Matrix theory Assignment 11

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***Abstract*—This document explains the concept of vector space over a binary field**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Let V be a vector space over the field $F = \{0, 1\}$. Suppose α, β and γ are linearly independent vectors in V. Comment on $(\alpha + \beta)$, $(\beta + \gamma)$ and $(\gamma + \alpha)$

2 Solution

The addition of elements in the field $\mathbf{F}$ is defined as,

$$\begin{aligned} 0 + 0 &= 0 \\ 1 + 1 &= 0 \end{aligned} \tag{2.0.1}$$

A set are vectors $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ are linearly independent if

$$a\mathbf{v_1} + b\mathbf{v_2} + c\mathbf{v_3} = 0 \tag{2.0.2}$$

has only one trivial solution

$$a = b = c = 0 \tag{2.0.3}$$

Now,

$$a(\alpha + \beta) + b(\beta + \gamma) + c(\gamma + \alpha) = 0 \tag{2.0.4}$$

$$\implies (a + c)\alpha + (a + b)\beta + (b + c)\gamma = 0 \tag{2.0.5}$$

Writing (2.0.5) in matrix form,

$$\begin{pmatrix} a+c & 0 & 0 \\ 0 & a+b & 0 \\ 0 & 0 & b+c \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.6}$$

In (2.0.6), since α, β and γ are linearly independent vectors, hence the only solution is

$$a + c = 0; \quad a + b = 0; \quad b + c = 0 \tag{2.0.7}$$

From (2.0.1), the possible values of a, b, c are,

$$a = b = c = 0; \tag{2.0.8}$$

$$a = b = c = 1; \tag{2.0.9}$$

If (2.0.9) is a solution, then (2.0.4)

$$a(\alpha + \beta) + b(\beta + \gamma) + c(\gamma + \alpha) = 0 \tag{2.0.10}$$

has a solution $a = b = c = 1$, which is not the trivial solution. Hence, $(\alpha + \beta)$, $(\beta + \gamma)$ and $(\gamma + \alpha)$ are linearly dependent.