

LABORATORY EXERCISE 5: ANOVA OF THE COMPLETELY RANDOMIZED DESIGN (CRD)

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Score:
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Objectives:

At the end of this exercises you should be able to:

1. Check the CRD model assumptions.
2. Perform ANOVA for CRD.
3. Perform multiple comparison tests of means.

Materials

1. Statistical software (Excel, PAST, R)
2. Ruler Activity

1. A manufacturer suspects that the batches of raw material furnished by his supplier differ significantly in calcium content. There are a large number of batches currently in the warehouse. Five of these are randomly selected for study. A chemist makes five determinations on each batch and obtains the following data:

Batch 1	Batch 2	Batch 3	Batch 4	Batch 5
23.46	23.59	23.51	23.28	23.29
23.48	23.46	23.64	23.40	23.46
23.56	23.42	23.46	23.37	23.37
23.39	23.49	23.52	23.46	23.32
23.40	23.50	23.49	23.39	23.38

- a. Present the plot of the residuals against the treatment means. Compute for the ratio between the largest and smallest variance. From the plot, ratio, and Levene's test, tell whether the homoscedasticity assumption has been met or not.

Compute for the ratio between the largest and smallest variance.

	Batch 1	Batch 2	Batch 3	Batch 4	Batch 5
N	5	5	5	5	5
Min	23.39	23.42	23.46	23.28	23.29
Max	23.56	23.59	23.64	23.46	23.46
Sum	117.29	117.46	117.62	116.9	116.82
Mean	23.458	23.492	23.524	23.38	23.364
Std. error	0.03072458	0.02817801	0.03075711	0.02915476	0.02908608
Variance	0.00472	0.00397	0.00473	0.00425	0.00423
Stand. dev	0.06870226	0.06300794	0.068775	0.06519202	0.06503845
Median	23.46	23.49	23.51	23.39	23.37
25 prcntil	23.395	23.44	23.475	23.325	23.305
75 prcntil	23.52	23.545	23.58	23.43	23.42
Skewness	0.7225357	0.8922949	1.609563	-0.7218494	0.6037527
Kurtosis	-0.07891051	1.530179	3.130439	1.767474	0.2949214
Geom. mean	23.45792	23.49193	23.52392	23.37993	23.36393
Coeff. var	0.2928735	0.2682102	0.292361	0.2788367	0.2783704

Based on our table, batch 3 have the largest variance and batch 2 have the smallest variance, so we will compute this by dividing: largest variance/smallest variance.

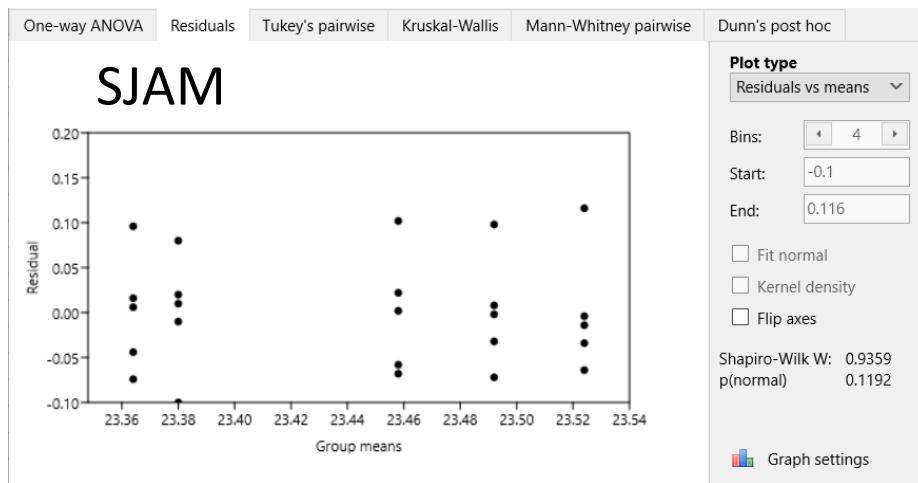
Computation:

SJAM	0.00473 / 0.00397 =
1.19143576826	

Conclusion:

First assumption is met because the results of the ratio between the largest and smallest variance is 1.191, which does not exceed 3.0.

Plot



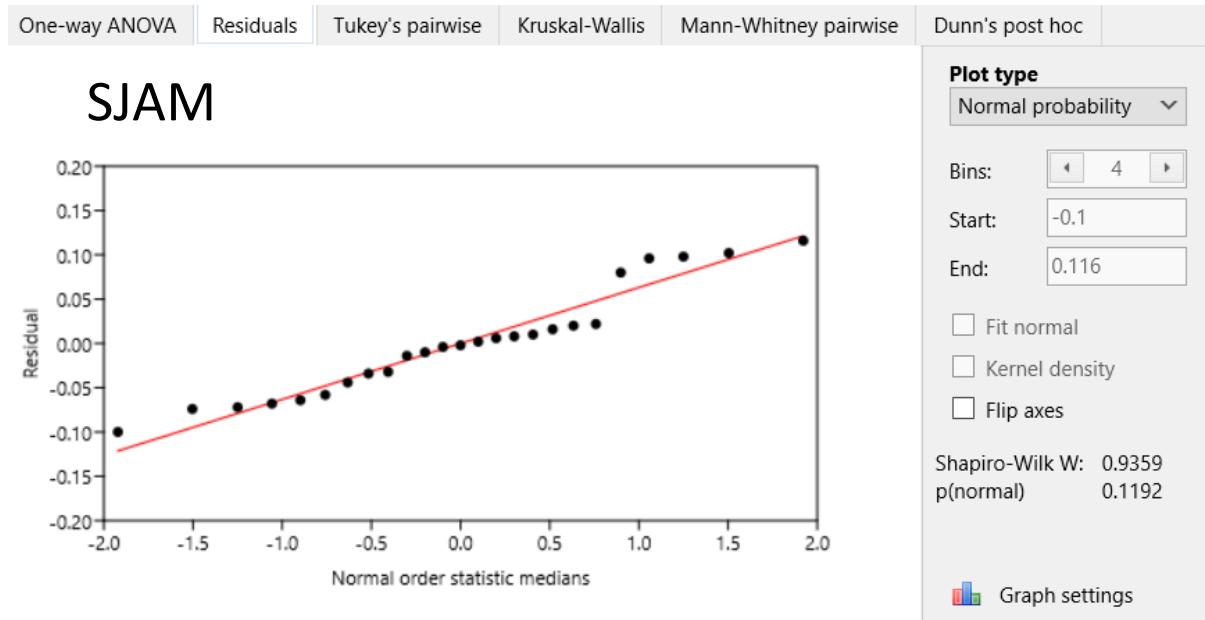
The residual plot shows no clear pattern, which suggests that the variances across the treatment means are uniform, this means that homoscedasticity assumption has been met in residual plot.

Lavene's test:

One-way ANOVA	Residuals	Tukey's pairwise	Kruskal-Wallis	Mann-Whitney pairwise	Dunn's post hoc				
Test for equal means									
	Sum of sqrs	df	Mean square	F	p (same)				
Between groups:	0.096976	4	0.024244	5.535	0.003626				
Within groups:	0.0876	20	0.00438		Permutation p (n=99999)				
Total:	0.184576	24			0.00316				
Components of variance (only for random effects):									
Var(group):	0.0039728	Var(error):	0.00438	ICC:	0.475625				
<i>omega</i> ² :	0.4205								
SJAM									
Levene's test for homogeneity of variance, from means <i>p</i> (same): 0.9984									
Levene's test, from medians <i>p</i> (same): 0.9978									
Welch F test in the case of unequal variances: F=4.633, df=9.997, p=0.02247									

To formally check the homogeneity of variance assumption, we used the Lavene's test. If the p value is greater than the level of significance, then the variances are homogenous. Since the p value is 0.9984 >0.5, the homogeneity of variances assumptions is met using Lavene's test.

b. Check the normality assumption by showing and describing the normal probability plot of the residuals and the Shapiro-Wilk test. From the plot and the formal test, tell whether the normality assumption has been met or not.



Since points in the normal probability plot nearly follows the diagonal red line and the residual plot has no obvious pattern, we can say that there is no violation in the normality assumption. To formally test this we use the Shapiro-Wilk test and that the p-value should be greater than the significance level. Since the $0.9984 > 0.5$, the homogeneity of variances assumptions is met.

c. Is there significant variation in calcium content from batch to batch? Perform the step by step procedure of the hypothesis testing. Use $\alpha = 0.05$. d. If necessary, perform Tukey's test. Write the summary table showing the mean per treatment and the appropriate superscripts. Write a short description about the test results.

Experiment Unit: Furnishes raw materials

Factor of Interest: Calcium content

Treatment Levels: Batch 1, Batch 2, Batch 3, Batch 4, Batch 5 (batches in a warehouse)

Blocking: None (CRD has no blocking factors)

Responsive variable: Each Batch's Calcium Content

1. Hypotheses:

H_0 : The mean calcium content is the same for all batches.

H_a : At least one batch produced a significantly different mean calcium content.

2. Decision Rule:

Reject H_0 if p-value < 0.05, otherwise, fail to reject H_0 .

Computation:

One-way ANOVA	Residuals	Tukey's pairwise	Kruskal-Wallis	Mann-Whitney pairwise	Dunn's post hoc
Test for equal means					
SJAM					
	Sum of sqrs	df	Mean square	F	
Between groups:	0.096976	4	0.024244	5.535	p (same) 0.003626
Within groups:	0.0876	20	0.00438		Permutation p (n=99999)
Total:	0.184576	24			0.00316
Components of variance (only for random effects):					
Var(group):	0.0039728	Var(error):	0.00438	ICC:	0.475625
omega²:	0.4205				
Levene's test for homogeneity of variance, from means			p (same):	0.9984	
Levene's test, from medians			p (same):	0.9978	
Welch Ftest in the case of unequal variances: F=4.633, df=9.997, p=0.02247					

Our p-value is 0.003626 which is less than 0.05, therefore we reject H_0 .

Conclusion:

There is sufficient evidence to conclude that at least one batch contains a significantly different mean calcium content, at 5% level of significance.

d. If necessary, perform Tukey's test. Write the summary table showing the mean per treatment and the appropriate superscripts. Write a short description about the test results.

Hypothesis Testing Using Tukey Test

State the null and alternative hypotheses.

$H_0: \bar{x}_i = \bar{x}_j$ (the mean of treatment i is equal to mean of treatment j)

$H_a: \bar{x}_i \neq \bar{x}_j$ (the mean of treatment i is not equal to mean of treatment j)

where $i=1, 2, \dots, k$, $j=1, 2, \dots, k$, and $i \neq j$ (k in the number of treatments)

Decision Rule

Reject if p-value > 0.05, otherwise fail to reject.

Compute the test statistics

	One-way ANOVA	Residuals	Tukey's pairwise	Kruskal-Wallis	Mann-Whitney pairwise	Dunn's post hoc	
	Tukey's Q below the diagonal, p(same) above the diagonal. Significant comparisons are pink.			SJAM	Copenhaver-Holland 1988		

	Batch 1	Batch 2	Batch 3	Batch 4	Batch 5
Batch 1		0.9238	0.528	0.3676	0.2039
Batch 2	1.149		0.9378	0.09375	0.04368
Batch 3	2.23	1.081		0.01942	0.008378
Batch 4	2.635	3.784	4.865		0.9951
Batch 5	3.176	4.325	5.406	0.5406	

Using the results from Tukey's pairwise test in PAST, we know that the highlighted cells in the table represent significant comparisons. The p-values in these cells are lower than 0.05, indicating that the highlighted pairs of batches have a statistically significant difference in mean values.

Batches (pair)	p-value	Decision
1&2	0.9238	p-value > 0.05, fail to reject
1&3	0.528	p-value > 0.05, fail to reject
1&4	0.3676	p-value > 0.05, fail to reject
1&5	0.2039	p-value > 0.05, fail to reject

2&3	0.9378	p-value > 0.05, fail to reject
2&4	0.09375	p-value > 0.05, fail to reject
2&5	0.04368	p-value < 0.05, reject the null
3&4	0.01942	p-value < 0.05, reject the null
3&5	0.008378	p-value < 0.05, reject the null
4&5	0.9951	p-value > 0.05, fail to reject

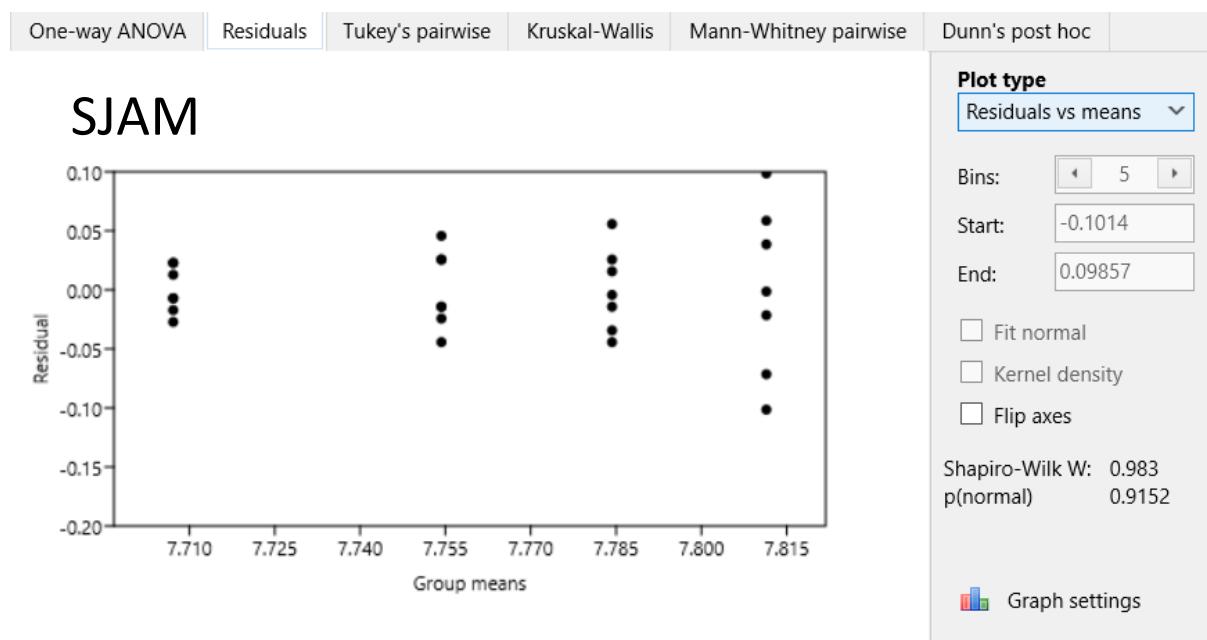
Conclusion:

In summary, the results of all our comparisons indicate significant differences in calcium content between the pairs of batches 2 and 5, 3 and 4, and 3 and 5. These three pairs have statistically different means, while the other batch comparisons do not show significant differences.

2. A limnologist obtained seven containers of water from each of the four pounds. The pH of each water sample was measured.

Pond 1	Pond 2	Pond 3	Pond 4
7.68	7.8	7.74	7.71
7.69	7.73	7.75	7.87
7.7	7.74	7.77	7.74
7.7	7.74	7.78	7.79
7.72	7.78	7.8	7.81
7.73	7.78	7.81	7.85
7.73	7.71	7.84	7.91

- a. Present the plot of the residuals against the treatment means. Compute for the ratio between the largest and smallest variance. From the plot, ratio, and Levene's test, tell whether the homoscedasticity assumption has been met or not.



	Pond 1	Pond 2	Pond 3	Pond 4
N	7	7	7	7
SJAM				
Min	7.68	7.71	7.74	7.71
Max	7.73	7.8	7.84	7.91
Sum	53.95	54.28	54.49	54.68
Mean	7.707143	7.754286	7.784286	7.811429
Std. error	0.007468756	0.01231668	0.01324803	0.02694161
Variance	0.0003904762	0.001061905	0.001228571	0.005080952
Stand. dev	0.01976047	0.03258688	0.03505098	0.0712808
Median	7.7	7.74	7.78	7.81
25 prcntil	7.69	7.73	7.75	7.74
75 prcntil	7.73	7.78	7.81	7.87
Skewness	0.007405768	0.1519223	0.3450124	-0.1299295
Kurtosis	-1.703986	-1.460528	-0.6375338	-1.053139
Geom. mean	7.707121	7.754227	7.784218	7.81115
Coeff. var	0.2563916	0.4202435	0.4502787	0.9125194

ratio

SJAM $0.005080952 \div 0.0003904762 =$

13.01219382897088

One-way ANOVA Residuals Tukey's pairwise Kruskal-Wallis Mann-Whitney pairwise Dunn's post hoc

Test for equal means

SJAM

	Sum of sqrs	df	Mean square	F	p (same)
Between groups:	0.0419143	3	0.0139714	7.2	0.001305
Within groups:	0.0465714	24	0.00194048		Permutation p (n=99999)
Total:	0.0884857	27			0.00149

Components of variance (only for random effects):

Var(group): 0.00171871 Var(error): 0.00194048 ICC: 0.469697

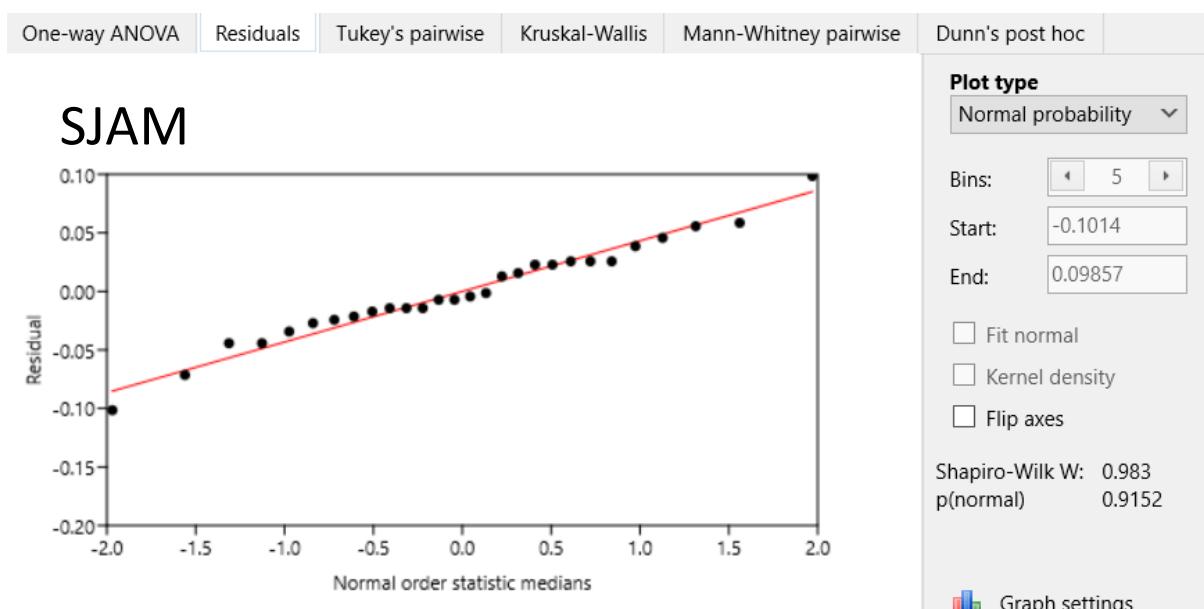
*omega*²: 0.3991

Levene's test for homogeneity of variance, from means p (same): 0.02023
 Levene's test, from medians p (same): 0.03914

Welch F test in the case of unequal variances: F=11.52, df=12.5, p=0.000661

As demonstrated in the images above, the ratio of the largest to the smallest treatment variance is 13.01, which is greater than 3. The residuals plotted against the treatment means display a funnel shape. Additionally, Levene's test indicated that the variances are not homogeneous, with a p-value of 0.02023, which is below the 0.05 threshold. Therefore, the assumption of homoscedasticity is not satisfied.

- b. Check the normality assumption by showing and describing the normal probability plot of the residuals and the Shapiro-Wilk test. From the plot and the formal test, tell whether the normality assumption has been met or not.



Since points in the normal probability plot nearly follows the diagonal red line and the residual plot has no obvious pattern, we can say that there is no violation in the normality assumption. To formally test this we use the Shapiro-Wilk test and that the p-value should be greater than the significance level. Since the 0.983 >0.5, the homogeneity of variances assumptions is met.

In this case, since one of the assumptions is not met, we will proceed with Kruskal Wallis.

Experimental Unit: Individual Water Container

Factor of Interest: Pond

Treatment Levels: Pond 1,2,3,4

Blocking: None (CRD has no blocking factors)

Response Variable: pH level of the water

Hypotheses

H_0 : The median pH level of the water is the same across all ponds. ($Md_1 = Md_2 = \dots = Md_k$ given k treatments)

H_a : At least one pond contains a statistically significant different pH level of water.

Decision Rule

Reject H_0 if p-value < a (0.05). Otherwise, fail to reject H_0 .

One-way ANOVA Residuals Tukey's pairwise **Kruskal-Wallis** Mann-Whitney pa

Kruskal-Wallis test for equal medians

$H (ch^2)$: 14.83
 H_C (tie corrected): 14.92
 p (same): 0.001888

SJAM

There is a significant difference between sample medians

Since p-value (0.001888) < 0.05, reject H_0 .

Conclusion:

The Kruskal-Wallis test results indicate that at least one pond has a significantly different pH level of water at the 5% significance level. Given the significant result, we will proceed with the post-hoc analysis.

POST HOC ANALYSIS

Several-sample tests					
SJAM					
	One-way ANOVA	Residuals	Tukey's pairwise	Kruskal-Wallis	Mann-Whitney pairwise
Raw p values, uncorrected significance					
	Pond 1	Pond 2	Pond 3	Pond 4	
Pond 1		0.01001	0.002117	0.007162	
Pond 2	0.01001		0.1378	0.1079	
Pond 3	0.002117	0.1378		0.4423	
Pond 4	0.007162	0.1079	0.4423		

Medians to be Compared	Stepwise Bonferroni corrected pvalues from Mann-Whitney pairwise tests	Decision	Conclusion
Md ₁ vs Md ₃	p-value = 0.002117 x 6 = 0.012702	Since 0.012702 < 0.05, reject H ₀	Statistically different medians
Md ₁ vs Md ₄	p-value = 0.002117 x 5 = 0.010585	Since 0.010585 < 0.05, reject H ₀	Statistically different medians
Md ₁ vs Md ₂	p-value = 0.01001 x 4 = 0.04004	Since 0.04004 < 0.05, reject H ₀	Statistically different medians

Md ₂ vs Md ₄	p-value = 0.1079 x 3 = 0.3237	Since 0.3237 > 0.05, fail to reject H ₀	Not statistically different medians
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The remaining pairwise comparison tests, specifically Md₂ vs. Md₃ and Md₃ vs. Md₄, are statistically insignificant.

Summary of the stepwise Bonferroni corrected p-values from Mann-Whitney pairwise test results

Pond	Median
1	7.70 ^a
2	7.74 ^b
3	7.78 ^b
4	7.79 ^b

Means with the same letter are not statistically different.

Conclusion:

	Pond 1	Pond 2	Pond 3	Pond 4
N	7	7	7	7
Min	7.68	7.71	7.74	7.71
Max	7.73	7.8	7.84	7.91
Sum	53.95	54.28	54.49	54.68
Mean	7.707143	7.754286	7.784286	7.811429
Std. error	0.007468756	0.01231668	0.01324803	0.02694161
Variance	0.0003904762	0.001061905	0.001228571	0.005080952
Stand. dev	0.01976047	0.03258688	0.03505098	0.0712808
Median	7.7	7.74	7.78	7.81
25 prcntil	7.69	7.73	7.75	7.74
75 prcntil	7.73	7.78	7.81	7.87
Skewness	0.007405768	0.1519223	0.3450124	-0.1299295
Kurtosis	-1.703986	-1.460528	-0.6375338	-1.053139
Geom. mean	7.707121	7.754227	7.784218	7.81115
Coeff. var	0.2563916	0.4202435	0.4502787	0.9125194

Make a Decision

Based on the results from the Mann-Whitney pairwise test in PAST, the highlighted cells in the table correspond to significant comparisons. The p-values in these cells are below 0.05, indicating that the highlighted batch pairs have a statistically significant difference in their mean values.

Summary of the stepwise Bonferroni corrected p-values from Mann-Whitney pairwise test results

Pond	Median
1	7.70 ^a
2	7.74 ^b
3	7.78 ^b
4	7.79 ^b

Means with the same letter are not statistically different.

Pond (pair)	p-value	Decision
1&2	0.01001	p-value < 0.05, reject the null
1&3	0.002117	p-value < 0.05, reject the null
1&4	0.007162	p-value < 0.05, reject the null
2&3	0.1378	p-value > 0.05, fail to reject
2&4	0.1079	p-value > 0.05, fail to reject
3&4	0.4423	p-value > 0.05, fail to reject

Conclusion:

The Stepwise Bonferroni corrected p-values from the Mann-Whitney pairwise test reveal significant differences in the pH levels of water between ponds 1 and 2, 1 and 3, and 1 and 4. These pairs have statistically different medians, while no significant differences are observed between the other pond pairs. Thus, we can conclude that the medians of ponds 1 and 2, 1 and 3, and 1 and 4 differ significantly at the 5% significance level. In contrast, the differences between ponds 2 and 3, and 3 and 4 are not statistically significant.