2023-24 EVEN SEMESTER TUTORIAL CONTINUOUS EVALUATION

SUBJECTCODE: 22MT2005
PROBABILITY STATISTICS AND QUEUING THEORY

Tutorial 1:

- Discussion on SAS software for solving PSQT course problems
- Demonstrate Probability: Sample Space and Events

Date of the Session: //15/12/2023 **Time of the Session:** 11 to 12:50.

Learning outcomes:

- Familiarity with SAS software.
- Outcome related to second session

Experiment 1

1. A pair of fair dice is tossed. Find the probability of getting
   (i) a total of 8 (ii) at most a total of 5
   (iii) Same number of dots on both dice (iv) at least a total of 10

   Generate the solution also using sas code

Solution: Total possible outcomes = 36.

i) posible outcomes are (2,6), (6,2), (3,5), (5,3), (4,4)

P(a total of 8) = 5/36.

ii) possible outcomes =

(1,1), (1,2), (2,1),

(1,3), (3,1), ~~(2,2)~~,

(2,3), (3,2), (1,4), (4,1).

$P = \boxed{10/36}$

(iii) Same no. of dots on both dice.

$\boxed{P = 6/36}$

(iv) possible outcomes = (4,6), (6,4)
(5,5), (5,6),
(6,5), (6,6).

$\boxed{P = 6/36}$

SAS code:-

```
data dice simulation:-
  do die1 = 1 to 6;
  do die2 = 1 to 6;
  total = die1 + die2;
/* probability of getting a total
   of 8 */
if total = 8 then event_i = 1;
/* probability of getting at most
   a total of 5 */
if total <= 5 then event_ii = 1;
/* probability of getting same no
   on both dice */
if die1 = die2 then event_iii = 1;
(iv) probability of getting at
   least a total of 10 */
if total >= 10 then event_iv = 1
output;
end;
end;
run;
```

2. A fair die is thrown. Write the set of outcomes associated with the following events:

(i) A: a number less than 7

(ii) B: a number greater than 7

(iii) C: a multiple of 3

(iv) D: a number less than 4

(v) E: an even number greater than 4

(vi) F: a number not less than 3

Generate the solution using SAS Code

Solution: (i) $A = \{1, 2, 3, 4, 5, 6\}$.

(ii) $B = \{\phi\}$.

(iii) $C = \{3, 6\}$.

(iv) $D = \{1, 2, 3\}$.

(v) $E = \{6\}$.

(vi) $F = \{3, 4, 5, 6\}$.

3. In a poker hand consisting of 5 cards, find the probability of holding
(i) 3 aces (ii) 4 hearts and 1 club (iii) Cards of same suit (iv) 2 aces and 3 jacks
Solution:

(i) $\dfrac{4C_3 * 48C_2}{52C_5}$

(iv) $\dfrac{4C_2 \times 4C_3}{52C_5}$.

(ii) $\dfrac{13C_4 \times 13C_1}{52C_5}$

(iii) $\dfrac{13C_5 + 13C_5 + 13C_5 + 13C_5}{52C_5}$

Experiment 2
Discussion on Statistical SAS software for solving PSQT course problems.

Solution:

→ It is an integrated system of software products provided by the SAS Institute.

→ It enables program to perform Satisfied analysis report writing, e conmetrics on data mining, information interval and data.

5) Weather Prediction

Background: Raj is working for a meteorological department, and his task is to predict the weather for a particular city. Raj has gathered historical data on weather conditions, and he wants to use probability concepts to make predictions help me doing this.

Solution: (i) A: Getting no heads.
B: Getting no Tails.

$A = \{T, T, T\}$. $B = \{H, H, H\}$ are disjoint.

$A = \{H, H, H\}$.

$B = \{HHH, HHT, HTH, THH\}$.

$A \cap B = \{HHH\} \in \phi$

6).The probability that an American industry will locate in shanghai, China is 0.7, the probability that it will locate in Beijing, China is 0.4, and the probability it will locate in either shanghai or Beijing or both is 0.8. Determine the probability that the industry will locate

i) in both cities ii) In neither city iii) in only one city iv) only in Shanghai
v) Only in Beijing

Solution:

(i) The probability that an American industry will locate
Both cities.

P(Shanghai and Beijing) = 0.7 x 0.4 x 0.8
= 0.3.

(ii) in Neither city.

P(neither shanghai and Beijing) = 1-0.8
= 0.2

(iii) P(only one city) = P(neither shanghai (or) Biejing)
= P(only in Biejing)
= 0.5-0.4
= 0.1.

(iv) only in Beijing

P(only in Beijing) = 0.5-0.1
= 0.4.

(v) only in shanghai.

P(only in shanghai) = 0.8 - 0.1
= 0.7.

Experiment 3

7.Following the example of information on students' performance in three different university courses, write an SAS code to 'print' the data of the marks of students obtained in different courses. The data should contain the course name, maximum mark, and mark obtained. The data includes the course name, the maximum mark achievable, and the mark obtained by each student

Solution:

```
data studentmarks;
  input studentID courseName $maxmarks
    markobtained;
   datalines;
1 Math 100 85
2 math 100 92
3 math 100 78
4 physics 90 88
5 physics 90 76
6 physics 90 82
7 chemistry 95 94
8 Chemistry 95 89
9 chemistry 95 91
;
run;
proc print data=studentmarks;
title 'students performance in
            Different courses';
    run;
```

Output:-

Math1 → 100 → 85

math2 → 100 → 92

math3 → 100 → 78

physics → 90 → 88

physics2 → 90 → 76

chemistry → 95 → 94.

8. The following frequency table shows the classification of 58 landfills in a state according to their concentration of the three hazardous chemicals arsenic, barium, and mercury.

		Barium			
		High		Low	
		Mercury		Mercury	
		High	Low	High	Low
Arsenic	High	1	3	5	9
	Low	4	8	10	18

If a landfill is selected at random, find the probability that it has

i) a high concentration of mercury;
ii) a high concentration of barium and low concentrations of arsenic and mercury;
iii) high concentrations of any two of the chemicals and low concentration of the third;
iv) A high concentration of any one of the chemicals and low concentrations of the other two.

Solution:

(i) A high concentration of mercury

$= \boxed{0.5}$

(ii) A high concentration of barium and low concentration of arsenic and mercury.

$\Rightarrow \boxed{0.42}$

(iii) high concentration of any two of the chemicals and low concentration of third

$\Rightarrow \boxed{0.39}$

(iv) A high con. of any one of the chemicals and low concentrations of the other two.

is $\boxed{0.65}$

VIVA QUESTIONS:

1. **What is a sample space in probability theory?**

A:- set of all possible outcomes in a random experiment.

2. **How do you calculate the number of outcomes in a sample space?**

A):- Using counting principle, Fractionals (or) combinations based on the nature of the experiment.

3. **Define a complementary event in probability.**

A:- Includes outcomes not in given event, its prob. is 1- minus the probability of original event.

4. **Define different types of events in the context of probability theory.**

Simple Event: single outcome.
Compound Event:- multiple outcomes.
mutually Exclusive - cannot happen together.
Independent:- one Event doesn't affect the other.

(For Evaluators use only)

Comment of the Evaluator (if Any)	Evaluator's Observation
	Marks Secured: 50 out of 50 Full Name of the Evaluator: Signature of the Evaluator: [signature] 10/12/24 Date of Evaluation

Experiment #	2	STUDENT ID :	2200032927
Date :	22/12/2023	STUDENT NAME :	K. Kavya.

SUBJECTCODE: 22MT2005

PROBABILITY STATISTICS AND QUEUING THEORY

Tutorial 2:

- Demonstrate Conditional Probabilities, Independent Events,
- Describe Bayes Formula and its applications.

Date of the Session: //22/12/2023. **Time of the Session:** 11:10 to 12:50

Learning outcomes:

- Understanding the Probabilities defined on Events Demonstrate Conditional Probabilities
- Understand the Conditional Probabilities Apply Bayes Formula to solve related problems

Experiment 1

1. Three cards are drawn in succession, without replacement, from an ordinary deck of playing cards. Find the probability that the events $A_1 \cap A_2 \cap A_3$ occurs where A_1 is the event that the first card is red ace, A_2 is the event that the second card is a 10 or a jack, and A_3 is the event that the third card is greater than 3 but less than 7.

Solution:

A):- A_1 = is the event that the first card is Red ace.

A_2 = is the event that the 2^{nd} card is 10 (or) Jack.

A_3 = is the event that the 3^{rd} card is >3 but <7.

1) $P(A_1)$: there are 2 red aces, and total of 52 cards.

$P(A_1) = 2/52 = \boxed{1/26}$ $\quad P(A_1) = \frac{2C_1}{52C_1}$

2) $P(A_2|A_1)$:- four 10s and 4 Jacks, two suits are red.

$P(A_2|A_1) = \boxed{8/51}$ $\quad \frac{4C_1 + 4C_1}{51C_1} = 2 \times \frac{4C_1}{51C_1}$

3) $P(A_3 | A_1 \cap A_2) = 12/50$

$= \boxed{6/25}$

There are four cards (4, 5, 6).

$P(A_1 \cap A_2 \cap A_3) \Rightarrow P(A_1) \cdot P(A_2 | A_1) \cdot P(A_3 | A_1 \cap A_2).$

$\Rightarrow \frac{2}{52} \qquad \Rightarrow 1/26 * 8/51 * 6/25$

$$P(A_3 | A_1 \cap A_2) = \frac{12C1}{50C1}.$$

2. A professor is interested in understanding the relationship between students' study habits and their performance on an exam. The professor collects data on whether students attended review sessions and whether they passed the exam.

Events of Interest:

1. **Event A:** The event that a student passes the exam.
2. **Event $B1$:** The event that a student attended review sessions.
3. **Event $B2$:** The event that a student did not attend review sessions.

Objective: Calculate the probabilities of the following events:

- $P(A)$ - Probability that a student passes the exam.
- $P(B1)$ - Probability that a student attended review sessions.
- $P(B2)$ - Probability that a student did not attend review sessions.
- $P(A|B1)$ - Probability that a student passes the exam given they attended review sessions.
- $P(A|B2)$ - Probability that a student passes the exam given they did not attend review sessions.

Provide the solution using SAS Code .

Solution:

```
/* Given dataset */
data students;
 input A B1 B2;
 datalines;
  1 1 0
  0 1 0
  1 0 1
  0 0 1
  1 1 0
  0 1 0
  1 0 1
  0 0 1
;
/* probability that a student passes the exam (p(A)) */
proc means data=students mean noprint;
 var A;
  output out=prob_pass_exam mean=prob_A;
  run;
/* probability that a student attended review sessions (p(B1)) */
 proc means data=students mean noprint;
     var B1;
     output
```

```
/* probability that a student did not attend review (p(B2)) */
proc means data= students mean noprint;
var B2;
output out= prob_not_attend-review mean= prob-B
run;
/* probability that a student passes the exam given they
did not attend review sessions (p(A|B2)) */
proc means data = students mean noprint;
var A;
where B2=1;
output out= prob-pass_exam-given-B2;
run;
/* Display the results */
proc print data = prob-pass_exam noobs;
var prob-A;
run;
proc print data = prob_attend_review noobs;
var prob-B1;
run;
proc print data = prob-not-attend-review noobs;
var prob-B2;
run;
proc print data = prob-pass-exam-given-B1 noobs;
var prob-A-given-B1;
run;

proc print data = prob-pass_exam-given-B2 noobs;
var prob-A-given-B2;
run;
```

3. Suppose there are 4 red, 6 blue and 2 green balls in a bag. Two balls are drawn at random from the bag "without replacement". Find the probability of that the two balls drawn are red using SAS code.

Solution: Red balls = 4
Blue balls = 6
Green balls = 2

$$P(A \cap B) = \frac{4C1}{12C1} \times \frac{3C1}{11C1}$$

SAS code:-

```
data ball-prob;
 /* Define the number of each color of balls */
   red-balls=4;
   blue-balls=6;
   green-balls=2;
 /* calculate the total number of balls */
   total_balls=red-balls+blue-balls+green-balls;
 /* probability of drawing the first red ball */
   prob-first-red=red-balls/total-balls;
 /* probability of drawing 2nd red ball without replacement */
   prob-second-red=(red-balls-1)/(total-balls-1);
 /* probability of drawing two red balls */
   prob-two-red=prob-first-red*prob-second-red;
  /* output the results to a dataset */
     output;
   /* stop the data step */
     stop;
   run;
```

```
/* Display the results using proc print */
proc print data=ball_prob;
run;
```

<u>Output:-</u>

p(both drawn as red) = p(first red) * p(2^{nd} red)

$= (1/3) * (3/11)$

$\Rightarrow 3/33$

$= 1/11.$

Experiment 2

4. Police plan to enforce speed limits by using radar traps at 4 different locations within the city limits. The radar traps at each of the location $L_1, L_2, L_3, and\ L_4$ are operated 40%, 30%, 20% and 10% of the time and if a person who is speeding on his way to work has probabilities of 0.2, 0.1, 0.5 and 0.2 respectively, of passing through these locations, Find out what is the probability that he will receive a speeding ticket using SAS Code.

Solution:

A: Receiving a speeding ticket.

L1, L2, L3, L4: passing through locations L1, L2, L3, L4.

* the probability of receiving a speeding ticket can be calculated using the law of total probability:-

$$p(A) = p(A|L1) * p(L1) + p(A|L2) * p(L2) + p(A|L3) * p(L3) + p(A|L4) * p(L4).$$

$\Rightarrow$ $p(A|L1) = 0.2$

$p(A|L2) = 0.1$

$p(A|L3) = 0.5$

$p(A|L4) = 0.2$

$p(L1) = 0.4$

$p(L2) = 0.3$

$p(L3) = 0.2$

$p(L4) = 0.1$

$P(A) = (0.2 * 0.4) + (0.1 * 0.3) + (0.5 * 0.2)$
$+ (0.2 * 0.1)$

$= 0.08 + 0.03 + 0.1 + 0.02$

$= 0.23$

* the probability that the person will receive a spending ticket is "0.23 or 23%."

Output:-

0.23 (or) 23%.

5. A Covid test kit was given to people in a particular area, then Covid positive was verified through blood tests. The following table shows the covid test results. Find

1. P (not Covid | positive test result)
2. P (positive test result | not Covid)

Table: Contingency/ Confusion matrix

	Positive test	Negative test	Total
Covid	70	4	74
Not Covid	5	14	19
Total	75	18	93

Solution:

(a) p(not covid | positive test result):-

the total no. of positive test results is 75.

* out of these 75 positive results, 5 are for individuals.

$= 5/75$

$= 1/15 \approx 0.0667.$

the probability of a person is "0.0667 or 6.67%".

(b) p(positive test result | not covid):-

* the total no. of individuals who do not have covid is 19.

* 19 individuals, 5 have received positive test results.

P(positive test result | not covid)

$= 5/19 \approx 0.2632.$

So, the probability of obtaining a positive test result given is "0.2632 or 26.32%"

Output:-

obtaining a positive test result = 0.2632 (or) 26.32%.

Experiment 3

6. Imagine a customer support department where customer queries are handled by three different teams: Team A, Team B, and Team C. These teams are responsible for 25%, 35%, and 40% of the customer support requests, respectively. Historical data reveals that 8%, 6%, and 4% of the responses provided by Teams A, B, and C contain errors. If a customer receives a response containing an error, what is the probability that the response originated from Team A?

Solution:

```
input p_A, p_B, p_C;
PB= p_B_A* p_A+ p_B* p_B + p_B_C*
  (1-p_A);
p_A-given_B_p_B_A * p_A(p.B);
datalines;
0.25, 0.08, 0.08, 0.04
proc print data= team_errors;
run;
```

Given probabilities.

$P(A) = 0.25$

$P(B|A) = 0.05$

$P(B) = P(B|(A))$

$= 0.05 + 0.25 + 0.04 + 0.35 + 0.02 + 0.40$

$\Rightarrow 0.0345$

Bayes theorem

$$P(A|B) = \frac{P(B|A) + P(A)}{P(B)}$$

$$= \frac{0.05 + 0.25}{0.345}$$

$$= \boxed{0.3623}$$

7. At an electronic plant, it is known from past experience that the probability is 0.83 that a new worker who has attended the company's training program will meet the production quota and that the corresponding probability is 0.35 for a new worker who has not attended the company's training program. If 80% of all new workers attend the training program. What is the probability that a new worker will meet the production quota using SAS Code?

Solution:

T: The new worker attends the training program.

NT:- the new worker does not attend program.

Q:- the new worker meets the production quota.

Given probabilities:-

$P(Q|T) = 0.83$

$P(Q|NT) = 0.35$

$P(T) = 0.80.$

$P(NT) = 1 - P(T) = 1 - 0.80 = 0.20.$

Using the law of total probability,

$P(Q) = P(Q|T) * P(T) + P(Q|NT) * P(NT).$

$\Rightarrow P(Q) = 0.83 * 0.80 + 0.35 * 0.20.$

$= 0.664 + 0.07$

$= 0.734.$

the probability that a new worker will meet the production quota is "0.734 or 73.4%"

VIVA QUESTIONS

1. **What is the definition of conditional probability?**

A) It is denoted by P(A|B), representing the probability of event A occuring given that event B has occured.

2. **Define Bayes' Formula.**

A) $p(A|B) = \frac{[P(B|A) * p(A)]}{P(B)}$.

3. **Can you describe a real-life example where Bayes' Formula is applied?**

A) example: Medical diagnosis.
* To update the probability of a patient having a disease based on results of diagnostic tests.

4. **Explain the difference between prior probability and posterior probability.**

A) prior probability:- the initial probability assigned to an event before condering new evidence.
posterior probability:- the event after taking into account the new evidence or information.

For Evaluators use only)

Comment of the Evaluator (if Any)	Evaluator's Observation
	Marks Secured: 50 out of 50
	Full Name of the Evaluator: 10/1/24
	Signature of the Evaluator:
	Date of Evaluation

SUBJECTCODE: 22MT2005
PROBABILITY STATISTICS AND QUEUING THEORY

Tutorial 3:
Demonstrate discrete and continuous Random Variables, Probability Distribution Functions

Date of the Session: //________________________ **Time of the Session:**______to______

Learning outcomes:

- Understanding concept of a random variables and its types.
- Verify the probability functions and their properties

Experiment 1

1. In a university, the scores of students in a particular exam are modeled as a normal distribution. The mean score is 75, and the standard deviation is 10. Analyze the probability distribution of exam scores using the normal distribution function and calculate the probability of scoring above a certain threshold using SAS code.

Solution:

```
Mean = 75;
stddev=10;
threshold=85;

/* the threshold score*/

/* calculate the probability using the
   normal distribution function*/
prob-above-threshold=1- cdf("Normal",
 thrashold, mean, stddev);
  print prob- above- threshold;
   quit;
```

We use the 'cdf' function to calculate the cummulative probability of scoring below the threshold, and then substract it from 1 to get the probability of scoring above the threshold.

Output:-

$$Z - \frac{\mu - x}{6}$$

$$= \frac{78 - x}{6}$$

$$= \frac{75 - 10}{6}$$

$$\Rightarrow \frac{65}{6}$$

$$\Rightarrow 1.8.$$

Output:-

obs	threshold-prob
1	15.9 %.

2. From a box containing 3 black balls and 2 green balls, 4 balls are drawn in succession, each ball being replaced in the box before the next draw is made. Write a SAS program to calculate the probability distribution for the number of green balls:

Solution:

* the probability of drawing 0 green balls in all 3 draws $= (4/6) * (4/6) * (4/6) = (4/6)^3 = 64/216$

$X1 \Rightarrow 0.2963.$

* probability of drawing 1 greenball in all 3 draws

$= 3 * (2/6) * (4/6) * (4/6) = 3 * (2/6) * (4/6)^2 = 48/216$

$X2 = 0.222$

* probability of drawing 2 greenballs in all 3 draws

$= 3 * (2/6) * (2/6) * (4/6) = 3 * (2/6)^2 * (4/6) = 24/216$

$X3 = 0.111$

* probability of drawing 3 green balls in all 3 draws $= (2/6) * (2/6) * (2/6) = (2/6)^3$

$X4 = 8/216 = 0.0370.$

$\rightarrow X1 + X2 + X3 + X4$

$\Rightarrow 0.2963 + 0.222 + 0.11 + 0.370 \approx 1$

$\Rightarrow$ the box containing 4 black balls and 2 green balls.

Output:-

Obs	k	probability
1	0	25.9%
2	1	34.6%
3	2	19.3%
4	3	3.84%
5	4	0.123%

Experiment 2

3. An online store sells electronic gadgets, and the inventory of a particular product is managed by the store. The store regularly receives shipments from suppliers, and each shipment may contain a certain number of defective items. The store wants to analyze the probability distribution of the number of defective items in each shipment to make informed decisions about quality control and inventory management. The store decides to collect data on the number of defective items in 50 recent shipments. They record the number of defective items in each shipment. Generate the SAS Code

Solution:

```
DATA Shipment_data;
INFILE "your_filename.csv" DSDELIMITERS ",";
INPUT shipment_number number_defective;
  RUN;
 /* 2. Descriptive statistics */
   PROC MEANS DATA=shipment_data;
     VAR number_defective;
    OUTPUT OUT=descriptive_stats MEAN STDDEV N;
    RUN;
  /* 3. Explore data distribution */
 PROC UNIVARIATE DATA=shipment_data PLOT HIST;
    VAR number_defective;
      RUN;
  /* 4. Fit probability distributions */
      PROC FREQ DATA=shipment_data;
        TABLES number_defective;
          CHISQ;
          RUN;
```

/* 5) Identify the best-fitting distribution */

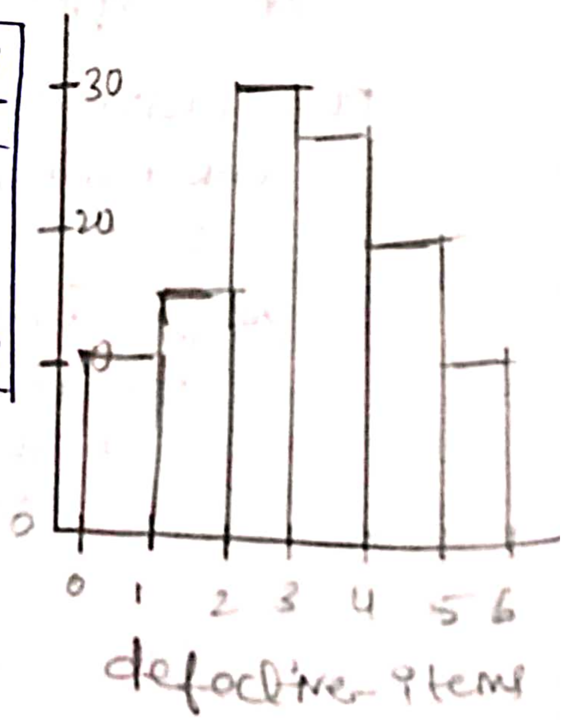
```
PROC LOGISTIC DATA= shipment-data;
  MODEL number_defective=_ALL_;
 OUTPUT OUT= logistic_modelp= PREDICT;
 RUN;
PROC GLMSELECT DATA=shipment-data;
  CLASS number-defective;
  SELECT BEST= 3 SCORR;
  RUN;
```

/* 6) visualize the selected distribution */

```
PROC UNIVARIATE DATA=shipment-data
      PLOT DENSITY FIT= logistic-model;
  VAR number-defective;
  RUN;
```

output:-

location		variability	
mean	6.60	S.D	1.7125
Median	15.00	Varia	1.341
mode	1.00	Range	4.00



4. A delivery service company is interested in analyzing the time it takes for their delivery trucks to travel between two distribution centers. The company wants to understand the distribution of delivery times to optimize scheduling and improve customer service. The company aims to model the delivery times as a continuous random variable and analyze its statistical properties. The company collects data on the travel times of 100 randomly selected delivery trucks between the two distribution centers.

Solution:

```
/* Sample data:- Travel times of 100 randomly selected
      delivery trucks */
   data delivery_times;
     input truck_id travel_time;
       datalines;
      1) 35.2
      2) 40.1
      3) 32.5
      4) 48.7
      5) 42.3
  /* Include data for the remaining trucks */
    ;
   run;
  /* Descriptive statistics */
  proc means data=delivery_times mean median
                    std min max;
       var travel_time;
    title 'Descriptive statistics of delivery Times';
       run;
```

B) $f(x) = \binom{2}{x}\binom{3}{3-x}$, for $x = 0,1,2.$

$CDF(0) = \sum f(x)$ for $x = 0$

$f(0)$

$= (1/30)(0^2+4)$

$= 4/30$

$= 2/15$

for $X = 1$

$CDF(1) = \sum f(x)$ for $x \le 1$

$= f(0) + f(1)$

$= (1/30)(0^2+4) + (1/30)(1^2+4)$

$= 4/30 + 5/30$

$\Rightarrow 9/30$

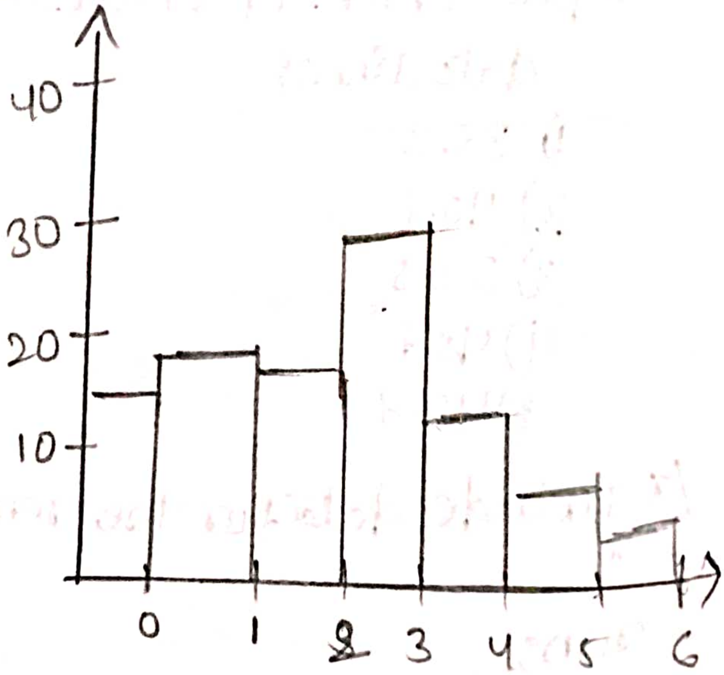
$= 3/10.$

$CDF(0) = 2/15$

$CDF(1) = 3/10$

$CDF(2) = 17/30$

$CDF(3) = 1.$



output :-

obs	delivery-time
1	15.2
2	19.5
3	18.3
4	13.2
5	13.2
6	12.1

Experiment 3

5. Determine the value of c so that each of the following functions can serves as a probability distribution of the discrete random variable X and also find Cumulative distribution function.

A) $f(x) = c(x^2 + 4), for\ x = 0,1,2,3;$

B) $f(x) = \binom{2}{x}\binom{3}{3-x}$, for $x = 0,1,2.$

Solution: ⓐ $f(x) = c(x^2+4)$ for $X = 0,1,2,3;$

$\Sigma f(x) = 1$

$f(0)+f(1)+f(2)+f(3) = c(0\text{^}2+4)+c(1^2+4)+c(2^2+4)+c(3^2+4$

$= c(4)+c(5)+c(8)+c(13)$

$= 4c+5c+8c+13c$

$= 30c$

$\boxed{c = 1/30}$

6. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} ax, 0 \le x \le 1 \\ a,\ 1 \le x \le 2 \\ -ax+3a,\ 2 \le x \le 3 \end{cases}$$

i) Determine a ii) Compute $P(X \le 1.5)$.

Solution: ⓘ $\int[0\text{ to }1]\, ax\, dx + \int[1\text{ to }2]\, a\, dx + \int[2\text{ to }3]\,(-ax+3a)\, dx = 1$

$(1/2)ax^2[0\text{ to }1] + ax[1\text{ to }2] - (1/2)ax^2 + 3ax[2\text{ to }3] = 1$

$(1/2)a + (1/2)a - (7/2)a = 1$

$(1/2)a - (4/2)a = 1$

$(-3/2)a = 1$

$\boxed{a = -2/3}$

ii) $p(X \leq 1.5) = \int [0 \text{ to } 1.5] (-2/3) x \, dx$

$(-2/3) * (1/2) * x^2 [0 \text{ to } 1.5]$

$(-2/3) * (1/2) * (1.5)^2$

$(-2/3) * (1/2) * 2.25$

$= -1.5 * 0.75$

$= \boxed{-1.125.}$

Experiment #		STUDENT ID :	
Date :		STUDENT NAME :	

VIVA QUESTIONS:

1. What is the key difference between a discrete and continuous random variable?

A) * Discrete random variable → distinct, seperate values.

* continuous random variable → Any value within a range.

2. Define the probability mass function (PMF) for a discrete random variable.

A):- the probability that the variables takes on a specific value. it is defined as $P(X=x)$, where 'X' is the random variable and x is particular value.

3. How do you calculate the cumulative distribution function (CDF) for a random variable?

A):- Calculated by summing the probabilities from the PMF up to a specified value. It gives the probability that the random variable is less than (or) equal to particular value.

4. Explain the concept of expected value for a random variable?

A):- the expected value for a variable is the average or mean value it is expected to over many repetitions of the random experiment.

(For Evaluators use only)

Comment of the Evaluator (if Any)	Evaluator's Observation Marks Secured: 50 out of 50 Full Name of the Evaluator: Signature of the Evaluator: 10/2/24 Date of Evaluation

Course Title :	Probability Statistics and Queueing Theory
Course Code :	22MT2005

SUBJECTCODE: 22MT2005
PROBABILITY STATISTICS AND QUEUING THEORY

Tutorial 4:
Demonstration of Discrete Probability distributions

Date of the Session: //____________________________________Time of the Session:______to_______

Learning outcomes:

- Outcome related to understanding of discrete random variables and Probability Mass Function
- Outcome related to understanding of continuous random variables and Probability Density Function

Experiment 1

1. A company conducts an online survey to gather customer feedback. The survey asks customers to rate their satisfaction on a scale of 1 to 5, where 1 represents "Very Dissatisfied" and 5 represents "Very Satisfied." The company is interested in analyzing the distribution of customer satisfaction ratings. Objective: The company aims to model the customer satisfaction ratings as a discrete random variable and analyze its PMF to understand the distribution of satisfaction levels. Data Collection: The company collects data on the satisfaction ratings from 200 randomly selected customers. Write SAS code Snipet

Solution:

```
/*Create a dataset with customer satisfaction ratings*/
data satisfaction_ratings;
input rating @@;
datalines;
4 5 3 2 5 4 3 5 4 2 1 3 4 5 3 4 5 2 3 4
/* include the rest of the ratings data */
;
run;
/* calculate the frequency of each rating */
proc freq data=satisfaction_ratings;
tables rating/out=satisfaction_freq (count percent);
```

```
run;
/* calculate the probability mass function(PMF)*/
data satisfaction_pmf;
set satisfaction_freq;
pmf = percent/100;
run;
/* print the pmf table */
proc print data = satisfaction_pmf;
title "probability mass function (PMF) for Ratings";
run;
.......
```

Output:-

Customer	probability
0	2.6 -33
1	2.00 →31
2	2.53 - 30
3	1.88 - 27
4	3.51 - 27

Mean	Variance
3.600	1.30000

2. An e-commerce platform allows customers to leave reviews for products they have purchased. The platform uses a rating system from 1 to 5, where 1 represents the lowest rating ("Not Satisfied") and 5 represents the highest rating ("Very Satisfied"). The platform is interested in understanding the distribution of product ratings to improve the user experience. The e-commerce platform aims to model the product ratings as a discrete random variable and analyze its PMF to gain insights into the overall satisfaction level of customers. Data is collected from 300 randomly selected product reviews. Each review includes a rating from 1 to 5.Write SAS code Snipet.

Solution:

```
/* Sample data: customer satisfaction ratings
   from 200 randomly selected customers */
data satisfaction_rating;
  input customer_id satisfaction_rating;
    datalines;
   1 4
   2 3
   3 5
   4 2
   5 4
/*Include data for the remaining customers*/
;
run;
/* Describe statistics */
proc freq data=satistafaction_ratings;
  tables satisfaction_rating / nocum;
title 'descriptive statistics of Customer Ratings';
run;
/* probability Mass Function (PMF) */
proc freq data=satisfaction_ratings;
```

```
tables satisfaction_rating/ plots(type=pmf);
title 'probability Mass function(pmf)
   of customer satisfaction Rating
run;
```

Output:-

the mean procedure

Analysis variable : product rating	
mean	variance
3.600	1.3000

procedure:-

rating	frequency
2	1
3	1
4	2
5	1
frequencyrating = 1	

Experiment 2

3. It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight on this problem, it is determined that some tests should be made. It is too expensive to test all of the wells in the area, so 10 were randomly selected for testing,

i) Using the binomial distribution, what is the probability that exactly 3 wells have the impurity assuming that the conjecture is correct?

ii) What is the probability that more than 3 wells are impure?

iii) What is the probability that not more than 2 wells are pure? Write the SAS Code and generate the solution.

Solution:

```
/* set the parameters */
%let n=10; /* Number of wells tested */
%let K_3=3; /* no.of wells with impurity */.
%let K_gt_3=3; /* no.of wells with impurity 2nd */.
let K_not_gt_2=2; /* no.of wells without impurity */
%let p=0.3; /* probability of a well having impurity */
/* Calculate the probabilities */
data binomial_probabilities;
   /* probability that exactly 3 wells impurity */
   prob_3 = pdf("BINOMIAL", &K_3, &n, &p);
  /* probability that more than 3 wells are impure */
  prob_gt_3=1-cdf("BINOMIAL", &K_gt_3-1, &n, &p);
```

```
/* probability that more than 3 wells are impure */
prob-gt-3=1-cdf("BINOMIAL", &K-not-gt-2,&n);
run;

/* Display the results */
proc print data=binomial-probabilities;
var prob-3 prob-gt-3 prob-not-gt-2;
run;
```

output:-

obs	n	p	K1	prob1	K2	prob2	K3	prob3
1	10	10.3	3	0.2683	3	0.350	3	0.0519

4. The number of customers arriving at a grocery store is a Poisson random variable. On average 10 customers arrive per hour. Let X be the number of customers arriving from 10am to 11:30am. What is P(10<X≤15)? Generate the SAS Code.

Solution:

To calculate the probability $p(10 < X < 15)$ for a poisson distribution.

(PMF)

$$P(X=K) = \frac{e^{-\lambda} \cdot \lambda^{K}}{K!}$$

SAS code:-

```
/* set the parameters */
%let lambda = 10; /* Average number of arrivals */
%let lower_bound = 10; /* Lower bound of the interval */
%let upper_bound = 15; /* upper bound of the interval */

/* calculate the probability */
data poission_probability;
  do x = &lower_bound to &upper_bound;
    prob = pdf("POISSON", x, lambda);
    output;
  end;
run;
```

```
/* Display the results */
proc print data= poission-probability;
  var x prob;
 run;.
```

Output:-

prob-freq	prob-x-freq-30	prob-10-u-15
0.583039	0.957568	0.368129

Experiment 3

5. A call center receives an average of 20 customer service calls per hour. The number of calls follows a Poisson distribution. The call center manager is interested in understanding the likelihood of receiving a specific number of calls within a given time frame. Determine the probabilities associated with different numbers of customer service calls within a 2-hour period using SAS code.

Solution:

$$P(X=k) = \frac{e^{-\lambda} \cdot \lambda^k}{k!}$$

SAS CODE:-

```
/* Set the parameters */
%let lambda=20; /* Avg no. of calls per Hour */
%let hours=2; /* Time period in hours */
/* calculate the probabilities for diffeent number
   of calls within a 2-hour period */
   data poission-probabilities;
  do calls = 0 to 40;
     prob= pdf("POISSON", calls, &lambda, &hours);
      output;
     end;
   run;
*/ Display the results */
   proc print data= poission-probabilities;
        var calls prob;
   run;
```

Output:-

obs	lambda	hours	T	calls	prob
1)	20	2	0	0	0.00
2)	20	2	1	1	0.00
3)	20	2	2	2	0.000
4)	20	2	3	3	0.000
5)	20	2	4	4	0.000

6. A transportation agency is responsible for managing traffic flow on a particular highway. The agency observes an average of 15 vehicles passing through a specific checkpoint per minute during peak hours. The number of vehicles passing through this checkpoint follows a Poisson distribution. Analyze the Poisson distribution to predict the likelihood of observing a certain number of vehicles passing through the checkpoint in a 5-minute interval.

Solution:

Probability Mass Function:-

$$P(X=k) = \frac{e^{-\lambda}.\lambda^{k}}{k!}$$

SAS CODE:-

```
/*Set the parameters */
%let lambda_per_minute=15; /*Avg no.of vehicles min*/
%let time_interval=5; /*Time interval in minutes*/
/*calculate the probabilities for different no.of
  vehicles in a 5-minute interval */
    data poisson_probabilities;
     do vehicles=0 to 100; /*Adjust the upper limit
      based on your needs */
      prob= pdf("POISSON", vehicles, &lambda_per_
       minute * &time_interval);
       output;
       end;
    run;
```

```
/* Display the results */
proc print data= poission-probabilities;
   var vehicles prob;
   run;
```

Output:-

obs	lambda	Mins	i	vehicle	prob
1	15	5	0	0	0.1678
2	15	5	1	1	1.8763
3	15	5	2	2	2.9698
4	15	5	3	3	3.2125
5	15	5	4	4	4.1577

7. A transportation agency is responsible for managing traffic flow on a particular highway. The agency observes an average of 15 vehicles passing through a specific checkpoint per minute during peak hours. The number of vehicles passing through this checkpoint follows a Poisson distribution. Analyze the Poisson distribution to predict the likelihood of observing a certain number of vehicles passing through the checkpoint in a 5-minute interval.

Solution:

```
data poission_vehicles;
  lambda=15%
  time_interval=5;
  do_vehicles=0 to 50;
  prob_vehicles=pdf("POISSON"; vehicles;
       lambda*time);
     output;
      end;
   run;
proc print data=poisson_vehicles;
    var vehicles <=30;
    run;.
```

Output:-

Vehicle	probability:-
0	2.6786-33
1	2.0089768-31
2	7.533e-30
3	1.883417e-28
4	3.5314e-27
5	5.2970e-29.

Experiment #		STUDENT ID :	
Date :		STUDENT NAME :	

Viva Questions

1. **What is a discrete probability distribution?**

A:- It is a function that Assign probabilities to each possible outcome of a discrete random variables.

2. **What is the expected value of a discrete random variable?**

A:- the expected value is the weighted average of all possible values, with weights as their probabilities.

3. **How is the probability mass function (PMF) related to a discrete probability distribution?**

A:- the probability mass function describe the probability distribution of a discrete random variable.

4. **How do you determine the cumulative distribution function (CDF) for a discrete probability distribution?**

A:- the CDF and a value 'x' is the sum of probabilities of all value less than or equal to x;

$CDF(x) = p(X \leq x) = \Sigma(p(X = x_i))$ for all $x_i - i \leq x$.

(For Evaluators use only)

Comment of the Evaluator (if Any)	Evaluator's Observation
	Marks Secured: 50 out of 50
	Full Name of the Evaluator:
	Signature of the Evaluator: 16/2/24
	Date of Evaluation

SUBJECTCODE: 22MT2005
PROBABILITY STATISTICS AND QUEUING THEORY

Tutorial 5:
Demonstration of Expectation of discrete and continuous random variables
Demonstration of Expectation of a function of a random variable.

Date of the Session: //_______________________________________Time of the Session:______to______

Learning outcomes:

- Understand Expectation of discrete and continuous random variables
- Understand Variance of a function of a random variable

Experiment 1

1. A professor conducts exams for a class, and students' scores follow a discrete distribution. The average score is known to be 75, and the distribution is modeled using the probability mass function. Analyze the expectation (mean) of the exam scores and understand its significance using SAS Code.

Solution:

```
data exam_scores;
  do score = 0 to 100;
    prob_score = 1/101;
    output;
    end;
  run;
proc sql;
 select sum(scored prob_score) as mean_score
  from exam_score;
   quit;
```

histogram
drawochayali

Output:-

the mean procedure.

Analysis : available : Score		
		mean
		81.0000

the univariate procedure

Variable:- daily-revenue.

2. A company's daily revenue follows a continuous distribution with an average of $10,000 per day. The distribution is modeled using the probability density function. Analyze the expectation (mean) of the daily revenue and interpret its significance for revenue forecasting Using SAS Code

Solution:

```
/* Set the parameters */
%let daily_mean= 10000; /* Avg daily revenue */
%let Sample_size=1000;
/* Generate random samples from a N.D */
   data revenue_Samples;
   do i=1 to & Sample_size;
    revenue=rand ("NORMAL", daily_mean);
     output;
      end;
       run;
/* calculate the sample mean */
 proc means data=revenue_Samples mean;
   var revenue;
   run;
```

Output

Moments:-

x1	5	Sum weight	5
mean	10400	sub observations	52000
std	1193.7864	variance	1425000
skewness	0.205728	Kurtosis	-1.117269
uncorrected	5465000	corrected	57000
coff-variation	11.478209	std	533.8889

Experiment 2

3. Six men and five women apply for an executive position in a small company. Two of the applicants are selected for an interview. Let X denote the number of women in the interview pool. We have found the probability mass function of X.

$X = x$	0	1	2
$P(x)$	$\frac{2}{11}$	$\frac{5}{11}$	$\frac{4}{11}$

How many women do you expect in the interview pool?

Solution:

* six men and five women apply for an executive position in a small company.

* Two of the applicants are selected for an interview.

Expectation of women in the Interview

$$\boxed{f(x) = \Sigma_x X p_x(x)}$$

$$= 0 \cdot \frac{2}{11} + \frac{1.5}{11} + 2 \cdot 4/11$$

$$= 0 + 0.454 + 0.727$$

$$\Rightarrow 1.181 \quad 13/11 = \boxed{1.181}$$

4. If A dealer's profit in units of \$5000, on a new automobile can be looked upon as a random variable X having the density function

$$F(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

Solution:

$$f(x) = \int_0^1 x.2(1-x)dx$$

$$= 2\left[x^2/2 - x^3/3\right]_0^1$$

$$= 2\left[\frac{1.0}{2} - \frac{1.0}{2}\right]$$

$$= 1/3.$$

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$\mu = \int_0^1 x^2(1-x)dx$$

$$\mu = 2\int_0^1 (2x - 2x^2)\, dx$$

$$\mu = 2\left(\frac{x^2}{2}\right)\Big|\frac{1}{0} - 2\left(\frac{x^3}{3}\right)\Big|_0^1$$

$$\mu = 2\left(\frac{1}{2}\right) - 2\left(\frac{1}{3}\right)$$

$$\mu = 1 - 2\left(\frac{1}{3}\right)$$

$$\Rightarrow \mu^2 = 1/9$$

$E(x^2) = 2(1/3) - 2(1/4)$

$E(x^2) = (1/6)$

$\sigma^2 = E(x^2) - \mu^2$

$\sigma^2 = 1/6 - 1/9$

$\sigma^2 = \frac{3}{54}$

$\sigma^2 = \frac{1}{18}$

As the unit is \$5000

$\sigma^2 = \frac{1}{18} \times 5000 = 277.77$

$\sigma^2 = \$277.77$

Experiment 3

5. Compute the expectation of X^2 where X is a random variable with the following probability density function:

$$f(x)=\begin{cases} 4x^3, if\ 0<x<1 \\ 0,\ otherwise \end{cases}$$

Solution:

the expectation of a continous random variable is computed using the following

$$f(g(x)) = \int_{-\infty}^{\infty} g(x).f(x)\,dx$$

$f(x^2)$ is computed as follows.

$$f(x^2) = \int_{-\infty}^{\infty} x^2(f(x))\,dx$$

$$= 4\left[\frac{x^6}{6}\right]_0^1$$

$$= 4\left[1/6\right]$$

$$= \boxed{2/3}$$

5. Determine the mean (i.e., E[X]) and $E[X^2]$ of a discrete random variable X whose Cumulative Probability Distribution (CDF) function is given below:

$X = x$	1	2	3	4	5	6
$F_x(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1

Solution: To compute expectation, we need the probability Mass Function (PMF) of discrete variable from CDF first the probability distribution of RV,

x	f(x)
1	$f(1) = 1/6$
2	$f(2) - f(1) = 1/6.$
3	$f(3) - f(2) = 3/6 - 2/6 = 1/6$
4	$f(4) - f(3) = 3/6 = 1/6.$
5	$f(5) - f(4) = 5/6 - 4/6 = 1/6.$
6	$f(6) - f(5) = 1 - 5/6 = 1/6.$

this PMF is

x	1	2	3	4	5	6
$p(x>m)$	1/6	1/6	1/6	1/6	1/6	1/6.

$$(\text{Mean}) = (1+2+3+4+5+6) \times 1/6$$
$$= \boxed{21/6}$$

7. The random variable X representing the number of errors per 100 lines of software code has the following probability distribution:

X	2	3	4	5	6
f(x)	0.01	0.25	0.4	0.3	0.04

i) Find Mean, standard deviation and Variance of X.

ii) Obtain the mean and variance of discrete random variable Z=3X-2, when X represents the number of errors per 100 lines of code.

Solution:

i) mean of $f(x) = \Sigma x f(x)$

$= 2(0.01)+3(0.25)+4(0.4)+5(0.3)+6(0.04)$

$= 4.11$

$= \sigma = \sqrt{\Sigma(x-\mu^2)+(x)}$

$\sigma^2 = (2-4.11)^2 * 0.01 + (3-4.11)^2 + 0.25 * (4-4.11)^2 +$

$0.4 + (5.4.11)^2 + 0.3 + (6-4.11)^2 * 0.04.$

$\sigma^2 = 0.779$

$Var(x) = 0.7379$

$S.D = \boxed{0.08590.}$

ii) $Z = 3x-2$

$\sigma(z) = 3F(x) - 2$

$= 10.33$

$Var(z) = 9\Sigma(x^2) - 12(x) + 4 - (3\Sigma(x)-2)^2$

$= 916.53 + 124.14 + 4 - (3*4.14-2)^2$

$= \boxed{36.95}$

the mean and variance of discrete random variable $Z = 3X - 2$,

$= \boxed{36.95}$

8. In a retail store, the arrival of customers follows a Poisson distribution with an average rate of 20 customers per hour. The store is interested in understanding the expected number of customers arriving in different time intervals. Analyze the expectation (mean) of the customer arrival rates for 1-hour, 2-hour, and 4-hour intervals.

Solution:

```
%let average_rate=20;
data customer_arrival_mean;
  lambda= &average_rate;
    time_intervals =1;
mean_1_hour = lambda *time_intervals;
   time intervals = 2;
mean_2_ hours =lambda *time_intervals;
    time_ intervals=4;
mean_4_hours =lambda*time_intervals;
     output;
    run;
proc print data= customer_arrival_mean;
   var mean_1_hour, mean_2_hour,
             mean_4_hour;
      run;.
```

Poission distribution:-

$$P(X=K) = \frac{e^{-\lambda} \cdot \lambda^{K}}{K!}$$

i) $E(x) = \lambda = 20$ - 1 hr inteval

ii) 2 hours of interval

$E(x) = 2 \cdot \lambda$.

$= 2 \times 20 = 40 \rightarrow$ 2 hr interval.

(iii) $E(x) = 4\lambda$..

4 hours of inteval

$E(X) = 4\lambda$ -

$= 4 \times 20 = 80 \rightarrow$ 4 hr interval.

VIVA QUESTIONS

1. **How does the concept of the expectation of a random variable relate to real-world applications?**

expectation is used in pending output like insurance claims stock.

2. **How do you calculate the expectation of a function of a discrete random variable?**

multiply each possible outcome of function by its probability the sum there products.

3. **Provide an example of a continuous random variable.**

the height of a person, it can take any value within a range.

4. **What is the expectation (expected value) of a discrete random variable X?**

the expectation of a discrete random variable 'x' is the sum of each possible value of "x" multiplied by its corresponding probabilities.

(For Evaluators use only)

Comment of the Evaluator (if Any)	Evaluator's Observation
	Marks Secured: 50 out of 50 Full Name of the Evaluator: 16/2/24 Signature of the Evaluator: Date of Evaluation

SUBJECTCODE: 22MT2005
PROBABILITY STATISTICS AND QUEUING THEORY

Tutorial 6:

Demonstrate Continuous probability distribution and joint random variables.

Date of the Session: //____________________ Time of the Session:______to______

Learning outcomes:

- Apply Continuous distributions to the realworld problems
- Define the Joint probability functions

Experiment 1

1. A company manufactures electronic devices, and the lifetime of a product follows an exponential distribution with an average lifetime of 5 years. The company is interested in understanding the probability of a product lasting a certain duration. Analyze the continuous probability distribution of product lifetimes and calculate the probability of a product lasting at least 8 years using SAS Code.

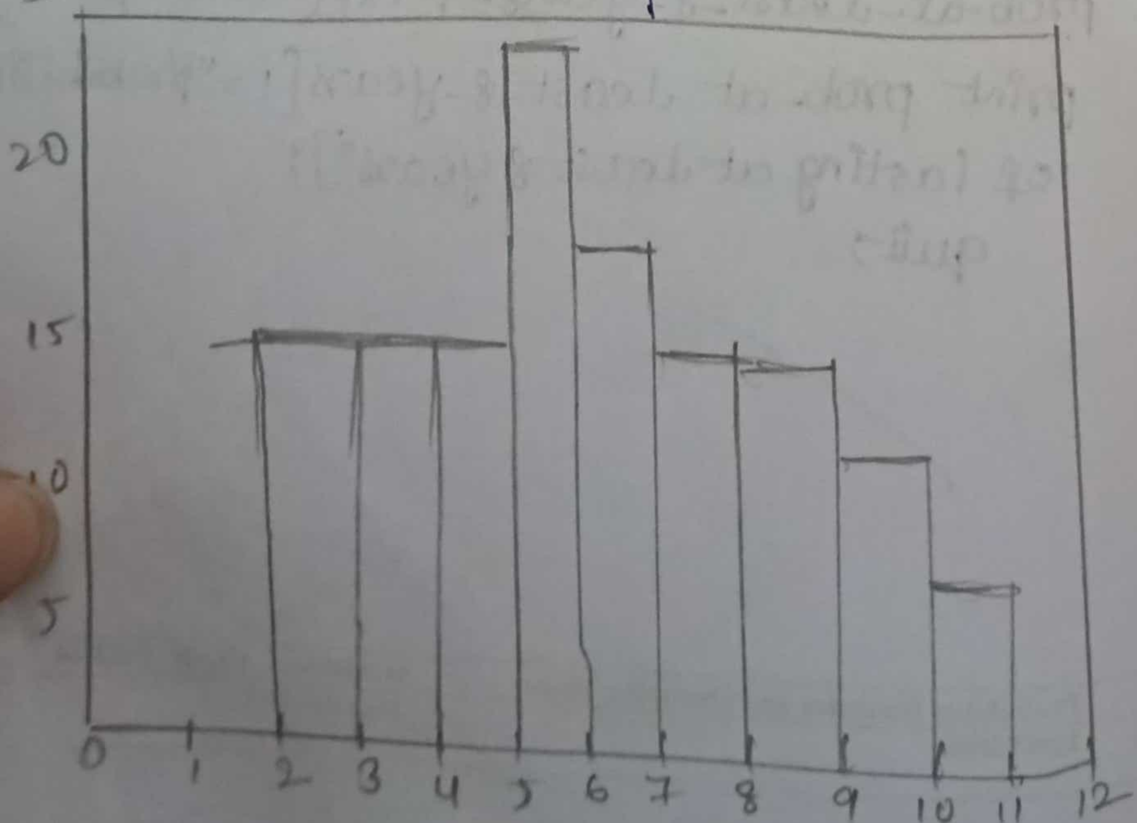
Solution:

```
%let average-lifetime=5;
proc iml;
lambda=1(&average-lifetime);
x=8;
prob-at-least_8-years=1-exp(-lambda*x)
print prob-at-least_8-years[L="probibility
of lasting at least 8 years"];
quit;
```

Output:-

Extreme	Observations		
lowest		highest	
value	obs	value	obs
2	2	6	4
3	7	7	3
4	5	8	6
5	10	9	8
5	1	10	9

Distribution of lifetime



2.The Heights of 1000 students are normally distributed with a mean of 174.5cm and a standard deviation of 6.9 cm. assuming that the heights are recorded to the nearest half-cm, how many of these students would you expect to have heights

a) Less than 160.0 cms?

b) Between 171.5 and 182.0 cms inclusive?

c) Equal to 175.0cm?

d) Greater than or equal to 188.0cms.

Solution: a) $\mu = 174.5$

$x = 160$

$\sigma = 6.9$

$Z = \frac{x-\mu}{\sigma} = -2.10$

$P(Z<160) = 0.0179$

$= \boxed{1.79}$

(b) $\mu = 174.5$

$\sigma = 16.9$

$x = 171.5$

$x = 182.0$

$\mu = 174.5$

$\sigma = 6.9$

$\Rightarrow \frac{171.5-174.5}{6.9} = 0.3336$

$\Rightarrow \frac{182.0-174.5}{6.9} = 0.8599$

0.526

(c) $Z = \frac{x-\mu}{\sigma} = \frac{175.0-174.5}{6.9} \Rightarrow \frac{0.5}{6.9}$

$\Rightarrow \boxed{0.0724}$

(d) $1-P(x \geq 188.0)$

$1-\frac{x-\mu}{\sigma}$

$\Rightarrow 1-\frac{188.0-174.5}{6.9}$

$\Rightarrow 1-0.9750$

$\Rightarrow 0.025 \times 1000 \Rightarrow \boxed{25}$

Experiment 2

3. In a manufacturing process, two variables, X and Y, represent the dimensions of a product. These dimensions are modeled as independent normal random variables with means of 10 cm and 15 cm, and standard deviations of 2 cm and 3 cm, respectively. Analyze the joint probability distribution of dimensions X and Y and calculate the probability that both dimensions are within specified ranges using SAS Code.

Solution:

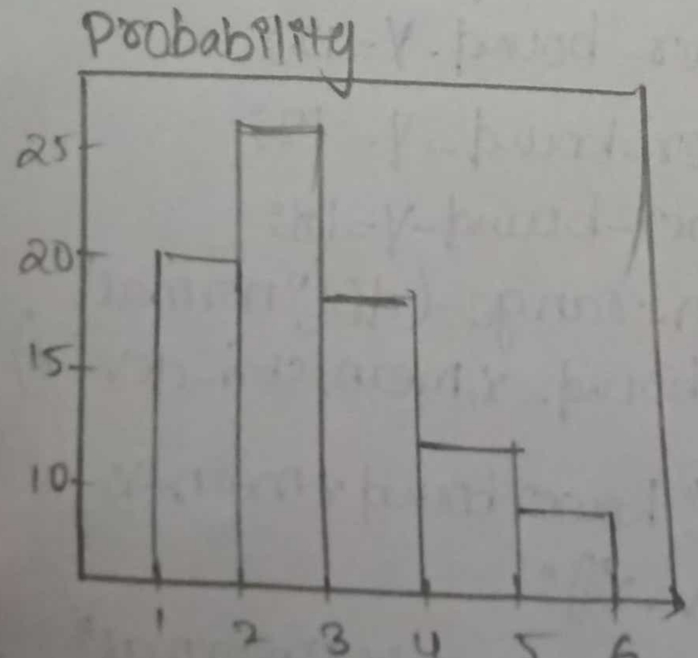
```
%let mean_x = 10;
%let std_dev_x = 2;
%let mean_y = 15;
%let std_dev_y = 3;
proc iml;
    mean_x = &mean_x;
    std_dev_x = &std_dev_x;
     mean_y = &mean_y;
      std_dev_y = &std dev_y;
       lower_bound_x = 8;
        upper_bound_x = 12;
         lower_bound_y = 12;
         upper_bound_y = 18;
prob_x_within_range = (cdf("normal",
      upperbound_x, mean, std_dev_x)
 - cdf("normal", lowerbound x, mean_x,
      std_dev_x));
prob_y_within_range = cdf("Normal",
      uperbound_y, mean_y; std_dev_y)
```

-cdf("normal", lower_bound-x, mean_x,
stddev_x);

print prob_both_within_range {
(="probability with both x & y within
specified ranges"];
quit;

<u>output:-</u>

obs	count_withinRanges	total count	joint prob
1	450	1000	0.45



4. An investor has a portfolio consisting of two stocks, A and B. The annual returns on these stocks are modeled as independent random variables with normal distributions. Stock A has a mean return of 8% and a standard deviation of 2%, while stock B has a mean return of 12% and a standard deviation of 3%. Analyze the joint probability distribution of the annual returns on stocks A and B and calculate the probability of achieving a positive overall portfolio return using SAS Code.

Solution:

```
%let mean_retuen_A=0.08;
%let md-dev-A=0.02;
%let mean-return-B=0.12;
%let std-dev-B=0.03;
proc iml;
mean-return_A=&mean-return-A;
std-dev-A=&std-dev-A;
mean-retuen-B=&mean-retuen-B;
std-dev-B=& std-dev-B;
mu={mean_retuen A, mean_retuen_B};
sigma={std-dev-A, std-dev-B};
condation={1};
prob-positive-retuen=1-cdf("normal", 0,
mu* correlation)
sigma* sqrt(correlation);
print prob-positive-retuen[L-"=probability
of probibility overall portfolio retuen"];
quit;
```

Output:-

property	value
label	returnA
Name	returnA
length	8
type	Neumeric
format	
informat	

No. of rows = 0
no. of columns = 3.

Experiment 3

5) In a manufacturing process, two quality control measurements, X and Y, are taken on each product. These measurements are modeled as joint random variables with a bivariate normal distribution. The means and standard deviations for X and Y are given. Analyze the joint probability distribution of quality control measurements X and Y and calculate the probability that both measurements are within specified ranges.

Solution:

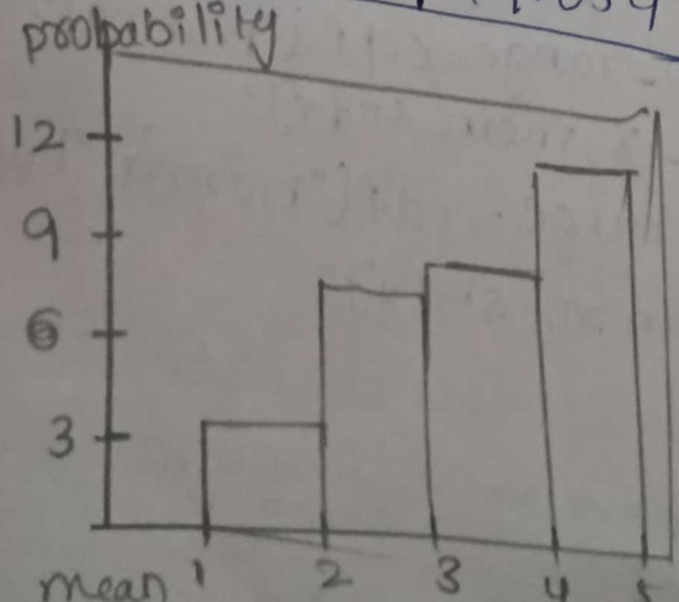
```
%let mean_x=----;
%let std-dev_x=---;
%let mean-y=---;
%let std-dev-y=----;
 proc imc;
  mean-x=&mean-x;
  std-dev-x=&std-dev_x;
   mean_y=&mean_y;
 std dev-y=&std-dev-y;
 correlation=&correlation;
   lower-boung-x=---;
   upper-boung-x=---;
   lower-boung_y=---;
   upper-boung-y=----;
prob-x-within_range=cdf("normal",
  upper-boung_x, mean, std x];
prob-y-below_range=cdf("normal",lower
 -boung-x, mean x1, std x];
```

prob_y_within_range = cdf("normal"_
upper_bound_x, mean_y, std_y);

prob_y_below_range = cdf("normal",
upper_bound_y, mean_y, std_y);

<u>output:-</u>

obs	x	y
1	3.1509	10.5224
2	8.3788	7.8489
3	9.6107	14.0770
4	10.287	21.4071
5	9.1867	14.054



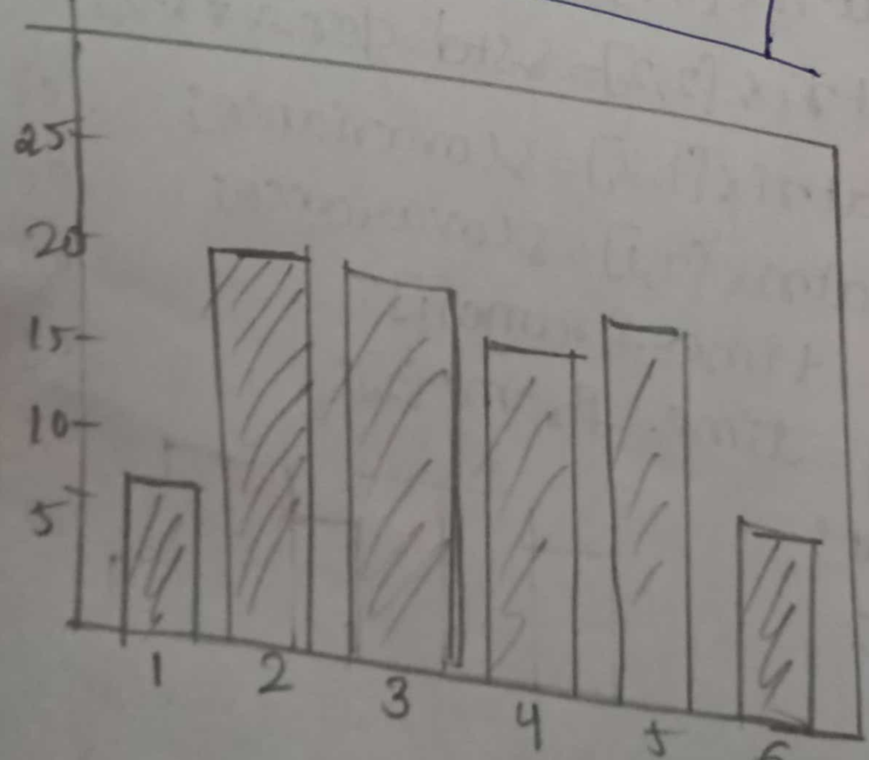
6. In a project management scenario, the completion time of two independent tasks, X and Y, is modeled as a bivariate normal distribution. Task X has a mean completion time of 20 days with a standard deviation of 3 days, while task Y has a mean completion time of 30 days with a standard deviation of 5 days. The correlation coefficient between the completion times of X and Y is 0.7. Analyze the joint probability distribution of the completion times of tasks X and Y. Calculate the probability of completing both tasks within a specified time frame.

Solution:

```
%let mean_x=20;
%let std-dev-x=3;
%let mean-y=30;
%let std-dev-y=5;
%let correlation=0.7;
data tasks-probabilities;
array means[2] &mean_x &mean_y;
array std-dev[2] &std-dev_x &std-dev-y;
array time-frame [2];
cor-matrix [1,1]= &std-der-x ** 2;
cor-matrix [2,2]= &std-der-y ** 2;
cor-matrix[1,2]=&covariance;
cor-matrix[2,1]=&covariance;
x= time-frame[1];
y= time-frame [2];
output;
run;
```

Output:-

obs	x	y
1	8.421	31.545
2	20.642	25.5680
3	20.801	26.8367
4	17.881	34.3034
5	20.110	32.8384
6	20.8686	30.447
7	12.8582	29.4729



Experiment #		STUDENT ID :	
Date :		STUDENT NAME :	

VIVA QUESTIONS:

1. **Define a continuous probability distribution.**

A continuous probability distribution is a standard concept used to describe the likehood of diffeent within values.

2. **How is the probability density function (PDF) used in a continuous probability distribution?**

It describes the probability distribution the CDF of C.D.V by assigning diffeent interval.

3. **What is the cumulative distribution function (CDF) for a continuous probability distribution?**

For a continuous probability the CDF provider into about the prob that random variable takes on a value.

4. **What is the concept of marginal probability in the context of joint random variables?**

Joint $p(X=x) = \sum p(X=x, Y=y)$ for all "y" for continuous random variable
$f_X(x) = \int f - f(xy(x,y))\,dxy$ for all y.

(For Evaluators use only)

Comment of the Evaluator (if Any)	Evaluator's Observation
	Marks Secured: 50 out of 50
	Full Name of the Evaluator: 10/1/24
	Signature of the Evaluator:
	Date of Evaluation