Matrix theory Assignment 5

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***Abstract*—This document explains the concept of a property regarding triangles**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Triangles on the same base(or equal bases) and between the same parallels are equal in area

2 solution

Consider 2 matrices,

$$\mathbf{A} = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \quad and \quad \mathbf{B} = \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}$$

The cross product of the 2 matrices is,

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{2.0.1}$$

Substituting $a_3 = b_3 = 0$ in (2.0.1) and simplifying,

$$\implies \mathbf{A} \times \mathbf{B} = \begin{pmatrix} 0 & a_1 \\ -a_2 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{2.0.2}$$

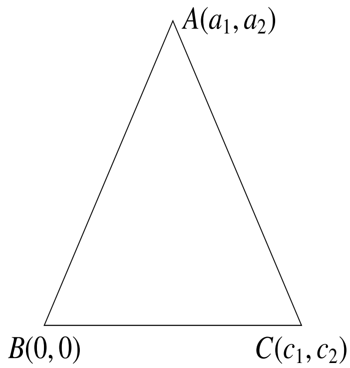

Fig. 1: $\triangle ABC$ with B at origin

Considering three points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ on a triangle and $\mathbf{B}$ at origin,

$$\mathbf{A} - \mathbf{B} = \mathbf{A} \tag{2.0.3}$$
$$\mathbf{C} - \mathbf{B} = \mathbf{C} \tag{2.0.4}$$

Area of triangle is,

$$Area(\triangle ABC) = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{C} - \mathbf{B})\| \tag{2.0.5}$$

Substituting (2.0.3), (2.0.4) in (2.0.5),

$$\implies Area(\triangle ABC) = \frac{1}{2} \|\mathbf{A} \times \mathbf{C}\| \tag{2.0.6}$$

Constructing another triangle DBC with base as BC,

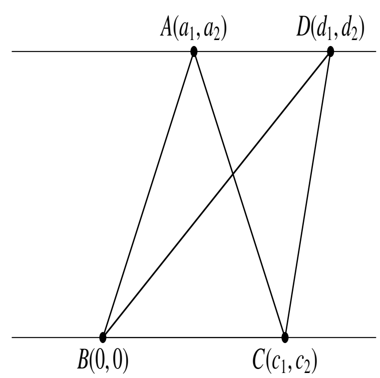

Fig. 2: $\triangle ABC$ and $\triangle DBC$ with BC as common base

Since AD || BC,

$$\mathbf{A} - \mathbf{D} = k(\mathbf{B} - \mathbf{C}) \tag{2.0.7}$$

Now calculating the area of $\triangle DBC$,

$$Area(\triangle DBC) = \frac{1}{2} \|(\mathbf{D} - \mathbf{B}) \times (\mathbf{C} - \mathbf{B})\| \tag{2.0.8}$$

Substituting (2.0.7) in (2.0.8),

$$\implies Area(\triangle DBC) = \frac{1}{2}\left\|(\mathbf{A} - k(\mathbf{B} - \mathbf{C})) \times (\mathbf{C} - \mathbf{B})\right\|$$

$$\implies Area(\triangle DBC) = \frac{1}{2}\left\|(\mathbf{A} + k\mathbf{C}) \times \mathbf{C}\right\|$$

$$(\because \mathbf{C} - \mathbf{B} = \mathbf{C})$$

$$\implies Area(\triangle DBC) = \frac{1}{2}\left\|\mathbf{A} \times \mathbf{C}\right\| (\because \mathbf{A} \times \mathbf{A} = 0) \tag{2.0.9}$$

It can be observed that (2.0.9) is same as (2.0.6) Hence, triangles on the same base(or equal bases) and between the same parallels are equal in area.