Matrix theory Assignment 14

K R Sai Pranav

***Abstract*—This document contains the concept of linear, one-one transformation**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Let $\mathbf{V}$ be the set of complex numbers regarded as a vector space over the field of real numbers. We define a function T from $\mathbf{V}$ into the space of 2×2 real matrices, as follows. If $z = x + iy$ with x and y real numbers, then

$$\mathbf{T}(z) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix}$$

Verify that T is a one-one (real) linear transformation of $\mathbf{V}$ into the space of 2×2 real matrices.

2 Solution

Given,

$$\mathbf{T} : \mathbf{C} \to \mathbf{R^2}$$
$$\mathbf{T}(x+iy) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{2.0.1}$$

Let,

$$z = x + iy; \quad w = a + ib; \quad z, w \in \mathbf{C}$$

$$\mathbf{T}(\alpha z + w) = \mathbf{T}((\alpha x + a) + i(\alpha y + b)) =$$
$$\begin{pmatrix} (\alpha x + a) + 7(\alpha y + b) & 5(\alpha y + b) \\ -10(\alpha y + b) & (\alpha x + a) - 7(\alpha y + b) \end{pmatrix} =$$
$$\begin{pmatrix} \alpha(x + 7y) + (a + 7b) & \alpha(5y) + 5b \\ \alpha(-10y) - 10b & \alpha(x - 7y) + (a - 7b) \end{pmatrix} =$$
$$\alpha \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} + \begin{pmatrix} a+7b & 5b \\ -10b & a-7b \end{pmatrix} =$$
$$\alpha \mathbf{T}(z) + \mathbf{T}(w) \tag{2.0.2}$$

(2.0.2) proves $\mathbf{T}$ is a linear operator. Suppose,

$$\mathbf{T}(z) = \mathbf{T}(w)$$
$$\implies \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} = \begin{pmatrix} a+7b & 5b \\ -10b & a-7b \end{pmatrix} \tag{2.0.3}$$

By comparing terms of (2.0.3),

$$x = a; \quad y = b; \quad \implies x + iy = a + ib \tag{2.0.4}$$

$$\implies z = w \tag{2.0.5}$$

Hence, $\mathbf{T}$ is a one-one transformation