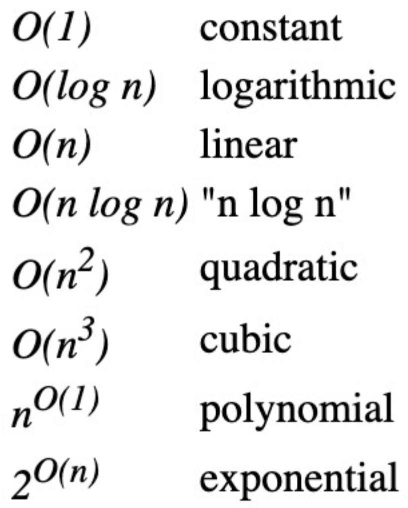
```
N = 1500450271 ( Prime number )
Time taken by A's program = 1 ms * number of divisions
                          = 1 ms * 1500450271
                          = approximately 1000000 seconds or 11.5 days

Time taken by B's program = 1ms * number of divisions
                          = 1ms * square root of 1500450271
                          = approximately 40000ms = 40 seconds.
```

- Computer scientists have developed a convenient notation for hiding the constant factor.
- We write *O(n)* (read: ''order *n*'') instead of ''*cn* for some constant *c*.''
- Thus, an algorithm is said to be *O(n)* or *linear time* if there is a fixed constant *c* such that for all sufficiently large *n*, the algorithm takes time at most *cn* on inputs of size *n*. An algorithm is said to be $O(n^2)$ or *quadratic time* if there is a fixed constant *c* such that for all sufficiently large *n*, the algorithm takes time at most cn^2 on inputs of size *n*.
- *O(1)* means *constant time*.
- *Polynomial time* means $n^{O(1)}$, or n^c for some constant *c*. Thus, any constant, linear, quadratic, or cubic ($O(n^3)$) time algorithm is a polynomial-time algorithm.
- One important advantage of big-O notation is that it makes algorithms much easier to analyze, since we can conveniently ignore low-order terms. For example, an algorithm that runs in time

  $10n^3 + 24n^2 + 3n \log n + 144$ is still a cubic algorithm, since

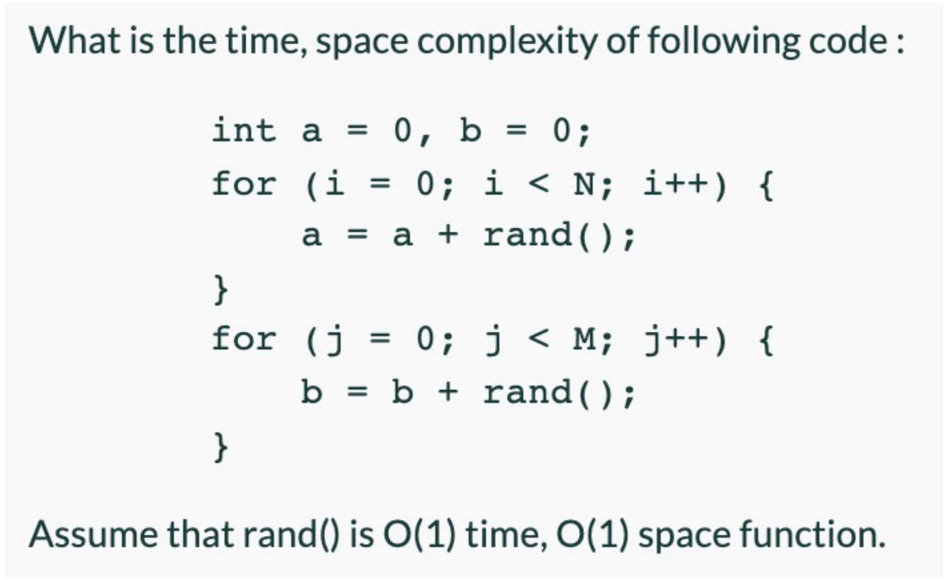
  $10n^3 + 24n^2 + 3n \log n + 144$
  $<= 10n^3 + 24n^3 + 3n^3 + 144n^3$
  $<= (10 + 24 + 3 + 144)n^3$
  $= O(n^3)$.

Some common orders of growth seen often in complexity analysis are,



$O(1)$	constant
$O(\log n)$	logarithmic
$O(n)$	linear
$O(n \log n)$	"n log n"
$O(n^2)$	quadratic
$O(n^3)$	cubic
$n^{O(1)}$	polynomial
$2^{O(n)}$	exponential

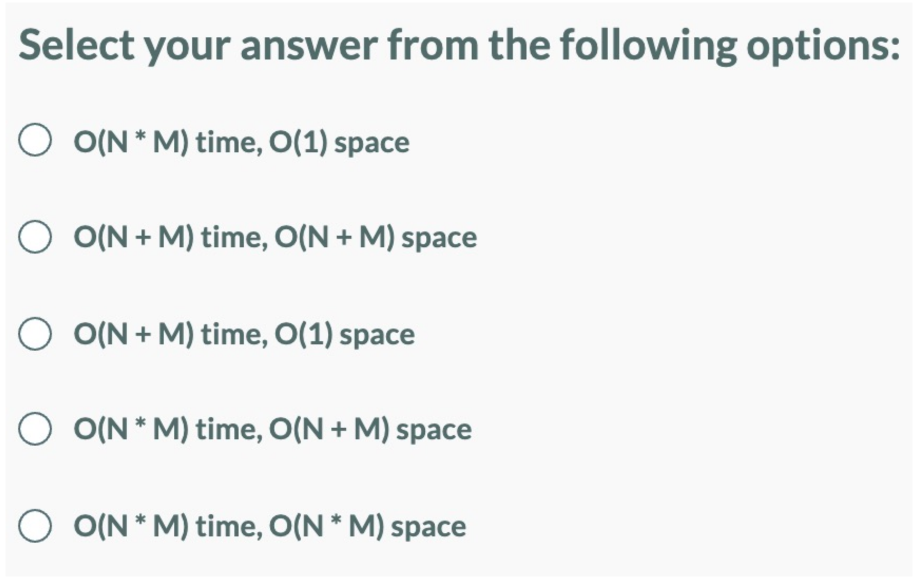
1)



What is the time, space complexity of following code :

```
int a = 0, b = 0;
for (i = 0; i < N; i++) {
    a = a + rand();
}
for (j = 0; j < M; j++) {
    b = b + rand();
}
```

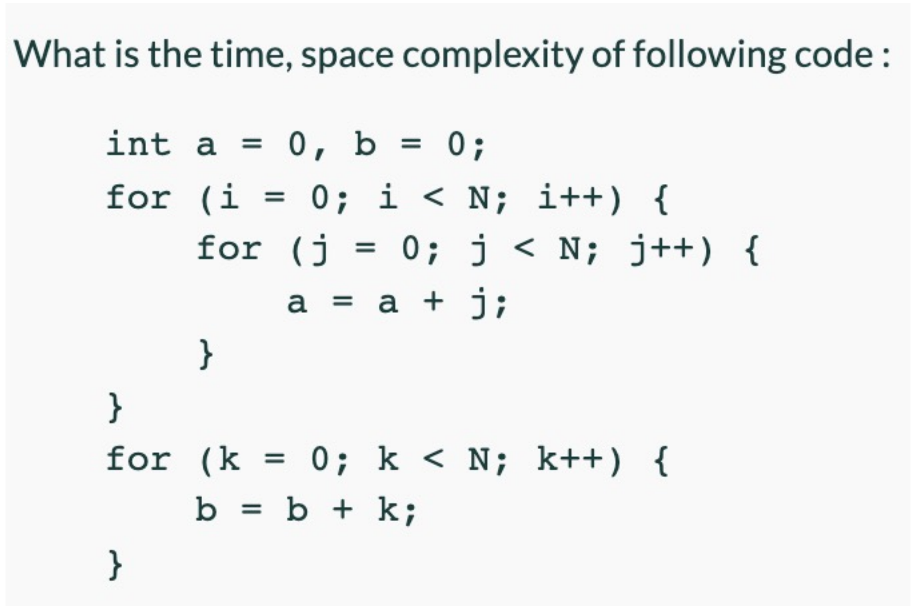
Assume that rand() is O(1) time, O(1) space function.



Select your answer from the following options:

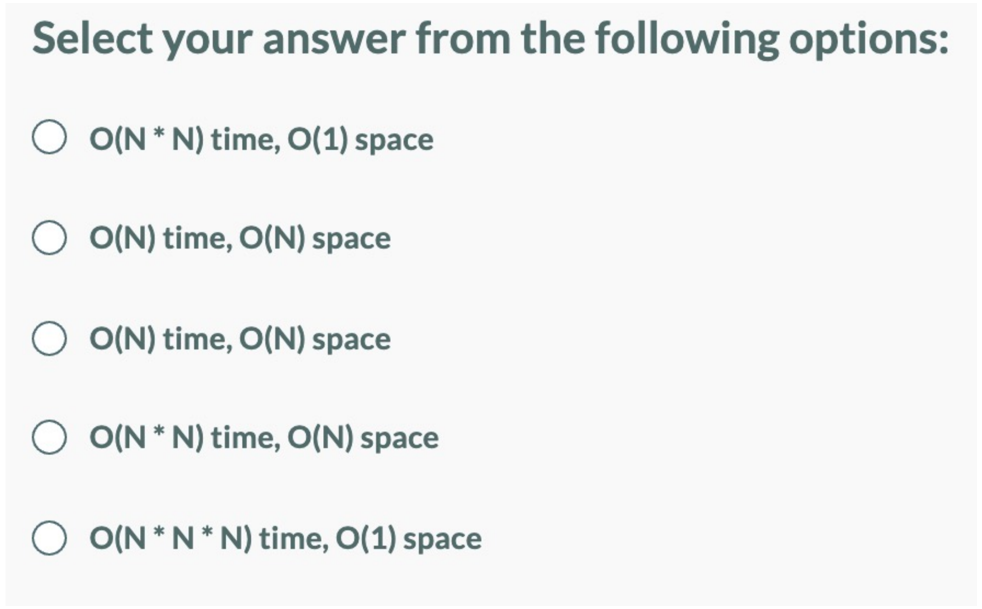
- ○ O(N * M) time, O(1) space
- ○ O(N + M) time, O(N + M) space
- ○ O(N + M) time, O(1) space
- ○ O(N * M) time, O(N + M) space
- ○ O(N * M) time, O(N * M) space

2)



What is the time, space complexity of following code :

```
int a = 0, b = 0;
for (i = 0; i < N; i++) {
    for (j = 0; j < N; j++) {
        a = a + j;
    }
}
for (k = 0; k < N; k++) {
    b = b + k;
}
```



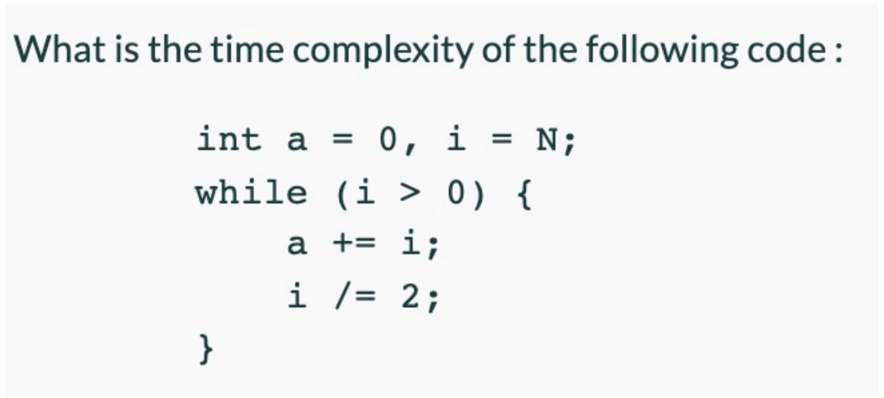
Select your answer from the following options:

- ○ O(N * N) time, O(1) space
- ○ O(N) time, O(N) space
- ○ O(N) time, O(N) space
- ○ O(N * N) time, O(N) space
- ○ O(N * N * N) time, O(1) space

3) What does it mean when we say that an algorithm X is asymptotically more efficient than Y?

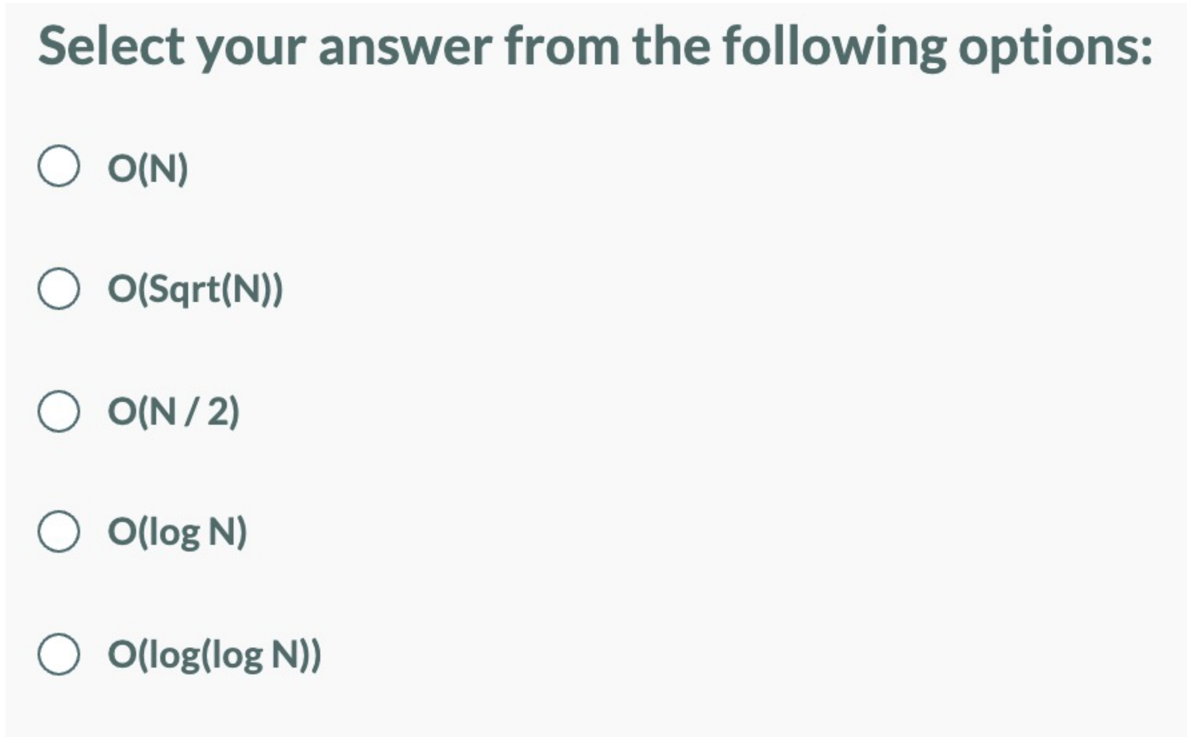
a) X will always be a better choice for all inputs
b) X will always be a better choice for large inputs
c) X will always be a better choice for small inputs
d) Y will always be a better choice for small inputs

4)



What is the time complexity of the following code :

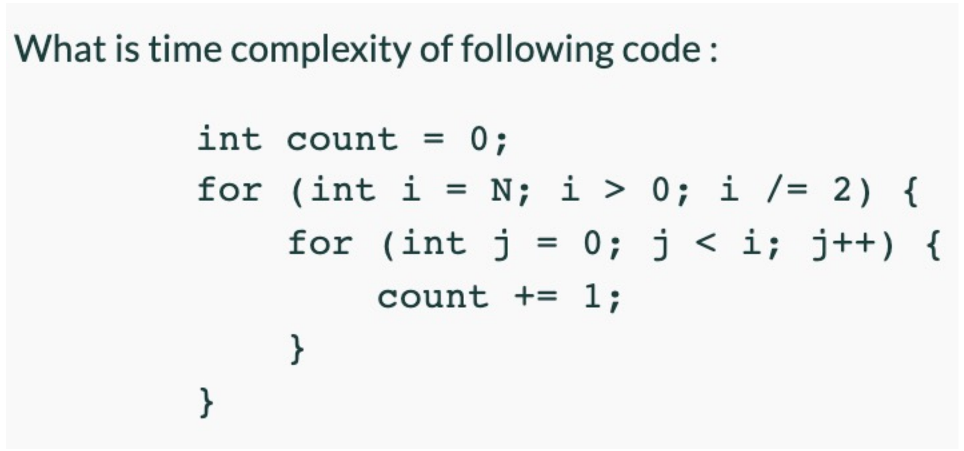
```
int a = 0, i = N;
while (i > 0) {
    a += i;
    i /= 2;
}
```



Select your answer from the following options:

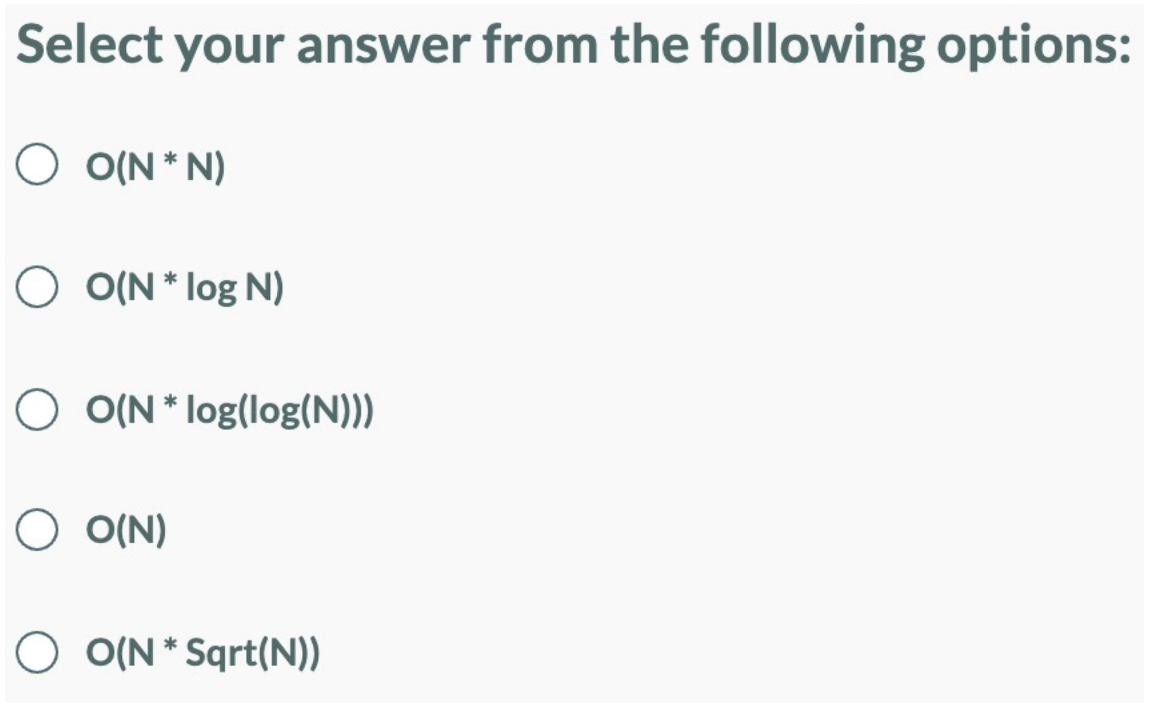
- ○ O(N)
- ○ O(Sqrt(N))
- ○ O(N / 2)
- ○ O(log N)
- ○ O(log(log N))

5)



What is time complexity of following code :

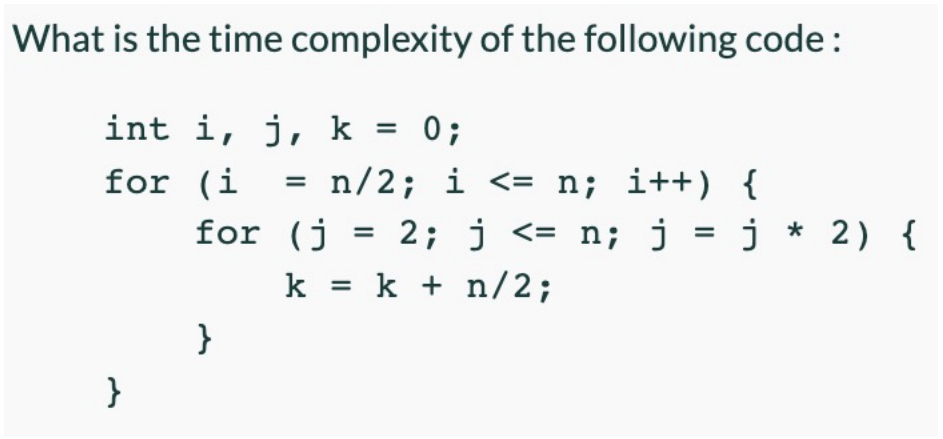
```
int count = 0;
for (int i = N; i > 0; i /= 2) {
    for (int j = 0; j < i; j++) {
        count += 1;
    }
}
```



Select your answer from the following options:

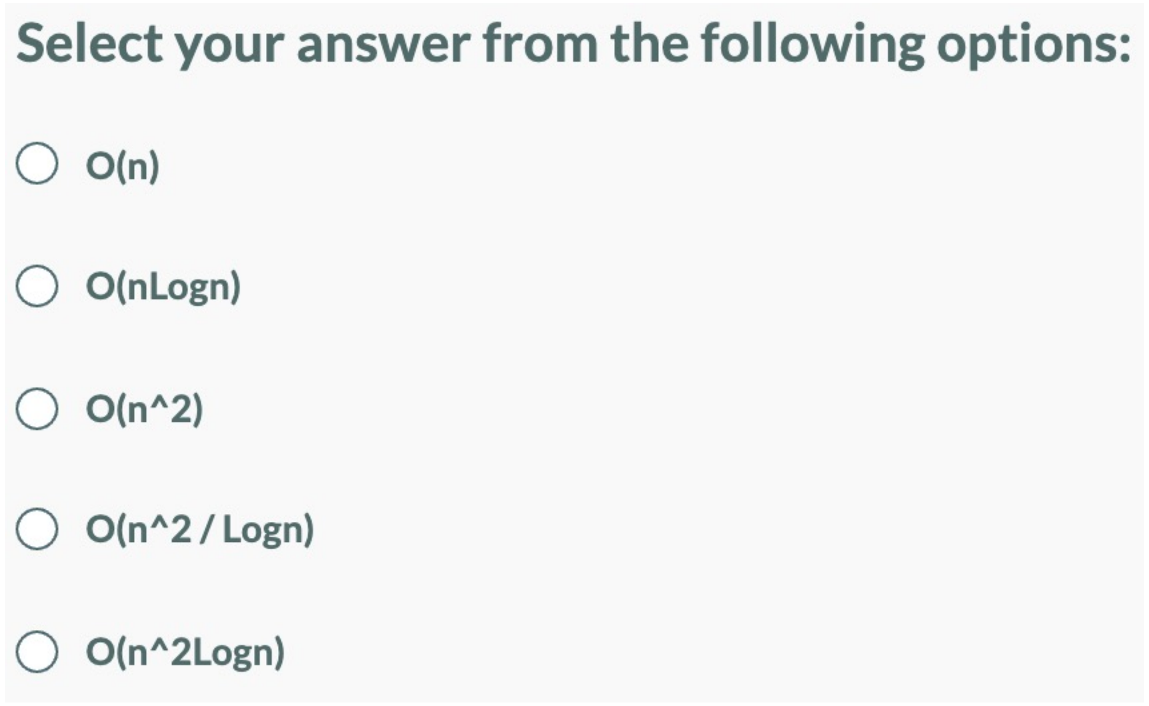
- O(N * N)
- O(N * log N)
- O(N * log(log(N)))
- O(N)
- O(N * Sqrt(N))

6)



What is the time complexity of the following code :

```
int i, j, k = 0;
for (i  = n/2; i <= n; i++) {
    for (j = 2; j <= n; j = j * 2) {
        k = k + n/2;
    }
}
```



Select your answer from the following options:

○ O(n)

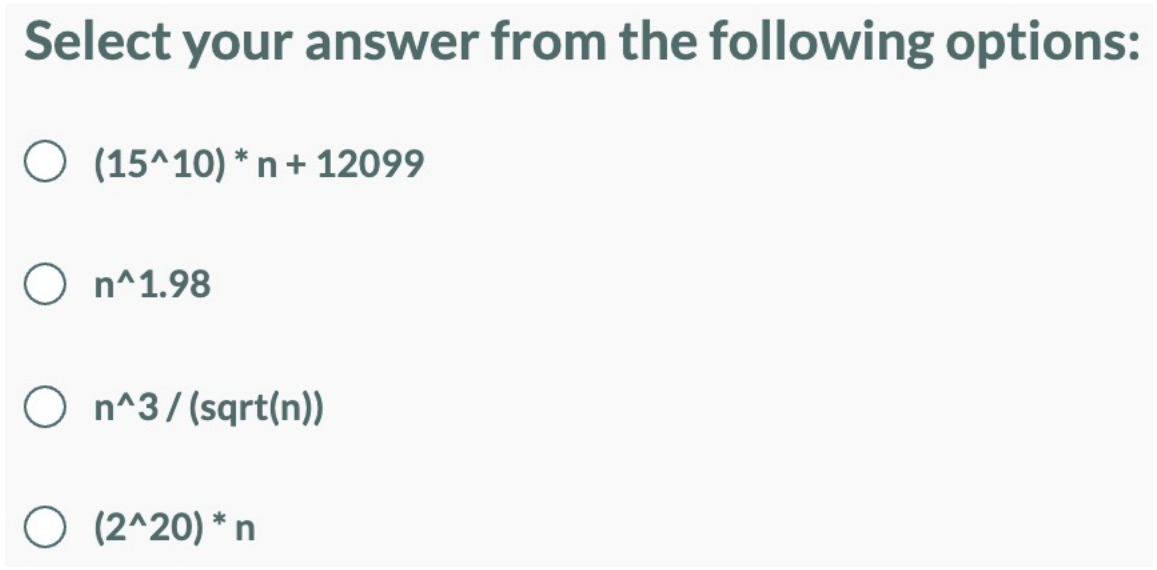
○ O(nLogn)

○ O(n^2)

○ O(n^2 / Logn)

○ O(n^2Logn)

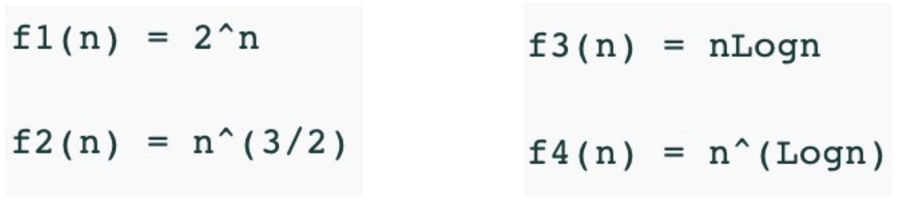
7) Which of the following is not bounded by O(n^2)?



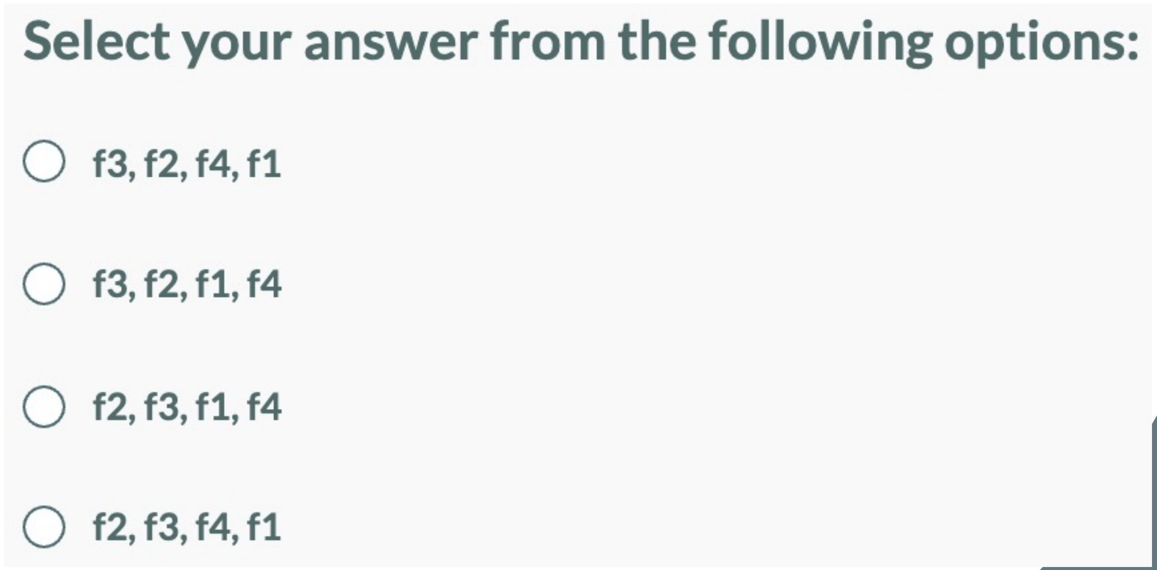
Select your answer from the following options:

- ○ (15^10) * n + 12099
- ○ n^1.98
- ○ n^3 / (sqrt(n))
- ○ (2^20) * n

8) Which of the given options provides the increasing order of complexity of functions f1, f2, f3 and f4:



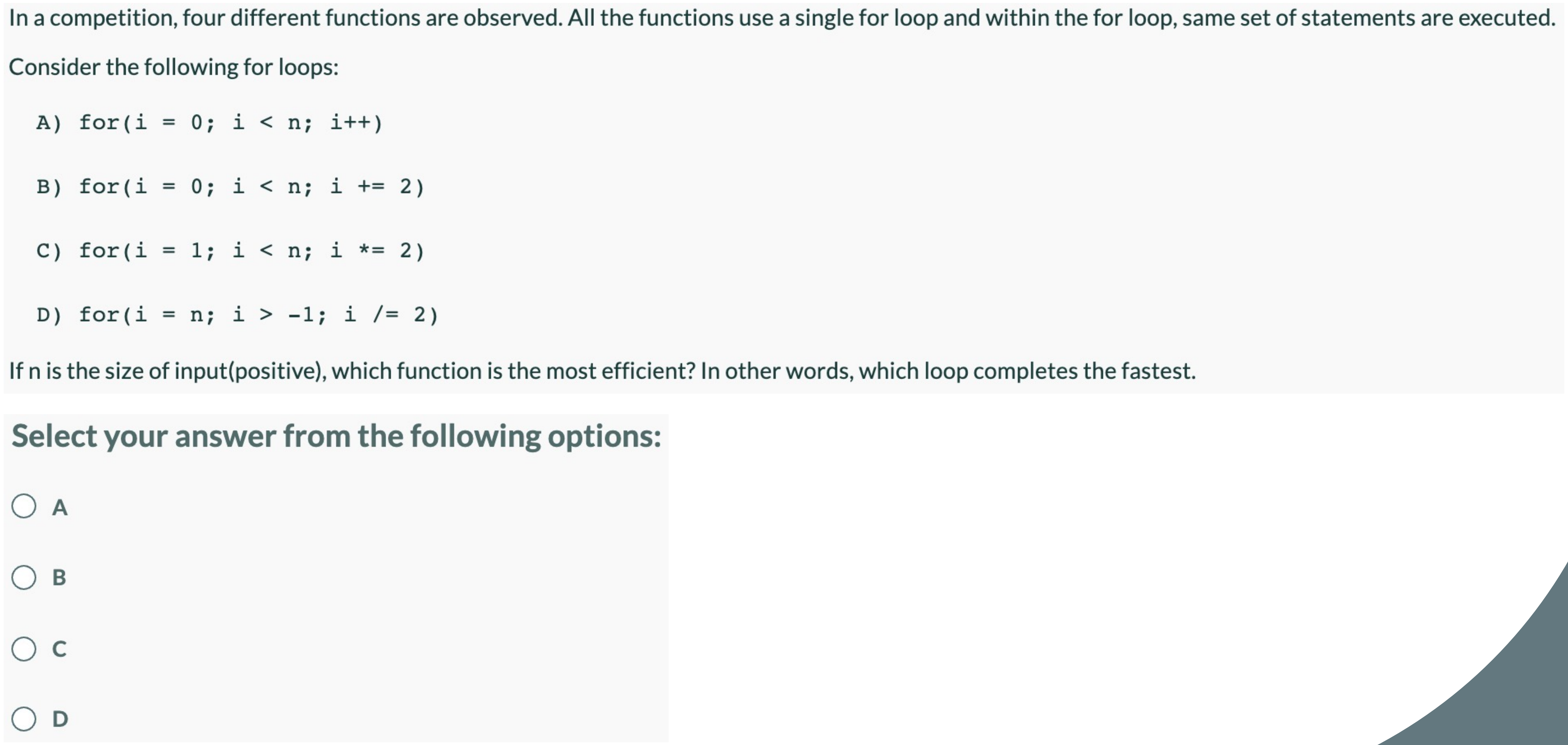
```
f1(n) = 2^n

f2(n) = n^(3/2)

f3(n) = nLogn

f4(n) = n^(Logn)
```



Select your answer from the following options:

- ○ f3, f2, f4, f1
- ○ f3, f2, f1, f4
- ○ f2, f3, f1, f4
- ○ f2, f3, f4, f1

9)



In a competition, four different functions are observed. All the functions use a single for loop and within the for loop, same set of statements are executed.

Consider the following for loops:

```
A) for(i = 0; i < n; i++)

B) for(i = 0; i < n; i += 2)

C) for(i = 1; i < n; i *= 2)

D) for(i = n; i > -1; i /= 2)
```

If n is the size of input(positive), which function is the most efficient? In other words, which loop completes the fastest.

Select your answer from the following options:

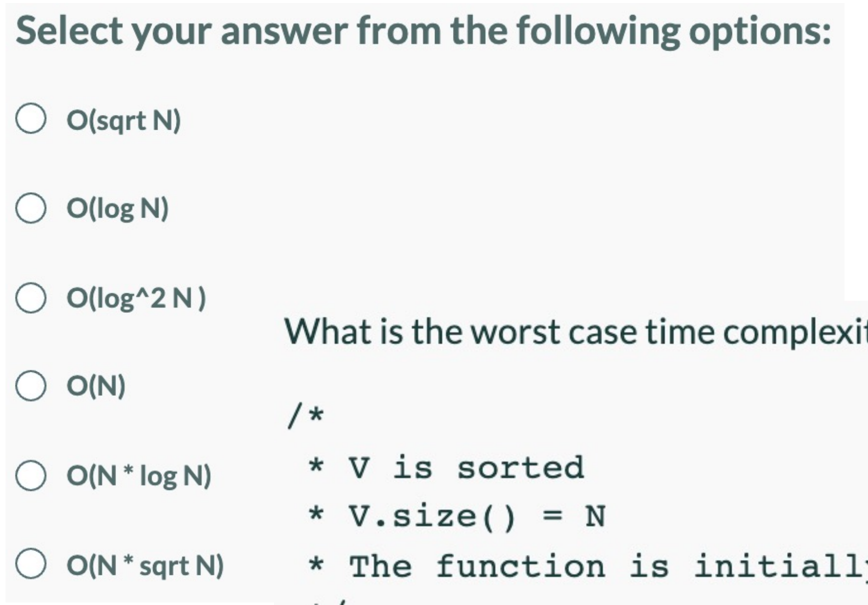
○ A

○ B

○ C

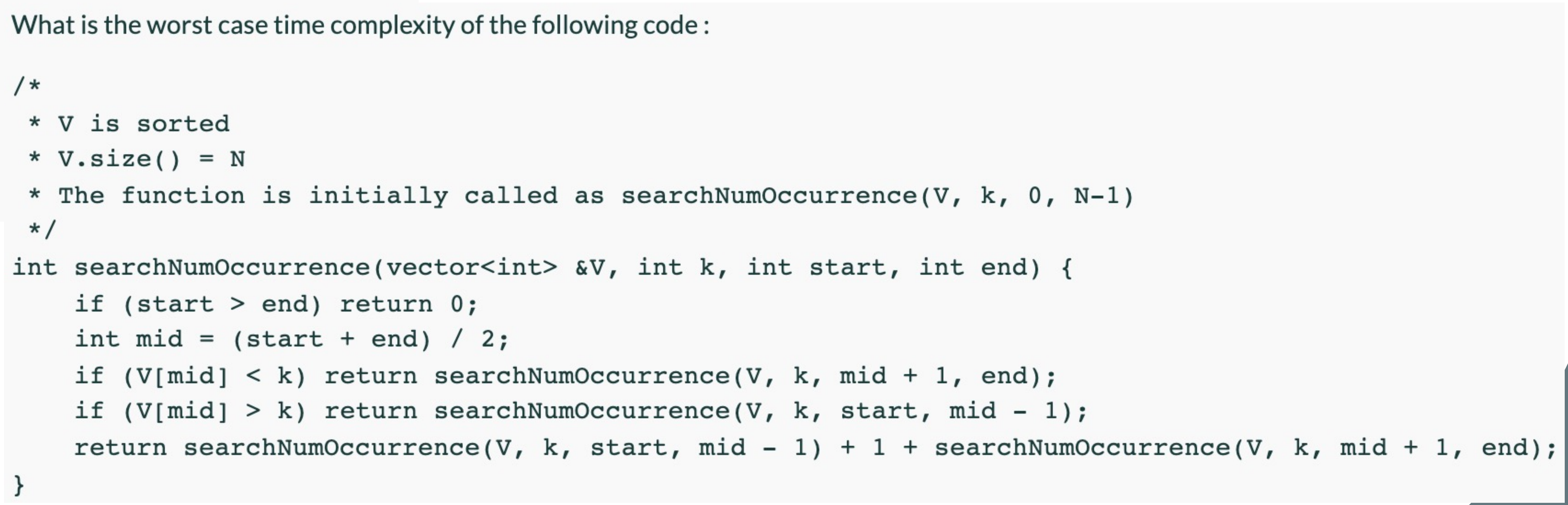
○ D

10)



Select your answer from the following options:

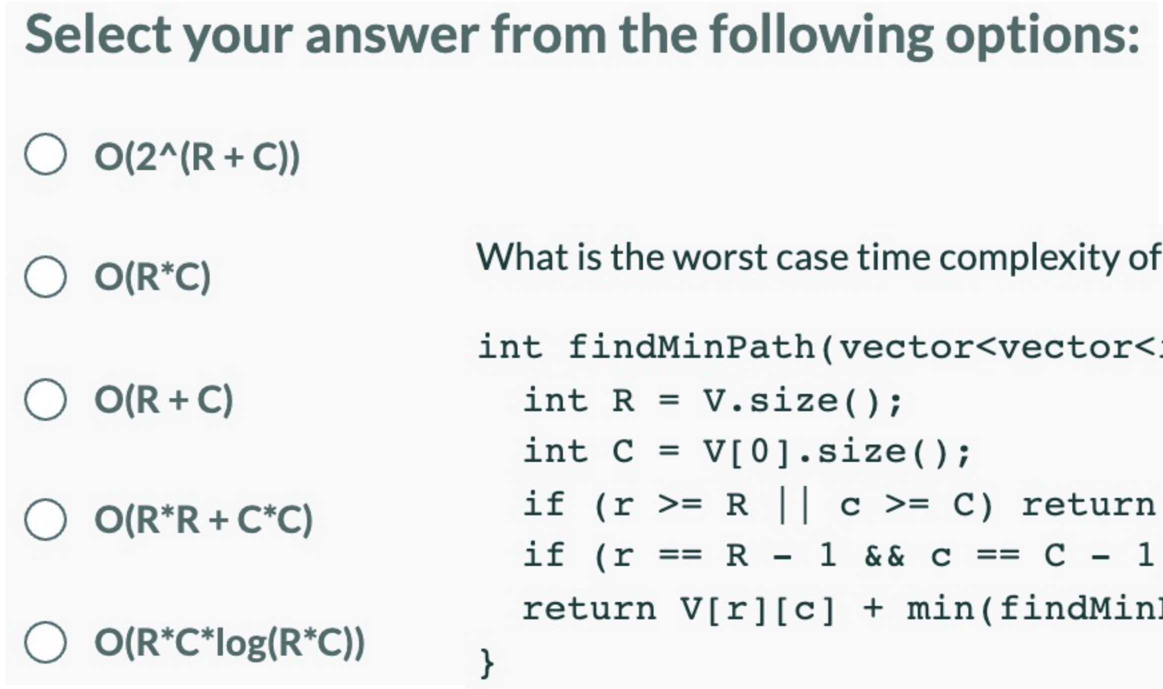
- ○ O(sqrt N)
- ○ O(log N)
- ○ O(log^2 N)
- ○ O(N)
- ○ O(N * log N)
- ○ O(N * sqrt N)



What is the worst case time complexity of the following code :

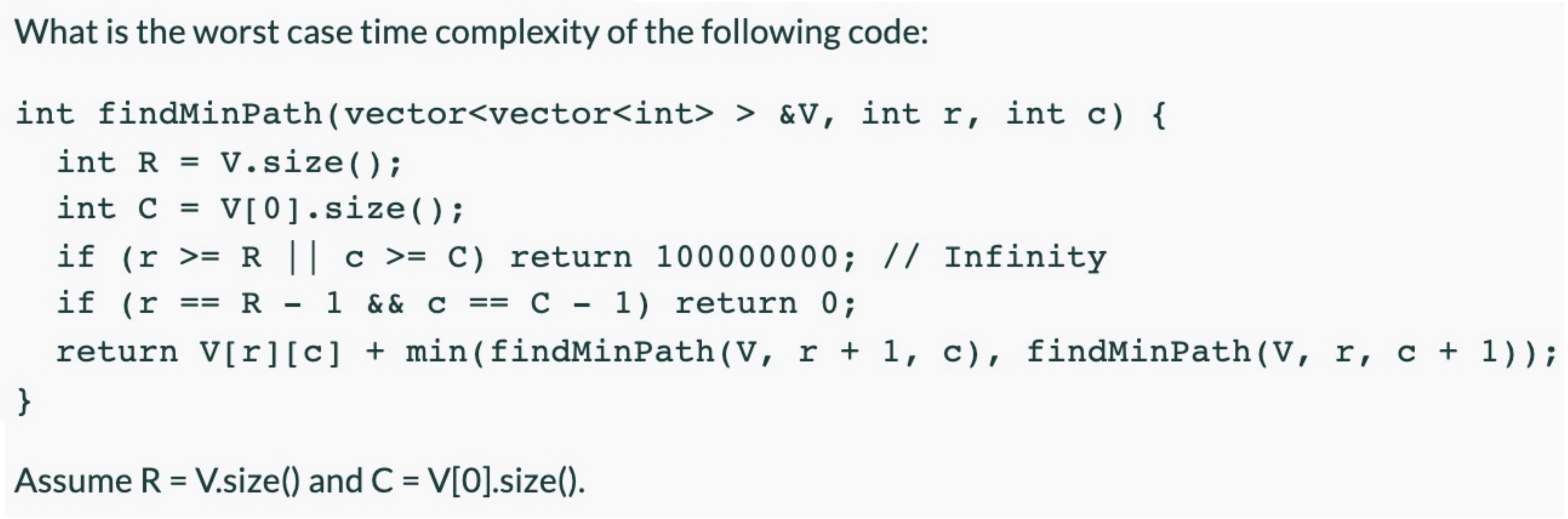
```
/*
 * V is sorted
 * V.size() = N
 * The function is initially called as searchNumOccurrence(V, k, 0, N-1)
 */
int searchNumOccurrence(vector<int> &V, int k, int start, int end) {
    if (start > end) return 0;
    int mid = (start + end) / 2;
    if (V[mid] < k) return searchNumOccurrence(V, k, mid + 1, end);
    if (V[mid] > k) return searchNumOccurrence(V, k, start, mid - 1);
    return searchNumOccurrence(V, k, start, mid - 1) + 1 + searchNumOccurrence(V, k, mid + 1, end);
}
```

11)



Select your answer from the following options:

- ○ O(2^(R + C))
- ○ O(R*C)
- ○ O(R + C)
- ○ O(R*R + C*C)
- ○ O(R*C*log(R*C))

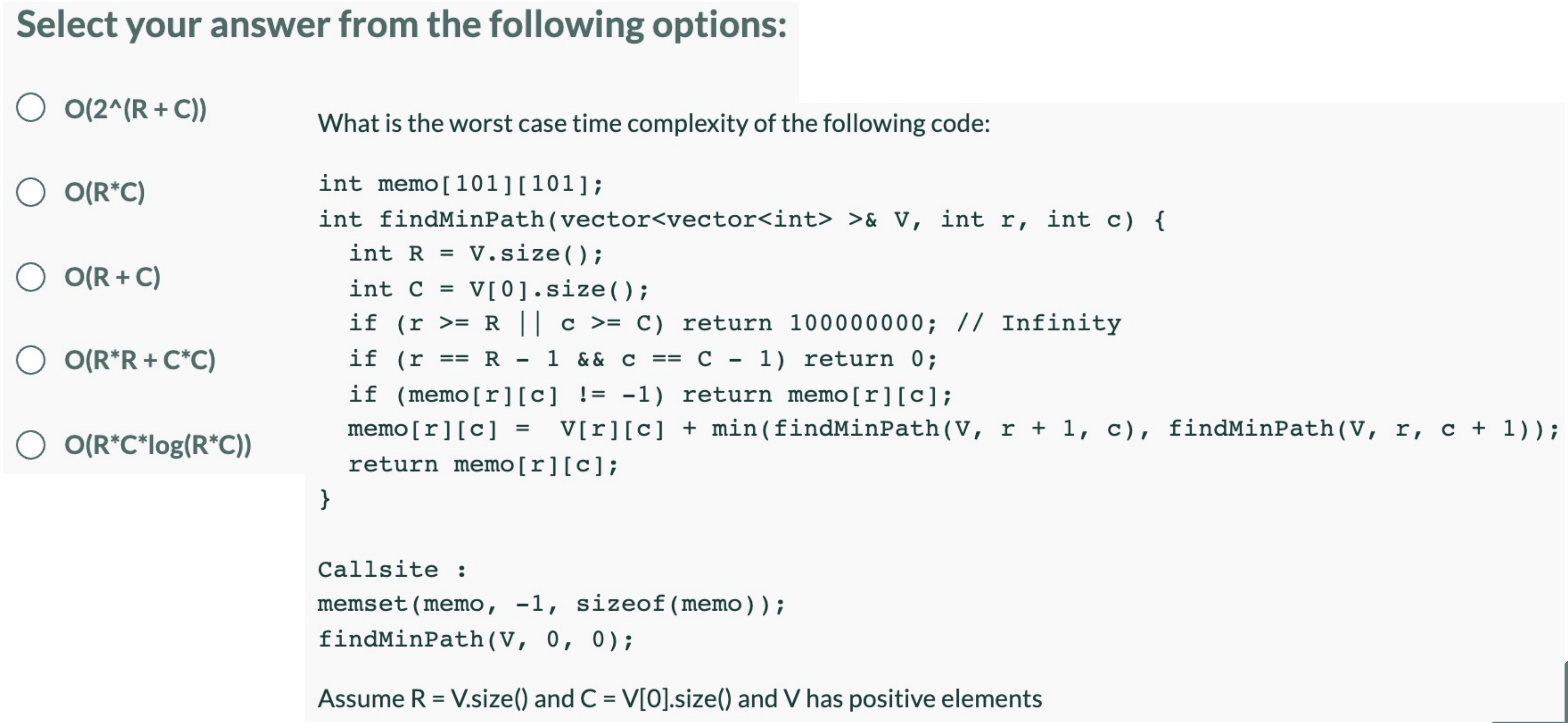


What is the worst case time complexity of the following code:

```
int findMinPath(vector<vector<int> > &V, int r, int c) {
  int R = V.size();
  int C = V[0].size();
  if (r >= R || c >= C) return 100000000; // Infinity
  if (r == R - 1 && c == C - 1) return 0;
  return V[r][c] + min(findMinPath(V, r + 1, c), findMinPath(V, r, c + 1));
}
```

Assume R = V.size() and C = V[0].size().

12)



Select your answer from the following options:

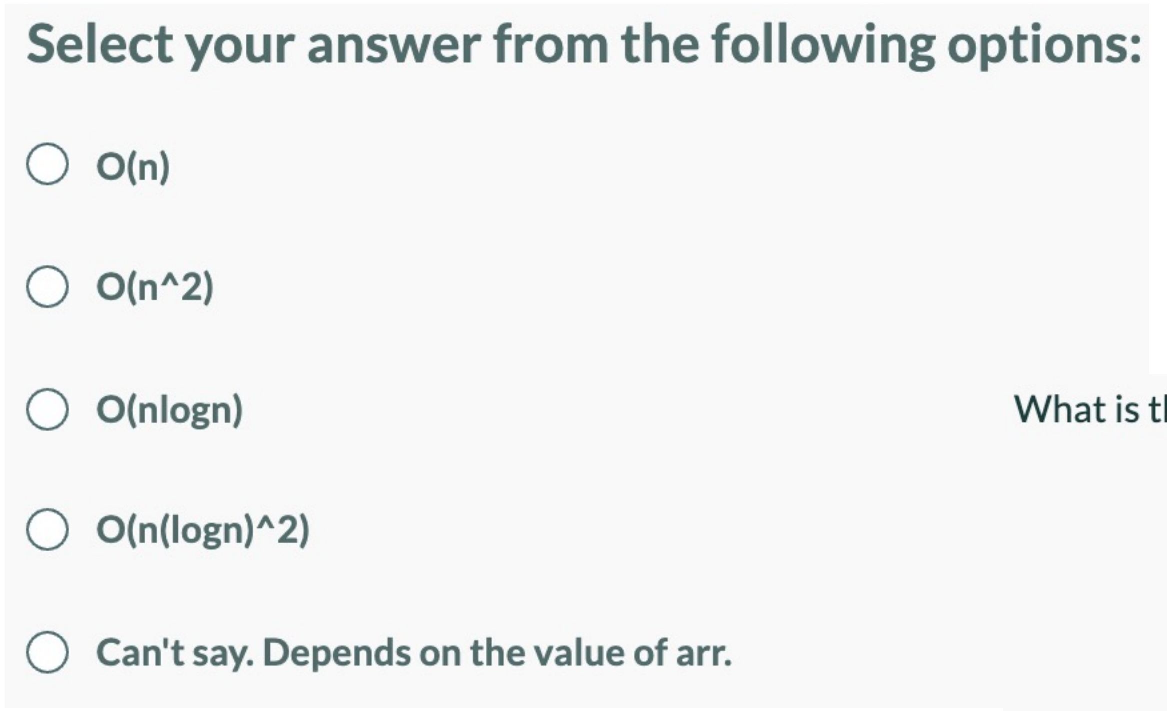
- ◯ O(2^(R + C))
- ◯ O(R*C)
- ◯ O(R + C)
- ◯ O(R*R + C*C)
- ◯ O(R*C*log(R*C))

What is the worst case time complexity of the following code:

```
int memo[101][101];
int findMinPath(vector<vector<int> >& V, int r, int c) {
  int R = V.size();
  int C = V[0].size();
  if (r >= R || c >= C) return 100000000; // Infinity
  if (r == R - 1 && c == C - 1) return 0;
  if (memo[r][c] != -1) return memo[r][c];
  memo[r][c] =  V[r][c] + min(findMinPath(V, r + 1, c), findMinPath(V, r, c + 1));
  return memo[r][c];
}

Callsite :
memset(memo, -1, sizeof(memo));
findMinPath(V, 0, 0);
```

Assume R = V.size() and C = V[0].size() and V has positive elements

13)



Select your answer from the following options:

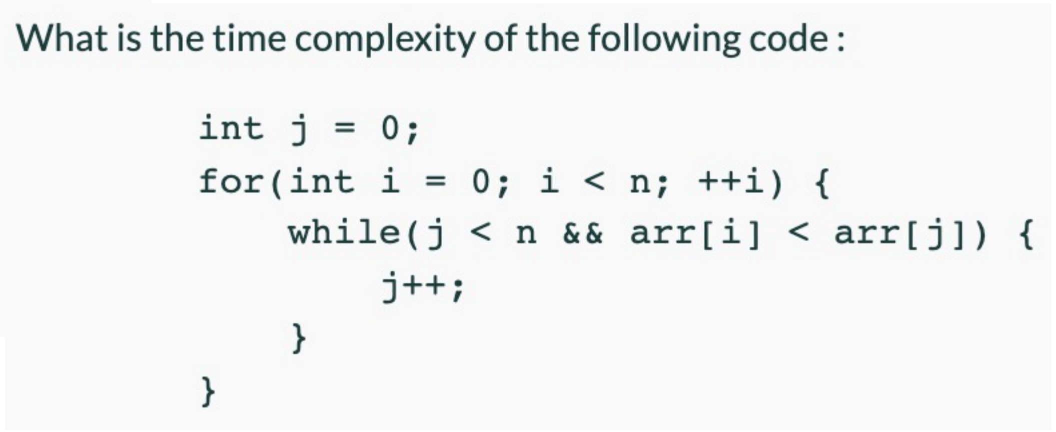
○ O(n)

○ O(n^2)

○ O(nlogn)

○ O(n(logn)^2)

○ Can't say. Depends on the value of arr.



What is the time complexity of the following code :

```
int j = 0;
for(int i = 0; i < n; ++i) {
    while(j < n && arr[i] < arr[j]) {
        j++;
    }
}
```