Matrix theory Assignment 14

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***Abstract*—This document contains the concept of linear, one-one transformation**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Let $\mathbf{V}$ be the set of complex numbers regarded as a vector space over the field of real numbers. We define a function T from $\mathbf{V}$ into the space of 2×2 real matrices, as follows. If $z = x + iy$ with x and y real numbers, then

$$\mathbf{T}(z) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix}$$

Verify that T is a one-one (real) linear transformation of $\mathbf{V}$ into the space of 2×2 real matrices.

2 Solution

Given,

$$\mathbf{T} : \mathbf{C} \to \mathbf{R^2}$$

$$\mathbf{T}(x+iy) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{2.0.1}$$

Let,

$$z = x + iy; \quad w = a + ib; \quad z, w \in \mathbf{C}$$

The representation of a complex number $z = x + iy$ in matrix form is:

$$\mathbf{z} = \begin{pmatrix} x & -y \\ y & x \end{pmatrix} \tag{2.0.2}$$

Also the RHS of (2.0.1) can be expressed as,

$$\begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} = \left(\begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \begin{pmatrix} 5 & 0 \\ -7 & -1 \end{pmatrix} \begin{pmatrix} y \\ -x \end{pmatrix} \right) \tag{2.0.3}$$

The transformation of (2.0.2) is (2.0.3)

$$\mathbf{T}(\mathbf{z}) = \left(\begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \begin{pmatrix} 5 & 0 \\ -7 & -1 \end{pmatrix} \begin{pmatrix} y \\ -x \end{pmatrix} \right) \tag{2.0.4}$$