Matrix theory Assignment 10

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Abstract—**This document explains the conversion of a matrix into both row and column reduced echelon form.**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Let A be a $m \times n$ matrix. Show that by a finite number of elementary row and/or column operations one can pass from A to a matrix R which is both row-reduced echelon and column-reduced echelon , i.e., $R_{ij} = 0$ if $i \neq j$, $R_{ii} = 1$, $1 \leq i \leq r$, $R_{ii} = 0$, if $i > r$. Show that $R = PAQ$, where P is an invertible $m \times m$ matrix and Q is an invertible $n \times n$ matrix.

2 Lemma

Every elementary matrix is invertible and the inverse is again an elementary matrix. If an elementary matrix E is obtained from I by using a certain row or column operation q, then E^{-1} is obtained from I by the "inverse" operation q^{-1}.

3 Solution

Given $\mathbf{A}$ is a $m \times n$ matrix. First converting $\mathbf{A}$ into row reduced echelon form by performing a series of elementary row operations. The elementary matrix that performs sequence of elementary operations is $\mathbf{P}$. Let $\mathbf{R}'$ be the row reduced echelon matrix. So, $\mathbf{P}$ is a $m \times m$ matrix. Also, by using the lemma we can tell that $\mathbf{P}$ is invertible.

$$\mathbf{R}' = \mathbf{PA} \tag{3.0.1}$$

A row of $\mathbf{R}'$ can be all zeroes, or can start with zeroes from the left then has a one, and can have non-zero entries after one. Suppose if we add a multiple of pivot column to any other column, it only effects the row containing the pivot(pivot is one) since all elements above and below the pivot are zeroes. So, performing a series of such actions can transform a row of all zeroes and single one(The pivot). Let $\mathbf{Q}$ be the sequence of elementary column operations performed on $\mathbf{R}'$. So, $\mathbf{Q}$ is a $n \times n$ matrix. Also, by using the lemma we can tell that $\mathbf{Q}$ is invertible.

$$\mathbf{R} = \mathbf{R}'\mathbf{Q} \tag{3.0.2}$$

Substituting (3.0.1) in (3.0.2),

$$\mathbf{R} = \mathbf{PAQ} \tag{3.0.3}$$

$\mathbf{R}$ in (3.0.3) is in both row and column reduced echelon form. Hence proved.