Matrix theory Assignment 4

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Abstract—This document solves for the determinant of a matrix using properties of determinants

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 PROBLEM

By using the properties of determinants, show that

$$
\begin{vmatrix}
1 + a^2 & ab & ac \\
ab & 1 + b^2 & bc \\
ac & bc & 1 + c^2
\end{vmatrix}
= 1 + a^2 + b^2 + c^2
$$

2 SOLUTION

$$
\begin{vmatrix}
1 + a^2 & ab & ac \\
ab & 1 + b^2 & bc \\
ac & bc & 1 + c^2
\end{vmatrix}
\xleftarrow{R_1 \leftarrow aR_1 ; R_2 \leftarrow bR_2 ; R_3 \leftarrow cR_3}
$$

$$
\frac{1}{abc}
\begin{vmatrix}
a(1 + a^2) & a^2 b & a^2 c \\
ab^2 & b(1 + b^2) & b^2 c \\
ac^2 & bc^2 & c(1 + c^2)
\end{vmatrix}
\xleftarrow{C_1 \leftarrow \frac{C_1}{a} ; C_2 \leftarrow \frac{C_2}{b} ; C_3 \leftarrow \frac{C_3}{c}}
$$

$$
\begin{vmatrix}
1 + a^2 & a^2 & a^2 \\
b^2 & 1 + b^2 & b^2 \\
c^2 & c^2 & 1 + c^2
\end{vmatrix}
\xleftarrow{R_1 \leftarrow R_1 + R_2 + R_3}
$$

$$
\begin{vmatrix}
1 + a^2 + b^2 + c^2 & 1 + a^2 + b^2 + c^2 & 1 + a^2 + b^2 + c^2 \\
b^2 & 1 + b^2 & b^2 \\
c^2 & c^2 & 1 + c^2
\end{vmatrix}
\tag{2.0.1}
$$

Taking $1 + a^2 + b^2 + c^2$ out from (2.0.1),

$$
\implies (1 + a^2 + b^2 + c^2)
\begin{vmatrix}
1 & 1 & 1 \\
b^2 & 1 + b^2 & b^2 \\
c^2 & c^2 & 1 + c^2
\end{vmatrix}
$$

$$
\xleftarrow{C_2 \leftarrow C_2 - C_1 ; C_3 \leftarrow C_3 - C_1}
(1 + a^2 + b^2 + c^2)
\begin{vmatrix}
1 & 0 & 0 \\
b^2 & 1 & 0 \\
c^2 & 0 & 1
\end{vmatrix}
$$

$= 1 + a^2 + b^2 + c^2$ ($\because$ Determinant of a lower triangle matrix is the product of it's diagonal elements)