Matrix theory Assignment 4

K R Sai Pranav

***Abstract*—This document solves for the determinant of a matrix using properties of determinants**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

By using the properties of determinants, show that

$$\begin{vmatrix} 1+a^2 & ab & ac \\ ab & 1+b^2 & bc \\ ac & bc & 1+c^2 \end{vmatrix} = 1+a^2+b^2+c^2$$

2 Solution

$$\begin{vmatrix} 1+a^2 & ab & ac \\ ab & 1+b^2 & bc \\ ac & bc & 1+c^2 \end{vmatrix} \xleftrightarrow[R_3 \leftarrow cR_3]{R_1 \leftarrow aR_1; R_2 \leftarrow bR_2}$$

$$\frac{1}{abc}\begin{vmatrix} a(1+a^2) & a^2b & a^2c \\ ab^2 & b(1+b^2) & b^2c \\ ac^2 & bc^2 & c(1+c^2) \end{vmatrix} \xleftrightarrow[C_3 \leftarrow \frac{C_3}{c}]{C_1 \leftarrow \frac{C_1}{a}; C_2 \leftarrow \frac{C_2}{b}}$$

$$\begin{vmatrix} 1+a^2 & a^2 & a^2 \\ b^2 & 1+b^2 & b^2 \\ c^2 & c^2 & 1+c^2 \end{vmatrix} \xleftrightarrow{R_1 \leftarrow R_1+R_2+R_3}$$

$$\begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & 1+b^2 & b^2 \\ c^2 & c^2 & 1+c^2 \end{vmatrix} \tag{2.0.1}$$

Taking $1+a^2+b^2+c^2$ out from (2.0.1),

$$\implies (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & 1+b^2 & b^2 \\ c^2 & c^2 & 1+c^2 \end{vmatrix}$$

$$\xleftrightarrow[C_3 \leftarrow C_3 - C_1]{C_2 \leftarrow C_2 - C_1} (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

$= 1+a^2+b^2+c^2$ ($\because$ Determinant of a lower triangle matrix is the product of it's diagonal elements)