

Matrix theory Assignment 18

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Abstract—This document contains the concept of 3-step transition

Download all python codes from

<https://github.com/saipranavkr/EE5609/codes>

and latex-tikz codes from

<https://github.com/saipranavkr/EE5609>

1 PROBLEM

Consider a Markov chain $\{X_n \mid n \geq 0\}$ with state space $\{1, 2, 3\}$ and transition matrix

$$\mathbf{P} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

Then, $P(X_3 = 1 \mid X_0 = 1)$ equals

2 THEORY

The three step transitional probabilities are given as,

$$P(X_3 = j \mid X_0 = i) = P(X_{n+3} = j \mid X_n = i) = (\mathbf{P}^3)_{ij} \text{ for any } n \quad (2.0.1)$$

3 SOLUTION

$$\mathbf{P}^3 = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}^3 = \begin{pmatrix} \frac{1}{4} & \frac{3}{8} & \frac{3}{8} \\ \frac{3}{8} & \frac{1}{4} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} & \frac{1}{4} \end{pmatrix} \quad (3.0.1)$$

From (3.0.1),

$$P(X_3 = 1 \mid X_0 = 1) = (\mathbf{P}^3)_{11} = \frac{1}{4} \quad (3.0.2)$$