

Matrix theory Assignment 5

K R Sai Pranav

Abstract—This document explains the concept of a property regarding triangles

Download all python codes from

<https://github.com/saipranavkr/EE5609/codes>

and latex-tikz codes from

<https://github.com/saipranavkr/EE5609>

1 PROBLEM

Triangles on the same base(or equal bases) and between the same parallels are equal in area

2 SOLUTION

Consider 2 matrices,

$$\mathbf{A} = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}$$

The cross product of the 2 matrices is,

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad (2.0.1)$$

Substituting $a_3 = b_3 = 0$ in (2.0.1) and simplifying,

$$\Rightarrow \mathbf{A} \times \mathbf{B} = \begin{pmatrix} 0 & a_1 \\ -a_2 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad (2.0.2)$$

Considering three points on a triangle as,

$$\mathbf{A} = \begin{pmatrix} a_1 & a_2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 & 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} c_1 & c_2 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \mathbf{A} \quad (2.0.4)$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \mathbf{C} \quad (2.0.5)$$

Area of triangle is,

$$Area(\triangle ABC) = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{C} - \mathbf{B})\| \quad (2.0.6)$$

Substituting (2.0.4), (2.0.5) in (2.0.6),

$$Area(\triangle ABC) = \frac{1}{2} \|\mathbf{A} \times \mathbf{C}\| \quad (2.0.7)$$

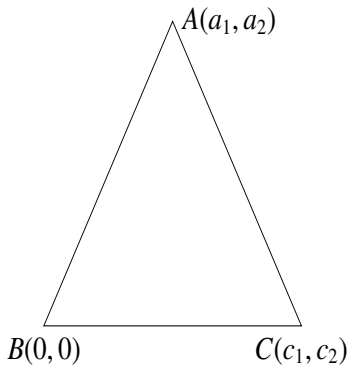


Fig. 1: $\triangle ABC$ with B at origin