HW1

Tuesday, September 21, 2021 2:15 PM

1. (1) under + and ×

- Id. for sum: $(a+b\sqrt{2})+(0+0\sqrt{2}) = a+b\sqrt{2}$
- Ass. for sum: $(a+b\sqrt{2}+c+d\sqrt{2})+(e+f\sqrt{2}) = a+c+e+(b+d+f)\sqrt{2} :=$

  $(a+b\sqrt{2})+(c+d\sqrt{2}+e+f\sqrt{2}) = a+c+e+(b+d+f)\sqrt{2}$
- Inv. for sum: $a+b\sqrt{2}+(c+d\sqrt{2}) = 0$ where $c=-a$, $d=-b$
- Com. for sum: $(a+b\sqrt{2})+(c+d\sqrt{2}) = a+c+(b+d)\sqrt{2} :=$

  $(c+d\sqrt{2})+(a+b\sqrt{2}) = a+c+(b+d)\sqrt{2}$
- Mult. Id.: $(a+b\sqrt{2})(c+d\sqrt{2}) = (c+d\sqrt{2})(a+b\sqrt{2}) = (a+b\sqrt{2})$

  where $c=1$, $d=0$
- Ass. for prod.: $[(a+b\sqrt{2})(c+d\sqrt{2})](e+f\sqrt{2}) = (ac+ad\sqrt{2}+bc\sqrt{2}+2bd)(e+f\sqrt{2})$

  $:= (a+b\sqrt{2})[(c+d\sqrt{2})(e+f\sqrt{2})] = (a+b\sqrt{2})(ce+cf\sqrt{2}+ed\sqrt{2}+2fd)$
- Dist.: $(a+b\sqrt{2})[(c+d\sqrt{2})+(e+f\sqrt{2})] = (a+b\sqrt{2})(c+d\sqrt{2})+(a+b\sqrt{2})(e+f\sqrt{2}) :=$

  $[(c+d\sqrt{2})+(e+f\sqrt{2})](a+b\sqrt{2}) = (c+d\sqrt{2})(a+b\sqrt{2})+(e+f\sqrt{2})(a+b\sqrt{2})$
- Com. for Prod.: $(a+b\sqrt{2})(c+d\sqrt{2}) = ac+ad\sqrt{2}+bc\sqrt{2}+2bd :=$

  $(c+d\sqrt{2})(a+b\sqrt{2}) = ca+cb\sqrt{2}+da\sqrt{2}+2db$
- Inv. for Prod.: $(a+b\sqrt{2})(c+d\sqrt{2}) = 1$

  where c, d solutions to $\left[\begin{array}{cc|c} a & 2b & 1 \\ b & a & 0 \end{array}\right]$

(2) $a+bi$ — same as above, except i in place of $\sqrt{2}$ and -1 in place of $\sqrt{2}^2$

2. ex:
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 7 \end{bmatrix} \neq \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 3 & 4 \end{bmatrix}$$

$$A \times B \neq B \times A$$

3.

+	[0]	[1]	[2]
[0]	[0]	[1]	[2]
[1]	[1]	[2]	[0]
[2]	[2]	[0]	[1]

×	[0]	[1]	[2]
[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]
[2]	[0]	[2]	[1]

4. See 1.(2)

5. B, C, D

6.
$$A+B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

7.
$$\begin{bmatrix} 6 & -1 & 1 \\ t & 0 & 1 \\ 0 & 1 & t \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -\frac{1}{6} & \frac{1}{6} \\ 0 & 1 & t \\ t & 0 & 1 \end{bmatrix}$$

8. a. $\left[\begin{array}{cc|c} 1 & h & 4 \\ 3 & 6 & 8 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & h & 4 \\ 0 & 6-3h & -4 \end{array}\right]$

b. $\left[\begin{array}{cc|c} -4 & 12 & h \\ 2 & -6 & -3 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -3 & -\frac{1}{4}h \\ 0 & 0 & -3+\frac{1}{2}h \end{array}\right]$

9. (1)

9. (1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 & \sim \\ 0 & 1 & \sim \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

10. $a = e = d = b = 0 \qquad c = 1$

11. $$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -3 & -2 \\ 0 & -4 & -5 & -6 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 7 & 2 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 2/7 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & 22/7 \\ 0 & 1 & 0 & 8/7 \\ 0 & 0 & 1 & 2/7 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 6/7 \\ 0 & 1 & 0 & 8/7 \\ 0 & 0 & 1 & 2/7 \end{bmatrix}$$

$A\vec{x} = \vec{0}$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 6/7 & 0 \\ 0 & 1 & 0 & 8/7 & 0 \\ 0 & 0 & 1 & 2/7 & 0 \end{array}\right]$ $x_4 = \text{free}$ $\quad x_2 = -8/7 \cdot x_4$

$x_1 = -6/7 \cdot x_4 \quad x_3 = -2/7 \cdot x_4$

$$\vec{x} = x_4 \begin{bmatrix} -6/7 \\ -8/7 \\ -2/7 \\ 1 \end{bmatrix}$$

(2)

12. Jupyter Notebook not functioning right

16. (1) $A^{-1} = BC \quad C^{-1} = AB$ B not necessarily invertible

(2) no

17.

18. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

19. (1) symm: $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$

skew symm: $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & 8 \\ 4 & -8 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -2 & -4 & -1 \\ 2 & 0 & 8 & -3 \\ 4 & -8 & 0 & 5 \\ 1 & 3 & -5 & 0 \end{bmatrix}$

(2) main diag is all 0

$a_{ij} = 0, \; i = j$

(3) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(4) $A + A^T$ for each a_{ij} in A, corresponding value in A^T is a_{ji}. In $A + A^T$, position

$a'_{ij} = a_{ij} + a_{ji} = a'_{ji} = a_{ji} + a_{ij}$

$a'_{ij} = a_{ij} + a_{ji} \quad = a'_{ji} = a_{ji} + a_{ij}$

$$AA^T = \begin{bmatrix} \text{row}_1 A \cdot \text{col}_1 A^T & \cdots & \text{row}_1 A \cdot \text{col}_n A^T \\ \vdots & & \\ \text{row}_n A \cdot \text{col}_1 A^T & & \end{bmatrix}$$

$\text{row}_1 A = \text{col}_1 A^T$

so AA^T is symmetric

A^TA follows same logic as AA^T

$A - A^T$ has diagonal of all 0

its ijth entry is $a_{ij} - a_{ji}$

and jith entry is $a_{ji} - a_{ij}$ so it is skew symmetric

(5) $A + A^T$ symm. $A - A^T$ skew symm.

$(A + A^T) + (A - A^T) = 2A$ so any $n \times n$ matrix

can be written as the sum of a symm.

and skew symm. matrix

20. a. surjective c. surjective

b. bijective d. injective

21. $$\begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 1 & 0 & 0 \\ 0 & \frac{15}{4} & 1 & 0 \\ 0 & 0 & \frac{56}{15} & 1 \\ 0 & 0 & 0 & \frac{209}{56} \end{bmatrix} = U$$

$\frac{1}{4} \rightarrow \frac{4}{15} \rightarrow \frac{15}{56}$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ 0 & \frac{4}{15} & 1 & 0 \\ 0 & 0 & \frac{15}{16} & 1 \end{bmatrix}$$

24,