

Matrix theory Assignment 11

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Abstract—This document explains the concept of vector space over a binary field

Download all python codes from

<https://github.com/saipranavkr/EE5609/codes>

and latex-tikz codes from

<https://github.com/saipranavkr/EE5609>

1 PROBLEM

Let V be a vector space over the field $F = \{0, 1\}$. Suppose α, β and γ are linearly independent vectors in V . Comment on $(\alpha + \beta)$, $(\beta + \gamma)$ and $(\gamma + \alpha)$

2 SOLUTION

The addition of elements in the field \mathbf{F} is defined as,

$$\begin{aligned} 0 + 0 &= 0 \\ 1 + 1 &= 0 \end{aligned} \quad (2.0.1)$$

A set are vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent if

$$a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3 = 0 \quad (2.0.2)$$

has only one trivial solution

$$a = b = c = 0 \quad (2.0.3)$$

Now,

$$a(\alpha + \beta) + b(\beta + \gamma) + c(\gamma + \alpha) = 0 \quad (2.0.4)$$

$$\implies (a + c)\alpha + (a + b)\beta + (b + c)\gamma = 0 \quad (2.0.5)$$

Writing (2.0.5) in matrix form,

$$\begin{pmatrix} a+c \\ a+b \\ b+c \end{pmatrix}^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = 0 \quad (2.0.6)$$

Let,

$$\mathbf{A}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \mathbf{A}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \mathbf{A}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\implies a\mathbf{A}_1 + b\mathbf{A}_2 + c\mathbf{A}_3 =$$

$$a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.7)$$

Now,

$$a\mathbf{A}_1 + b\mathbf{A}_2 + c\mathbf{A}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.8)$$

$$\implies a = b = c = 0 \quad (2.0.9)$$

From (2.0.8), $\mathbf{A}_1, \mathbf{A}_2$ and \mathbf{A}_3 are linearly independent. Now, rewriting (2.0.5) as,

$$\begin{aligned} (a + c)\mathbf{A}_1 + (a + b)\mathbf{A}_2 + (b + c)\mathbf{A}_3 &= \\ (a + c) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + (a + b) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \\ (b + c) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} &= \end{aligned}$$

$$a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} +$$

$$a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} +$$

$$b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\implies (a + c)\mathbf{A}_1 + (a + b)\mathbf{A}_2 + (b + c)\mathbf{A}_3 =$$

$$a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} +$$

$$c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.10)$$

For $a = b = c = 1$, using properties from (2.0.1), (2.0.10) becomes,

$$(a + c)\mathbf{A}_1 + (a + b)\mathbf{A}_2 + (b + c)\mathbf{A}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.11)$$

Since it is not the trivial solution. Hence, $(\alpha + \beta)$, $(\beta + \gamma)$ and $(\gamma + \alpha)$ are linearly dependent.