

# Matrix theory Assignment 11

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**Abstract**—This document explains the concept of vector space over a binary field

Download all python codes from

<https://github.com/saipranavkr/EE5609/codes>

and latex-tikz codes from

<https://github.com/saipranavkr/EE5609>

## 1 PROBLEM

Let  $V$  be a vector space over the field  $F = \{0, 1\}$ . Suppose  $\alpha, \beta$  and  $\gamma$  are linearly independent vectors in  $V$ . Comment on  $(\alpha + \beta)$ ,  $(\beta + \gamma)$  and  $(\gamma + \alpha)$

## 2 SOLUTION

The addition of elements in the field  $\mathbf{F}$  is defined as,

$$\begin{aligned} 0 + 0 &= 0 \\ 1 + 1 &= 0 \end{aligned} \quad (2.0.1)$$

A set are vectors  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  are linearly independent if

$$a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3 = 0 \quad (2.0.2)$$

has only one trivial solution

$$a = b = c = 0 \quad (2.0.3)$$

Now,

$$a(\alpha + \beta) + b(\beta + \gamma) + c(\gamma + \alpha) = 0 \quad (2.0.4)$$

$$\implies (a + c)\alpha + (a + b)\beta + (b + c)\gamma = 0 \quad (2.0.5)$$

Writing (2.0.5) in matrix form,

$$\begin{pmatrix} \alpha & \beta & \gamma \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.6)$$

where,

$$\mathbf{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\mathbf{x}^T \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = 0 \quad (2.0.7)$$

Since  $\alpha, \beta$  and  $\gamma$  are linearly independent vectors,

$$\mathbf{x}^T \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \quad (2.0.8)$$

Transposing on both sides,

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.9)$$

By using the properties from (2.0.1) and reducing (2.0.9) to row echelon form,

$$\begin{aligned} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} &\xrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \\ &\xrightarrow{R_3 \leftarrow R_2 + R_3} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (2.0.10)$$

From (2.0.10),

$$a + c = 0; \quad b + c = 0 \quad (2.0.11)$$

The solutions to (2.0.11) are,

$$a = b = c = 0; \quad a = b = c = 1 \quad (2.0.12)$$

Since there is no trivial solution,  $(\alpha + \beta)$ ,  $(\beta + \gamma)$  and  $(\gamma + \alpha)$  are linearly dependent