Matrix theory Assignment 3

K R Sai Pranav

***Abstract*—This document solves for X and Y matrices based on the properties of matrix addition**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Find X and Y, if

$$(i) X + Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix} \quad and \quad X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$(ii) 2X + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} \quad and \quad 3X + 2Y = \begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix}$$

2 Solution

(i) Let,

$$X + Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix} = A \tag{2.0.1}$$

$$X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = B \tag{2.0.2}$$

Now, expressing the matrices (2.0.1), (2.0.2) in vector form,

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = A$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = B$$

Combining both the equations into a single matrix equation and constructing the augmented matrix,

$$\begin{pmatrix} 1 & 1 & A \\ 1 & -1 & B \end{pmatrix} \tag{2.0.3}$$

Transforming the equation (2.0.3) using row reduction,

$$\begin{pmatrix} 1 & 1 & A \\ 1 & -1 & B \end{pmatrix} \xleftrightarrow{R2 \leftarrow \frac{R1+R2}{2}} \begin{pmatrix} 1 & 1 & A \\ 1 & 0 & \frac{A+B}{2} \end{pmatrix} \xleftrightarrow{R1 \leftarrow R1-R2}$$

$$\begin{pmatrix} 0 & 1 & \frac{A-B}{2} \\ 1 & 0 & \frac{A+B}{2} \end{pmatrix} \tag{2.0.4}$$

From (2.0.4),

$$X = \frac{A+B}{2} = \begin{pmatrix} 5 & 0 \\ 1 & 4 \end{pmatrix} \tag{2.0.5}$$

$$Y = \frac{A-B}{2} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \tag{2.0.6}$$

(ii) Let,

$$2X + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} = A \tag{2.0.7}$$

$$3X + 2Y = \begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix} = B \tag{2.0.8}$$

Now, expressing the matrices (2.0.7), (2.0.8) in vector form,

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = A$$

$$\begin{pmatrix} 3 & 2 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = B$$

Combining both the equations into a single matrix equation and constructing the augmented matrix,

$$\begin{pmatrix} 2 & 3 & A \\ 3 & 2 & B \end{pmatrix} \tag{2.0.9}$$

Transforming the equation (2.0.9) using row reduction,

$$\begin{pmatrix} 2 & 3 & A \\ 3 & 2 & B \end{pmatrix} \xleftrightarrow{R2 \leftarrow \frac{3R1-2R2}{5}} \begin{pmatrix} 2 & 3 & A \\ 0 & 1 & \frac{3A-2B}{5} \end{pmatrix} \xleftrightarrow{R1 \leftarrow \frac{R1-3R2}{2}}$$

$$\begin{pmatrix} 1 & 0 & \frac{3B-2A}{5} \\ 0 & 1 & \frac{3A-2B}{5} \end{pmatrix} \tag{2.0.10}$$

From (2.0.10),

$$X = \frac{3B - 2A}{5} = \begin{pmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{pmatrix} \tag{2.0.11}$$

$$Y = \frac{3A - 2B}{5} = \begin{pmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{pmatrix} \tag{2.0.12}$$