Matrix theory Assignment 14

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Abstract—**This document contains the concept of linear, one-one transformation**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Let $\mathbf{V}$ be the set of complex numbers regarded as a vector space over the field of real numbers. We define a function T from $\mathbf{V}$ into the space of 2×2 real matrices, as follows. If $z = x + iy$ with x and y real numbers, then

$$\mathbf{T}(z) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix}$$

Verify that T is a one-one (real) linear transformation of $\mathbf{V}$ into the space of 2×2 real matrices.

2 Solution

Given,

$$\mathbf{T} : \mathbf{C} \to \mathbf{R^2}$$

$$\mathbf{T}(x+iy) = \begin{pmatrix} x+7y & 5y \\ -10y & x-7y \end{pmatrix} \tag{2.0.1}$$

Let,

$$z = x + iy; \quad w = a + ib; \quad z, w \in \mathbf{C}$$

Also the RHS of (2.0.1) can be expressed as,

$$\mathbf{T}(\mathbf{z}) = \left(\begin{pmatrix} 1 & 7 \\ 0 & -10 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \begin{pmatrix} 0 & 5 \\ 1 & -7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right)$$

$$= \begin{pmatrix} 1 & 7 & 0 & 5 \\ 0 & -10 & 1 & -7 \end{pmatrix} \begin{pmatrix} x & 0 \\ y & 0 \\ 0 & x \\ 0 & y \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \mathbf{x} & 0 \\ 0 & \mathbf{x} \end{pmatrix} \tag{2.0.2}$$

where $\mathbf{A}$ and $\mathbf{B}$ are block matrices and,

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Consider,

$$\mathbf{T}(\alpha\mathbf{z} + \mathbf{w}) = \begin{pmatrix} 1 & 7 & 0 & 5 \\ 0 & -10 & 1 & -7 \end{pmatrix} \begin{pmatrix} \alpha x + a & 0 \\ \alpha y + b & 0 \\ 0 & \alpha x + a \\ 0 & \alpha y + b \end{pmatrix}$$

$$= \begin{pmatrix} (\alpha x + a) + 7(\alpha y + b) & 5(\alpha y + b) \\ -10(\alpha y + b) & (\alpha x + a) - 7(\alpha y + b) \end{pmatrix}$$

$$= \alpha \begin{pmatrix} 1 & 7 & 0 & 5 \\ 0 & -10 & 1 & -7 \end{pmatrix} \begin{pmatrix} x & 0 \\ y & 0 \\ 0 & x \\ 0 & y \end{pmatrix} +$$

$$\begin{pmatrix} 1 & 7 & 0 & 5 \\ 0 & -10 & 1 & -7 \end{pmatrix} \begin{pmatrix} a & 0 \\ b & 0 \\ 0 & a \\ 0 & b \end{pmatrix}$$

So,

$$\mathbf{T}(\alpha\mathbf{z} + \mathbf{w}) = \alpha \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \mathbf{x} & 0 \\ 0 & \mathbf{x} \end{pmatrix} + \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \mathbf{w} & 0 \\ 0 & \mathbf{w} \end{pmatrix}$$

$$= \alpha\mathbf{T}(\mathbf{z}) + \mathbf{T}(\mathbf{w}) \tag{2.0.3}$$

From (2.0.3), $\mathbf{T}$ is a linear operator.