Matrix theory Assignment 5

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***Abstract*—This document explains the concept of a property regarding triangles**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Triangles on the same base(or equal bases) and between the same parallels are equal in area

2 solution

Consider 2 matrices,

$$\mathbf{A} = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \quad and \quad \mathbf{B} = \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}$$

The cross product of the 2 matrices is,

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{2.0.1}$$

Substituting $a_3 = b_3 = 0$ in (2.0.1) and simplifying,

$$\implies \mathbf{A} \times \mathbf{B} = \begin{pmatrix} 0 & a_1 \\ -a_2 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{2.0.2}$$

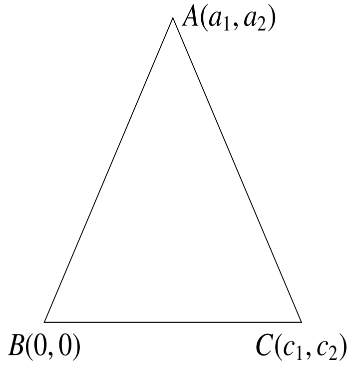

Fig. 1: $\triangle ABC$ with B at origin

Considering three points on a triangle as,

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \mathbf{A} \tag{2.0.4}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \mathbf{C} \tag{2.0.5}$$

Area of triangle is,

$$Area(\triangle ABC) = \frac{1}{2} \left\| (\mathbf{A} - \mathbf{B}) \times (\mathbf{C} - \mathbf{B}) \right\| \tag{2.0.6}$$

Substituting (2.0.4), (2.0.5) in (2.0.6),

$$\implies Area(\triangle ABC) = \frac{1}{2} \left\| \mathbf{A} \times \mathbf{C} \right\| \tag{2.0.7}$$

Constructing another triangle DBC with base as BC,

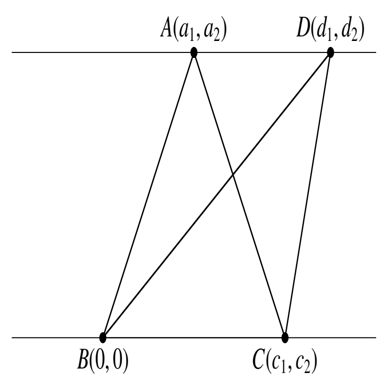

Fig. 2: $\triangle ABC$ and $\triangle DBC$ with BC as common base