# Matrix theory Assignment 13

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***Abstract*—This document explains the commutativity of transformations**

Download all python codes from

```
https://github.com/saipranavkr/EE5609/codes
```

and latex-tikz codes from

```
https://github.com/saipranavkr/EE5609
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## 1 Problem

Find two linear operators $\mathbf{T}$ and $\mathbf{U}$ on $\mathbf{R}^2$ such that $\mathbf{TU} = 0$ but $\mathbf{UT} \neq 0$

## 2 Solution

Let a vector,

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbf{R^2} \quad x, y \neq 0 \tag{2.0.1}$$

Let,

$$\mathbf{T}(\mathbf{X}) = \mathbf{AX}; \quad \mathbf{U}(\mathbf{X}) = \mathbf{BX}; \tag{2.0.2}$$

such that

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.3}$$

Consider,

$$\mathbf{TU} : \mathbf{R^2} \to \mathbf{R^2}$$

$$(\mathbf{TU})(\mathbf{X}) = \mathbf{T}(\mathbf{U}(\mathbf{X})) = \mathbf{T}(\mathbf{BX}) = \mathbf{ABX} \tag{2.0.4}$$

Simplifying (2.0.4),

$$\mathbf{ABX} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \implies \mathbf{TU} = 0 \tag{2.0.5}$$

Now since,

$$\mathbf{UT} : \mathbf{R^2} \to \mathbf{R^2}$$

$$(\mathbf{UT})(\mathbf{X}) = \mathbf{U}(\mathbf{T}(\mathbf{X})) = \mathbf{U}(\mathbf{AX}) = \mathbf{BAX} \tag{2.0.6}$$

Simplifying (2.0.6),

$$\mathbf{BAX} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \implies \mathbf{UT} \neq 0 \tag{2.0.7}$$