# Matrix theory Assignment 13

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***Abstract*—This document contains the concept of linear transformations**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

## 1 Problem

Find two linear operators $\mathbf{T}$ and $\mathbf{U}$ on $\mathbf{R}^2$ such that $\mathbf{TU} = 0$ but $\mathbf{UT} \neq 0$

## 2 Solution

Let,

$$\mathbf{x}, \mathbf{y} \in \mathbf{R^2} \tag{2.0.1}$$

Let $\mathbf{T}$ and $\mathbf{U}$ be given by the matrices

$$\mathbf{T}(\mathbf{x}) = \mathbf{Ax}; \quad \mathbf{U}(\mathbf{x}) = \mathbf{Bx} \tag{2.0.2}$$

where,

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{T}(a\mathbf{x} + \mathbf{y}) = a\mathbf{Tx} + \mathbf{Ty} \tag{2.0.4}$$

$$\mathbf{U}(a\mathbf{x} + \mathbf{y}) = a\mathbf{Ux} + \mathbf{Uy} \tag{2.0.5}$$

From (2.0.4) and (2.0.5), we can tell that $\mathbf{T}$ and $\mathbf{U}$ are linear operators. Now,

$$\mathbf{TU} = \mathbf{AB} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \mathbf{0} \tag{2.0.6}$$

$$\mathbf{UT} = \mathbf{BA} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq \mathbf{0} \tag{2.0.7}$$

From (2.0.6) and (2.0.7) it can be observed that $\mathbf{TU}$ = 0 but $\mathbf{UT} \neq 0$