Camera Calibration and Epipolar Geometry

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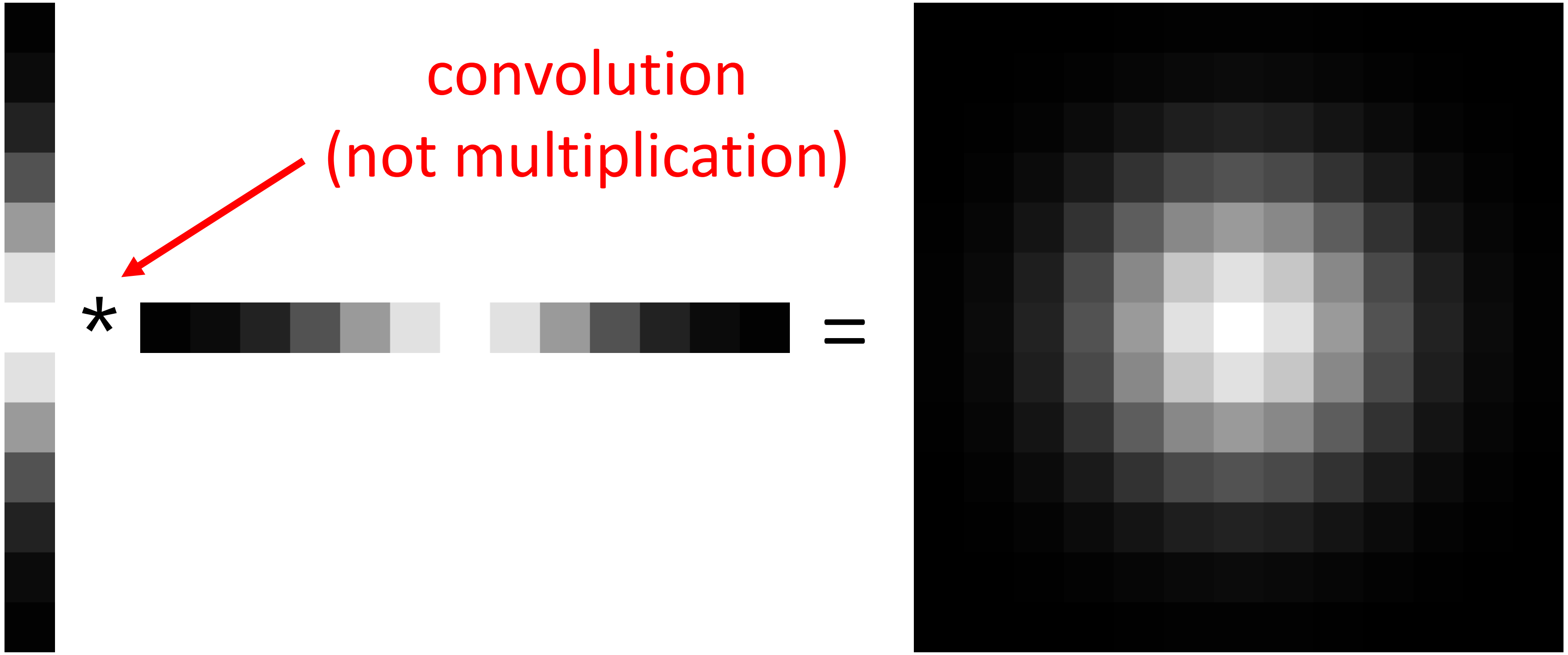
Fall 2021, Northeastern University

Separability of the Gaussian filter

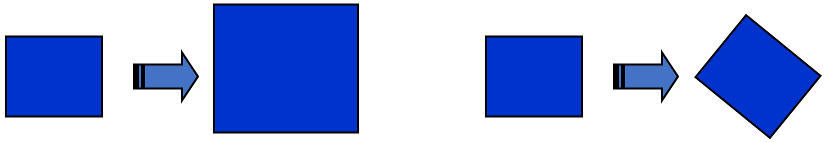


Unit Tests in PA2

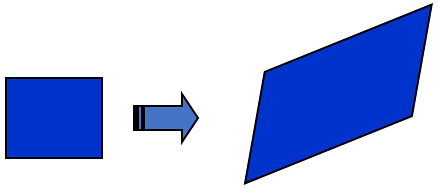
- 1.7
  - The confidence value should be ~4.8
  - The one that is used in the current unit test is local minimum
- 4.4
  - Working on it
- The deadline extended by two days until midnight of Oct 14
  - Start early, more challenging than pa1
  - Use your late days if necessary

2D transformation models

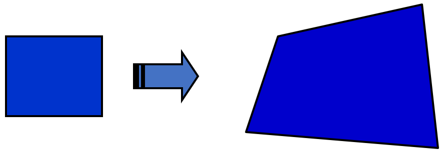
- Similarity (translation, scale, rotation)



- Affine



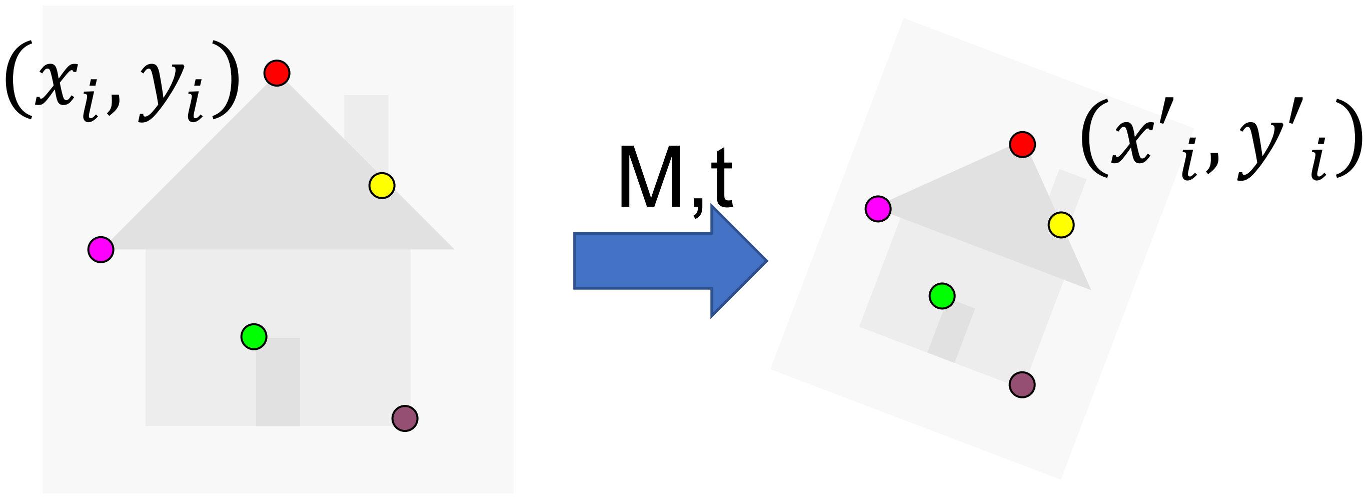
- Projective (homography)



Slide credit: S Lazebnik

Fitting Transformations

Setup: have pairs of correspondences



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \boldsymbol{M} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \boldsymbol{t}$$

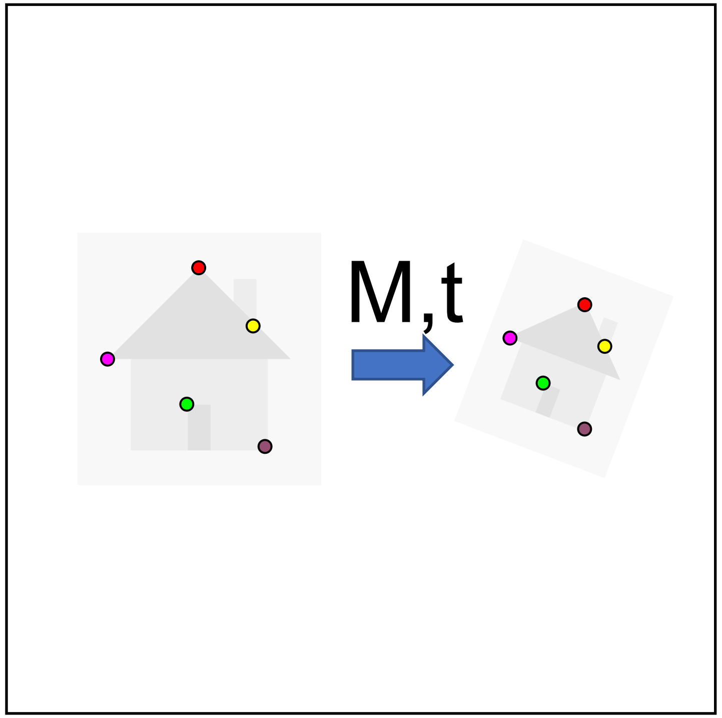
Fitting Transformation

Affine Transformation: M,t

Data: (x_i,y_i,x'_i,y'_i) for $i=1,\ldots,k$

Model:
$[x'_i,y'_i] = M[x_i,y_i]+t$

Objective function:
$||[x'_i,y'_i] - (M[x_i,y_i]+t)||^2$



Slide Credit: D Fouhey & J Johnson

Fitting Transformations

Given correspondences: $[x'_i, y'_i] \leftrightarrow [x_i, y_i]$

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Set up two equations per point

$$\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix} = \begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_x \\ t_y \end{bmatrix}$$

Fitting Transformations

$$2k\left\{\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix}\right. = \overbrace{\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix}}^{6} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_x \\ t_y \end{bmatrix}$$

2 equations per point, 6 unknowns

How many points do we need to properly constrain the problem?

Fitting Transformations

$$2k\;\underbrace{\begin{bmatrix}\vdots\\ x_i'\\ y_i'\\ \vdots\end{bmatrix}}_{b} = \underbrace{\overbrace{\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0\\ 0 & 0 & x_i & y_i & 0 & 1\\ & & \cdots & & & \end{bmatrix}}^{6}}_{A}\;\underbrace{\begin{bmatrix} m_1\\ m_2\\ m_3\\ m_4\\ t_x\\ t_y\end{bmatrix}}_{x}$$

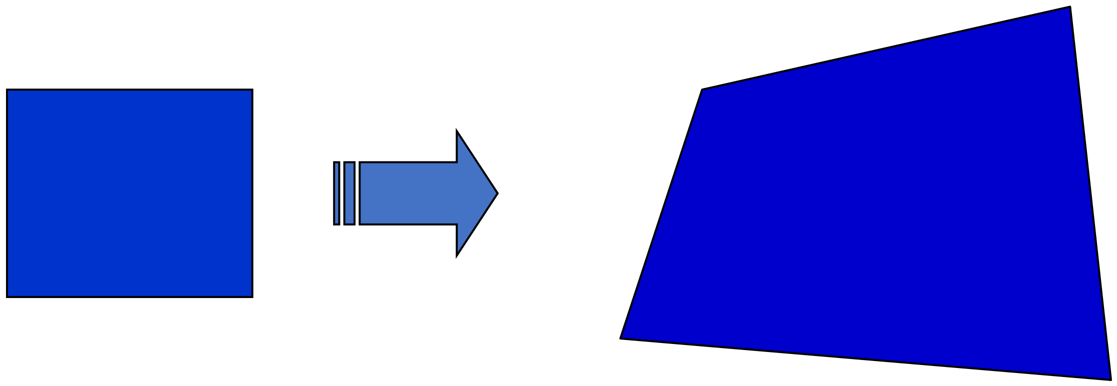
Want: b = Ax (x contains all parameters)

Overconstrained, so solve $\arg\min ||\boldsymbol{Ax} - \boldsymbol{b}||$

How?

Fitting a plane projective transformation

- **Homography:** plane projective transformation (transformation taking a quad to another arbitrary quad)



Slide credit: S Lazebnik

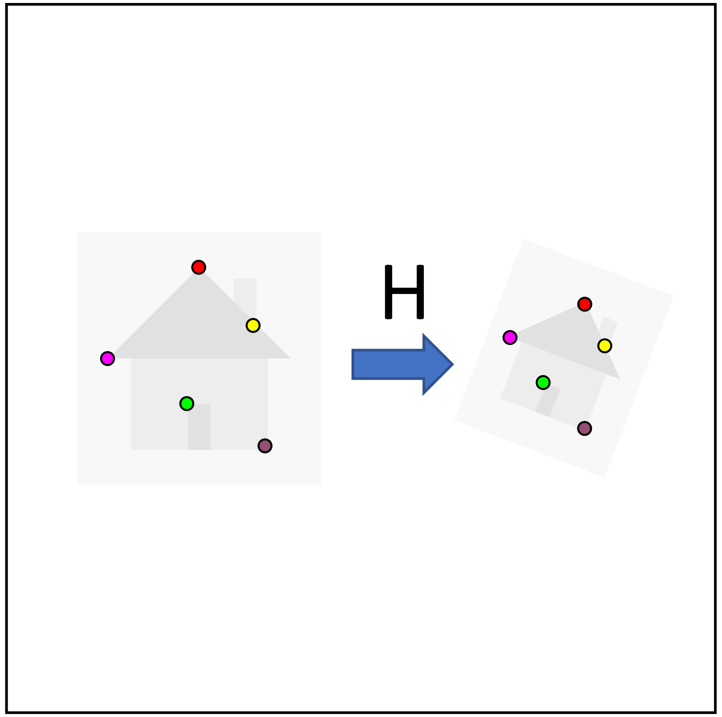
Fitting Transformation

Homography: H

Data: (x_i, y_i, x'_i, y'_i) for $i=1,\ldots,k$

Model:
$[x'_i, y'_i, 1] \equiv \mathbf{H}[x_i, y_i, 1]$

Objective function:
It's complicated



Slide Credit: D Fouhey & J Johnson

Want: $\begin{bmatrix} x_i' \\ y_i' \\ w_i' \end{bmatrix} \equiv \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ w_i \end{bmatrix}$

$\boldsymbol{p}_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ w_i \end{bmatrix} \equiv \boldsymbol{H}\boldsymbol{p}_i \equiv \begin{bmatrix} \boldsymbol{h}_1^T \\ \boldsymbol{h}_2^T \\ \boldsymbol{h}_3^T \end{bmatrix} \boldsymbol{p}_i \equiv \begin{bmatrix} \boldsymbol{h}_1^T\boldsymbol{p}_i \\ \boldsymbol{h}_2^T\boldsymbol{p}_i \\ \boldsymbol{h}_3^T\boldsymbol{p}_i \end{bmatrix}$$

Recall: $\boldsymbol{a} \equiv \boldsymbol{b} \rightarrow \boldsymbol{a} = \lambda \boldsymbol{b}$ **Fun Fact:** $\rightarrow \boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$

In the end want: $\begin{bmatrix} x_i' \\ y_i' \\ w_i' \end{bmatrix} \times \begin{bmatrix} \boldsymbol{h}_1^T\boldsymbol{p}_i \\ \boldsymbol{h}_2^T\boldsymbol{p}_i \\ \boldsymbol{h}_3^T\boldsymbol{p}_i \end{bmatrix} = \boldsymbol{0}$

Why?
Cross products have explicit forms

Fitting Transformation

$$\mathbf{p}_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\text{k points} \rightarrow \quad \underbrace{\overbrace{\begin{bmatrix} \mathbf{0}^T & -w'_1\mathbf{p}_1^T & y'_1\mathbf{p}_1^T \\ w'_1\mathbf{p}_1^T & \mathbf{0}^T & -x'_1\mathbf{p}_1^T \\ & \vdots & \\ \mathbf{0}^T & -w'_n\mathbf{p}_n^T & y'_n\mathbf{p}_n^T \\ w'_n\mathbf{p}_n^T & \mathbf{0}^T & -x'_n\mathbf{p}_n^T \end{bmatrix}}^{9}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix}}_{\boldsymbol{h}} = \mathbf{0}$$

2k

$$\boldsymbol{A}\boldsymbol{h} = \mathbf{0}$$

If h is up to scale, what do we use from last time?

$$h^* = \arg\min_{\|h\|=1} \|Ah\|^2 \rightarrow$$ Eigenvector of A^TA with smallest eigenvalue

Slide Credit: D Fouhey & J Johnson

Fitting Transformation – In Practice

$$\boldsymbol{p}_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\begin{array}{cc} & 9 \\ \text{N points} \rightarrow 2n & \begin{bmatrix} \mathbf{0}^T & -\boldsymbol{p}_1^T & y_1'\boldsymbol{p}_1^T \\ \boldsymbol{p}_1^T & \mathbf{0}^T & -x_1'\boldsymbol{p}_1^T \\ & \vdots & \\ \mathbf{0}^T & -\boldsymbol{p}_n^T & y_n'\boldsymbol{p}_n^T \\ \boldsymbol{p}_n^T & \mathbf{0}^T & -x_n'\boldsymbol{p}_n^T \end{bmatrix} \begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \\ \boldsymbol{h}_3 \end{bmatrix} = \mathbf{0} \end{array}$$

$$\boldsymbol{A}\boldsymbol{h} = \mathbf{0}$$

What is the minimum number of matches needed for a solution?

Four: A has 8 degrees of freedom (9 parameters, but scale is arbitrary), one match gives us two linearly independent equations

Slide adapted from: D Fouhey & J Johnson

Perspective & 3D Geometry

	Perspective and 3D Geometry	
Week 4	Thur, Oct 7	Camera Calibration, Epipolar Geometry
	Mon, Oct 11	No class, Indigenous Peoples Day
Week 5	Thu, Oct 14	Stereo Vision
	Mon, Oct 18	Optical Flow
Week 6	Thu, Oct 21	Structure from Motion

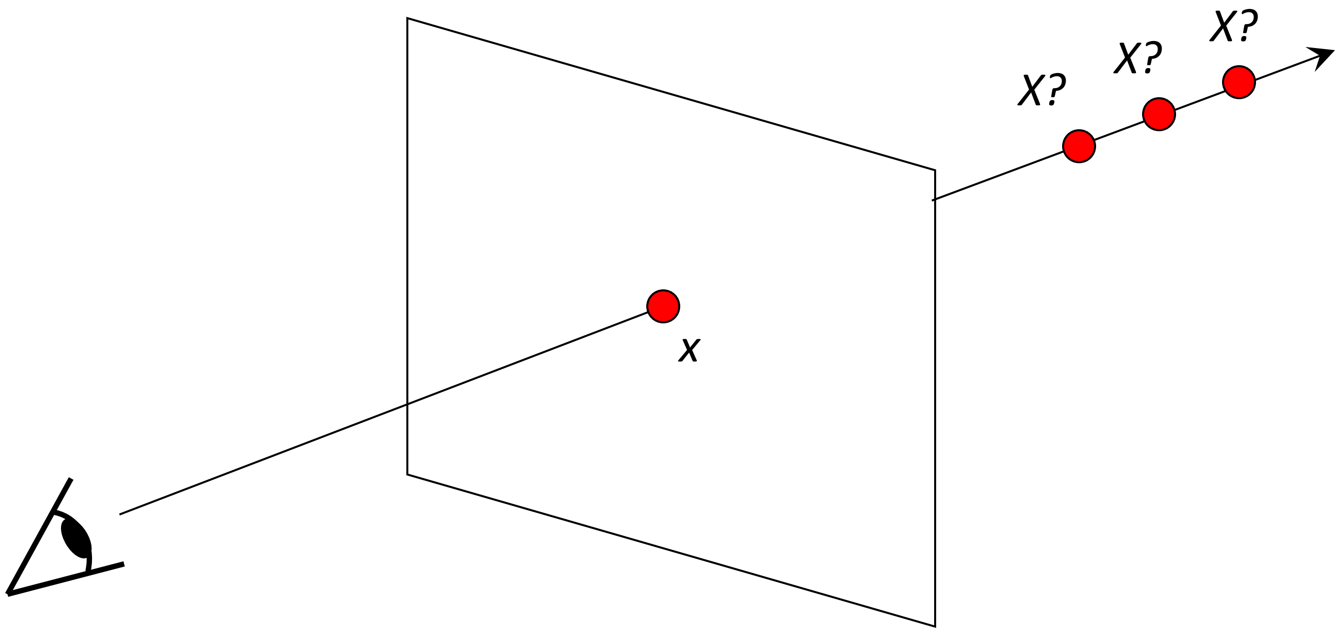
Our goal: Recovery of 3D structure

J. Vermeer, *Music Lesson*, 1662

A. Criminisi, M. Kemp, and A. Zisserman, [Bringing Pictorial Space to Life: computer techniques for the analysis of paintings](), *Proc. Computers and the History of Art*, 2002

Slide credit: S Lazebnik

Single-view ambiguity



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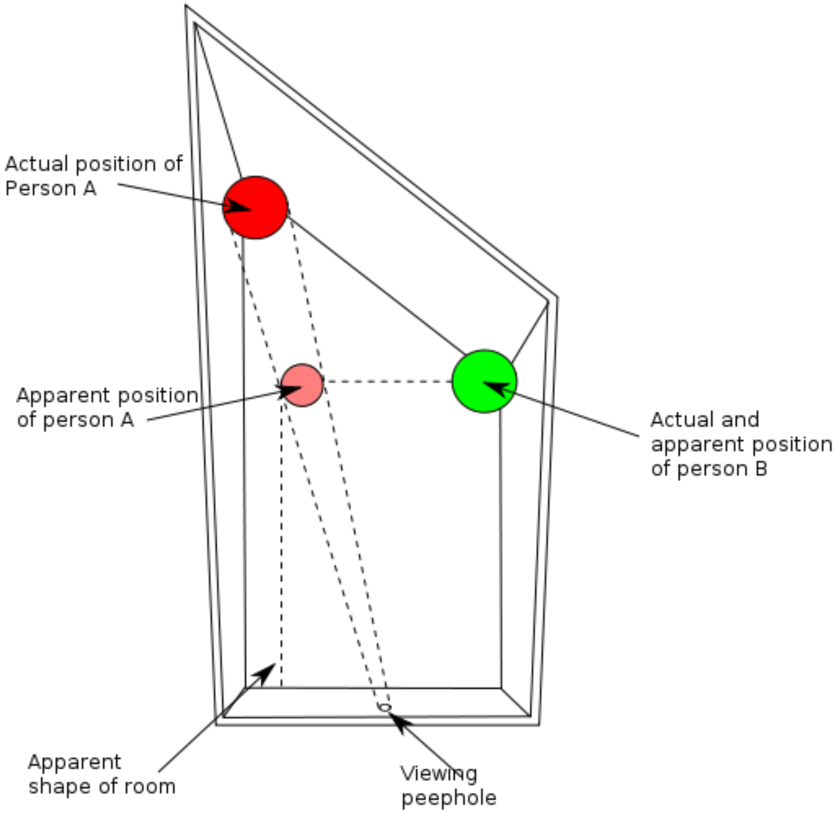
Things aren't always as they appear…



http://en.wikipedia.org/wiki/Ames_room

Slide credit: S Lazebnik

Single-view ambiguity

Slide credit: S Lazebnik

Single-view ambiguity

[Rashad Alakbarov shadow sculptures]

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Anamorphic perspective

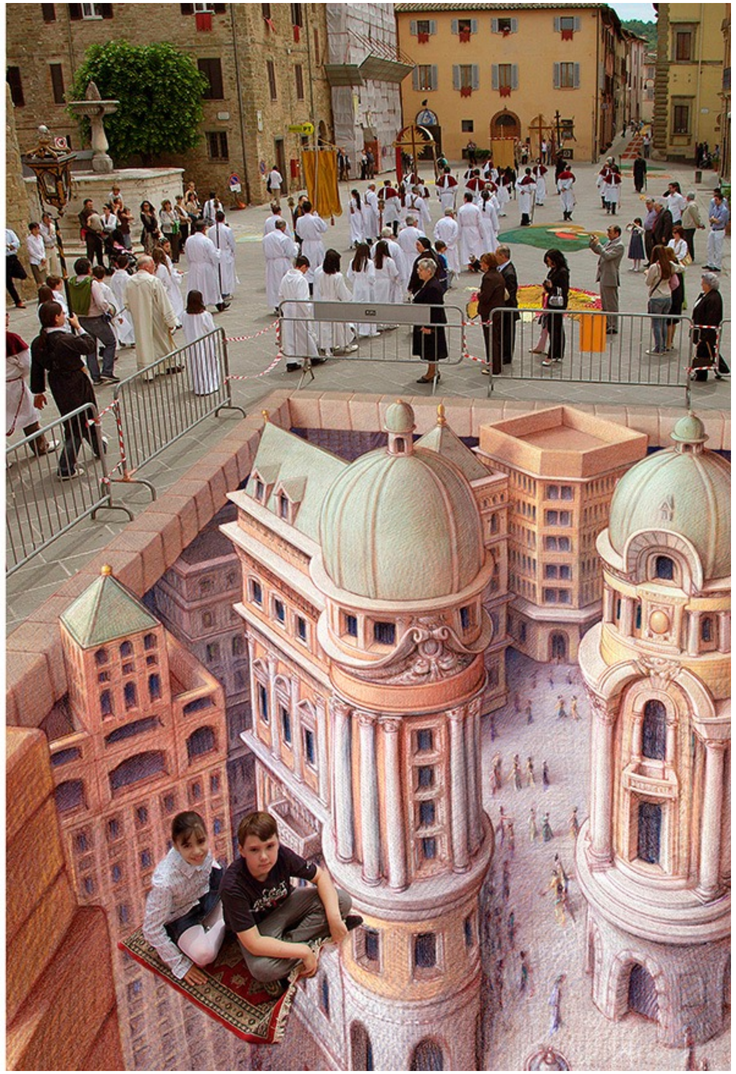

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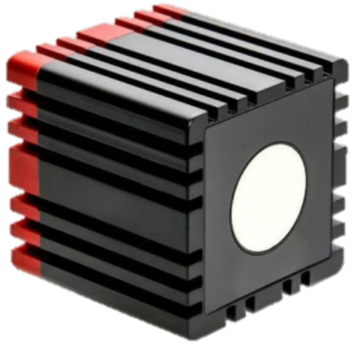
Image source

Resolving Single-view Ambiguity



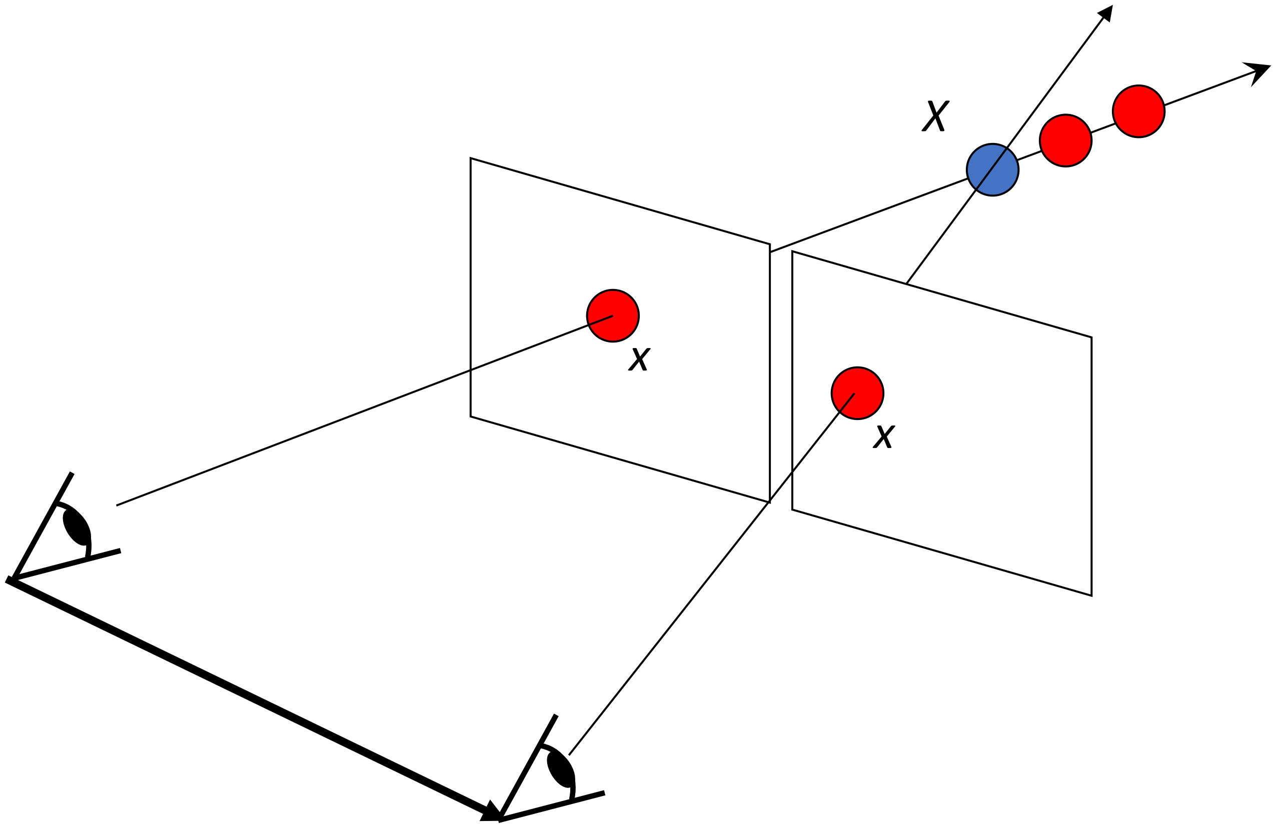
- Shoot light (lasers etc.) out of your eyes!
- Con: not so biologically plausible, dangerous?

Slide credit: D Fouhey & J Johnson

Resolving Single-view Ambiguity

- Shoot light (lasers etc.) out of your eyes!
- Con: not so biologically plausible, dangerous?

Resolving Single-view Ambiguity

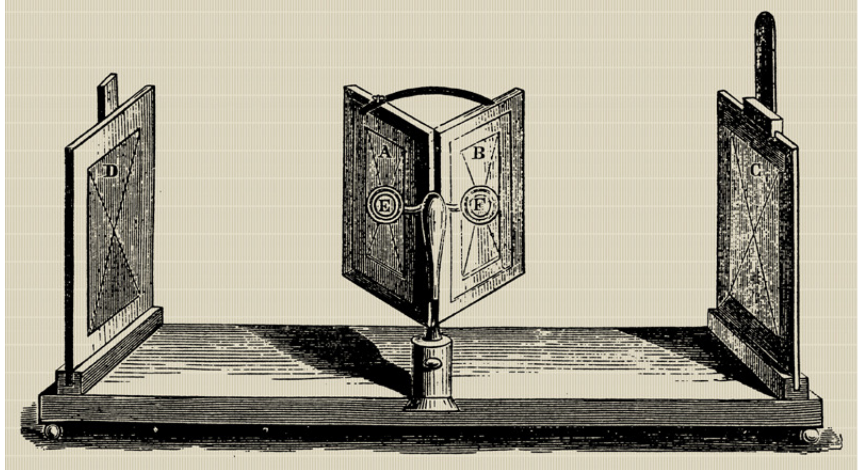


- Stereo: given 2 calibrated cameras in different views and correspondences, can solve for X

Slide credit: D Fouhey & J Johnson   Original diagram credit: S. Lazebnik

Stereo photography and stereo viewers

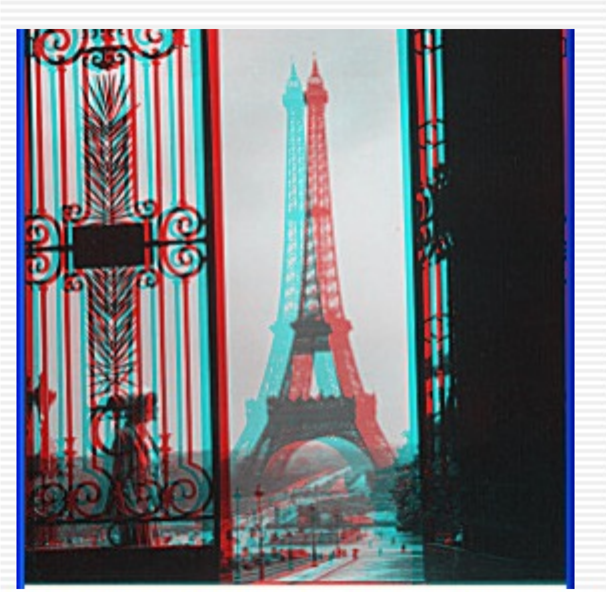
Take two pictures of the same subject from two slightly different viewpoints and display so that each eye sees only one of the images.



Invented by Sir Charles Wheatstone, 1838

Image from fisher-price.com

Slide credit: J. Hays



© Copyright 2001 Johnson-Shaw Stereoscopic Museum

http://www.johnsonshawmuseum.org

Slide credit: J. Hays

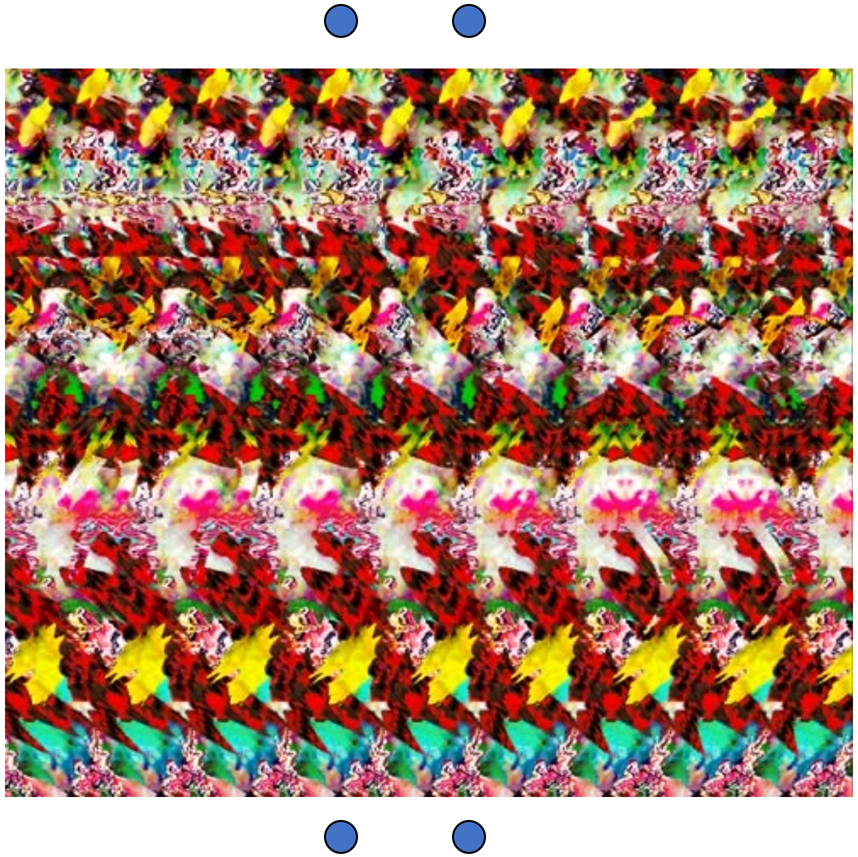
http://www.well.com/~jimg/stereo/stereo_list.html

Slide credit: J. Hays

http://www.well.com/~jimg/stereo/stereo_list.html

Slide credit: J. Hays

Autostereograms



Exploit disparity as depth cue using single image.

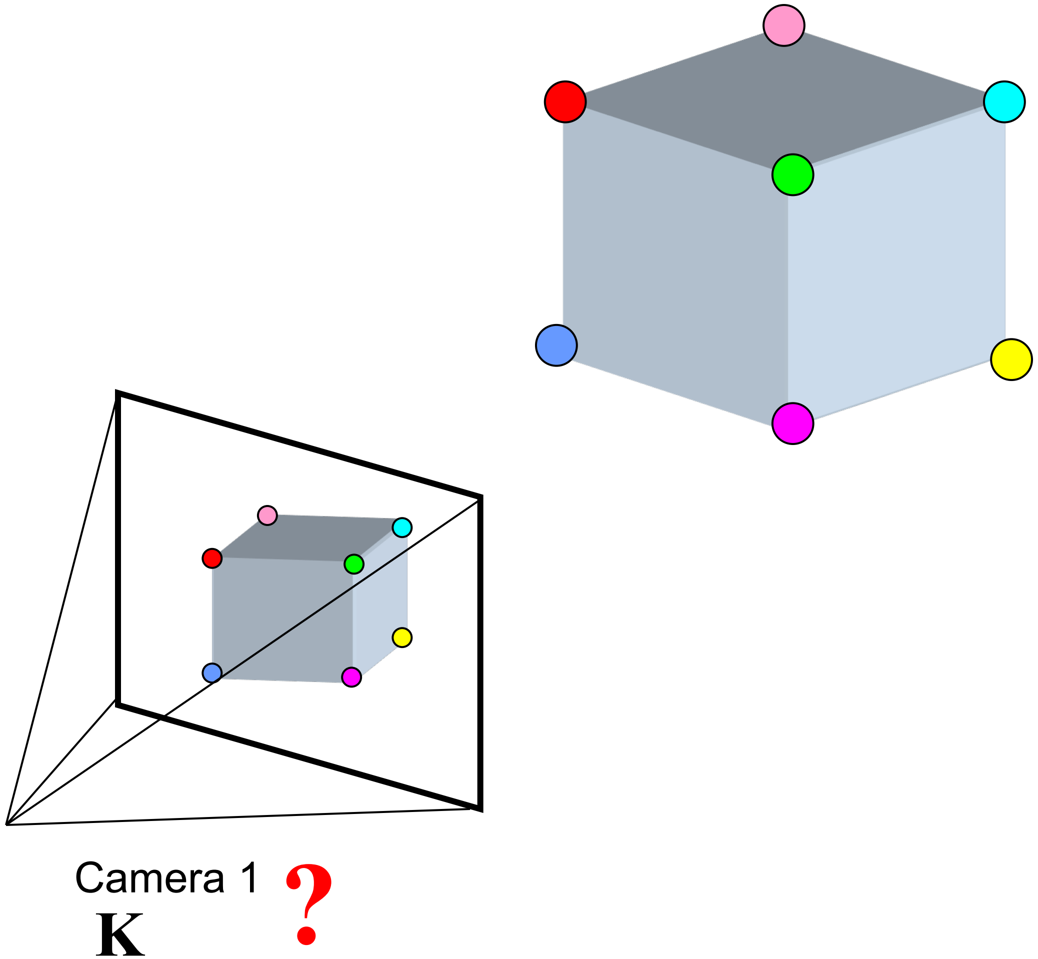
(Single image random dot stereogram, Single image stereogram)

Slide credit: J. Hays, Images from magiceye.com

Autostereograms

Slide credit: J. Hays, Images from magiceye.com

Multi-view geometry problems



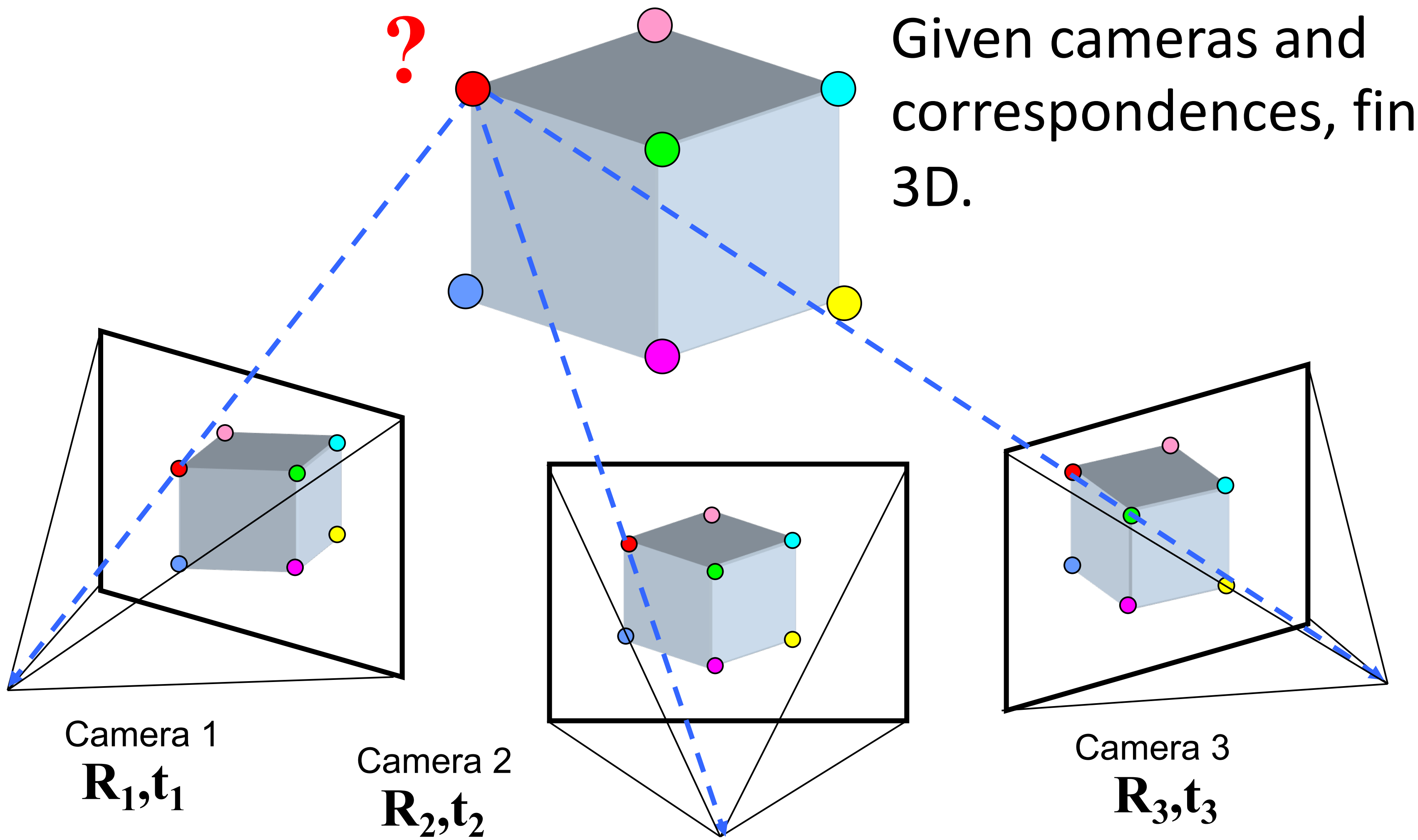
Calibration:
Figure out intrinsics of camera (K).

We need camera intrinsics / K in order to figure out where the rays are

Slide credit:
Noah Snavely

Multi-view geometry problems

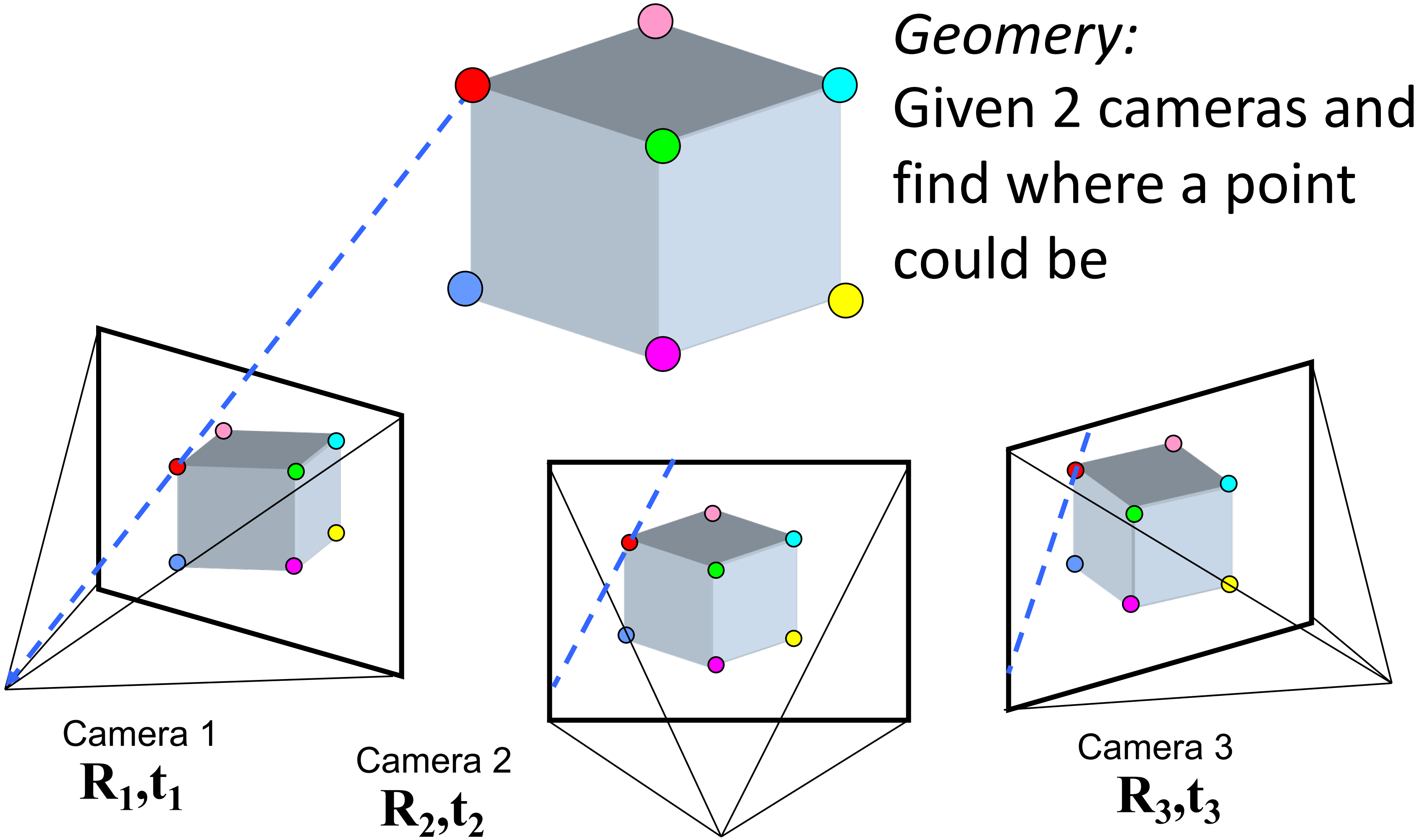
Recovering structure:
Given cameras and correspondences, find 3D.



Slide credit:
Noah Snavely

Multi-view geometry problems

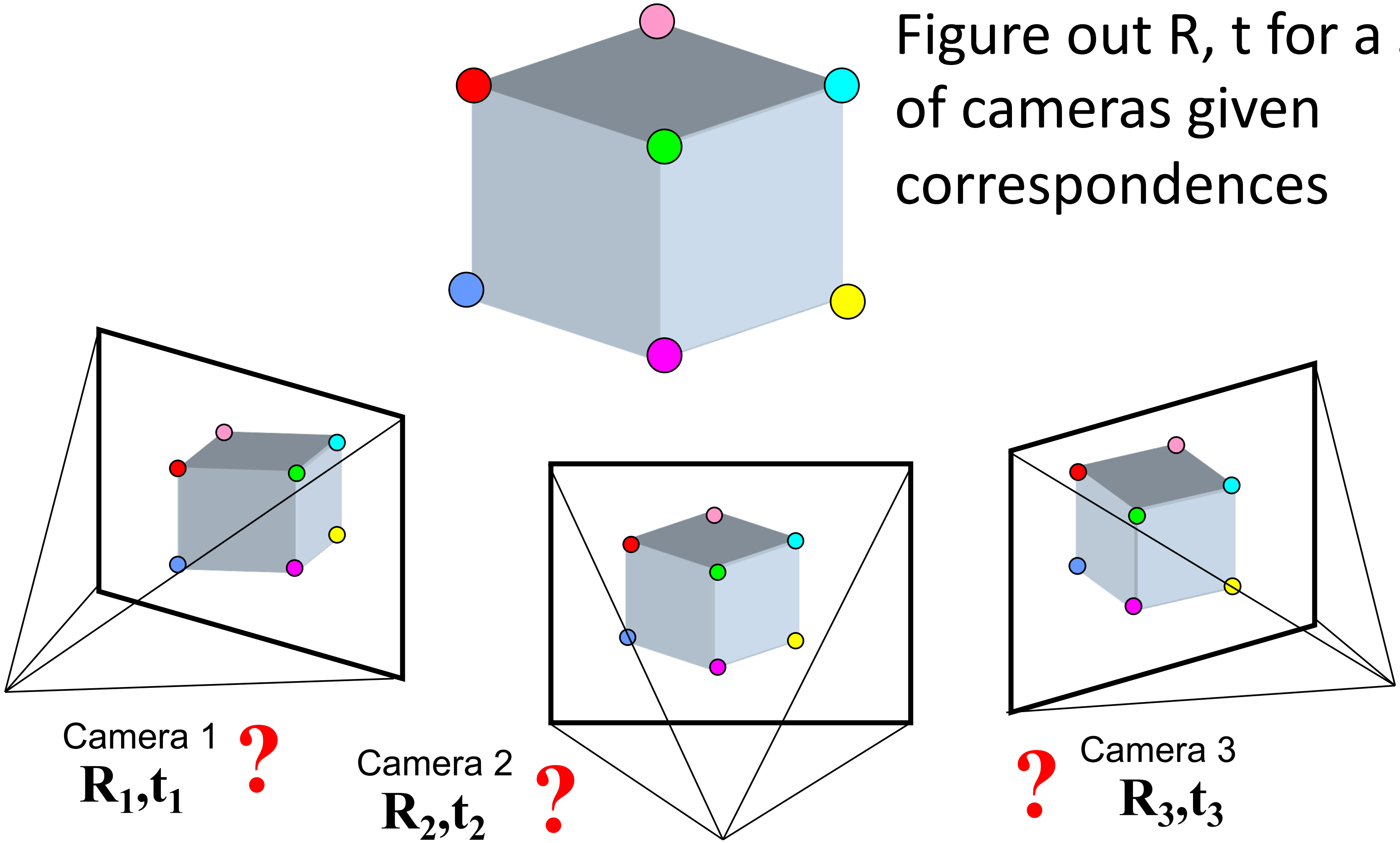
Stereo/Epipolar Geomery:
Given 2 cameras and find where a point could be



Slide credit: Noah Snavely

Multi-view geometry problems

Motion:
Figure out R, t for a set of cameras given correspondences



Slide credit:
Noah Snavely

Outline

- (Today) Calibration:
  - Getting intrinsic matrix/K
- (Today) Epipolar geometry:
  - 2 pictures -> depth map
- Stereo
  - Rectified 2 pictures -> depth map
- Optical flow
  - Finding correspondences in 2 pictures (similar to stereo)
- Structure from motion (SfM):
  - 2+ pictures -> cameras, pointcloud

Slide inspired by: D Fouhey & J Johnson

Our goal: Recovery of 3D structure

- When certain assumptions hold, we can recover structure from a single view

- In general, we need *multi-view geometry*

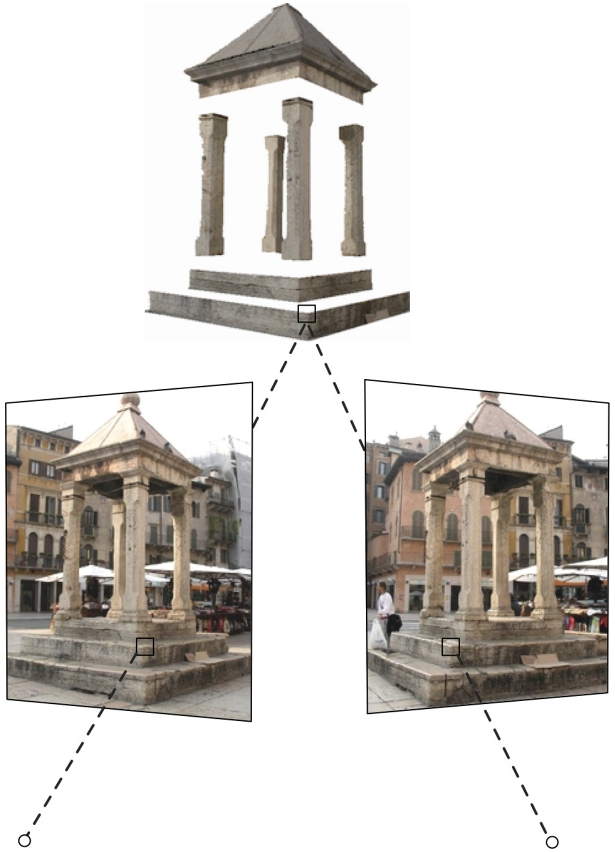
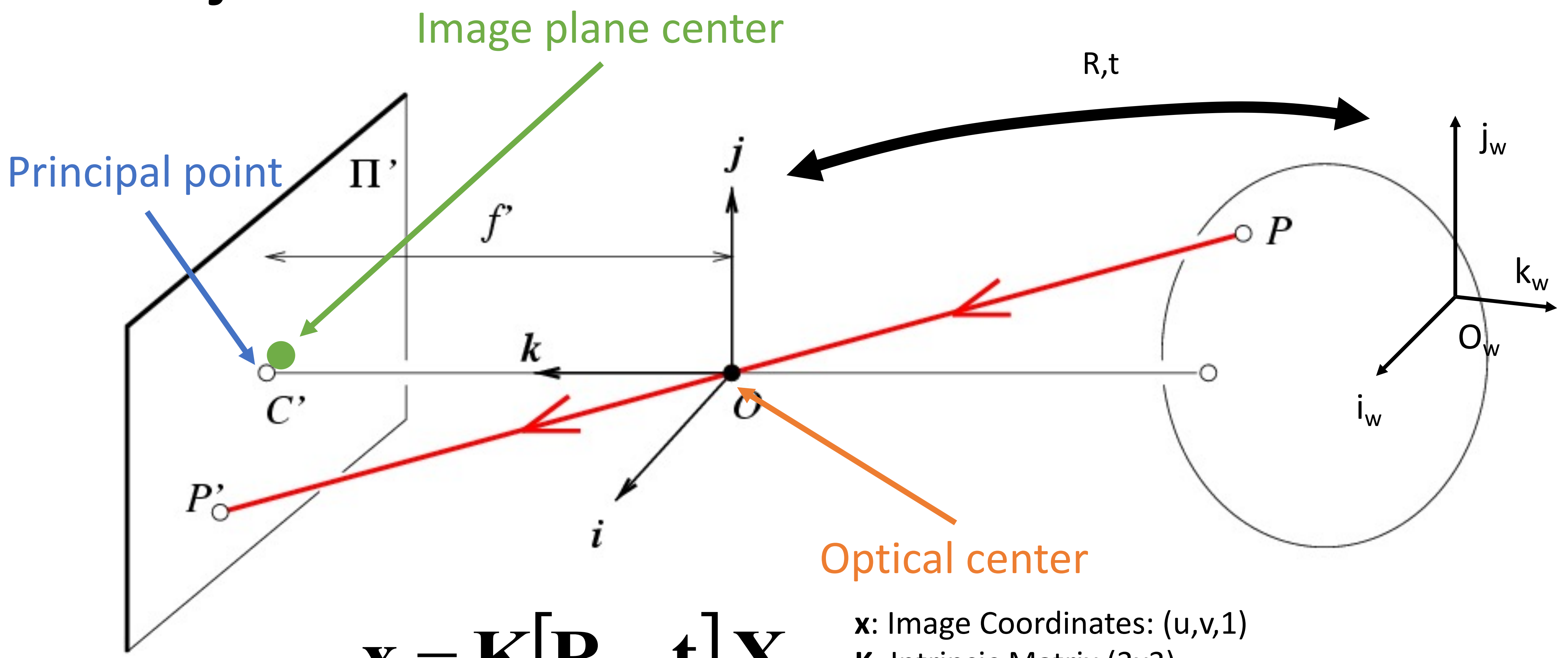


Image source

- But first, we need to understand the geometry of a single camera…

Slide credit: S Lazebnik

Projection matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Typical Perspective Model

principal point (image coords
of camera origin on retina)
Just moves camera origin

focal length

rotation translation

$$\boldsymbol{p} \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{R}_{3x3} & \boldsymbol{t}_{3x1} \end{bmatrix} \boldsymbol{X}_{4x1}$$

2D Projection of X

3D point

Slide credit: D Fouhey & J Johnson

Camera Calibration

$$\boldsymbol{p} \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{R}_{3x3} & \boldsymbol{t}_{3x1} \end{bmatrix} \boldsymbol{X}_{4x1}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \boldsymbol{M}_{3x4} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

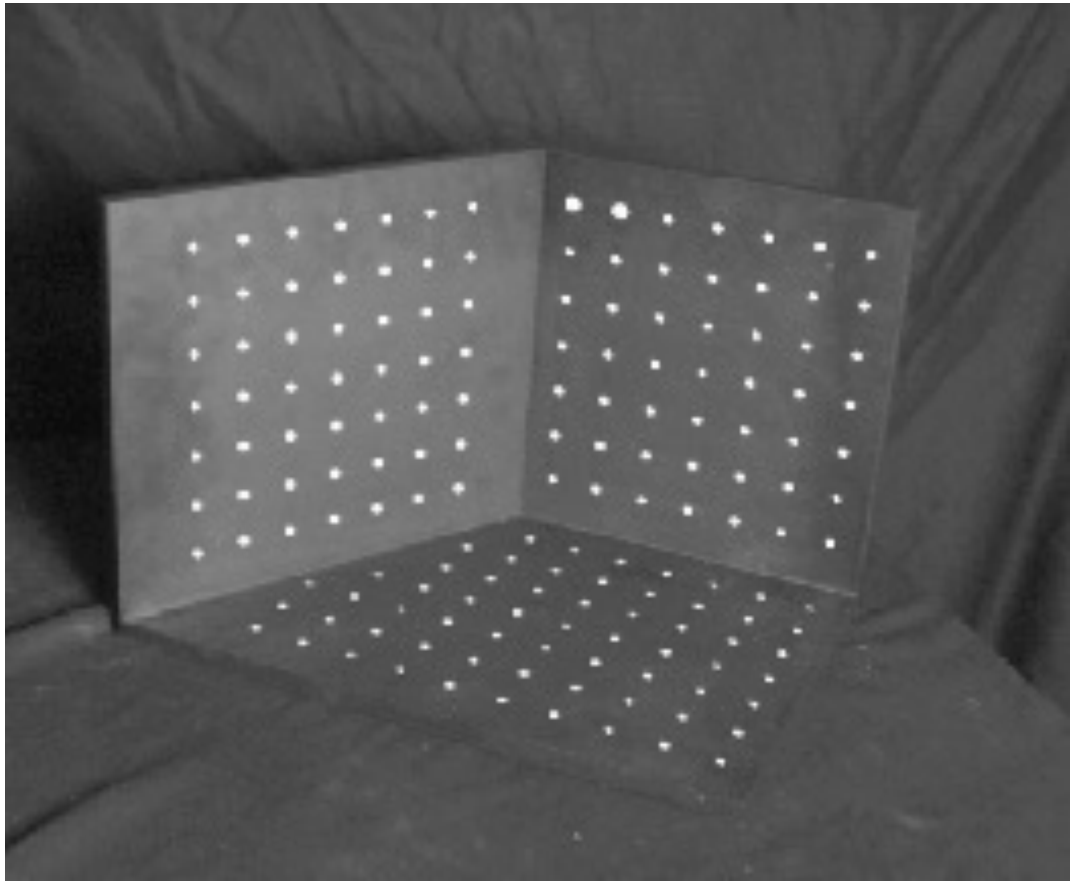
Pairs of [X,Y,Z] and [u,v] → eqns to constrain **M**

How do I get [X,Y,Z], [u,v]?

Slide credit: D Fouhey & J Johnson

Camera Calibration

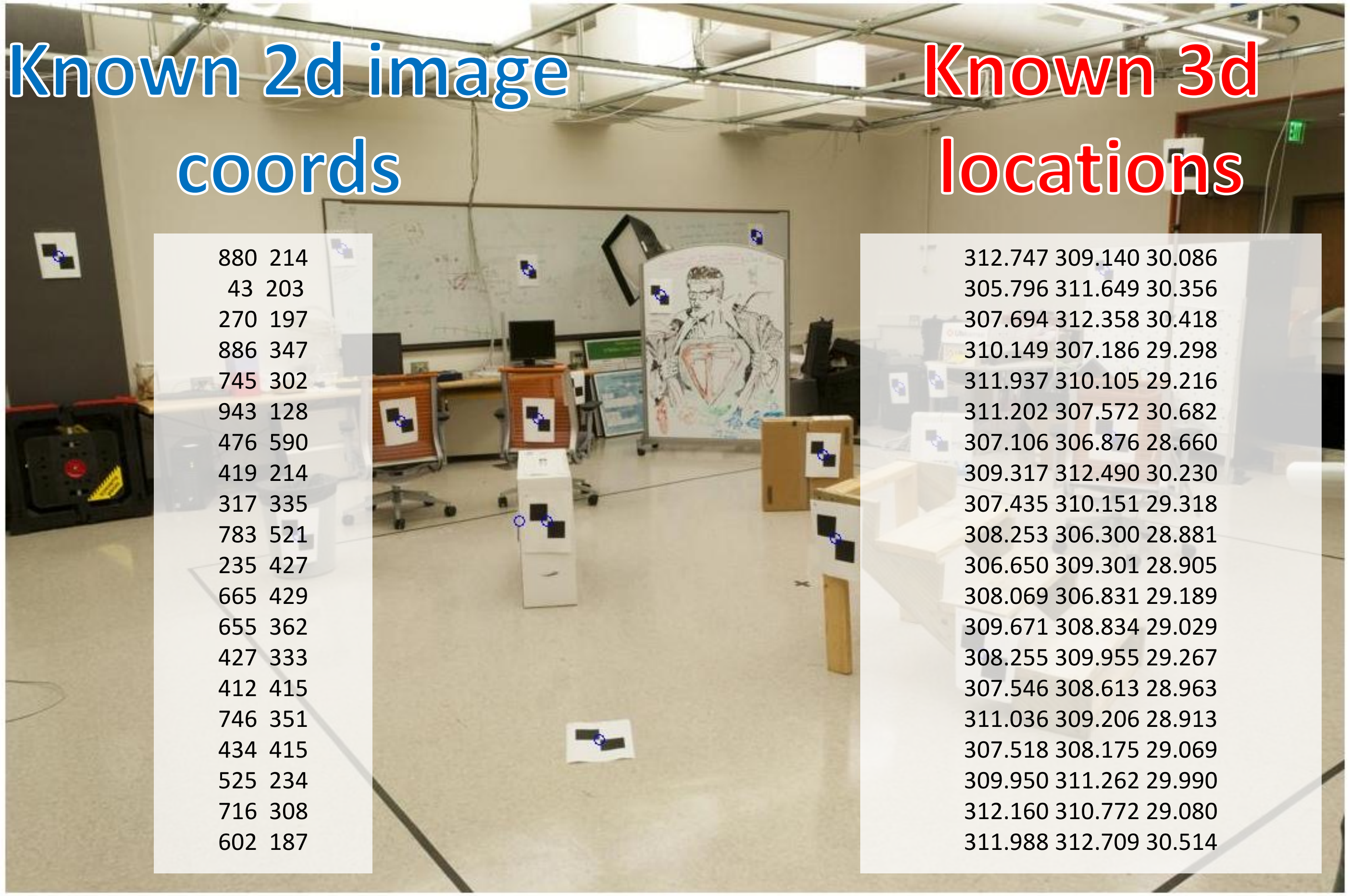
A funny object with multiple planes.



Slide credit: D Fouhey & J Johnson

Camera Calibration Targets

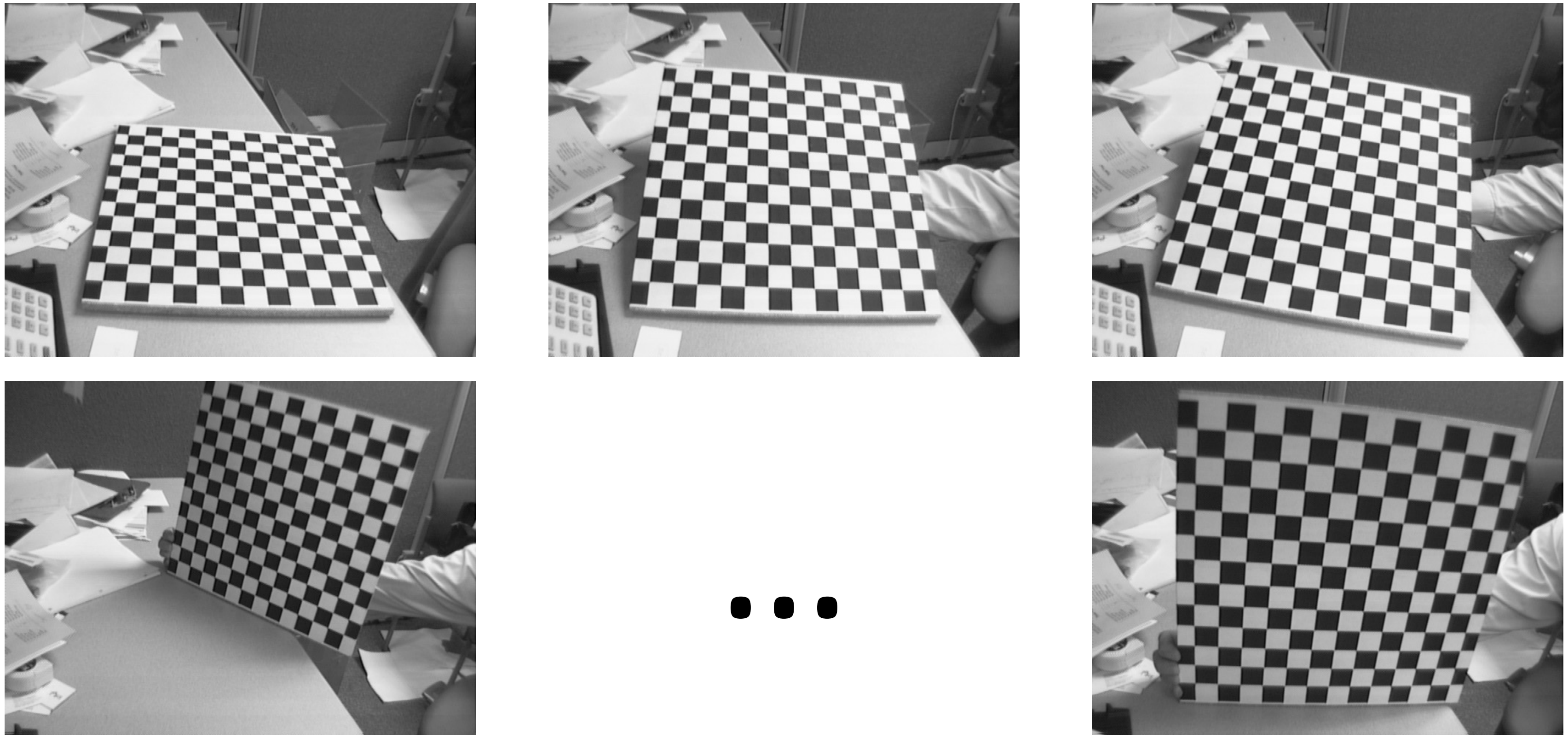
Using a tape measure



Slide credit: D Fouhey & J Johnson Image credit: J. Hays

Camera Calibration Targets

A set of views of a plane (not covered today)



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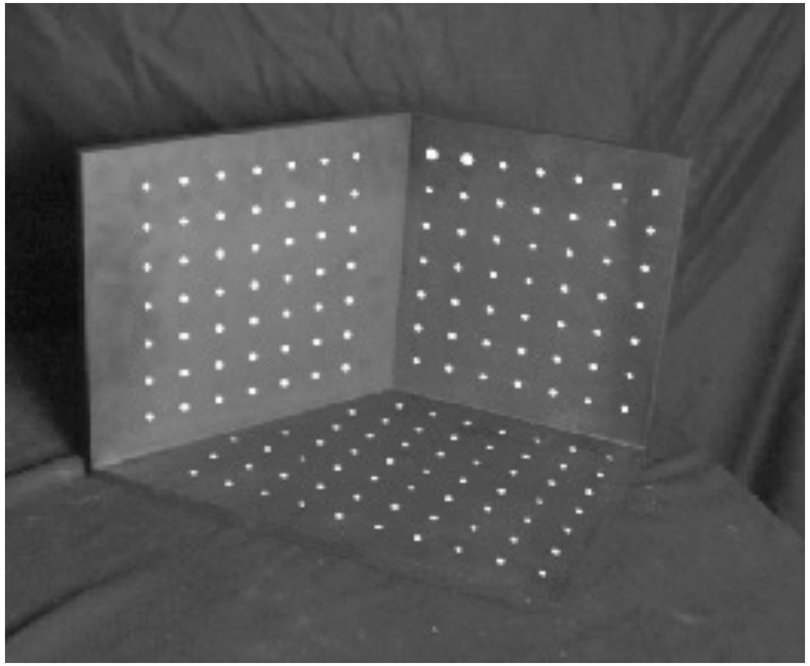
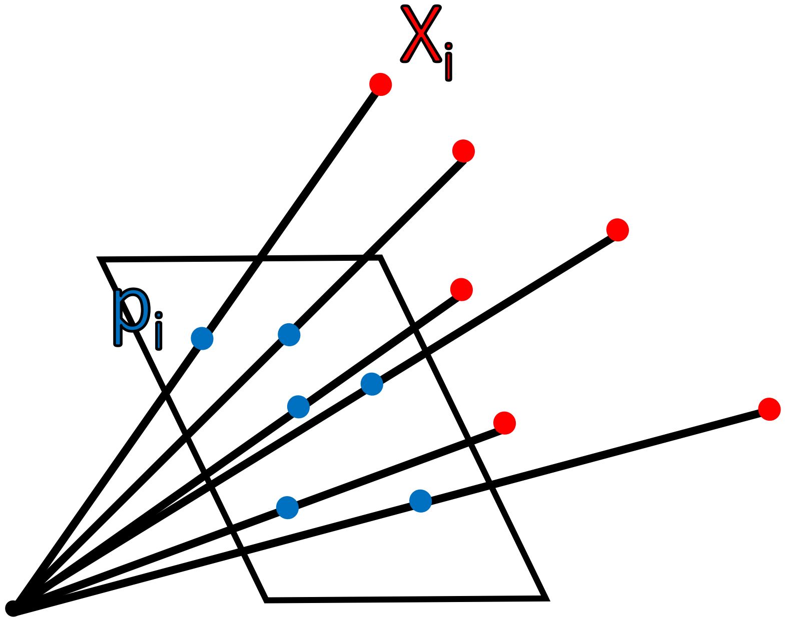
Camera calibration

- Given *n* points with known 3D coordinates X_i and known image projections $\mathbf{p}_i$, estimate the camera parameters





Camera Calibration: Linear Method

$$\boldsymbol{p_i} \equiv \boldsymbol{MX_i}$$

Remember (from geometry): this implies **MX_i &** **p_i** are proportional/scaled copies of each other

$$\boldsymbol{p_i} = \lambda \boldsymbol{MX_i}, \lambda \neq 0$$

Remember (from homography fitting): this implies their cross product is **0**

$$\boldsymbol{p_i} \times \boldsymbol{MX_i} = \mathbf{0}$$

Slide credit: D Fouhey & J Johnson

Camera Calibration: Linear Method

$$\boldsymbol{p}_i \times \boldsymbol{M}\boldsymbol{X}_i = \boldsymbol{0}$$

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \times \begin{bmatrix} \boldsymbol{M}_1\boldsymbol{X}_i \\ \boldsymbol{M}_2\boldsymbol{X}_i \\ \boldsymbol{M}_3\boldsymbol{X}_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

…Some tedious math occurs…

(see Homography deriviation)

$$\begin{bmatrix} \boldsymbol{0}^T & -\boldsymbol{X}_i^T & v_i\boldsymbol{X}_i^T \\ \boldsymbol{X}_i^T & \boldsymbol{0}^T & -u_i\boldsymbol{X}_i^T \\ -v_i\boldsymbol{X}_i^T & u_i\boldsymbol{X}_i^T & \boldsymbol{0}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{M}_1^T \\ \boldsymbol{M}_2^T \\ \boldsymbol{M}_3^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Slide credit: D Fouhey & J Johnson

Camera Calibration: Linear Method

$$\begin{bmatrix} \mathbf{0}^T & -\boldsymbol{X}_i^T & v_i \boldsymbol{X}_i^T \\ \boldsymbol{X}_i^T & \mathbf{0}^T & -u_i \boldsymbol{X}_i^T \\ -v_i \boldsymbol{X}_i^T & u_i \boldsymbol{X}_i^T & \mathbf{0}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{M}_1^T \\ \boldsymbol{M}_2^T \\ \boldsymbol{M}_3^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

How many linearly independent equations?

2

How many equations per [u,v] + [X,Y,Z] pair?

2

If M is 3x4, how many degrees of freedom?

11

Slide credit: D Fouhey & J Johnson

Camera Calibration: Linear Method

$$\begin{bmatrix} \mathbf{0}^T & \boldsymbol{X}_1^T & -v_1\boldsymbol{X}_1^T \\ \boldsymbol{X}_1^T & \mathbf{0}^T & -u_1\boldsymbol{X}_1^T \\ \cdots & \cdots & \cdots \\ \mathbf{0}^T & \boldsymbol{X}_n^T & -v_1\boldsymbol{X}_n^T \\ \boldsymbol{X}_n^T & \mathbf{0}^T & -u_n\boldsymbol{X}_n^T \end{bmatrix} \begin{bmatrix} \boldsymbol{M}_1^T \\ \boldsymbol{M}_2^T \\ \boldsymbol{M}_3^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

How do we solve problems of the form

$\arg\min \|\boldsymbol{A}\boldsymbol{n}\|_2^2, \|\boldsymbol{n}\|_2^2 = 1$ **?**

Eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue

Camera calibration: Linear method

$$\begin{bmatrix} \mathbf{0}^T & \boldsymbol{X}_1^T & -v_1\boldsymbol{X}_1^T \\ \boldsymbol{X}_1^T & \mathbf{0}^T & -u_1\boldsymbol{X}_1^T \\ \cdots & \cdots & \cdots \\ \mathbf{0}^T & \boldsymbol{X}_n^T & -v_1\boldsymbol{X}_n^T \\ \boldsymbol{X}_n^T & \mathbf{0}^T & -u_n\boldsymbol{X}_n^T \end{bmatrix} \begin{bmatrix} \boldsymbol{M}_1^T \\ \boldsymbol{M}_2^T \\ \boldsymbol{M}_3^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- Note: for coplanar points that satisfy $\mathbf{\Pi}^T\mathbf{X}=0$, we will get degenerate solutions $(\mathbf{\Pi},\mathbf{0},\mathbf{0})$, $(\mathbf{0},\mathbf{\Pi},\mathbf{0})$, or $(\mathbf{0},\mathbf{0},\mathbf{\Pi})$

Slide credit: S. Lazebnik

Camera calibration: Linear vs. nonlinear

- Linear calibration is easy to formulate and solve, but it doesn't directly tell us the camera parameters

$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \text{vs.} \quad \mathbf{x} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X}$$

- In practice, non-linear methods are preferred
  - Write down objective function in terms of intrinsic and extrinsic parameters
  - Define error as sum of squared distances between measured 2D points and estimated projections of 3D points
  - Minimize error using Newton's method or other non-linear optimization
  - Can model radial distortion and impose constraints such as known focal length and orthogonality

Slide credit: S. Lazebnik

What Does This Get You?

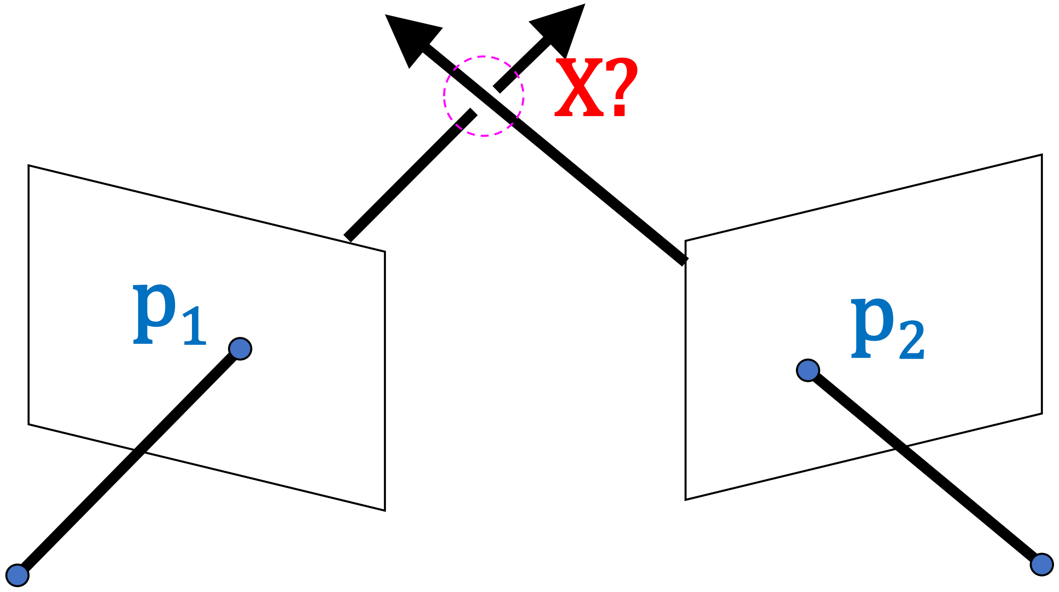
Given projection $\mathbf{p}_i$ of unknown 3D point $\mathbf{X}$ in two or more images (with known cameras $\mathbf{M}_i$), find $\mathbf{X}$

Slide credit: D Fouhey & J Johnson

Triangulation

Given projection $\mathbf{p}_i$ of unknown 3D point $\mathbf{X}$ in two or more images (with known cameras $\mathbf{M}_i$), find $\mathbf{X}$

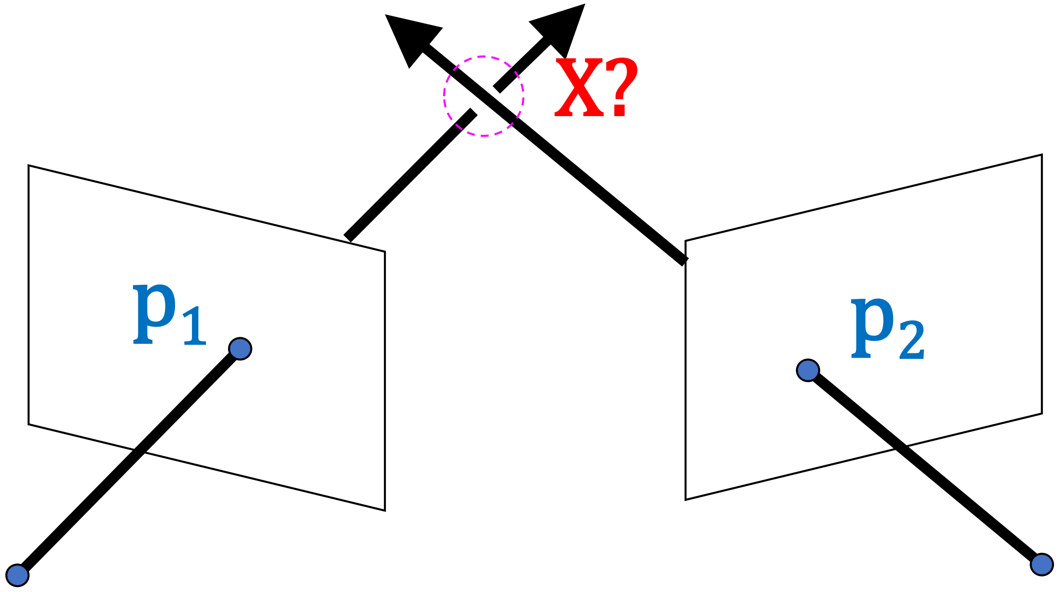
Why is the calibration here important?



Slide credit: D Fouhey & J Johnson

Triangulation

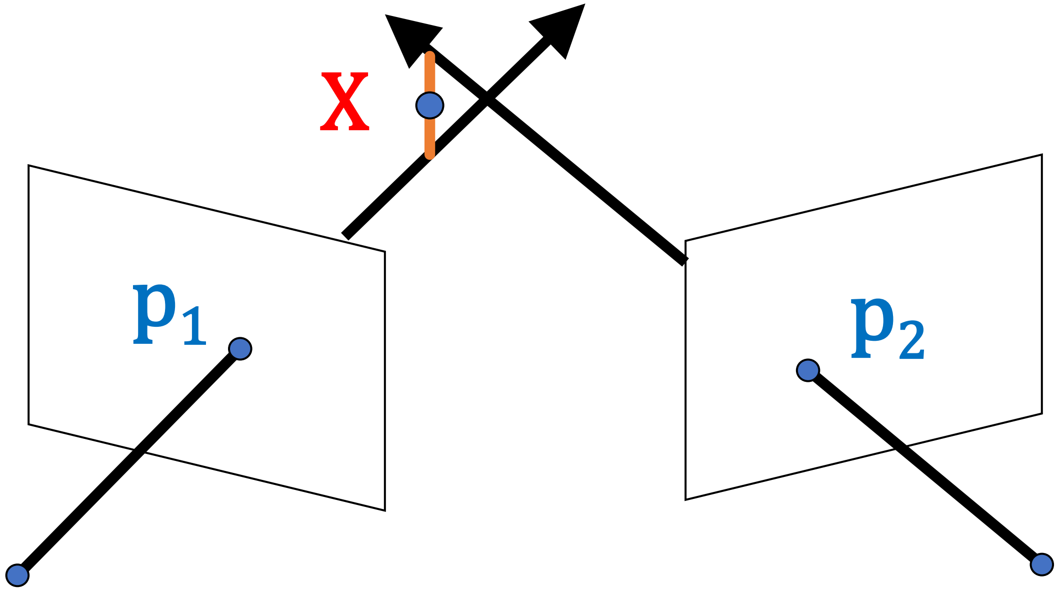
Rays in principle should intersect, but in practice usually don't exactly due to noise, numerical errors.



Slide credit: D Fouhey & J Johnson

Triangulation – Geometry

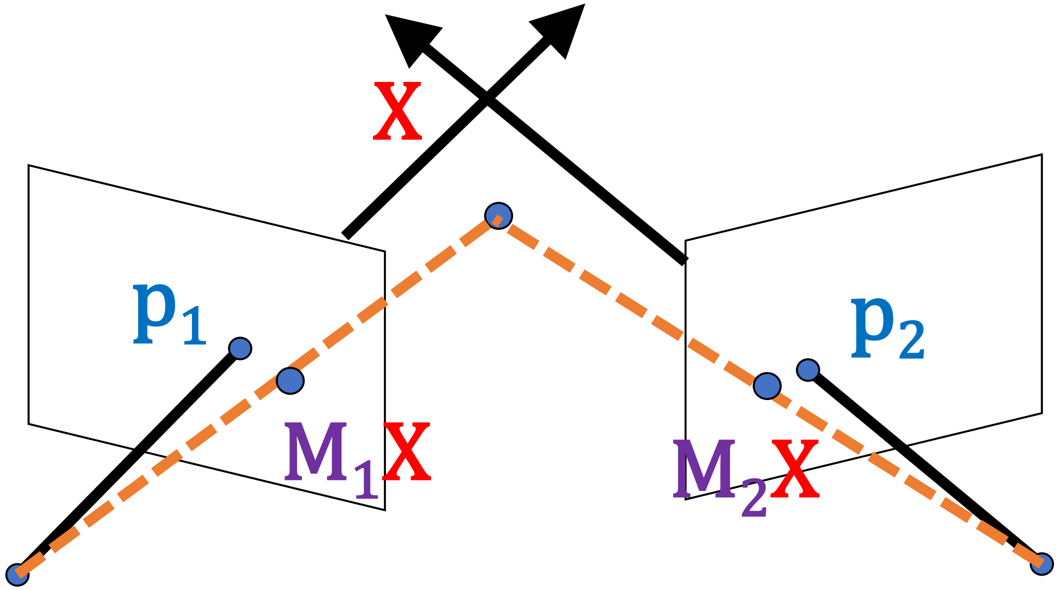
Find shortest segment between viewing rays, set **X** to be the midpoint of the segment.



Slide credit: D Fouhey & J Johnson

Triangulation – Non-linear Optim.

Find X minimizing $d(\boldsymbol{p}_1, \boldsymbol{M}_1\boldsymbol{X})^2 + d(\boldsymbol{p}_2, \boldsymbol{M}_2\boldsymbol{X})^2$



Triangulation – Linear Optimization

$$\boldsymbol{p}_1 \equiv \boldsymbol{M}_1\boldsymbol{X} \qquad \boldsymbol{p}_1 \times \boldsymbol{M}_1\boldsymbol{X} = \boldsymbol{0} \qquad [\boldsymbol{p}_{1x}]\boldsymbol{M}_1\boldsymbol{X} = \boldsymbol{0}$$

$$\boldsymbol{p}_2 \equiv \boldsymbol{M}_2\boldsymbol{X} \rightarrow \boldsymbol{p}_2 \times \boldsymbol{M}_2\boldsymbol{X} = \boldsymbol{0} \rightarrow [\boldsymbol{p}_{2x}]\boldsymbol{M}_2\boldsymbol{X} = \boldsymbol{0}$$

Cross Prod. as matrix

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = [\boldsymbol{a}_x]\boldsymbol{b}$$

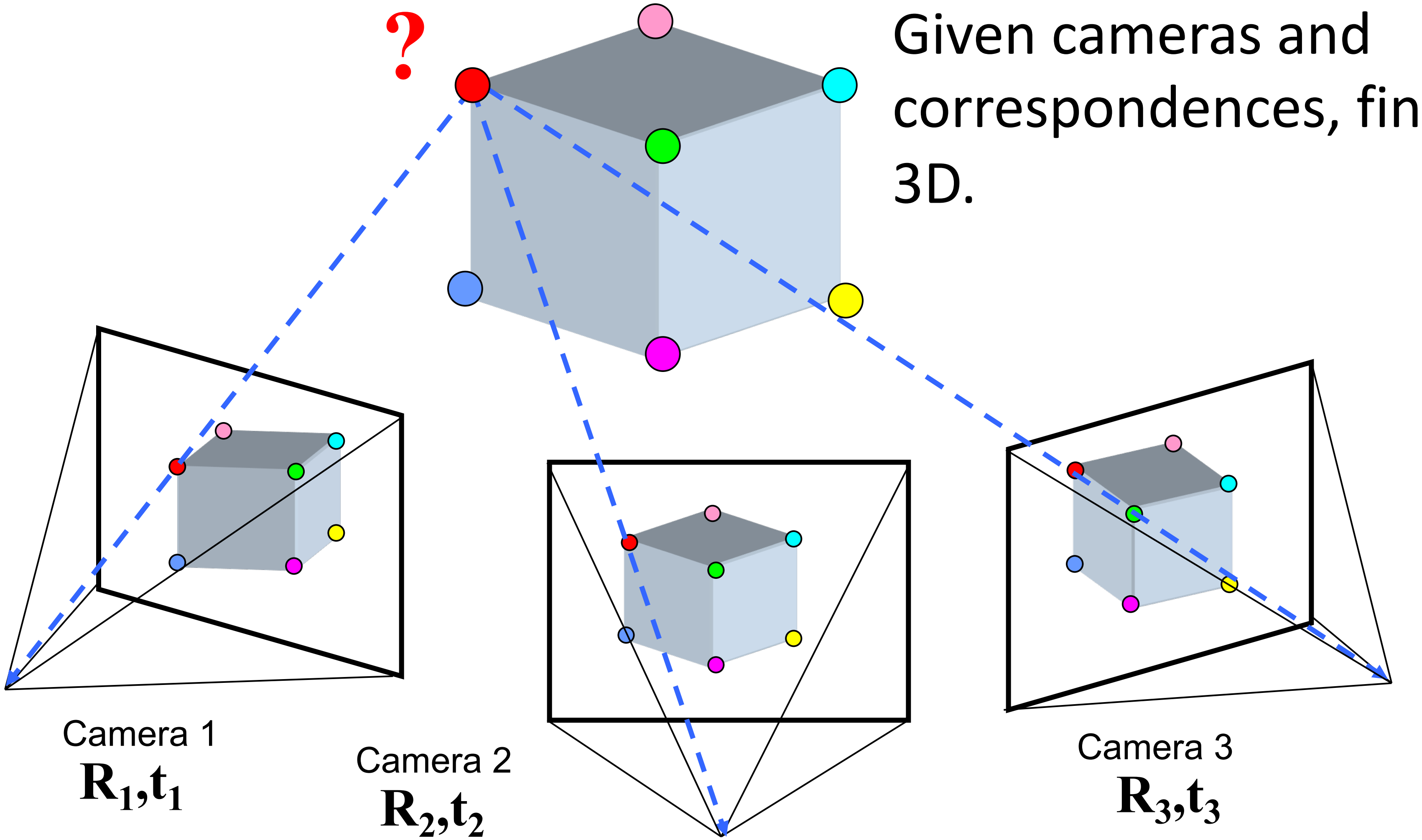
$$[\boldsymbol{p}_{1x}]\boldsymbol{M}_1\boldsymbol{X} = \boldsymbol{0} \qquad ([\boldsymbol{p}_{1x}]\boldsymbol{M}_1)\boldsymbol{X} = \boldsymbol{0}$$

$$[\boldsymbol{p}_{2x}]\boldsymbol{M}_2\boldsymbol{X} = \boldsymbol{0} \rightarrow ([\boldsymbol{p}_{2x}]\boldsymbol{M}_2)\boldsymbol{X} = \boldsymbol{0} \rightarrow$$

Two eqns per camera for 3 unkn. in X

Slide credit: D Fouhey & J Johnson

Multi-view geometry problems

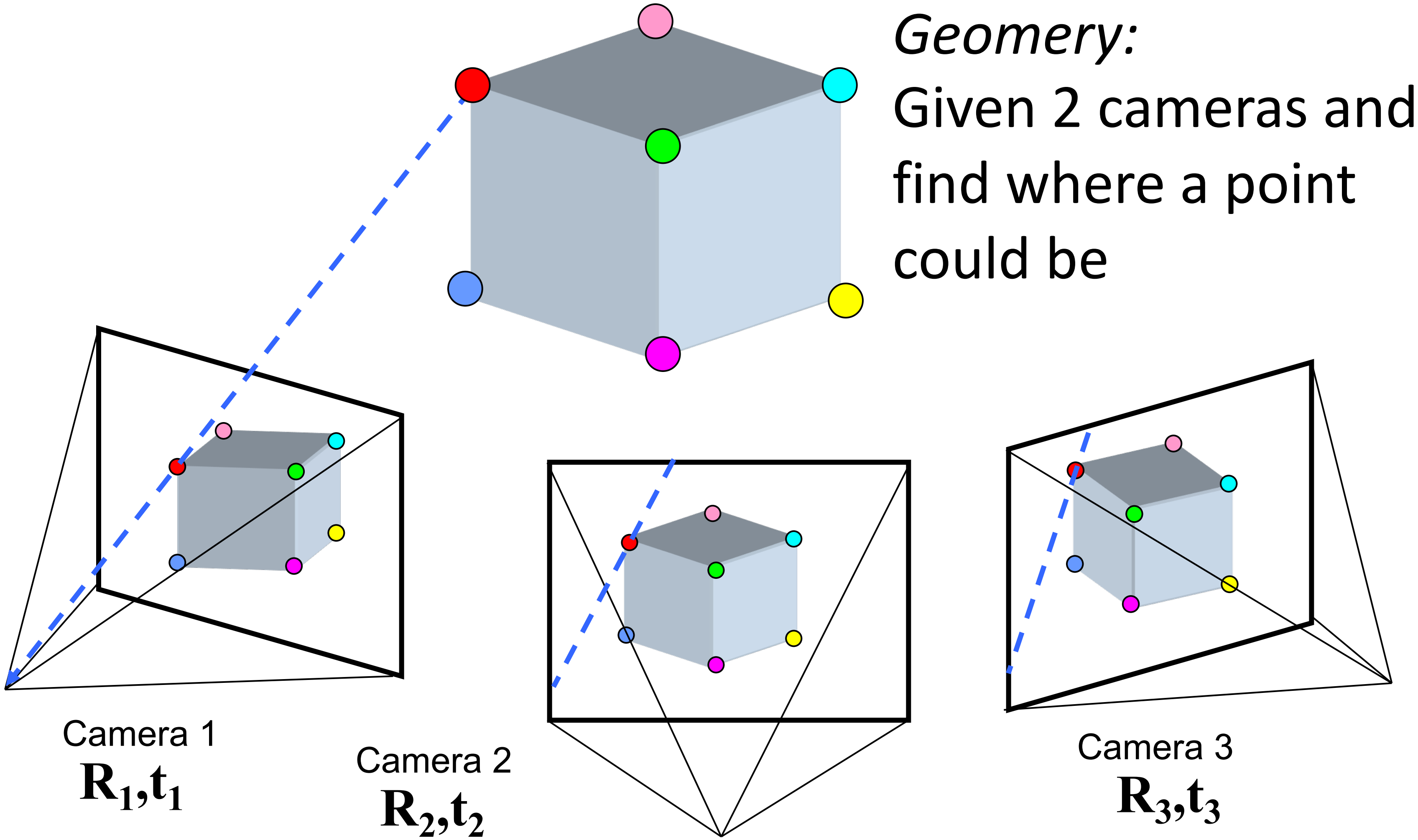


Recovering structure: Given cameras and correspondences, find 3D.

Slide credit: Noah Snavely

Multi-view geometry problems

Stereo/Epipolar Geomery:
Given 2 cameras and find where a point could be



Slide credit: Noah Snavely

Two-view geometry

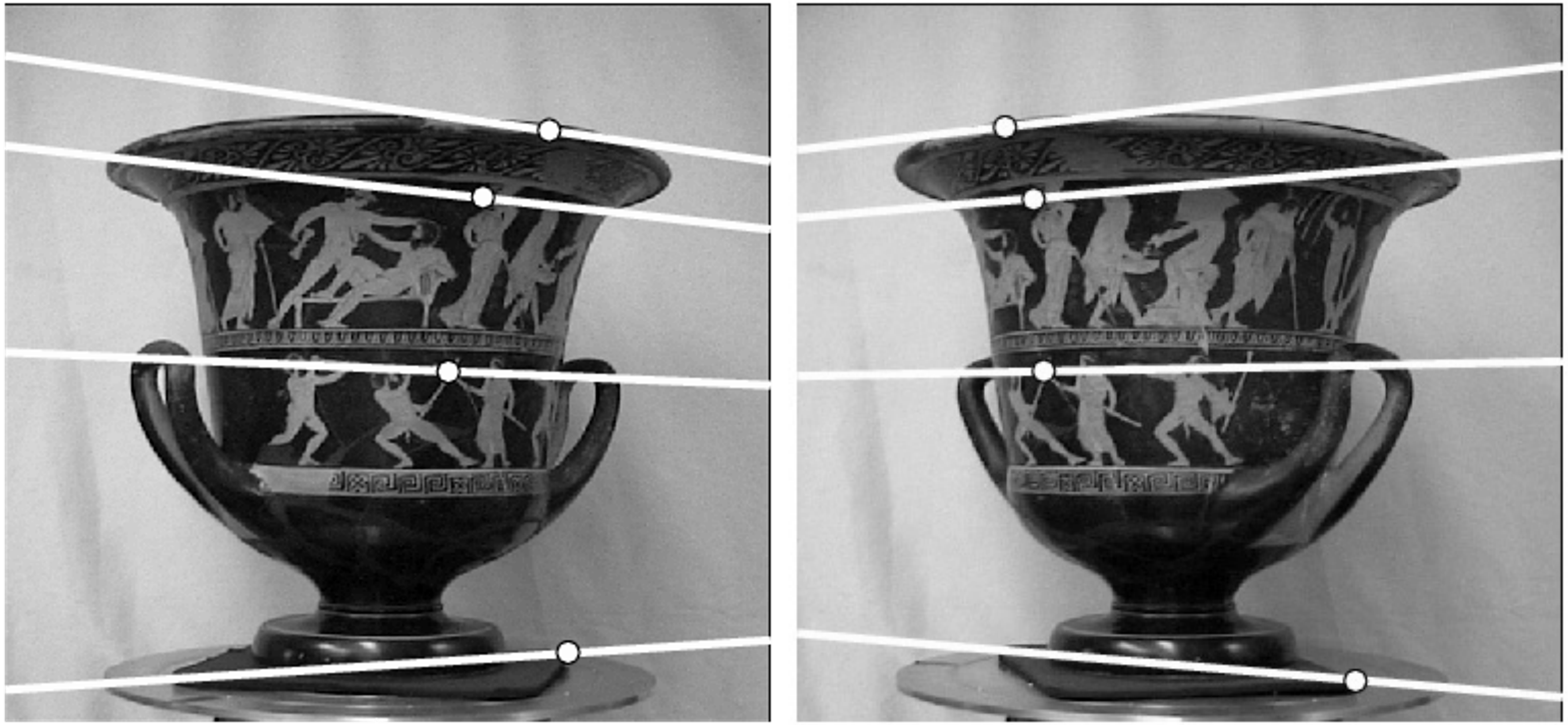
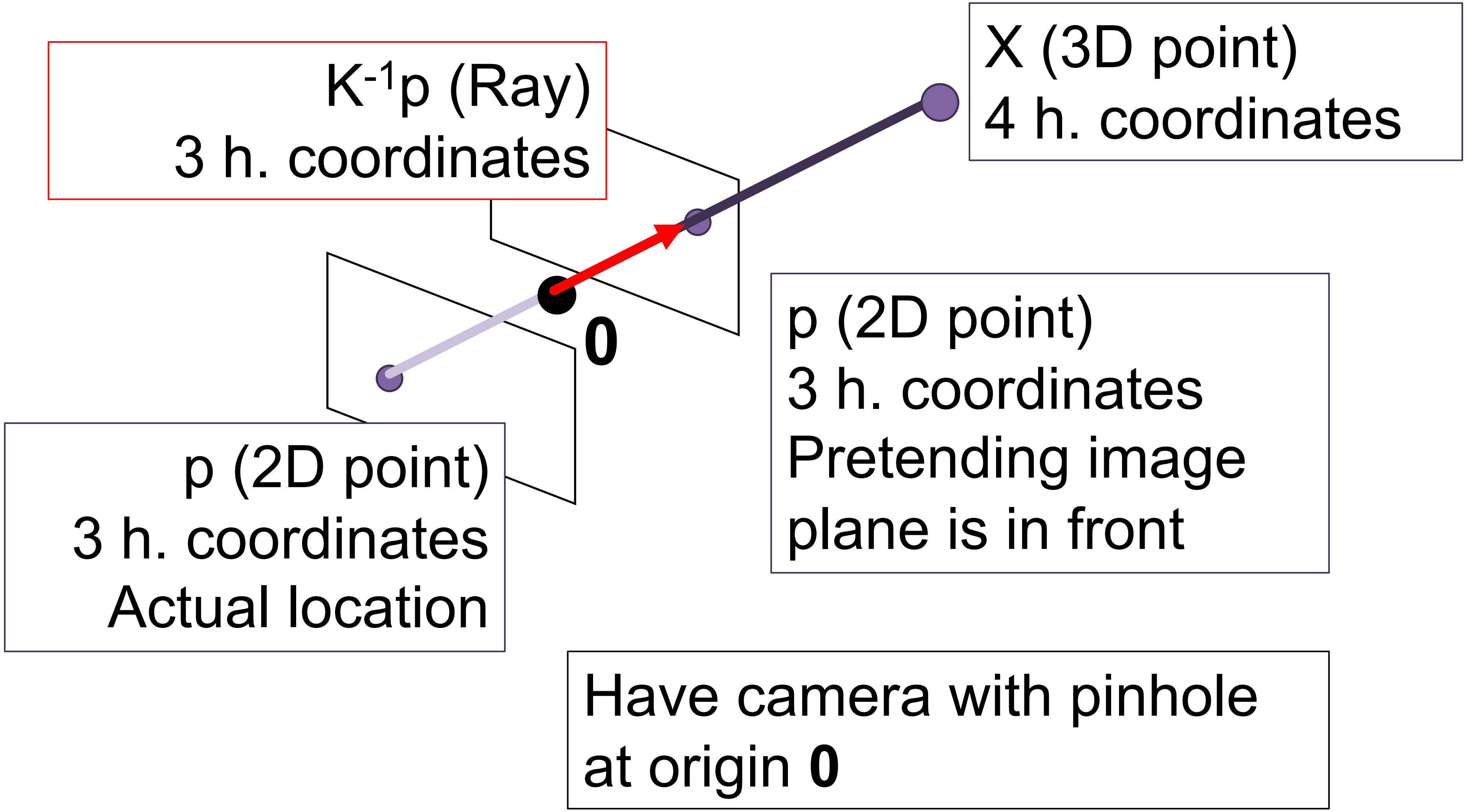
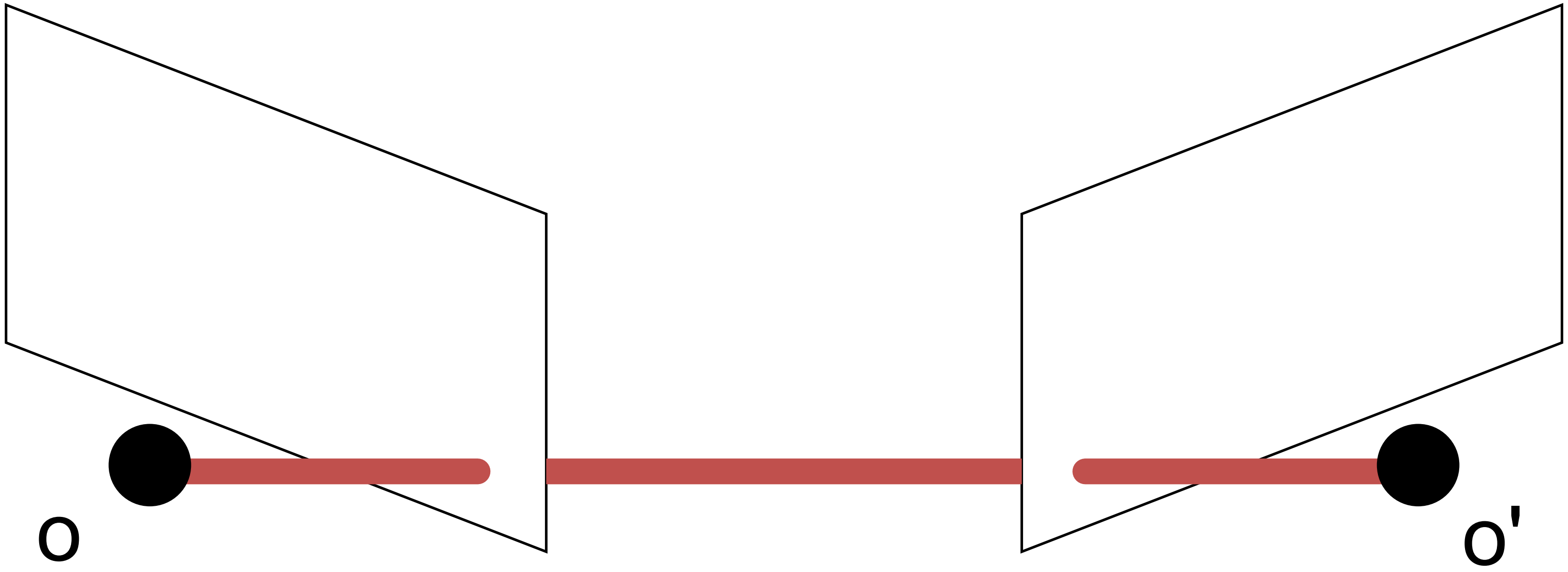


Image Credit: Hartley & Zisserman

Camera Geometry Reminder



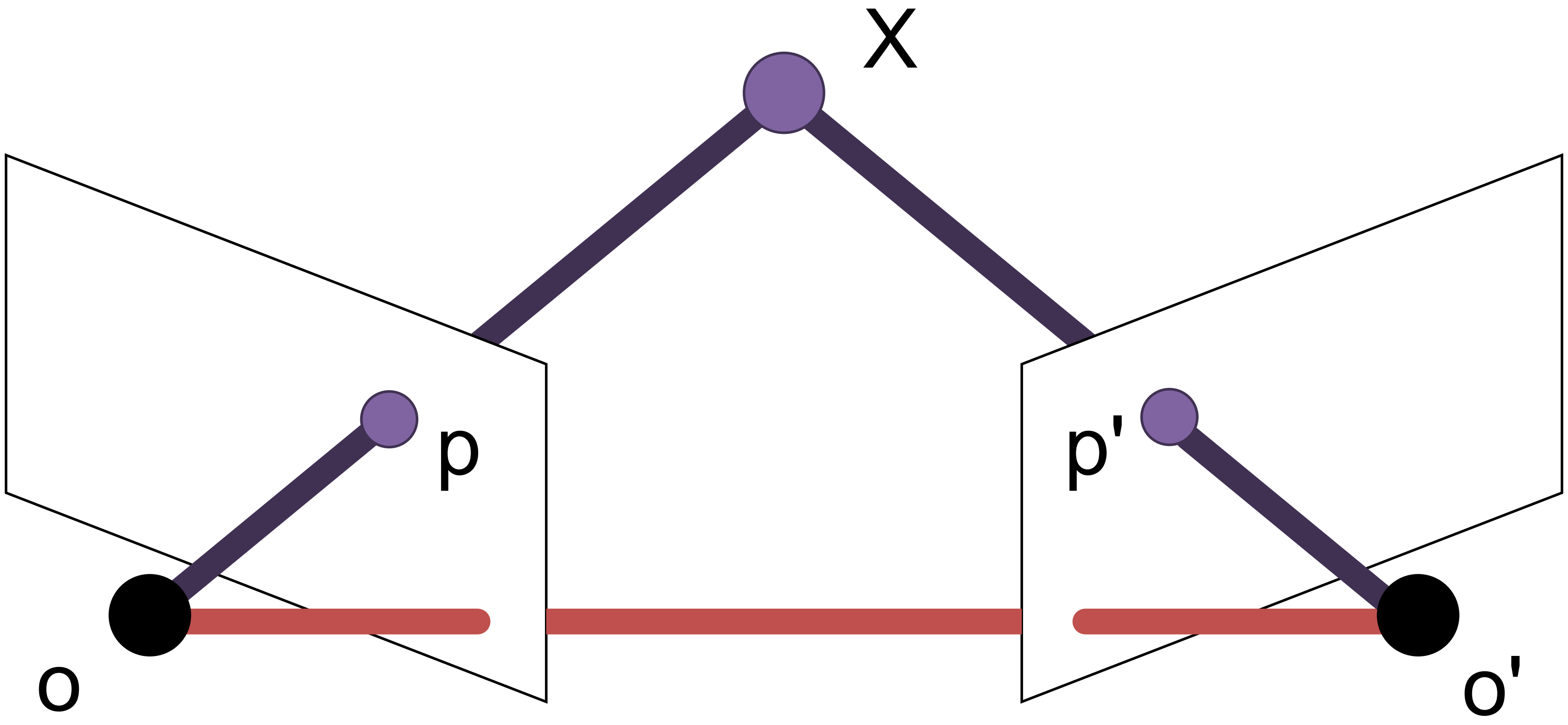
Epipolar Geometry



Suppose we have two cameras at origins o, o’

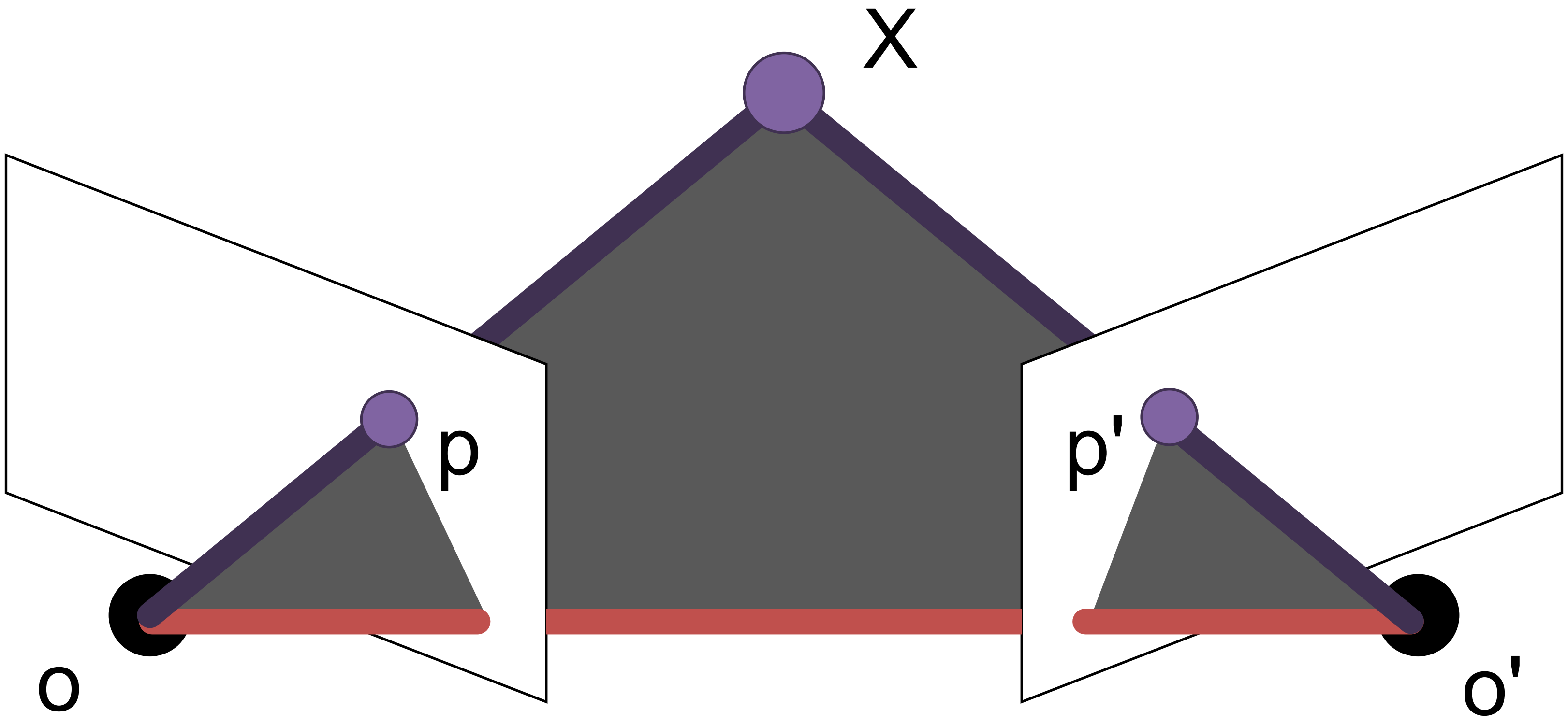
Baseline is the line connecting the origins

Epipolar Geometry



Now add a **point X**, which projects to p and p’

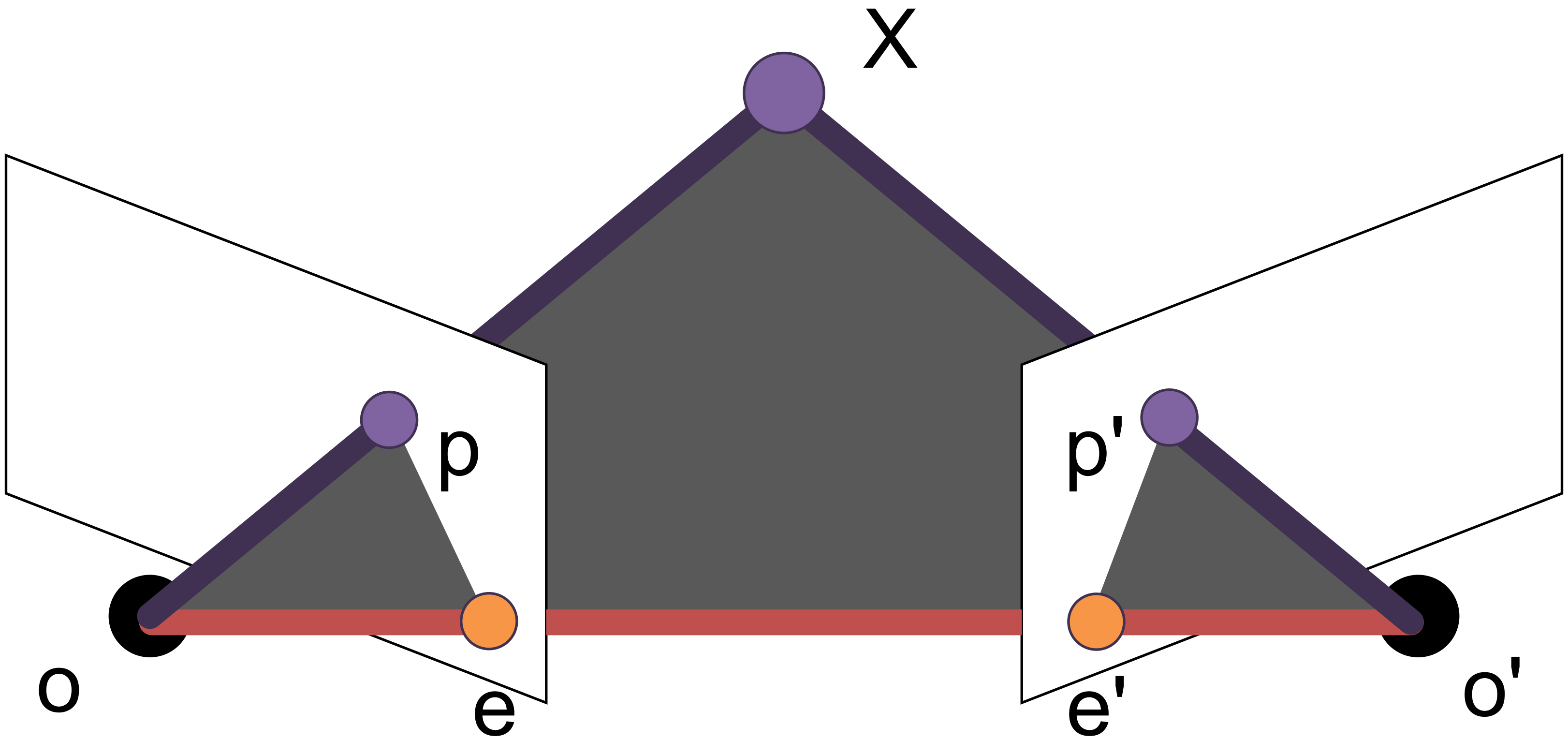
Epipolar Geometry



The plane formed by X, o, and o’ is called the epipolar plane

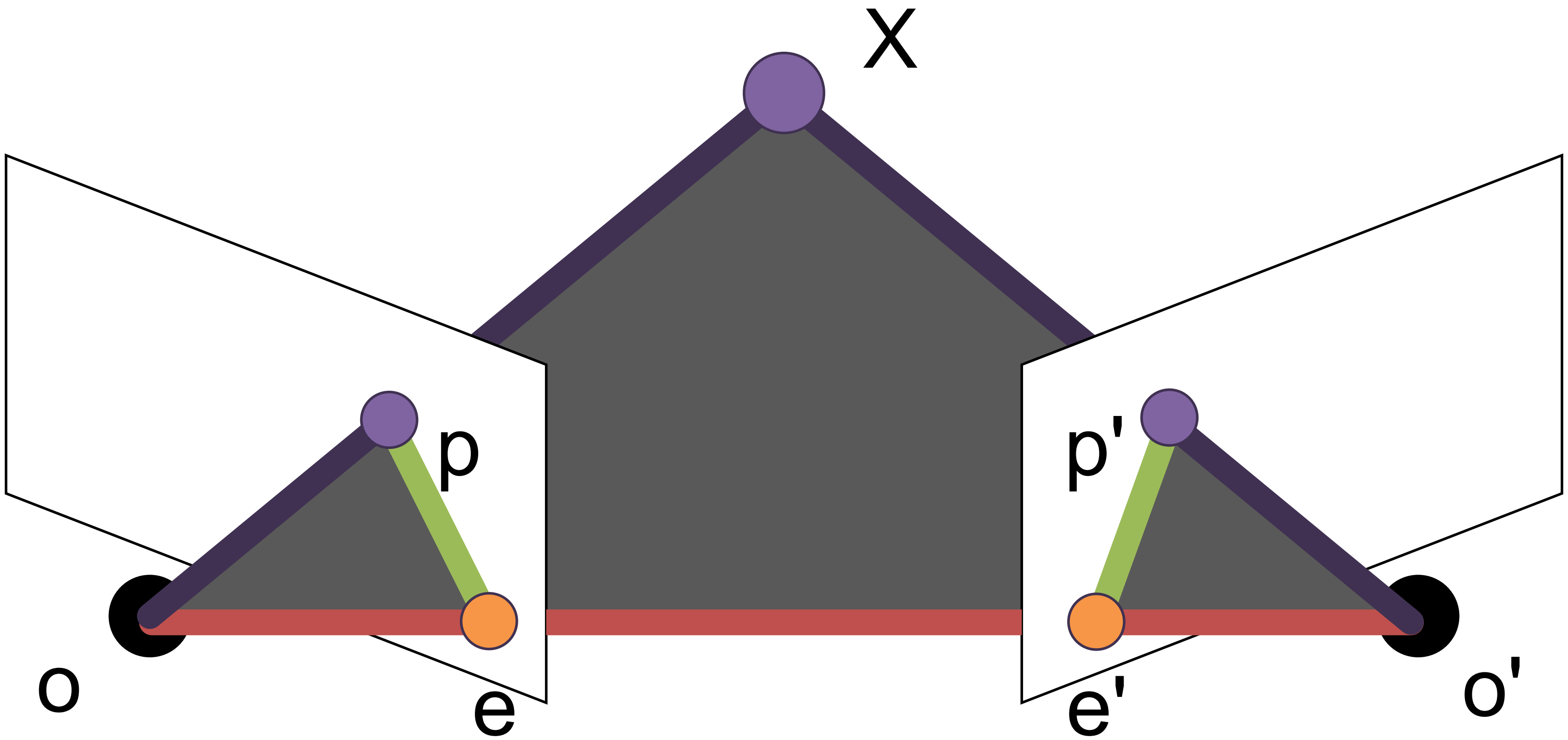
There is a family of planes per o, o’

Epipolar Geometry



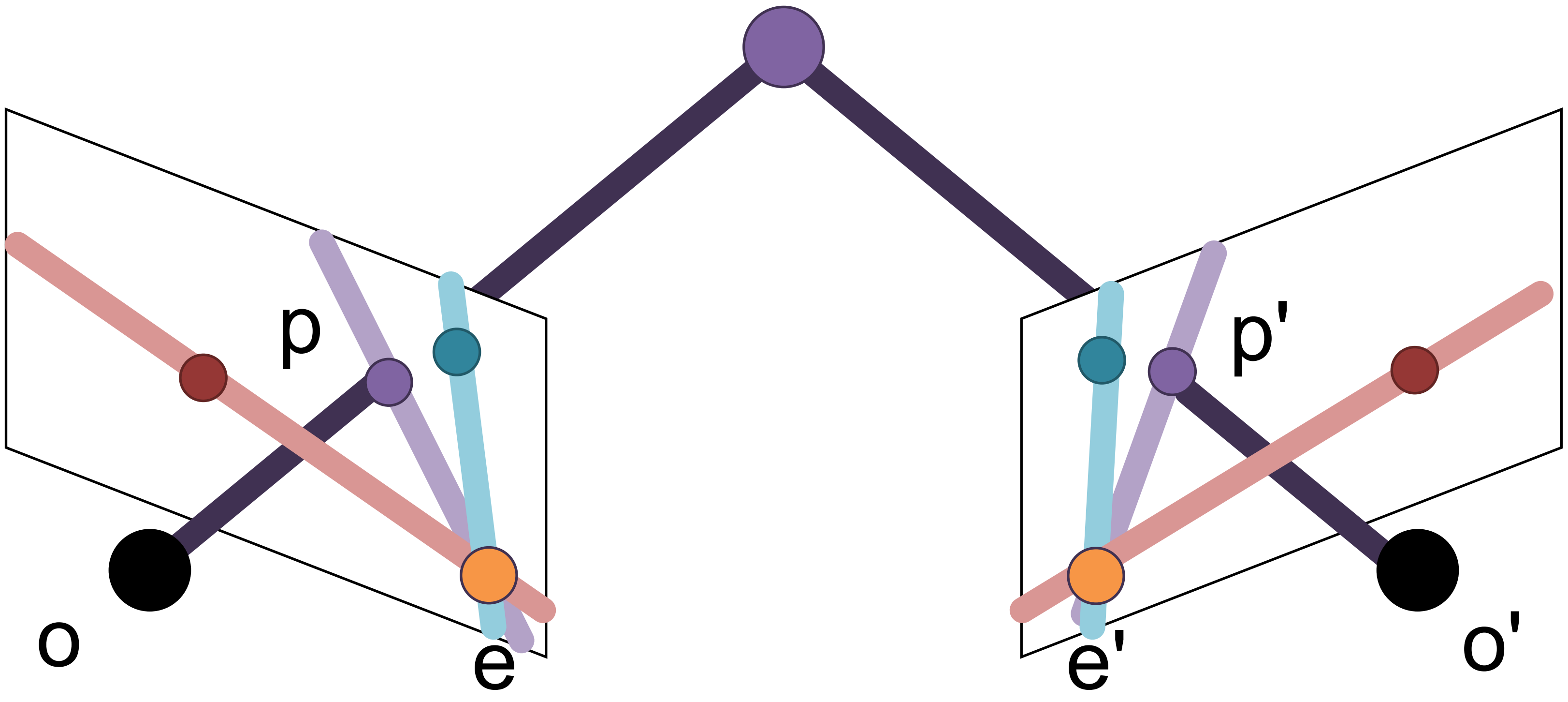
- Epipoles e, e’ are where the baseline intersects the image planes
- Projection of other camera in the image plane

Epipolar Geometry



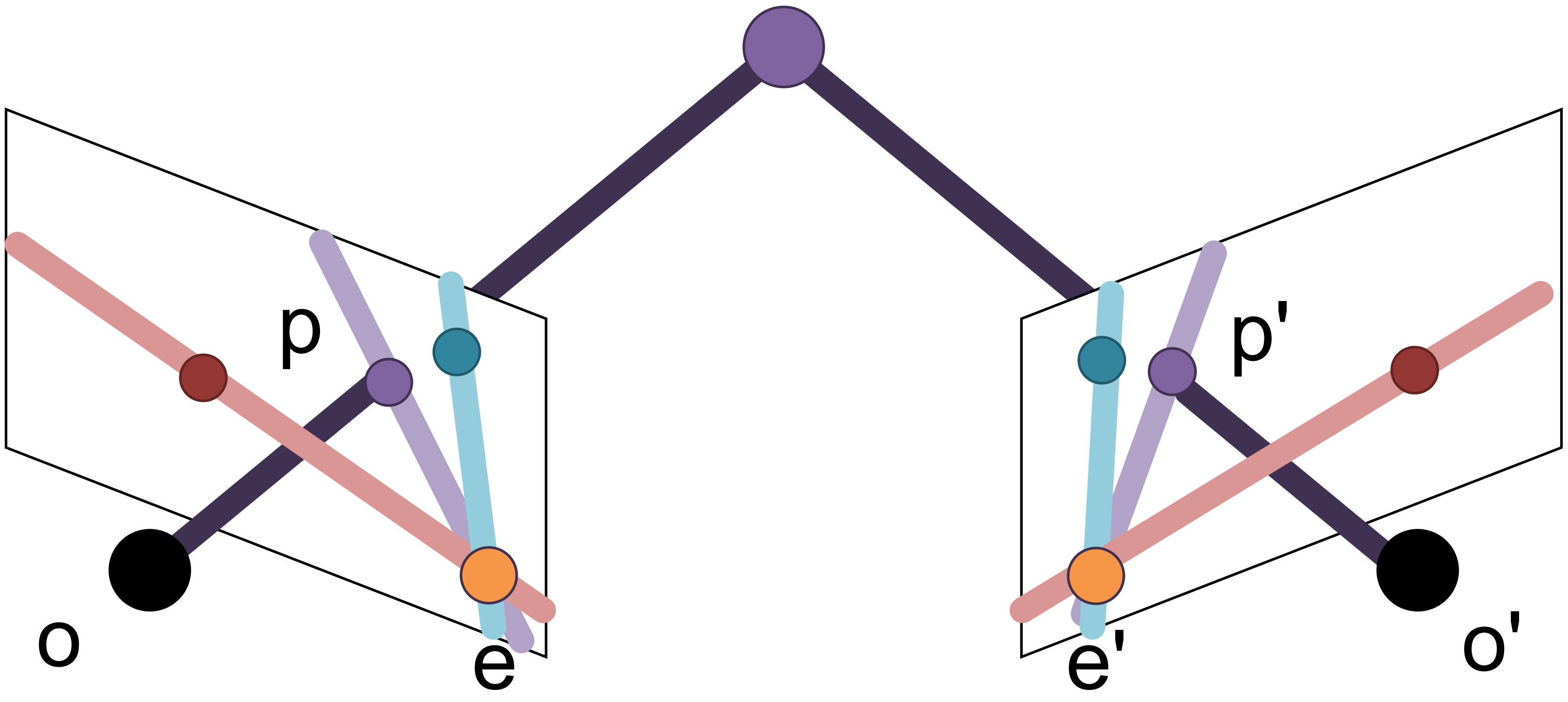
- Epipolar lines go between the epipoles and the projections of the points.
- Intersection of epipolar plane with image plane

Example: Converging Cameras



Epipoles finite, maybe in image; epipolar lines converge

Example: Converging Cameras



Epipolar lines come in pairs: given a point p, we can construct the epipolar line for p’.

Example 1: Converging Cameras

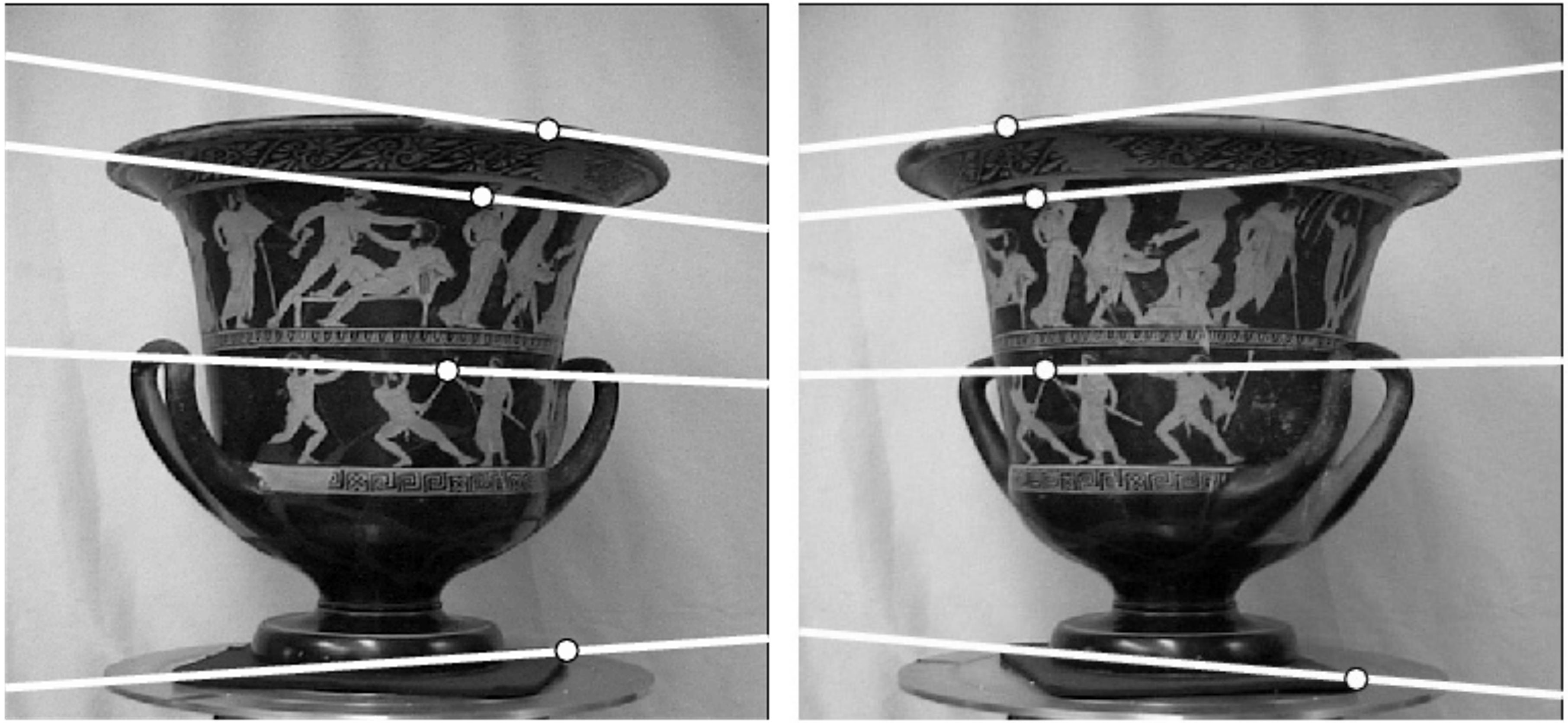
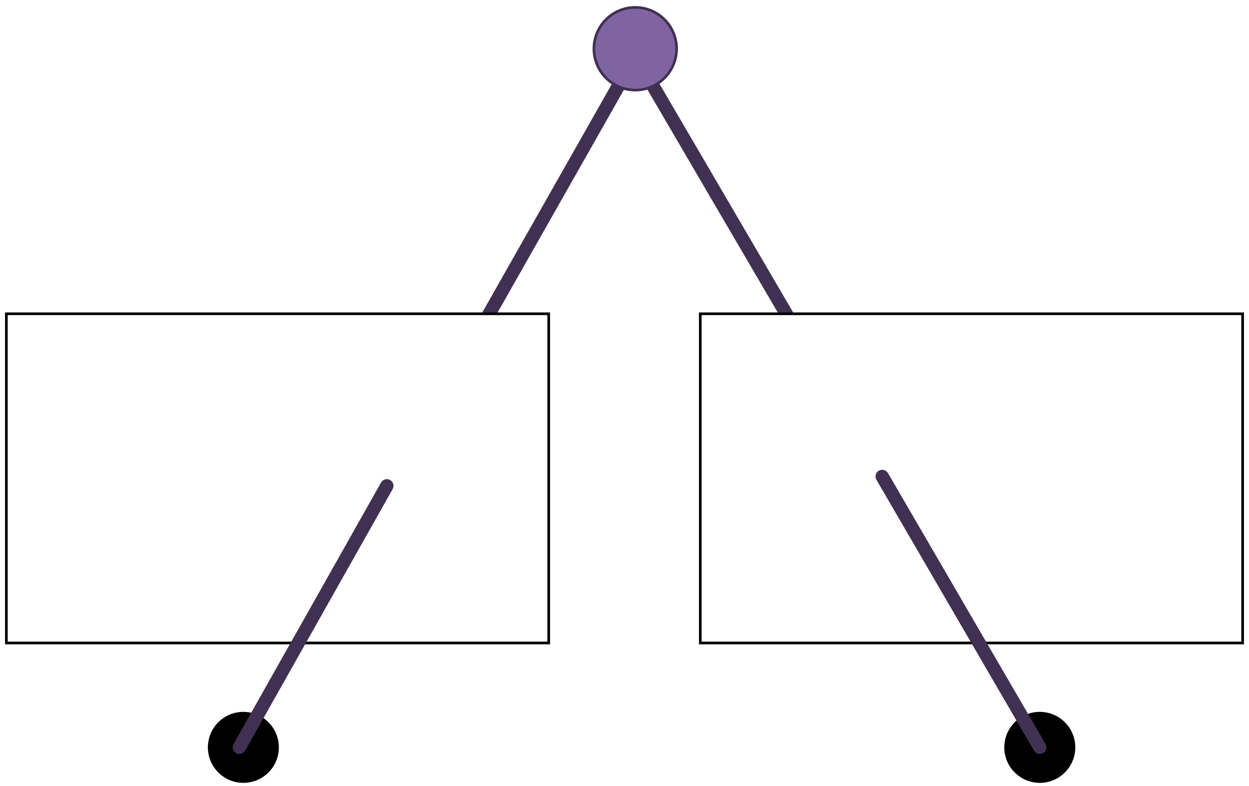


Image Credit: Hartley & Zisserman

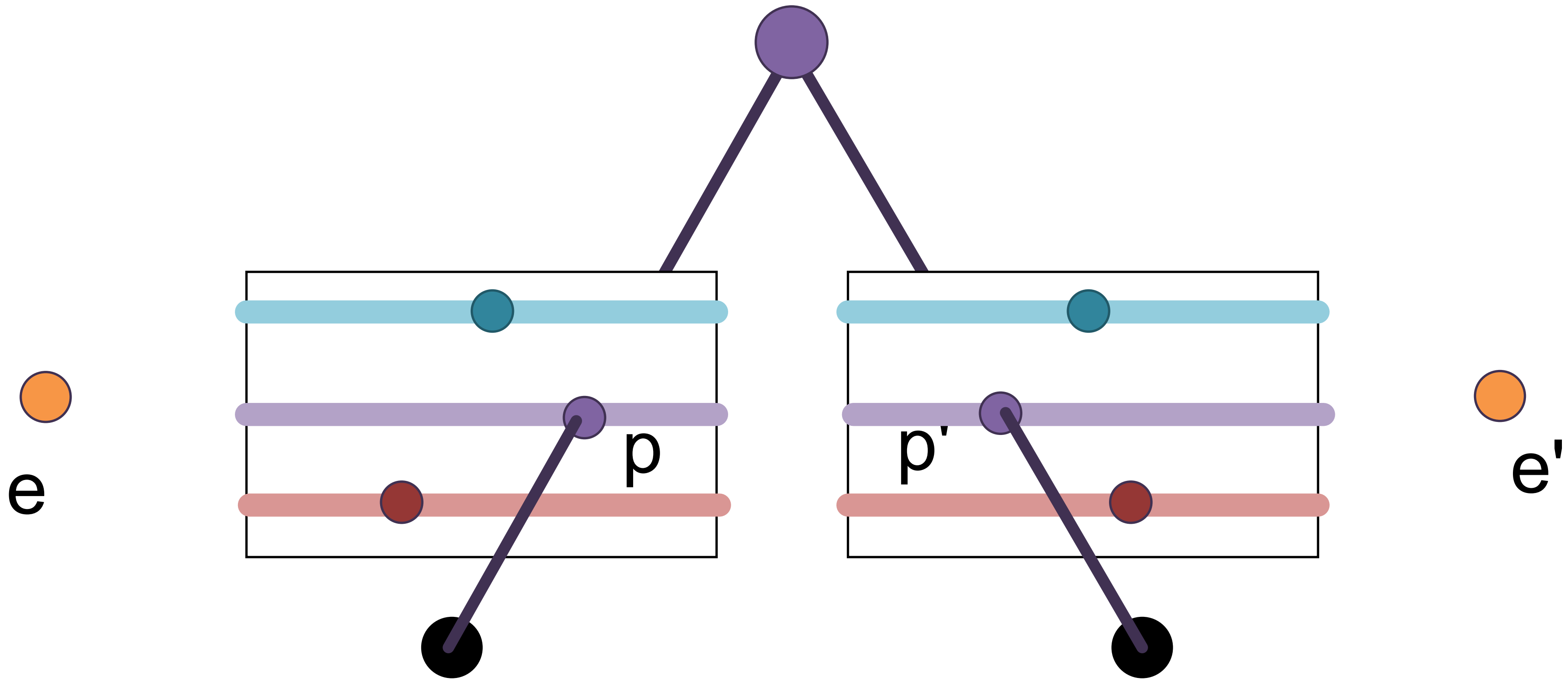
Example: Parallel to Image Plane



Suppose the cameras are both facing outwards.

Where are the epipoles (proj. of other camera)?

Example: Parallel to Image Plane



Epipoles *infinitely* far away, epipolar lines parallel

Example: Parallel to Image Plane

Image Credit: Hartley & Zisserman

Example: Forward Motion

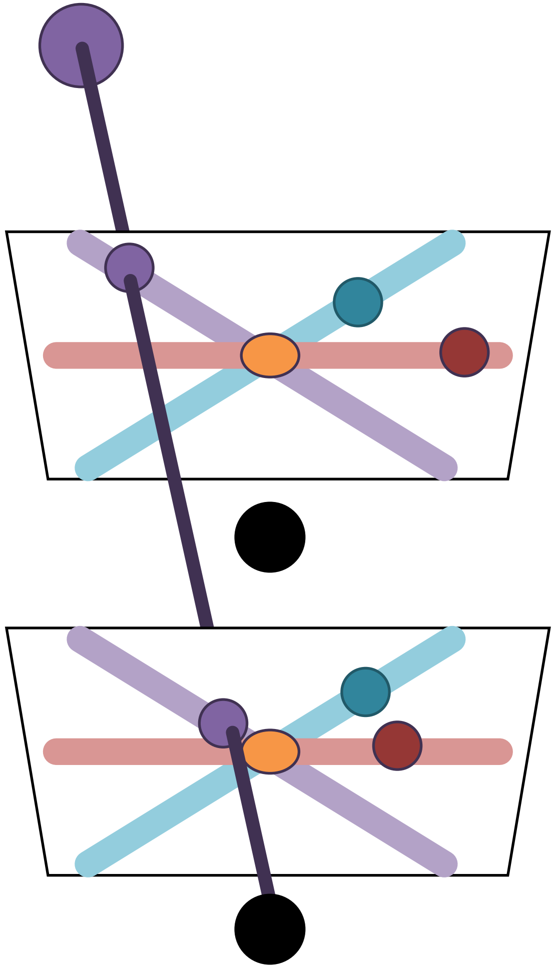
Image Credit: Hartley & Zisserman

Example: Forward Motion

Image Credit: Hartley & Zisserman

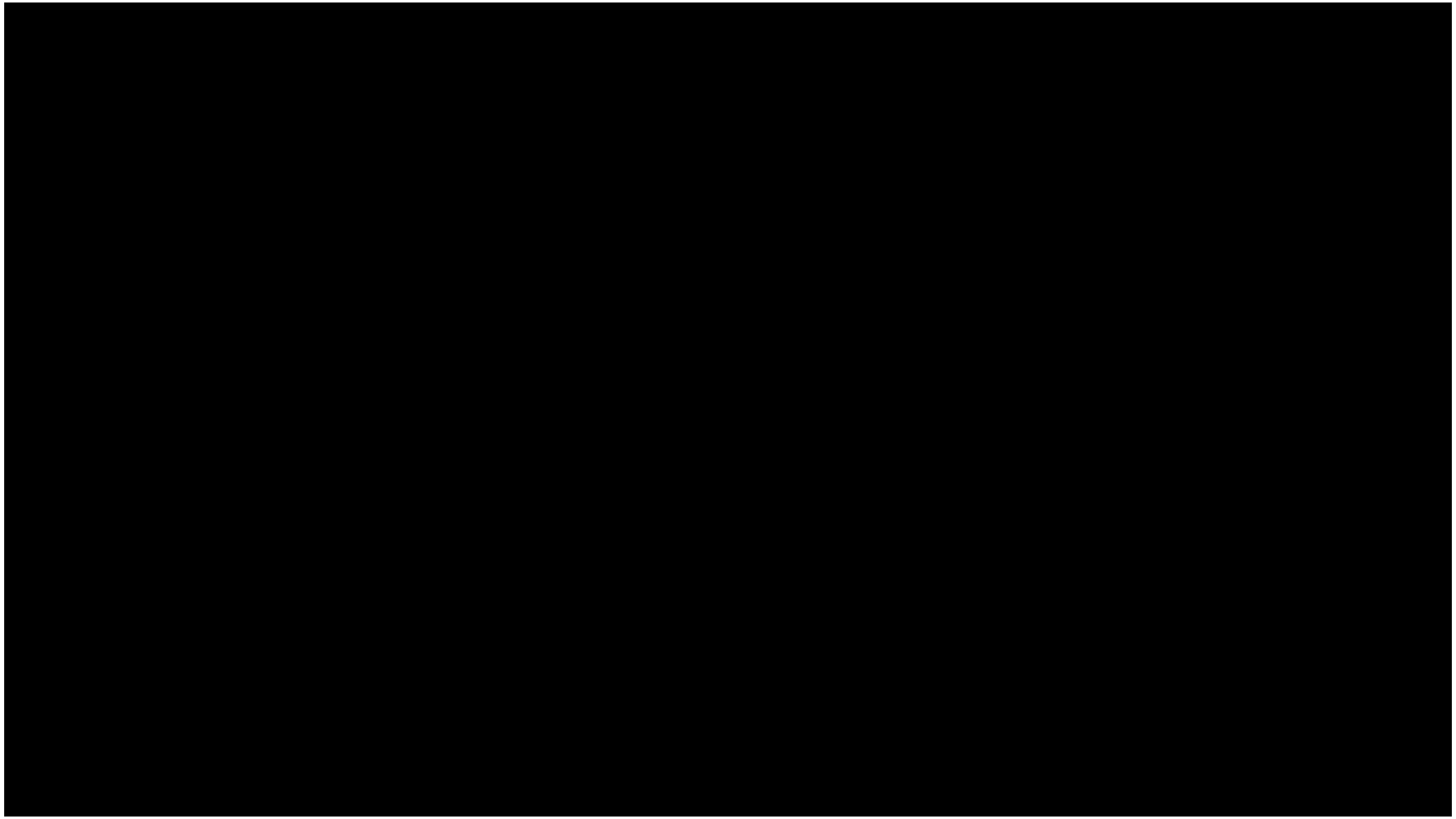
Example: Forward Motion

Epipole is focus of expansion / principal point of the camera.



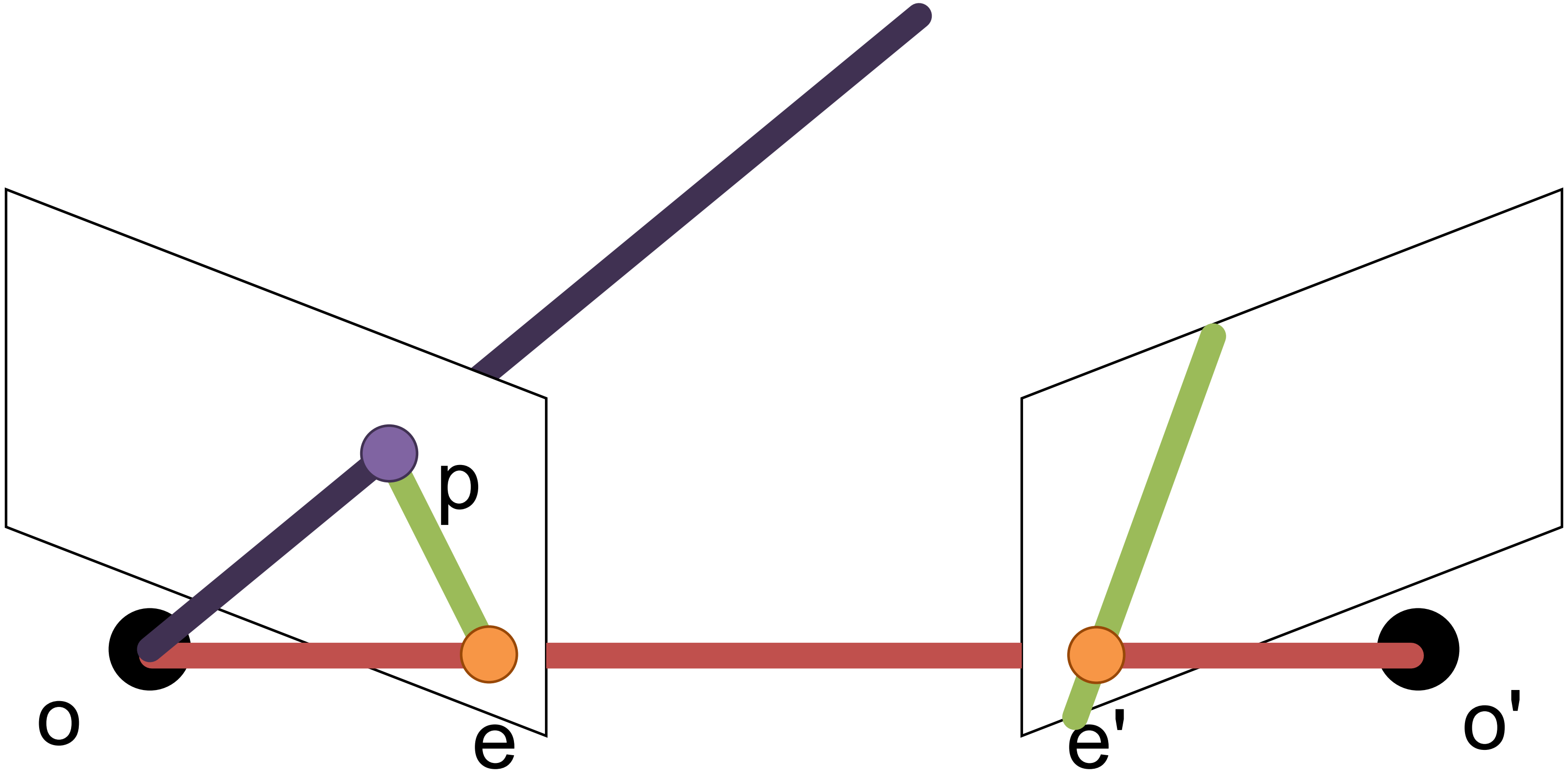
Epipolar lines go out from principal point

Motion perpendicular to image plane



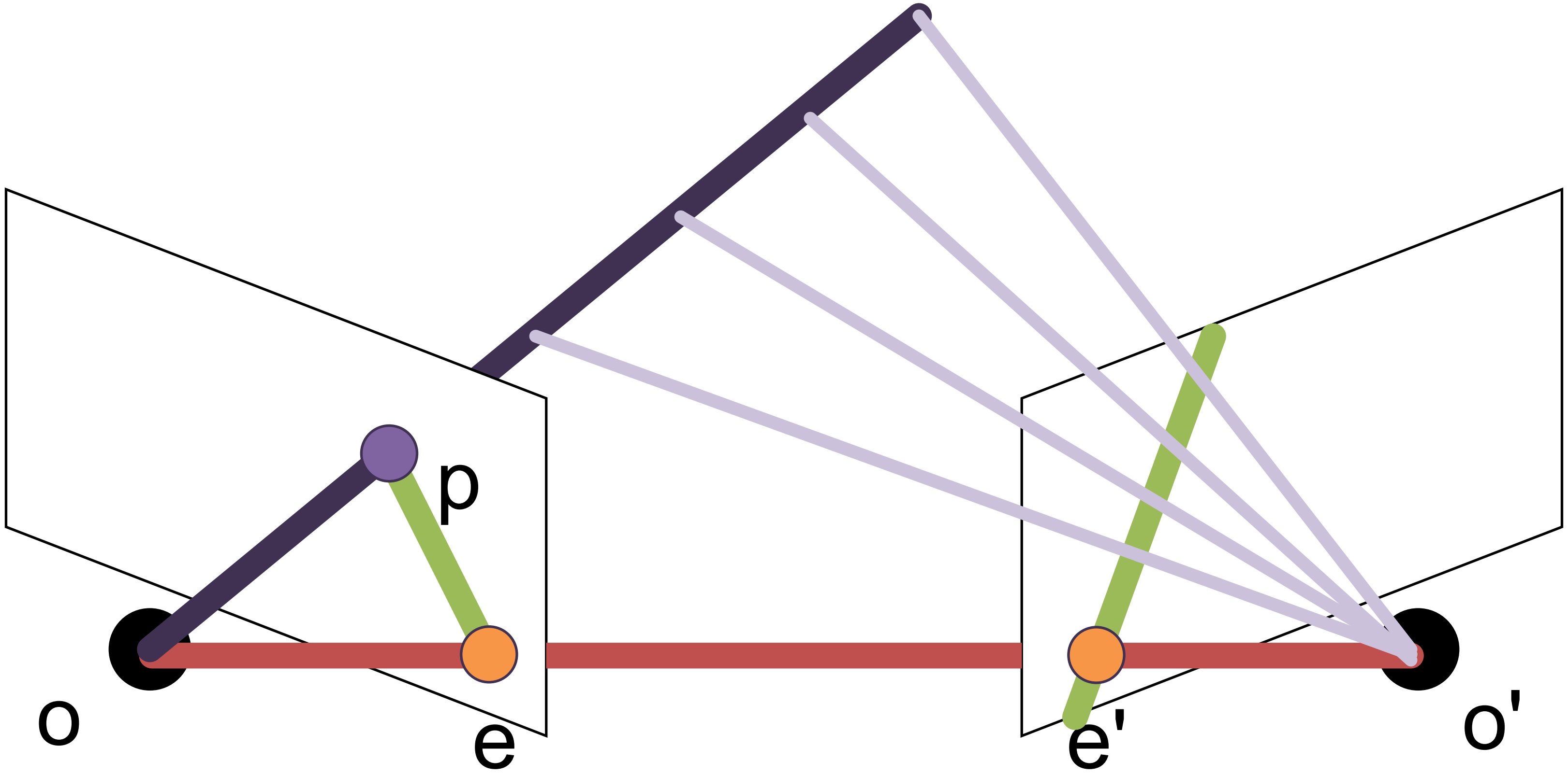
http://vimeo.com/48425421

Epipolar Geometry



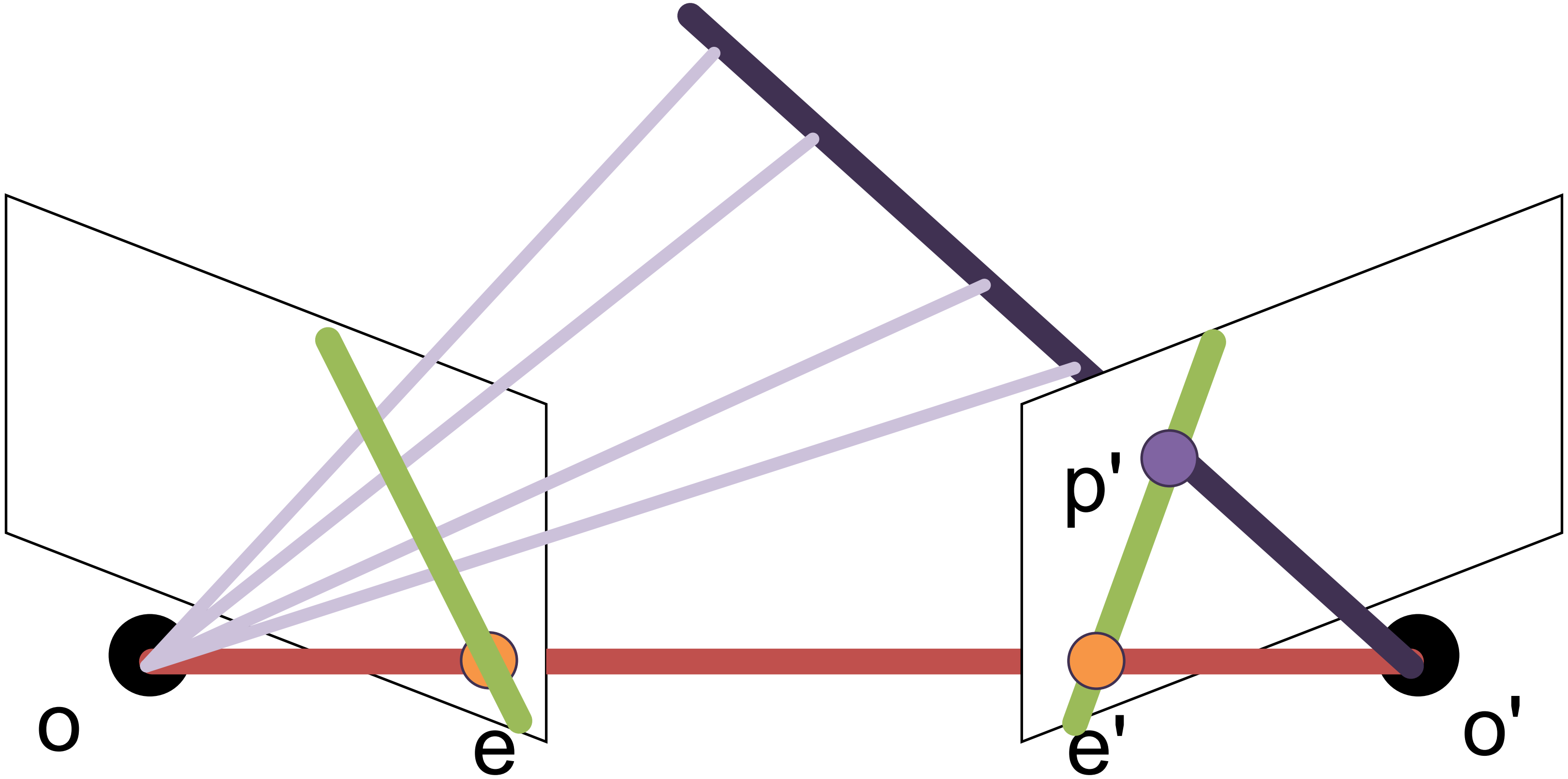
- Suppose we don't know X and just have p
- Can construct the epipolar line in the other image

Epipolar Geometry



- Suppose we don’t know X and just have p
- Corresponding p’ is on corresponding epipolar line

Epipolar Geometry

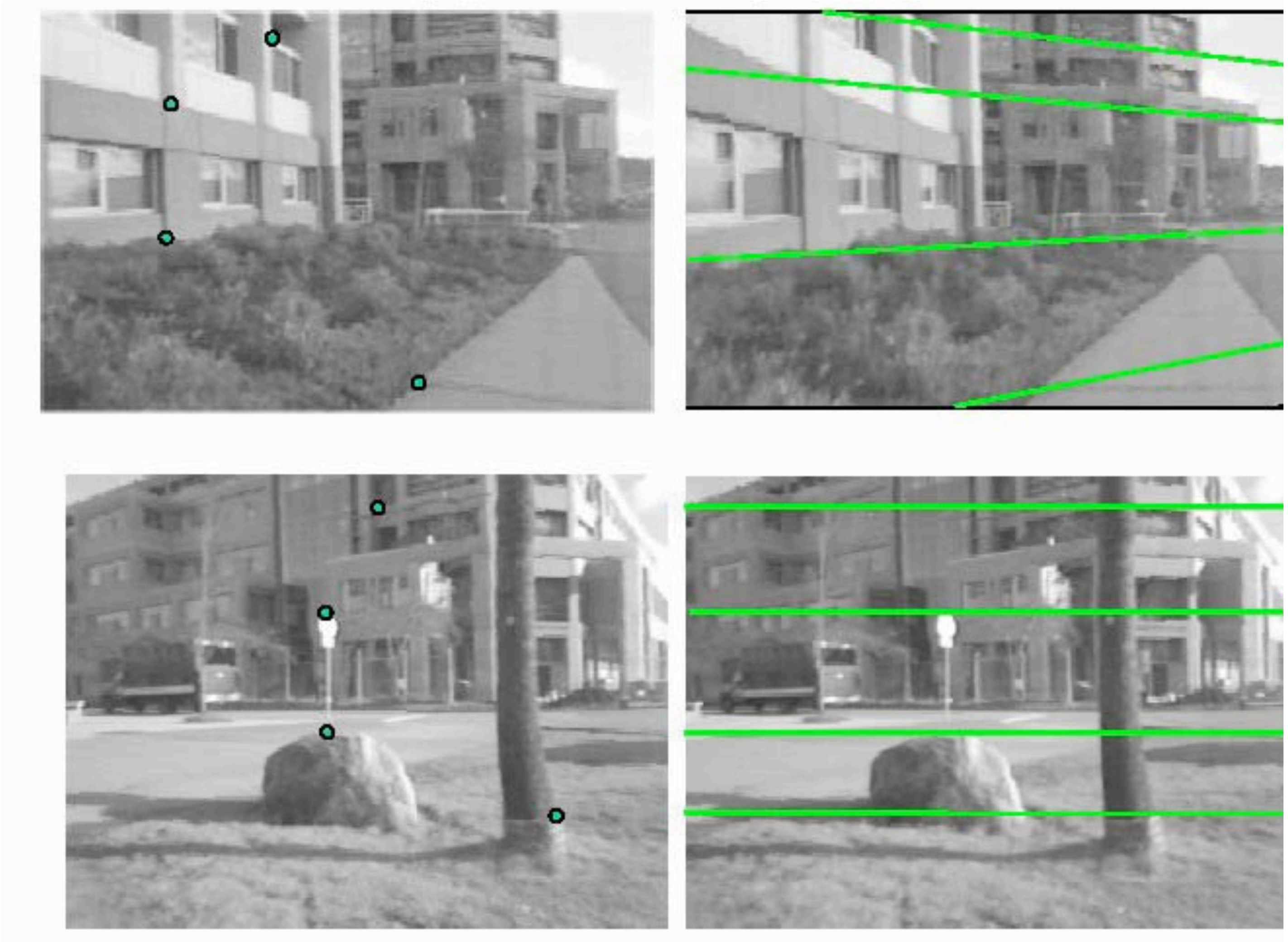


- Suppose we don't know X and just have p'
- Corresponding p is on corresponding epipolar line

Epipolar Geometry

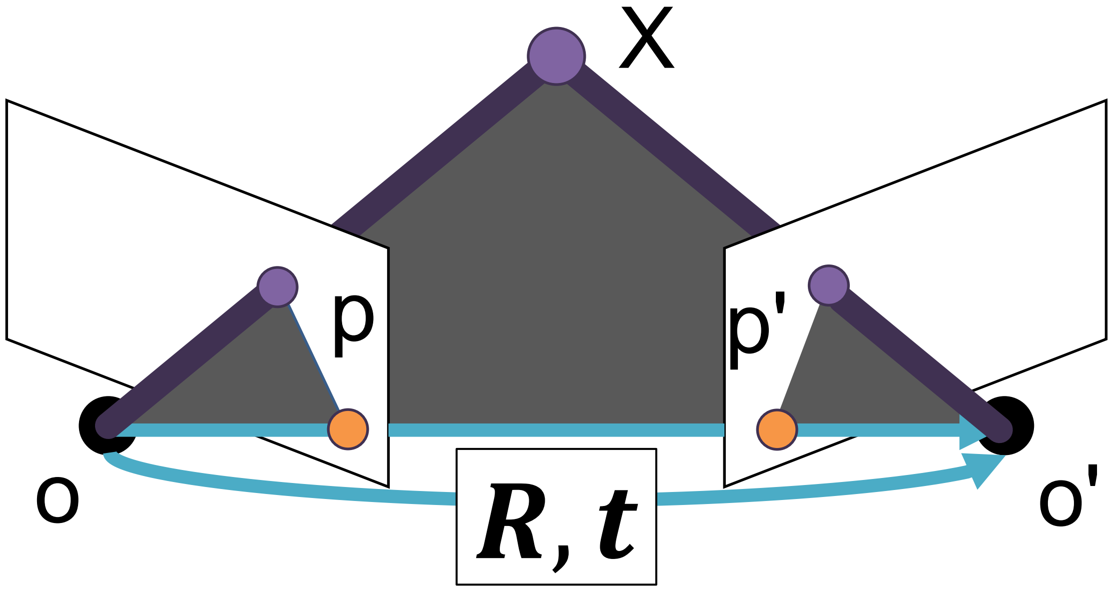
- If I want to do stereo, I want to find a corresponding pixel for each pixel in the image:
- Naïve search:
  - For each pixel, search every other pixel
- With epipolar geometry:
  - For each pixel, search along each line (1D search)

Epipolar constraint example



Slide Credit: S. Lazebnik

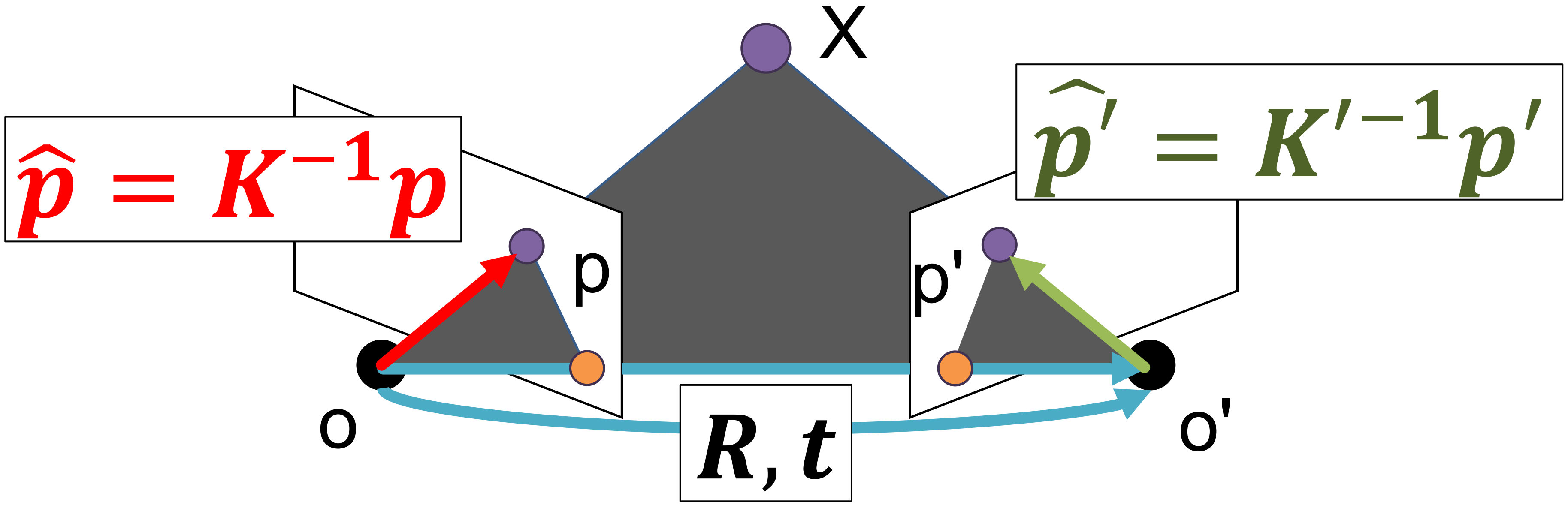
Epipolar Constraint: Calibrated Case



- If we know intrinsic and extrinsic parameters, set coordinate system to first camera
- Projection matrices: $\boldsymbol{M}_1 = \boldsymbol{K}[\boldsymbol{I}, \boldsymbol{0}]$ and $\boldsymbol{M}_2 = \boldsymbol{K}'[\boldsymbol{R}, \boldsymbol{t}]$
- **What are:**

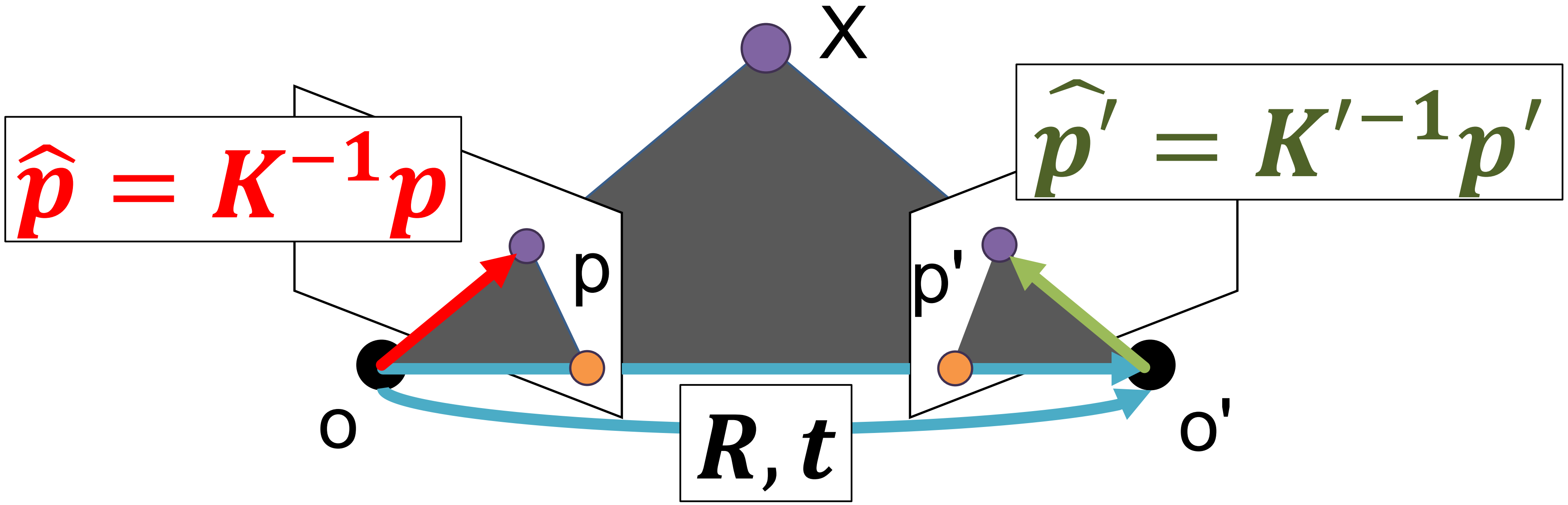
$$\boldsymbol{M}_1 \boldsymbol{X} \qquad \boldsymbol{M}_2 \boldsymbol{X} \qquad \boldsymbol{K}^{-1} \boldsymbol{p} \qquad \boldsymbol{K}'^{-1} \boldsymbol{p}'$$

Epipolar Constraint: Calibrated Case



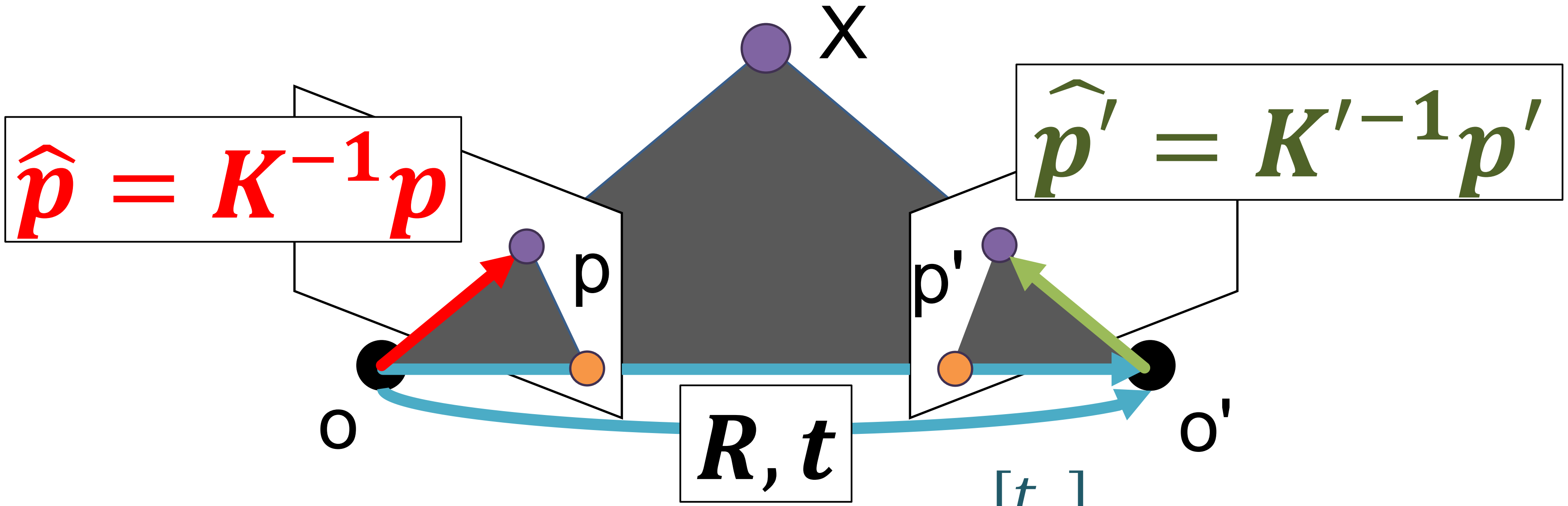
- Given calibration, $\hat{p} = K^{-1}p$ and $\widehat{p'} = K'^{-1}p'$ are "normalized coordinates"
- Note that $\widehat{p'}$ is actually translated and rotated to o'

Epipolar Constraint: Calibrated Case



- The following are all co-planar: $\mathbf{R}\hat{p}$, t, $\hat{p}'$ (can ignore translation for co-planarity here)
- One way to check co-planarity (triple product): $\hat{p}'^T(t \times R\hat{p}) = 0$

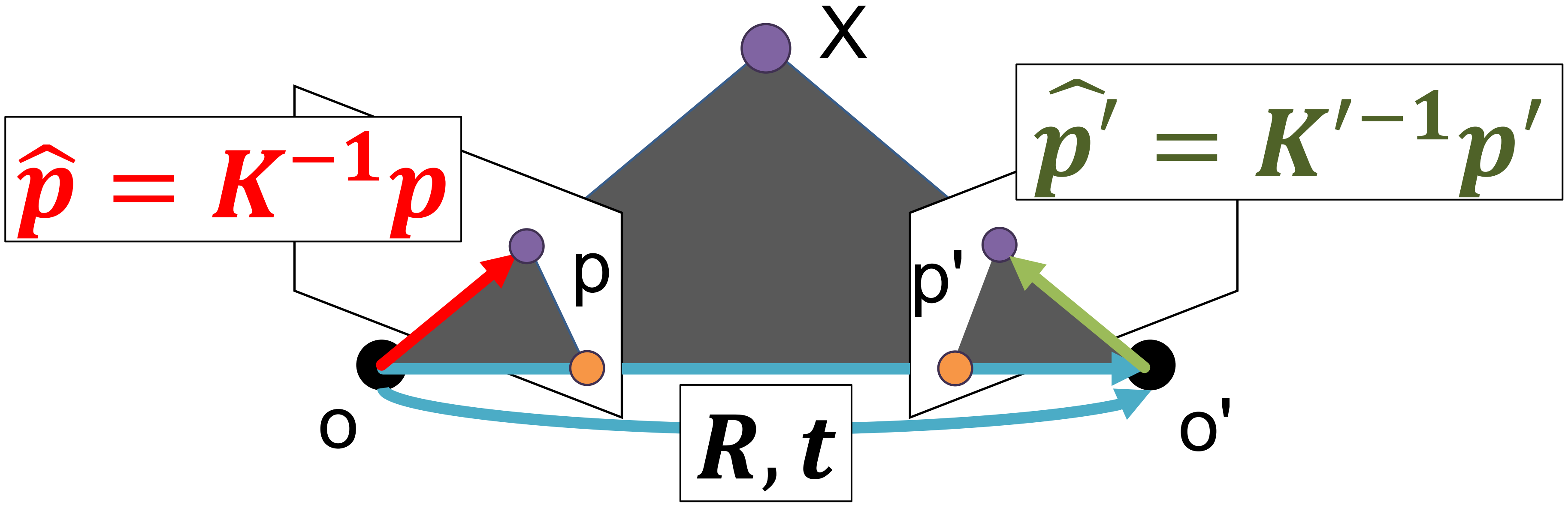
Epipolar Constraint: Calibrated Case



$$\widehat{p}'^{T}(t \times R\widehat{p}) = 0 \quad \rightarrow \quad \widehat{p}'^{T}\underbrace{\begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}}_{}\!\overset{[t_x]}{\phantom{\begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}}} R\widehat{p} = 0$$

Want something like $\mathbf{x}^T\mathbf{A}\mathbf{y}$=0. **What's A?**

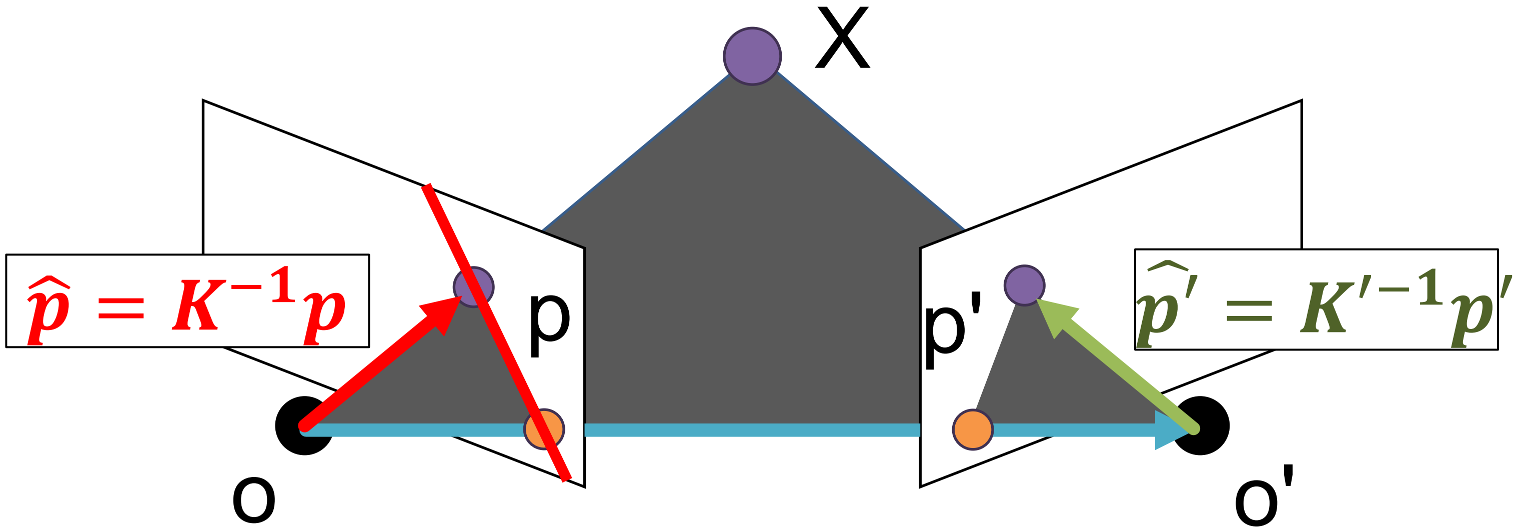
Epipolar Constraint: Calibrated Case



Essential matrix (Longuet-Higgins, 1981): $E = [t_x]R$

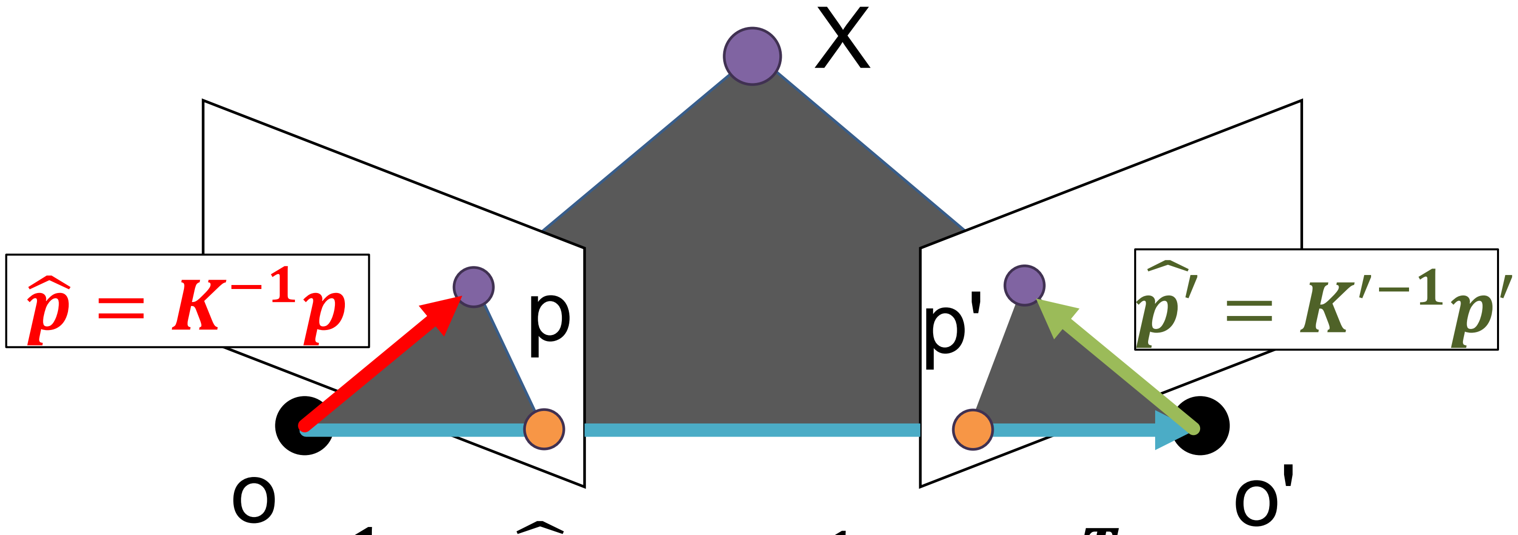
If you have a normalized point $\widehat{p}$, its correspondence $\widehat{p}'$ **must** satisfy $\widehat{p}'^T E \widehat{p} = 0$

Essential Essential Matrix Facts



- Suppose we know **E** and $\widehat{\boldsymbol{p}}'^T\boldsymbol{E}\widehat{\boldsymbol{p}} = 0$. What is the set $\{\boldsymbol{x}: \widehat{\boldsymbol{p}'}^T\boldsymbol{E}\boldsymbol{x} = \boldsymbol{0}\}$?
- $\widehat{\boldsymbol{p}'}^T\boldsymbol{E}$ gives equation of the epipolar line (in ax+by+c=0 form) in image for o.
- What's $\boldsymbol{E}^T\widehat{\boldsymbol{p}}'$?

What if we don't know K?



Have: $\hat{p} = K^{-1}p$, $\widehat{p'} = K'^{-1}p'$, $\hat{p}'^T E \hat{p} = 0$

$$\left(K'^{-1}p'\right)^T E\left(K^{-1}p\right) = 0 \;\longrightarrow\; p'^T K'^{-T} E K^{-1} p = 0$$

Set: $\underbrace{F = K'^{-T} E K^{-1}}_{\text{Fundamental Matrix (Faugeras and Luong, 1992)}}$ Then: $p'^T F p = 0$

The Fundamental Matrix Song

http://danielwedge.com/fmatrix/

Next class

- A bit more about epipolar geometry
  - Properties of the essential and fundamental matrices
  - How to estimate the fundamental matrix from images
- Stereo vision