

# Matrix theory Assignment 10

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**Abstract**—This document explains the conversion of a matrix into both row and column reduced echelon form.

Download all python codes from

<https://github.com/saipranavkr/EE5609/codes>

and latex-tikz codes from

<https://github.com/saipranavkr/EE5609>

## 1 PROBLEM

Let  $A$  be a  $m \times n$  matrix. Show that by a finite number of elementary row and/or column operations one can pass from  $A$  to a matrix  $R$  which is both row-reduced echelon and column-reduced echelon, i.e.,  $R_{ij} = 0$  if  $i \neq j$ ,  $R_{ii} = 1$ ,  $1 \leq i \leq r$ ,  $R_{ii} = 0$ , if  $i > r$ . Show that  $R = PAQ$ , where  $P$  is an invertible  $m \times m$  matrix and  $Q$  is an invertible  $n \times n$  matrix.

## 2 LEMMA

Every elementary matrix is invertible and the inverse is again an elementary matrix. If an elementary matrix  $E$  is obtained from  $I$  by using a certain row or column operation  $q$ , then  $E^{-1}$  is obtained from  $I$  by the "inverse" operation  $q^{-1}$ .

## 3 SOLUTION

Given  $A$  is a  $m \times n$  matrix. Converting  $A$  into row reduced echelon form by performing a series of elementary row operations  $P$ . Let  $R'$  be the row reduced echelon matrix. Also, by using the lemma we can tell that  $P$  is invertible and order  $m \times m$ .

$$R' = PA \quad (3.0.1)$$

where,

$$R' = \begin{pmatrix} I & F \\ 0 & 0 \end{pmatrix}$$

$I$  is an identity matrix,  $F$  is Free variables matrix and  $0$  represents a block of zeroes

$R'$  is in row-reduced echelon form. To perform column operations, elementary matrices should be multiplied on the right side in order to convert the  $R'$  into column-reduced echelon form

$$R = R'Q \quad (3.0.2)$$

But performing column operations on a matrix is equivalent to performing row operations on the transposed matrix.

$$\begin{aligned} R^T &= (R'Q)^T \\ \Rightarrow R^T &= Q^T R'^T \end{aligned} \quad (3.0.3)$$

Hence, by using lemma it can be observed that  $Q^T$  is invertible and of the order  $n \times n$ . Converting  $R^T$  to row-reduced echelon is equivalent to converting  $R$  to column-reduced echelon.

$$R = PAQ \quad (3.0.4)$$

where,

$$R = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

$I$  is an identity matrix and  $0$  represents a block of zeroes.  $Q$  is a upper triangular matrix.  $R$  in (3.0.4) is in both row and column reduced echelon form. Hence proved.