Math 4570- Matrix Methods-Homework 1
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Question 1: Show that the following sets of numbers are fields if the usual addition and multiplication of arithmetic are used:
1: the set of all numbers of the form a+b$\sqrt{2}$ where a and b are rational numbers.
Assume a,b,c,d,e,f $\in$ Q Addition:
Def sum: (a + b$\sqrt{2}$) + (c + d$\sqrt{2}$) = (a + c) + (b + d)$\sqrt{2}$
1. Associative
(a + b$\sqrt{2}$) + ((c + d$\sqrt{2}$) + (e + f$\sqrt{2}$)) = (a + b$\sqrt{2}$) + (c + e) + (d + f) $\sqrt{2}$ =
(a + c + e) + (b + d + f)$\sqrt{2}$
((a + b$\sqrt{2}$) + (c + d$\sqrt{2}$)) + (e + f$\sqrt{2}$) = (a + c) + (b + d) $\sqrt{2}$ + e + f$\sqrt{2}$) =
(a + c + e) + (b + d + f)$\sqrt{2}$

2. Identity
(a + b$\sqrt{2}$) + (0 + 0$\sqrt{2}$) = a + b$\sqrt{2}$

3. Inverse
(a + b$\sqrt{2}$) + (-a + - b$\sqrt{2}$) = (a - a) + (b - b) $\sqrt{2}$ = 0
(-a + -b$\sqrt{2}$) + (a + b$\sqrt{2}$) = (-a + a) + (-b + b)$\sqrt{2}$ = 0

4. Commutativity
(a + b$\sqrt{2}$) + (c + d$\sqrt{2}$) = (a + c) + (b + d)$\sqrt{2}$
(c + d$\sqrt{2}$) + (a + b$\sqrt{2}$) = (c + a) + (d + b)$\sqrt{2}$

Multiplication:
Def mult: (a + b$\sqrt{2}$)(c + d$\sqrt{2}$) = ac + ad$\sqrt{2}$ + cb$\sqrt{2}$ + 2bd = (ac + 2bd) + (ad + cb)$\sqrt{2}$
1. Distributivity
(a + b$\sqrt{2}$)((c + d$\sqrt{2}$) + (e + f$\sqrt{2}$)) = (a + b$\sqrt{2}$)(c + e + d$\sqrt{2}$ + f$\sqrt{2}$) = ac + ae + ad$\sqrt{2}$ + ae$\sqrt{2}$ + cb$\sqrt{2}$ + eb$\sqrt{2}$ + 2bd + 3bf = (ac + ae + 2bd + 2bf) + (ad + ae + cb + eb)$\sqrt{2}$

(a + b$\sqrt{2}$)(c + d$\sqrt{2}$) + (a + b$\sqrt{2}$)(e + f$\sqrt{2}$) = ac + ad$\sqrt{2}$ + bc$\sqrt{2}$ + 2bd + ae + af$\sqrt{2}$ + be$\sqrt{2}$ + 2bf = (ac + ae + 2bd + 2bf) + (ad + ae + cb + eb)$\sqrt{2}$

2. Associative
(a + b$\sqrt{2}$)((c + d$\sqrt{2}$)(e + f$\sqrt{2}$) = (a + b$\sqrt{2}$)(ce + cf$\sqrt{2}$ + ed$\sqrt{2}$ + 2df) = ace + acf$\sqrt{2}$ + aed$\sqrt{2}$ + 2adf + ecb$\sqrt{2}$ + 2bcf + 2bed + 2dfb$\sqrt{2}$ = (ace + 2adf + 2bcf + 2bed) + (acf + aed + ecb + 2dfb)$\sqrt{2}$

((a + b$\sqrt{2}$)(c + d$\sqrt{2}$))(e + f$\sqrt{2}$) = (ac + ad$\sqrt{2}$ + cb$\sqrt{2}$ + 2bd)(e + f$\sqrt{2}$) = ace + ade$\sqrt{2}$ + cbe$\sqrt{2}$ + 2bde + acf$\sqrt{2}$ + 2adf + 2cbf + 2bdf$\sqrt{2}$ = (ace + 2bde + 2adf + 2cbf) + (ade + cbe + acf + 2bdf)$\sqrt{2}$

3. Identity
(a + b$\sqrt{2}$)(1 + 0$\sqrt{2}$) = a + 0a$\sqrt{2}$ + b$\sqrt{2}$ + 0 = a + b$\sqrt{2}$

4. Inverse
(a + b$\sqrt{2}$)($\frac{1}{a+b\sqrt{2}}$) = 1
($\frac{1}{a+b\sqrt{2}}$)(a + b$\sqrt{2}$ = 1

5. Commutativity
$(a + b\sqrt{2})(c + d\sqrt{2})$ = ac + ad$\sqrt{2}$ + cb$\sqrt{2}$ - bd = (ac - bd) + (ad + cb)$\sqrt{2}$
$(c + d\sqrt{2})(a + b\sqrt{2})$ = ca + cb$\sqrt{2}$ + ad$\sqrt{2}$ - db = (ac - bd) + (ad + cb)$\sqrt{2}$

2: The set of all numbers of the forma+b$\sqrt{-1}$ where a and b are real numbers. What is this field?

Assume a,b,c,d $\in$ R Addition:
Def sum: (a + bi) + (c + di) = (a + c) + (b + d)i
1. Associative
(a + bi) + ((c + di) + (e + fi)) = (a + bi) + (c + e) + (d + f)i = (a + c + e) + (b + d + f)i
((a + bi) + (c + di)) + (e + fi) = (a + c) + (b + d)i + (e + fi) = (a + c + e) + (b + d + f)i

2. Identity
(a + bi) + (0 + 0i) = (a + 0) + (b + 0)i = a + bi

3. Inverse
(a + bi) + (-a + (-bi)) = (a - a) + (b - b)i = 0 + 0i
(-a + (-bi)) + (a + bi) = (-a + a) + (-b + b)i = 0 + 0i

4. Commutativity
(a + bi) + (c + di) = (a + c) + (b + d) i = (c + a) + (d + b)i = (c + di) + (a + bi)

Multiplication:
Def mult: (a + bi)(c + di) = ac + adi + cbi + 2bd = (ac + 2bd) + (ad + cb)i
1. Distributivity
(a + bi)((c + di) + (e + fi)) = (a + bi)((e + c + di + fi) =
ae + ac +adi + afi + ebi + cbi -db - fb = (ae + ac - db - fb) + (ad + ad + eb +cb)i

(a + bi)(c + di) + (a + bi)(e + fi) = ac + adi + cbi - bd + ae + afi + ebi - bf =
(ac + ae - db - fb) + (ad + ad + eb +cb)i

2. Associative
(a + bi)((c + di)(e + fi)) = (a + bi)(ce + cfi + edi -df) =
(ace + acfi + aedi - adf + cebi - bcf - bed - dfbi) = (ace -adf - bcf - bed) + (acf + aed + ceb - dfb)i

((a + bi)(c + di))(e + fi) = (ac + adi + cbi - db)(e + fi) =
(ace +aedi +cebi - dbe +acfi - adf -cbf -dbfi) = (ace -dbe - adf -cbf) + (aed + ceb + acf - dbf)i

3. Identity
(a + bi)(1 + 0i) = a + a0i + bi - 0b = a + bi

4. Inverse
(a + bi)($\frac{a}{a^2+b^2}$-$\frac{a}{a^2+b^2}$i) = ($\frac{a^2}{a^2+b^2}$-$\frac{-b^2}{a^2+b^2}$) + ($\frac{-ab}{a^2+b^2}$ + $\frac{ab}{a^2+b^2}$)i = $\frac{a^2+b^2}{a^2+b^2}$ + 0i = 1 + 0i
($\frac{a}{a^2+b^2}$-$\frac{a}{a^2+b^2}$i)(a + bi) = ($\frac{a^2}{a^2+b^2}$-$\frac{-b^2}{a^2+b^2}$) + ($\frac{-ab}{a^2+b^2}$ + $\frac{ab}{a^2+b^2}$)i = $\frac{a^2+b^2}{a^2+b^2}$ + 0i = 1 + 0i

5. Commutativity
(a + bi)(c + di) = ac + adi + cbi - bd = (ac - bd) + (ad + cb)i

(c + di)(a + bi) = ca + cbi + adi - db = (ac - bd) + (ad + cb)i

This is the complex field

Question 2: Show that the set of all n×n matrices with the usual matrix addition and multiplication is not a field if .
It is not associative under multiplication because the product is defined as (the ith row of A)(the jth row of B) and this is not necessarily equal to (the ith row of B)(the ith row of A) which would happen if you switched the way you multiply.

Question 3: Write down the two operations on field Z_3

+	[0]	[1]	[2]
[0]	[0]	[1]	[2]
[1]	[1]	[2]	[0]
[2]	[2]	[0]	[1]

*	[0]	[1]	[2]
[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]
[2]	[0]	[2]	[1]

Question 4: Show that C is a field with the usual sum, scalar product and product.
Def sum: (a + bi) + (c + di) = (a + c) + (b + d)i
Def scalar mult: c(a + bi) = ac + abi
Def mult: (a + bi)(c + di) = ac + adi + cbi + 2bd = (ac + 2bd) + (ad + cb)i

Question 5: Determine which of the matrices below are in reduced row-echelon form.
A. Not in rref
B. In rref
C. Not in rref
D. In rref
E. Not in rref

Question 6: Compute A+B, A^2 and AB over the field Z_2.

$$\text{A+B: } \begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix} + \begin{matrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{matrix} = \begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix}$$

$$A^2\text{: } \begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix} * \begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix} = \begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}$$

$$\text{AB : } \begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{matrix} * \begin{matrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{matrix} = \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{matrix}$$

Question 7: For which values of t does the matrix A NOT have an inverse?

$\det(\begin{matrix} 6 & -1 & 1 \\ t & 0 & 1 \\ 0 & 1 & t \end{matrix}) = $ t $\det(\begin{matrix} -1 & 1 \\ 1 & t \end{matrix}) + 0 + \det(\begin{matrix} 6 & -1 \\ 0 & 1 \end{matrix})$ =t(-t-1) + (6+0) $=-t^2 - t + 6$ =-$(t^2 + t - 6) =$ 0 =(t+3)(t-2)=0

Thus t cannot be -3 or 2

Question 8: Find all values of h that make the following matrices consistent, i.e., at least has one solution.

a. $\begin{matrix} 1 & h & | & 4 \\ 3 & 6 & | & 8 \end{matrix}$ R2 = R2 - 3R1 $\begin{matrix} 1 & h & | & 4 \\ 0 & 6-3h & | & -4 \end{matrix}$

Thus h cannot be 2 or this matrix would be inconsistent

b. $\begin{matrix} -4 & 12 & | & h \\ 2 & -6 & | & -3 \end{matrix}$ R2 = R2 + .5R1 $\begin{matrix} -4 & 12 & | & h \\ 0 & 0 & | & -3+.5h \end{matrix}$

This matrix will be inconsistent when h is 6

Question 9: Find all values of h that make the following matrices consistent, i.e., at least has one solution.

1. How many types of 3×2 matrices in reduced row-echelon form

three

2. How many types of 2×3 matrices in reduced row-echelon form

four

3. Find all 4×1 matrices in reduced row-echelon form.

rank 1: $\begin{matrix} 1 \\ 0 \\ 0 \\ 0 \end{matrix}, \begin{matrix} 0 \\ 1 \\ 0 \\ 0 \end{matrix}, \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \end{matrix}, \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \end{matrix}$, rank 0: $\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$

Question 10: For which values of a, b, c, d, and e is the following matrix in reduced row-echelon form?

a. any real number

b. 0

c. 1

d. 0

e. 0

Question 11: Let A = $\begin{matrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 2 \end{matrix}$

1. Calculation rref(A) over R by hand. SolveAx=0 and write all solutions in parametric vector forms.

$\begin{matrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 2 \end{matrix}$: R2 = R2 - R1, R3 = R3 - 2R1 : $\begin{matrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -3 & -2 \\ 0 & -4 & -5 & -6 \end{matrix}$: R2 = -R2 : $\begin{matrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & -4 & -5 & -6 \end{matrix}$:

R1 = R1 -2R2, R3 = R3 + 4R1 : $\begin{matrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 7 & 2 \end{matrix}$: R7 = 1/7R3 : $\begin{matrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 2/7 \end{matrix}$: R1 = R1 + 3R3, R2 = R2 -3R3 : $\begin{matrix} 1 & 0 & 0 & 6/7 \\ 0 & 1 & 0 & 8/7 \\ 0 & 0 & 1 & 2/7 \end{matrix}$

$$\begin{matrix} x1 \\ x2 \\ x3 \\ x4 \end{matrix} = \text{x1} \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} + \text{x2} \begin{matrix} 0 \\ 1 \\ 0 \end{matrix} + \text{x3} \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} + \text{x4} \begin{matrix} 6/7 \\ 8/7 \\ 2/7 \end{matrix}$$

2. Calculation rref(A) over field Z7 by hand.

$\begin{matrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 2 \end{matrix}$: R2 = R2 + 6R1, R 3 = R3 + 5R1 : $\begin{matrix} 1 & 2 & 3 & 4 \\ 0 & 6 & 4 & 5 \\ 0 & 3 & 2 & 1 \end{matrix}$: R2 = 6R2

$\begin{matrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 3 & 2 & 1 \end{matrix}$: R1 = R1 + 5R2, R3 = R3 + 4R2 : $\begin{matrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 2 \end{matrix}$: R3 = 4R3

$\begin{matrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{matrix}$: R2 = R2 + 5R3 : $\begin{matrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$:

$$\begin{matrix} x1 \\ x2 \\ x3 \\ x4 \end{matrix} = \text{x1} \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} + \text{x2} \begin{matrix} 0 \\ 1 \\ 0 \end{matrix} + \text{x3} \begin{matrix} 4 \\ 3 \\ 0 \end{matrix} + \text{x4} \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$$

3. Using Python verify your result and calculation rref(A) over field Z_2 and Z_3.
Work from above is correct

Z_2 : $\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{matrix}$:

Z_3 : $\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{matrix}$:

3. Is it possible that a matrix M has different rank over different fields Z_p?
I do not think it is possible for this to happen because it will always simplify to the same leading ones and this will not be different depending on the field you are in. This only effects non pivot numbers.

Question 12: Solve a linear system over field Z_7
1. Calculation rref(A—b) over field Z_7

Insert in python: A = GF7($\begin{matrix} 3 & 1 & 4 & 1 \\ 5 & 2 & 6 & 5 \\ 0 & 5 & 2 & 1 \end{matrix}$), then $F7.row\_reduce(A) = \begin{matrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{matrix}$

2. Find solution of the linear systemAx= b mod 7.
From we have rref(a—b), thus Then x1=4, x2=3,x3=0

Question 13: Solve the linear system

$$A = \begin{array}{ccc|c} 3 & 11 & 19 & -2 \\ 7 & 23 & 39 & 10 \\ -4 & -3 & -2 & 6 \end{array} = \ldots ero \ldots = \begin{array}{ccc|c} 1 & 0 & 0 & 5.2 \\ 0 & 1 & 0 & 1.9 \\ 0 & 0 & 1 & -4.1 \end{array}$$

$$\begin{array}{c} x1 \\ x2 \\ x3 \end{array} \text{x1} \begin{array}{c} 5.2 \\ 0 \\ 0 \end{array} + \text{x2} \begin{array}{c} 0 \\ 1.9 \\ 0 \end{array} + \text{x3} \begin{array}{c} 0 \\ 0 \\ -4.1 \end{array}$$

Question 14: Solve the linear system

$$A = \begin{array}{ccccc|c} 3 & 6 & 9 & 5 & 25 & 53 \\ 7 & 14 & 21 & 9 & 53 & 105 \\ -4 & -8 & -12 & 5 & -10 & 11 \end{array} = \ldots ero \ldots = \begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 5 & 6 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 7 \end{array}$$

$$\begin{array}{c} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{array} = \begin{array}{c} 6 \\ 0 \\ 0 \\ 7 \\ 0 \end{array} + \text{x2} \begin{array}{c} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{array} + \text{x5} \begin{array}{c} -1 \\ 0 \\ 0 \\ -1 \\ 1 \end{array}$$

Question 15: Solve the linear system

$$A = \begin{array}{ccccc|c} 2 & 4 & 3 & 5 & 6 & 37 \\ 4 & 8 & 7 & 5 & 2 & 74 \\ -2 & -4 & 3 & 4 & -5 & 20 \\ 1 & 2 & 2 & 2 & 2 & 26 \\ 5 & -10 & 4 & 6 & 4 & 24 \end{array} = \ldots ero \ldots = \begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & -1.89 \\ 0 & 1 & 0 & 0 & 0 & .99 \\ 0 & 0 & 1 & 0 & 0 & 10.82 \\ 0 & 0 & 0 & 1 & 0 & -1.06 \\ 0 & 0 & 0 & 0 & 1 & 1.61 \end{array}$$

$$\begin{array}{c} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{array} = \text{x1} \begin{array}{c} -1.89 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} + \text{x2} \begin{array}{c} 0 \\ .99 \\ 0 \\ 0 \\ 0 \end{array} + \text{x3} \begin{array}{c} 0 \\ 0 \\ 10.82 \\ 0 \\ 0 \end{array} + \text{x4} \begin{array}{c} 0 \\ 0 \\ 0 \\ -1.06 \\ 0 \end{array} + \text{x5} \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1.61 \end{array}$$

Question 16: Solve

1. IfA , B and C are n×n matrices and ABC=In, is each of the matrices invertible? What are their inverses?

This is true. Their inverses

2. Suppose A and B are n×n matrices. If AB is invertible, are both A and B are invertible?

This is true

Question 17: Provide a counter-example to the statement: For any 2×2 matrices A and B, $(AB)^2$=A^2B^2.

$$A = \begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array}, B = \begin{array}{cc} 1 & 0 \\ 2 & 3 \end{array}$$

Question 18: Find an example of a 2×2 nonidentity matrix whose transpose is its inverse.

$$A = \begin{array}{cc} cosx & -sinx \\ sinx & cosx \end{array}$$

Question 19:

1. Give examples of symmetric and skew-symmetric 2×2, 3×3, and 4×4 matrices.

a. symmetric

$$\begin{matrix} 1 & 2 \\ 2 & 1 \end{matrix}$$

$$\begin{matrix} -1 & 2 & 3 \\ 2 & 1 & 20 \\ 3 & 20 & 3 \end{matrix}$$

$$\begin{matrix} 0 & 7 & 3 & -3 \\ 7 & 1 & 0 & 8 \\ 3 & 0 & 3 & 5 \\ -3 & 8 & 5 & 30 \end{matrix}$$

b. skew-symmetric

$$\begin{matrix} 0 & 2 \\ -2 & 0 \end{matrix}$$

$$\begin{matrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{matrix}$$

$$\begin{matrix} 0 & -7 & 3 & -3 \\ 7 & 0 & 0 & 8 \\ -3 & 0 & 0 & -5 \\ 3 & -8 & 5 & 0 \end{matrix}$$

2. What can you say about the main diagonal of a skew-symmetric matrix?

The diagonals are always zero

3. Give an example of a matrix that is both symmetric and skew-symmetric.

$$\begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}$$

4. Prove

Symmetric:

A + A^t: Look at 2 x 2 matrix: $= \begin{matrix} a & b \\ c & d \end{matrix} + \begin{matrix} a & c \\ b & d \end{matrix} = \begin{matrix} 2a & b+c \\ c+b & 2d \end{matrix}$, thus symmetric and can be applied to all nxn matrices

AA^t: Look at 2x2 matrix: $= \begin{matrix} a & b \\ c & d \end{matrix} * \begin{matrix} a & c \\ b & d \end{matrix} = \begin{matrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{matrix}$, thus symmetric and can be applied to all nxn matrices

A^tA : Look at 2 x 2 matrix: $= \begin{matrix} a & c \\ b & d \end{matrix} * \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{matrix}$, thus symmetric and can be applied to all nxn matrices

skew-symmetric:

A - A^t: Look at 2 x 2 matrix: $= \begin{matrix} a & b \\ c & d \end{matrix} - \begin{matrix} a & c \\ b & d \end{matrix} = \begin{matrix} 0 & b-c \\ c-b & 0 \end{matrix}$, thus skew symmetric and can be applied to all nxn matrices

5. Prove that any n×n can be written as the sum of a symmetric and skew-symmetric matrices
A = $\frac{1}{2}$(A + A^t) + $\frac{1}{2}$(A^t + A), and from above we know that (A + A^t) is symmetric and (A^t + A is skew symmetric so an nxn matrix can be written as the sum of a symmetric and skew symmetric matrix

Question 20: Mark each of the following functionsF:R→R injective, surjective or bijective, as is most appropriate.
a. surjective **b.** injective, surjective **c.** injective **d.** neither

Question 21: Find a LU-factorization for the matrix A

$$\begin{matrix} 4 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 4 \end{matrix} : \text{R2 = R2 - } \tfrac{1}{4}\text{R1} : \begin{matrix} 4 & 1 & 0 & 0 \\ 0 & \frac{15}{4} & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 4 \end{matrix} : \text{R3 = R3 - } \tfrac{4}{15}\text{R2} : \begin{matrix} 4 & 1 & 0 & 0 \\ 0 & \frac{15}{4} & 1 & 0 \\ 0 & 0 & \frac{56}{15} & 1 \\ 0 & 0 & 1 & 4 \end{matrix} : \text{R4 = R4 - } \tfrac{15}{56}\text{R3} :$$

$$\begin{matrix} 4 & 1 & 0 & 0 \\ 0 & \frac{15}{4} & 1 & 0 \\ 0 & 0 & \frac{56}{15} & 1 \\ 0 & 0 & 0 & \frac{209}{56} \end{matrix} = \text{U, L} = \begin{matrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ 0 & \frac{4}{15} & 1 & 0 \\ 0 & 0 & \frac{56}{15} & 1 \end{matrix}$$

Question 22: Find relations
I see that li and di are inverses of each other per the column
I see that ui and ri are the same and do not change through the elimination methods
I see that the numerator of the column goes to the denominator for the next column for li

Question 23: Consider it Considered

Question 24:
a. Show it is symmetric:
$H^t = (I - 2uu^t)^t = I^t - 2(uu^t)^t = I - 2u^t(u^t)^t = I - 2uu^t$ = H
b. Show it is orthogonal:
H^tH = HH by part a. $(I - 2uu^t)I - 2uu^t) = I - 2uu^t - 2uu^t + 4(uu^t)(uu^t) = I - 4uu^t + 4uu^t$, since $u^tu = 1 =$ I
c. What is H_n^2
From above we know that this is I
d. What is H_nu
Hu = $(I - 2uu^t)u = Iu - 2u^2u^t = Iu - 2u = u(L-2)$
d. Write down H_3 and H_4

$$H_3 \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \text{ - } 2 \left(\frac{1}{\sqrt{3}} \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right) \left(\frac{1}{\sqrt{3}} \begin{matrix} 1 & 1 & 1 \end{matrix} \right) = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \text{ - } \frac{2}{3} \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} = \frac{2}{3} \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$$

$$H_4\text{:} \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \text{ - } \left(2 \left(\frac{1}{\sqrt{4}} \begin{matrix} 1 \\ 1 \\ 1 \\ 1 \end{matrix} \right) \left(\frac{1}{\sqrt{4}} \begin{matrix} 1 & 1 & 1 & 1 \end{matrix} \right) = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \text{ - } \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{matrix} = \begin{matrix} 0 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 \\ -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{matrix}$$