Matrix theory Assignment 19

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***Abstract*—This document contains the concept of Jordan canonical form**

Download all python codes from

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https://github.com/saipranavkr/EE5609/codes
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and latex-tikz codes from

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https://github.com/saipranavkr/EE5609
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1 Problem

If A is a complex 5×5 matrix with characteristic polynomial $f = (x-2)^3(x+7)^2$ and minimal polynomial $p = (x-2)^2(x+7)$, what is the Jordan form for A?

2 Theory

For an eigenvalue λ, (1) the multiplicity of λ in the characteristic polynomial determines the size of the Jordan block for that eigenvalue. (2) the multiplicity of λ in the minimal polynomial determines the size of the largest sub-block (Elementary Jordan block). Let f and p be the characteristic and minimal polynomials of a matrix,

$$f = (x-\lambda_1)^3 \tag{2.0.1}$$

$$p = (x-\lambda_1)^2 \tag{2.0.2}$$

To satisfy both (2.0.1) and (2.0.2), the size of Jordan block must be 3 and the largest block should be of size 2. Hence, Jordan form is written as,

$$\mathbf{J} = \begin{pmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_1 \end{pmatrix} \tag{2.0.3}$$

3 Solution

Based on (2.0.1) and (2.0.2), the Jordan blocks for eigenvalues of $\mathbf{A}$ can be written as,

$$\mathbf{J_1} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}; \quad \mathbf{J_2} = \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} \tag{3.0.1}$$

Where $\mathbf{J_1}$ and $\mathbf{J_1}$ are the Jordan blocks corresponding to $\lambda_1 = 2$ and $\lambda_1 = -7$ respectively. The Jordan form for $\mathbf{A}$ can be written as,

$$\mathbf{J} = \begin{pmatrix} \mathbf{J_1} & 0 \\ 0 & \mathbf{J_2} \end{pmatrix} = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -7 & 0 \\ 0 & 0 & 0 & 0 & -7 \end{pmatrix} \tag{3.0.2}$$

4 Inference

An $n \times n$ matrix with with λ as diagonal elements, ones on the super diagonal and zeroes in all other entries is nilpotent with minimal polynomial $(A - \lambda I)^n$

5 Example

Let,

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{5.0.1}$$

(5.0.1) is nilpotent for minimal polynomial A^3