Question 1. Show that the following sets of numbers are fields if the usual addition and multiplication of arithmetic are used:

- (1) the set of all numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers
- (2) the set of all numbers of the form $a + b\sqrt{-1}$ where a and b are real numbers. What is this field?

inverse multiplication

| a 26 | | x = | 0 | Ax = 1

A A 7 = A 6

A= (a) | a-26 | 10

= A-12

 $\frac{1}{9^{2}-2b^{2}}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$

 $\frac{1}{a^{2}-\lambda b^{2}} \left\{ \begin{array}{c} a \\ -b \end{array} \right\} = \left[\begin{array}{c} a \\ a^{2}-\lambda b^{2} \end{array} \right]$

(a+652) (7+452)=1 ax+ax12 +612+264

(9x+2by)+(9y+bx)727

Question 2. Show that the set of all $n \times n$ matrices $\mathbb{R}^{n \times n}$ with the usual matrix addition and multiplication is not a field if n > 1.

The complex conjugate of
$$z=a$$
 is the C is $z=a-bi$.

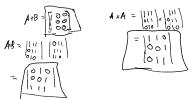
The complex conjugate of z is $z=b$.

The complex conjugate of $z=b$ is $z=a-bi$.

The complex conjugate of $z=a$ is $z=a-bi$.

Question 5. Determine which of the matrices below are in reduced row-celebro form.

$$\begin{vmatrix}
1 & 2 & 0 & 2 & 0 \\
0 & 0 & 1 & 3 & 0 \\
0 & 0 & 1 & 3 & 0 \\
0 & 0 & 1 & 0 & 0
\end{vmatrix}
= \begin{vmatrix}
0 & 1 & 0 & 2 & 4 \\
0 & 0 & 1 & 3 & 0 \\
0 & 0 & 0 & 0 & 0
\end{vmatrix}
= \begin{vmatrix}
0 & 1 & 0 & 2 & 4 \\
0 & 0 & 1 & 3 & 5 \\
0 & 0 & 0 & 0 & 0
\end{vmatrix}
= \begin{vmatrix}
0 & 1 & 0 & 2 & 4 \\
0 & 0 & 1 & 3 & 5 \\
0 & 0 & 0 & 0 & 0
\end{vmatrix}
= \begin{vmatrix}
0 & 1 & 0 & 2 & 4 \\
0 & 0 & 1 & 3 & 5 \\
0 & 0 & 1 & 3 & 5
\end{vmatrix}$$
Question 6. Let $A = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1
\end{vmatrix}
= \begin{vmatrix}
0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1
\end{vmatrix}
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0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1
\end{vmatrix}
= \begin{vmatrix}$



Question 7. For which values of t does the matrix $A = \begin{bmatrix} 6 & -1 & 1 \\ t & 0 & 1 \\ 0 & 1 & t \end{bmatrix}$ NOT have an inverse?

$$\frac{\partial^{2} A = 0}{\partial (0 - 1)} - 1 \left(\frac{4^{2} - 0}{4^{2} - 0}\right) + 1 \left(\frac{6 - 0}{4^{2} - 0}\right) \\
- \left(\frac{4^{2} - 4}{4^{2} - 4}\right) + 6 \\
\left(\frac{4^{2} - 4}{4^{2} - 3}\right) \left(\frac{4^{2} - 0}{4^{2} - 3}\right) \\
\left(\frac{1}{4^{2} - 4}\right) \left(\frac{4^{2} - 0}{4^{2} - 3}\right) + 1 \left(\frac{6 - 0}{4^{2} - 3}\right)$$

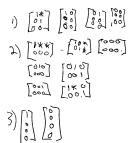
Question 8. Find all values of h that make the following matrices consistent, i.e., at least has one solution.

a)
$$\begin{bmatrix} 1 & h & 1 & 4 \\ 3 & 6 & 1 & 8 \end{bmatrix}$$

b) $\begin{bmatrix} -4 & 12 & 1 & h \\ 2 & -6 & 1 & -3 \end{bmatrix}$
9) $\begin{bmatrix} 1 & h & 1 & 4 \\ 3 & 6 & 1 & 9 \end{bmatrix}$
6 -3h=0
(60,1) (120)
6) $\begin{bmatrix} -4 & 12 & 1 & h \\ 3 & 6 & 1 & 9 \end{bmatrix}$
R₂=R₂+ $\frac{1}{2}$ R₁
0 = -3+ $\frac{1}{2}$ R₂

Question 9. We says that two $m \times n$ matrices in reduced row-echelon form are of the same type if they have the same number of leading 1's in the same position.

- (1) How many types of 3 × 2 matrices in reduced row-echelon form.
- (2) How many types of 2×3 matrices in reduced row-echelon form.
- (3) Find all 4×1 matrices in reduced row-echelon form.



Question 10. For which values of a, b, c, d, and e is the following matrix in reduced row-echelon form?

$$A = \begin{bmatrix} 1 & a & b & 3 & 0 & -2 \\ 0 & 0 & c & 1 & d & 3 \\ 0 & e & 0 & 0 & 1 & 1 \end{bmatrix}$$

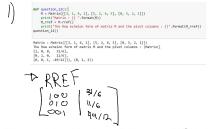
Question 11. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 2 \end{bmatrix}$

- (1) Calculation $\mathbf{rref}(A)$ over $\mathbb R$ by hand. Solve $A\vec x'=\vec 0$ and write all solutions in parametric vector forms
- (3) Using Python verify your result and calculation $\mathbf{rref}(A)$ over field \mathbb{Z}_2 and \mathbb{Z}_3 . (Python suggestion is uploaded on Canvas.)
- (4) Is it possible that a matrix M has different rank over different fields \mathbb{Z}_n ? (By calculation in (3))

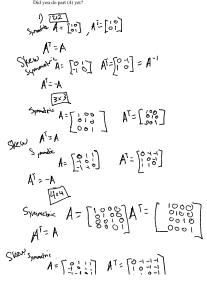


Question 12. (Solve a linear system over field \mathbb{Z}_7 .) Let $A = \begin{bmatrix} 3 & 1 & 4 \\ 5 & 2 & 6 \\ 0 & 5 & 2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

- (1) Calculation $\mathbf{rref}(A|\vec{b})$ over field \mathbb{Z}_7 .
- (2) Find solution of the linear system $A\vec{x} = \vec{b} \mod 7$.



- Give examples of symmetric and skew-symmetric 2 × 2, 3 × 3, and 4 × 4 matrices.
 What can you say about the main diagonal of a skew-symmetric matrix?
 Give an example of a matrix that is both symmetric and skew-symmetric.
 Prove that for any n × n matrix A, the matrices A + A^T, AA^T, and A^TA are symmetric and A A^T is skew-symmetry.
- skew-symmetric.
 (5) Prove that any n × n can be written as the sum of a symmetric and skew-symmetric matrices. Hint: Did you do part (4) yet?





Size Symmetric AT =
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

AT = $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

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= Int-25

5) 4= I3-20

 $\begin{array}{lll} \mathcal{H}_{3} = I_{3} - 2 & \sqrt[3]{3} & \text{ } \\ & = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - 2 \begin{bmatrix} \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} \end{bmatrix} \begin{bmatrix} \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} \end{bmatrix} & = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - 7 \begin{bmatrix} \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} \end{bmatrix} \begin{bmatrix} \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} & = \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} & = \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} & = \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} & = \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} & = \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} & = \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} & = \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} & = \begin{bmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12}$

Question 20. Mark each of the following functions $F: \mathbb{R} \to \mathbb{R}$ injective, surjective or bijective, as is most appropriate. (You may wish to draw the graph of the function in some cases.)

(a)
$$F(x) = x^2$$
; Surjective
(b) $F(x) = x^3/(x^2 + 1)$; by jective
(c) $F(x) = x(x - l)(x - 2)$; Surjective

(b)
$$F(x) = x^3/(x^2 + 1)$$
; b "ject we

(d)
$$F(x) = e^x + 2$$
. Suggestive

Question 16. (1) If A, B and C are $n \times n$ matrices and $ABC = I_n$, is each of the matrices invertible? What are

(2) Suppose A and B are $n \times n$ matrices. If AB is invertible, are both A and B are invertible?

Question 17. Provide a counter-example to the statement: For any 2×2 matrices A and B, $(AB)^2 = A^2B^2$.

$$A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} AB = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} (AB)^{2} = \begin{bmatrix} 16 & 8 \\ 56 & 37 \end{bmatrix}$$

$$A^{2} = \begin{bmatrix} 16 & 24 \\ 40 & 64 \end{bmatrix} B^{2} = \begin{bmatrix} 10 \\ 01 \end{bmatrix} AB^{2} \begin{bmatrix} 14 & 24 \\ 40 & 64 \end{bmatrix} AB = \begin{bmatrix} 16 & 8 \\ 40 & 64 \end{bmatrix}$$

Question 18. Find an example of a 2×2 nonidentity matrix whose transpose is its inverse.

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$Cos^{2}\theta + \sin^{2}\theta - \sin^{2}\theta & \cos \theta \end{bmatrix}$$

$$Cos^{2}\theta + \sin^{2}\theta - \sin^{2}\theta & \cos \theta \end{bmatrix}$$

$$Cos^{2}\theta + \sin^{2}\theta - \sin^{2}\theta & \cos \theta \end{bmatrix}$$

Question 12. (Solve a linear system over field
$$\mathbb{Z}_7$$
.) Let $A = \begin{bmatrix} 3 & 1 & 4 \\ 5 & 2 & 6 \\ 0 & 5 & 2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

- (1) Calculation $\mathbf{rref}(A|\vec{b})$ over field \mathbb{Z}_7 .
- (2) Find solution of the linear system $A\vec{x} = \vec{b} \mod 7$.

Question 13. (Use Python) Solve the linear system

$$\begin{cases} 3x_1 + 11x_2 + 19x_3 &= -2 \\ 7x_1 + 23x_2 + 39x_3 &= 10 \\ -4x_1 - 3x_2 - 2x_3 &= 6 \end{cases}$$

and write solutions in parametric vector forms.

Question 14. (Use Python) Solve the linear system

$$\begin{cases} 3x_1 + 6x_2 + 9x_3 + 5x_4 + 25x_5 &= 53 \\ 7x_1 + 14x_2 + 21x_3 + 9x_4 + 53x_5 &= 105 \\ -4x_1 - 8x_2 - 12x_3 + 5x_4 - 10x_5 &= 11 \end{cases}$$

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and write solutions in parametric vector forms.

 ${\bf Question~15.}~{\rm (Use~Python)}$ Solve the linear system

$$\begin{pmatrix} 2x_1 + 4x_2 + 3x_3 + 5x_4 + 6x_5 & = 37 \\ 4x_1 + 8x_2 + 7x_3 + 5x_4 + 2x_5 & = 74 \\ -2x_1 - 4x_2 + 3x_3 + 4x_4 - 5x_5 & = 20 \\ x_1 + 2x_2 + 2x_3 - x_4 + 2x_5 & = 26 \\ 5x_1 - 10x_2 + 4x_3 + 6x_4 + 4x_5 & = 24 \end{pmatrix}$$

and write solutions in parametric vector forms. (Hint: In Python, if you want precise value, use symbolic