Matrix theory Assignment 8

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***Abstract*—This document explains QR decomposition of a matrix with an example**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Find the QR decomposition of

$$\mathbf{A} = \begin{pmatrix} 4 & 7 \\ 3 & 5 \end{pmatrix}$$

2 Solution

If $\mathbf{A} \in \mathbf{R}^{m\times n}$ has linearly independent columns then it can be factored as

$$\mathbf{A} = \mathbf{QR}$$

where $\mathbf{Q}$ is a orthogonal matrix and $\mathbf{R}$ is a upper triangular matrix with non zero diagonal elements

$$\mathbf{A} = \begin{pmatrix} 4 & 7 \\ 3 & 5 \end{pmatrix} \tag{2.0.1}$$

The column vectors of $\mathbf{A}$ are,

$$\mathbf{a} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}; \quad \mathbf{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{2.0.2}$$

(2.0.1) can be written as,

$$\mathbf{QR} = \begin{pmatrix} \mathbf{p_1} & \mathbf{p_2} \end{pmatrix} \begin{pmatrix} u_1 & u_3 \\ 0 & u_2 \end{pmatrix} \tag{2.0.3}$$

Now,

$$u_1 = \|a\| = \sqrt{4^2 + 7^2} = \sqrt{65} \tag{2.0.4}$$

$$\mathbf{q_1} = \frac{\mathbf{a}}{u_1} = \begin{pmatrix} \frac{4}{\sqrt{65}} \\ \frac{7}{\sqrt{65}} \end{pmatrix} \tag{2.0.5}$$

$$u_3 = \frac{\mathbf{q_1}^T \mathbf{b}}{\|\mathbf{q_1}\|^2} = \begin{pmatrix} \frac{4}{\sqrt{65}} & \frac{7}{\sqrt{65}} \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \frac{47}{\sqrt{65}} \tag{2.0.6}$$

$$\mathbf{q_2} = \frac{\mathbf{b} - u_3\mathbf{q_1}}{\|\mathbf{b} - u_3\mathbf{q_1}\|} = \begin{pmatrix} \frac{7}{\sqrt{65}} \\ -\frac{4}{\sqrt{65}} \end{pmatrix} \tag{2.0.7}$$

$$u_2 = \mathbf{q_2}^T \mathbf{b} = \begin{pmatrix} \frac{7}{\sqrt{65}} & -\frac{4}{\sqrt{65}} \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \frac{1}{\sqrt{65}} \tag{2.0.8}$$

Substituting (2.0.4) to (2.0.8) in (2.0.3),

$$\begin{pmatrix} 4 & 7 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} \frac{4}{\sqrt{65}} & \frac{7}{\sqrt{65}} \\ \frac{7}{\sqrt{65}} & -\frac{4}{\sqrt{65}} \end{pmatrix} \begin{pmatrix} \sqrt{65} & \frac{47}{\sqrt{65}} \\ 0 & \frac{1}{\sqrt{65}} \end{pmatrix} \tag{2.0.9}$$

Which can also be written as,

$$\begin{pmatrix} 4 & 7 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} -\frac{4}{\sqrt{65}} & -\frac{7}{\sqrt{65}} \\ -\frac{7}{\sqrt{65}} & \frac{4}{\sqrt{65}} \end{pmatrix} \begin{pmatrix} -\sqrt{65} & -\frac{47}{\sqrt{65}} \\ 0 & -\frac{1}{\sqrt{65}} \end{pmatrix} \tag{2.0.10}$$