## RELATION OF ANGULAR FUNCTIONS IN TERMS OF ONE ANOTHER

## TRIGONOMETRIC FUNCTIONS

Function	$\sin \alpha$	cos α	tan α	cot a	sec a	csc α
$\sin \alpha$	$\sin \alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\frac{\tan\alpha}{\pm\sqrt{1+\tan^2\alpha}}$	$\frac{1}{\pm\sqrt{1+\cot^2\alpha}}$	$\frac{\pm\sqrt{\sec^2\alpha-1}}{\sec\alpha}$	$\frac{1}{\csc \alpha}$
$\cos \alpha$	$\pm\sqrt{1-\sin^2\alpha}$	$\cos \alpha$	$\frac{1}{\pm\sqrt{1+\tan^2\alpha}}$	$\frac{\cot \alpha}{\pm \sqrt{1 + \cot^2 \alpha}}$	$\frac{1}{\sec \alpha}$	$\frac{\pm\sqrt{\csc^2\alpha-1}}{\csc\alpha}$
$\tan \alpha$	$\frac{\sin \alpha}{\pm \sqrt{1 - \sin^2 \alpha}}$	$\frac{\pm\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$\tan \alpha$	$\frac{1}{\cot \alpha}$	$\pm\sqrt{\sec^2\alpha-1}$	$\frac{1}{\pm\sqrt{\csc^2\alpha-1}}$
$\cot \alpha$	$\frac{\pm\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	$\frac{\cos\alpha}{\pm\sqrt{1-\cos^2\alpha}}$	$\frac{1}{\tan \alpha}$	cot α	$\frac{1}{\pm\sqrt{\sec^2\alpha-1}}$	$\pm\sqrt{\csc^2 \alpha - 1}$
sec α	$\frac{1}{\pm\sqrt{1-\sin^2\alpha}}$	$\frac{1}{\cos \alpha}$	$\pm\sqrt{1+\tan^2\alpha}$	$\frac{\pm\sqrt{1+\cot^2\alpha}}{\cot\alpha}$	$\sec \alpha$	$\frac{\csc \alpha}{\pm \sqrt{\csc^2 \alpha} - 1}$
csc α	$\frac{1}{\sin \alpha}$	$\frac{1}{\pm\sqrt{1-\cos^2\alpha}}$	$\frac{\pm\sqrt{1+\tan^2\alpha}}{\tan\alpha}$	$\pm\sqrt{1+\cot^2\alpha}$	$\frac{\sec \alpha}{\pm \sqrt{\sec^2 \alpha - 1}}$	$\csc \alpha$

Note: The choice of sign depends upon the quadrant in which the angle terminates.

## HYPERBOLIC FUNCTIONS

Function	sinh x	$\cosh x$	tanh x
$\sinh x =$	$\sinh x$	$\pm\sqrt{\cosh^2 x - 1}$	$\frac{\tanh x}{\sqrt{1-\tanh^2 x}}$
$\cosh x =$	$\sqrt{1+\sinh^2 x}$	$\cosh x$	$\frac{1}{\sqrt{1-\tanh^2 x}}$
$\tanh x =$	$\frac{\sinh x}{\sqrt{1+\sinh^2 x}}$	$\pm \frac{\sqrt{\cosh^2 x - 1}}{\cosh x}$	tanh x
$\operatorname{cosech} x =$	$\frac{1}{\sinh x}$	$\pm \frac{1}{\sqrt{\cosh^2 x - 1}}$	$\frac{\sqrt{1-\tanh^2 x}}{\tanh x}$
$\operatorname{sech} x =$	$\frac{1}{\sqrt{1+\sinh^2 x}}$	$\frac{1}{\cosh x}$	$\sqrt{1-\tanh^2 x}$
$\coth x =$	$\frac{\sqrt{1+\sinh^2 x}}{\sinh x}$	$\frac{\pm \cosh x}{\sqrt{\cosh^2 x - 1}}$	$\frac{1}{\tanh x}$
Function	cosech x	sech x	coth x
Function $\sinh x =$	$\frac{1}{\operatorname{cosech} x}$		$\frac{\coth x}{\pm 1}$ $\frac{\pm 1}{\sqrt{\coth^2 x - 1}}$
		sech x	
$\sinh x =$	$\frac{1}{\operatorname{cosech} x}$	$\pm \frac{\operatorname{sech} x}{\frac{\sqrt{1 - \operatorname{sech}^2 x}}{\operatorname{sech} x}}$	$\frac{\pm 1}{\sqrt{\coth^2 x - 1}}$
$\sinh x = \cosh x =$	$\frac{1}{\operatorname{cosech} x}$ $\pm \frac{\sqrt{\operatorname{cosech}^2 x + 1}}{\operatorname{cosech} x}$	$\pm \frac{\text{sech } x}{\pm \frac{\sqrt{1 - \operatorname{sech}^2 x}}{\operatorname{sech } x}}$ $\pm \frac{1}{\operatorname{sech } x}$	$\frac{\pm 1}{\sqrt{\coth^2 x - 1}}$ $\pm \frac{\coth x}{\sqrt{\coth^2 x - 1}}$
$\sinh x = \cosh x = \tanh x =$	$\pm \frac{1}{\operatorname{cosech} x}$ $\pm \frac{\sqrt{\operatorname{cosech}^2 x + 1}}{\operatorname{cosech} x}$ $\frac{1}{\sqrt{\operatorname{cosech}^1 x + 1}}$	$\pm \frac{\sqrt{1 - \operatorname{sech}^2 x}}{\frac{1}{\operatorname{sech} x}}$ $\pm \sqrt{1 - \operatorname{sech}^2 x}$ $\pm \sqrt{1 - \operatorname{sech}^2 x}$	$\frac{\pm 1}{\sqrt{\coth^2 x - 1}}$ $\pm \frac{\coth x}{\sqrt{\coth^2 x - 1}}$ $\frac{1}{\coth x}$

Whenever two signs are shown, choose + sign if x is positive, - sign if x is negative.