Derivatives*

In the following formulas u, v, w represent functions of x, while a, c, n represent fixed real numbers. All arguments in the trigonometric functions are measured in radians, and all inverse trigonometric and hyperbolic functions represent principal values.

1. $\dfrac{d}{dx}(a) = 0$

2. $\dfrac{d}{dx}(x) = 1$

3. $\dfrac{d}{dx}(au) = a\dfrac{du}{dx}$

4. $\dfrac{d}{dx}(u + v - w) = \dfrac{du}{dx} + \dfrac{dv}{dx} - \dfrac{dw}{dx}$

5. $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

6. $\dfrac{d}{dx}(uvw) = uv\dfrac{dw}{dx} + vw\dfrac{du}{dx} + uw\dfrac{dv}{dx}$

7. $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2} = \dfrac{1}{v}\dfrac{du}{dx} - \dfrac{u}{v^2}\dfrac{dv}{dx}$

8. $\dfrac{d}{dx}(u^n) = nu^{n-1}\dfrac{du}{dx}$

9. $\dfrac{d}{dx}\left(\sqrt{u}\right) = -\dfrac{1}{2\sqrt{u}}\dfrac{du}{dx}$

10. $\dfrac{d}{dx}\left(\dfrac{1}{u}\right) = -\dfrac{1}{u^2}\dfrac{du}{dx}$

11. $\dfrac{d}{dx}\left(\dfrac{1}{u^n}\right) = -\dfrac{n}{u^{n+1}}\dfrac{du}{dx}$

12. $\dfrac{d}{dx}\left(\dfrac{u^n}{v^m}\right) = \dfrac{u^{n-1}}{v^{m+1}}\left(nv\dfrac{du}{dx} - mu\dfrac{dv}{dx}\right)$

13. $\dfrac{d}{dx}(u^n v^m) = u^{n-1}v^{m-1}\left(nv\dfrac{du}{dx} + mu\dfrac{dv}{dx}\right)$

14. $\dfrac{d}{dx}[f(u)] = \dfrac{d}{du}[f(u)] \cdot \dfrac{du}{dx}$

* Let $y = f(x)$ and $\dfrac{dy}{dx} = \dfrac{d[f(x)]}{dx} = f'(x)$ define respectively a function and its derivative for any value x in their common domain. The differential for the function at such a value x is accordingly defined as

$$dy = d[f(x)] = \frac{dy}{dx}dx = \frac{d[f(x)]}{dx}dx = f'(x)\,dx$$

Each derivative formula has an associated differential formula. For example, formula 6 above has the differential formula

$$d(uvw) = uv\,dw + vw\,du + uw\,dv$$

15. $\dfrac{d^2}{dx^2}[f(u)] = \dfrac{df(u)}{du}\cdot\dfrac{d^2u}{dx^2} + \dfrac{d^2f(u)}{du^2}\cdot\left(\dfrac{du}{dx}\right)^2$

16. $\dfrac{d^n}{dx^n}[uv] = \binom{n}{0}v\dfrac{d^nu}{dx^n} + \binom{n}{1}\dfrac{dv}{dx}\dfrac{d^{n-1}u}{dx^{n-1}} + \binom{n}{2}\dfrac{d^2v}{dx^2}\dfrac{d^{n-2}u}{dx^{n-2}}$
$+\cdots+\binom{n}{k}\dfrac{d^kv}{dx^k}\dfrac{d^{n-k}u}{dx^{n-k}}+\cdots+\binom{n}{n}u\dfrac{d^nv}{dx^n}$

where $\binom{n}{r} = \dfrac{n!}{r'(n-r)'}$ the binomial coefficient, n non-negative integer and $\binom{n}{0} = 1$.

17. $\dfrac{du}{dx} = \dfrac{1}{\dfrac{dx}{du}}$ if $\dfrac{dx}{du} \neq 0$

18. $\dfrac{d}{dx}(\log_a u) = (\log_a e)\dfrac{1}{u}\dfrac{du}{dx}$

19. $\dfrac{d}{dx}(\log_e u) = \dfrac{1}{u}\dfrac{du}{dx}$

20. $\dfrac{d}{dx}(a^u) = a^u(\log_e a)\dfrac{du}{dx}$

21. $\dfrac{d}{dx}(e^u) = e^u\dfrac{du}{dx}$

22. $\dfrac{d}{dx}(u^v) = vu^{v-1}\dfrac{du}{dx} + (\log_e u)\,u^v\dfrac{dv}{dx}$

23. $\dfrac{d}{dx}(\sin u) = \dfrac{du}{dx}(\cos u)$

24. $\dfrac{d}{dx}(\cos u) = -\dfrac{du}{dx}(\sin u)$

25. $\dfrac{d}{dx}(\tan u) = \dfrac{du}{dx}(\sec^2 u)$

26. $\dfrac{d}{dx}(\cot u) = -\dfrac{du}{dx}(\csc^2 u)$

27. $\dfrac{d}{dx}(\sec u) = \dfrac{du}{dx}\sec u\cdot\tan u$

28. $\dfrac{d}{dx}(\csc u) = -\dfrac{du}{dx}\csc u\cdot\cot u$

29. $\dfrac{d}{dx}(\text{vers}\, u) = \dfrac{du}{dx}\sin u$

30. $\dfrac{d}{dx}(\arcsin u) = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad \left(-\dfrac{\pi}{2} \leq \arcsin u \leq \dfrac{\pi}{2}\right)$

31. $\dfrac{d}{dx}(\arccos u) = -\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \qquad (0 \le \arccos u \le \pi)$

32. $\dfrac{d}{dx}(\arctan u) = \dfrac{1}{1+u^2}\dfrac{du}{dx}, \qquad \left(-\dfrac{\pi}{2} < \arctan u < \dfrac{\pi}{2}\right)$

33. $\dfrac{d}{dx}(\operatorname{arc\,cot} u) = -\dfrac{1}{1+u^2}\dfrac{du}{dx}, \qquad (0 \le \operatorname{arc\,cot} u \le \pi)$

34. $\dfrac{d}{dx}(\operatorname{arc\,sec} u) = \dfrac{1}{u\sqrt{u^2-1}}\dfrac{du}{dx}, \qquad \left(0 \le \operatorname{arc\,sec} u < \dfrac{\pi}{2},\ -\pi \le \operatorname{arc\,sec} u < -\dfrac{\pi}{2}\right)$

35. $\dfrac{d}{dx}(\operatorname{arc\,csc} u) = -\dfrac{1}{u\sqrt{u^2-1}}\dfrac{du}{dx}, \qquad \left(0 < \operatorname{arc\,csc} u \le \dfrac{\pi}{2},\ -\pi < \operatorname{arc\,csc} u \le -\dfrac{\pi}{2}\right)$

36. $\dfrac{d}{dx}(\operatorname{arc\,vers} u) = \dfrac{1}{\sqrt{2u-u^2}}\dfrac{du}{dx}, \qquad (0 \le \operatorname{arc\,vers} u \le \pi)$

37. $\dfrac{d}{dx}(\sinh u) = \dfrac{du}{dx}(\cosh u)$

38. $\dfrac{d}{dx}(\cosh u) = \dfrac{du}{dx}(\sinh u)$

39. $\dfrac{d}{dx}(\tanh u) = \dfrac{du}{dx}(\operatorname{sech}^2 u)$

40. $\dfrac{d}{dx}(\coth u) = -\dfrac{du}{dx}(\operatorname{csch}^2 u)$

41. $\dfrac{d}{dx}(\operatorname{sech} u) = -\dfrac{du}{dx}(\operatorname{sech} u \cdot \tanh u)$

42. $\dfrac{d}{dx}(\operatorname{csch} u) = -\dfrac{du}{dx}(\operatorname{csch} u \cdot \coth u)$

43. $\dfrac{d}{dx}(\sinh^{-1} u) = \dfrac{d}{dx}[\log(u + \sqrt{u^2+1})] = \dfrac{1}{\sqrt{u^2+1}}\dfrac{du}{dx}$

44. $\dfrac{d}{dx}(\cosh^{-1} u) = \dfrac{d}{dx}[\log(u + \sqrt{u^2-1})] = \dfrac{1}{\sqrt{u^2-1}}\dfrac{du}{dx}, \qquad (u > 1, \cosh^{-1} u > 0)$

45. $\dfrac{d}{dx}(\tanh^{-1} u) = \dfrac{d}{dx}\left[\dfrac{1}{2}\log\dfrac{1+u}{1-u}\right] = \dfrac{1}{1-u^2}\dfrac{du}{dx}, \qquad (u^2 < 1)$

46. $\dfrac{d}{dx}(\coth^{-1} u) = \dfrac{d}{dx}\left[\dfrac{1}{2}\log\dfrac{u+1}{u-1}\right] = \dfrac{1}{1-u^2}\dfrac{du}{dx}, \qquad (u^2 > 1)$

47. $\dfrac{d}{dx}(\operatorname{sech}^{-1} u) = \dfrac{d}{dx}\left[\log\dfrac{1+\sqrt{1-u^2}}{u}\right] = -\dfrac{1}{u\sqrt{1-u^2}}\dfrac{du}{dx}, \qquad (0 < u < 1, \operatorname{sech}^{-1} u > 0)$

48. $\dfrac{d}{dx}(\operatorname{csch}^{-1} u) = \dfrac{d}{dx}\left[\log\dfrac{1+\sqrt{1+u^2}}{u}\right] = -\dfrac{1}{|u|\sqrt{1+u^2}}\dfrac{du}{dx}$

49. $\dfrac{d}{dq}\displaystyle\int_p^q f(x)\,dx = f(q), \qquad [p \text{ constant}]$

50. $\dfrac{d}{dp}\displaystyle\int_p^q f(x)\,dx = -f(p), \qquad [q \text{ constant}]$

51. $\dfrac{d}{da}\displaystyle\int_p^q f(x,a)\,dx = \int_p^q \frac{\partial}{\partial a}[f(x,a)]\,dx + f(q,a)\frac{dq}{da} - f(p,a)\frac{dp}{da}$