The Beta Function*

Definition: $B(m,n) = \int_0^1 t^{m-1}(1-t)^{m-1}\,dt \qquad m > 0,\ n > 0$

Relationship with

Gamma Function: $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Properties:

$$B(m,n) = B(n,m)$$

$$B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\,d\theta$$

$$B(m,n) = \int_0^{\infty} \frac{t^{m-1}}{(1+t)^{m+n}}\,dt$$

$$B(m,n) = r^n(r+1)^m \int_0^1 \frac{t^{m-1}(1-t)^{n-1}}{(r+t)^{m+n}}\,dt$$