Matrix theory Assignment 4

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Abstract—This document solves for the determinant of a matrix using properties of determinants

1 PROBLEM

By using the properties of determinants, show that

$$\begin{vmatrix} 1+a^2 & ab & ac \\ ab & 1+b^2 & bc \\ ac & bc & 1+c^2 \end{vmatrix} = 1+a^2+b^2+c^2$$

2 SOLUTION

$$\begin{vmatrix} 1+a^2 & ab & ac \\ ab & 1+b^2 & bc \\ ac & bc & 1+c^2 \end{vmatrix} \xleftarrow{R1 \leftarrow aR1;R2 \leftarrow bR2;R3 \leftarrow cR3}$$

$$\frac{1}{abc} \begin{vmatrix} a(1+a^2) & a^2b & a^2c \\ ab^2 & b(1+b^2) & b^2c \\ ac^2 & bc^2 & c(1+c^2) \end{vmatrix} \xleftarrow{C1 \leftarrow \frac{C1}{a};C2 \leftarrow \frac{C2}{b};C3 \leftarrow \frac{C3}{c}}$$

$$\begin{vmatrix} 1+a^2 & a^2 & a^2 \\ b^2 & 1+b^2 & b^2 \\ c^2 & c^2 & 1+c^2 \end{vmatrix} \xleftarrow{R1 \leftarrow R1+R2+R3}$$

$$\begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & 1+b^2 & b^2 \\ c^2 & c^2 & 1+c^2 \end{vmatrix}$$

$$(2.0.1)$$

Taking $1+a^2+b^2+c^2$ out from (2.0.1),

$$\implies (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & 1+b^2 & b^2 \\ c^2 & c^2 & 1+c^2 \end{vmatrix}$$

$$\xleftarrow{C2 \leftarrow C2-C1;C3 \leftarrow C3-C1} (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

$= 1+a^2+b^2+c^2$ ($\because$ Determinant of a lower triangle matrix is the product of it's diagonal elements)