SAIR

Spatial AI & Robotics Lab

CSE 473/573-A

L5: FILTERING

Chen Wang

Spatial AI & Robotics Lab

Department of Computer Science and Engineering

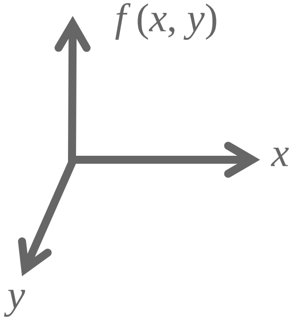
University at Buffalo The State University of New York

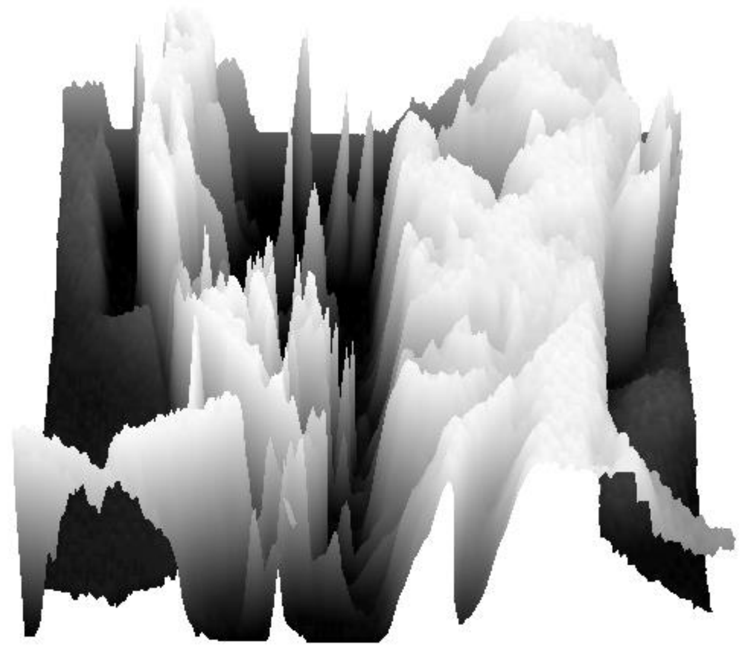
Content

- Filtering
  - Linear filters
  - Correlation and Convolution
  - Equivariance, Invariance
  - Smoothing, Gaussian Filter, Median filter

Recap: Image representation

- A (grayscale) image as a **function**, *f*, from R^2 to R:
  - $f\,(x, y)$ gives the **intensity** at position (x, y).
  - A **digital image** is a discrete (**sampled**, **quantized**) version of this function.





Recap: Images as functions

- Take an image as a function, f, from R^2 to R:
  - $f(x, y)$ gives the intensity at position (x, y)
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
    - f: $[a, b] \times [c, d] \rightarrow [0, 255]$
    - Important: we often convert [0, 255] to **[0,1.0]**.
- A color image is three functions pasted together, a "vector-valued" function

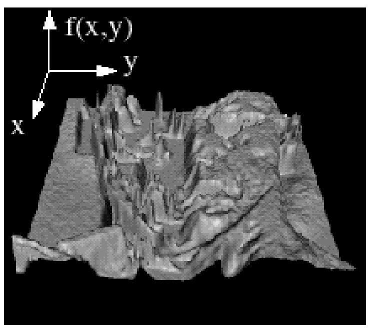
$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

SAIR
Spatial AI & Robotics Lab

Source: S. Seitz

Recap: Digital images

- In computer vision, we operate on **digital (discrete)** images:
  - **Sample** the 2D space on a regular grid
  - **Quantize** each sample (round to nearest integer)
- Image thus represented as a matrix of integer values.

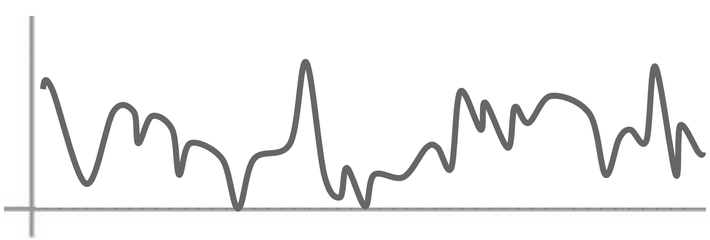


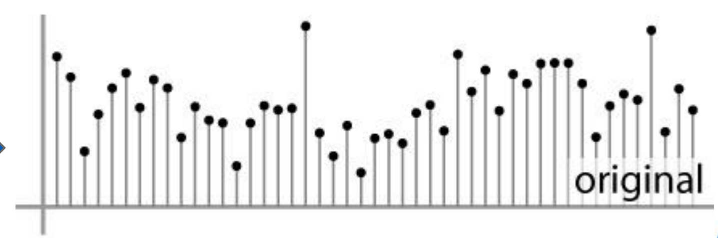
j

i

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

2D





1D

Recap: Warping v.s. Filtering

image warping: change ***domain*** of image

$$g(x) = f(T(x))$$

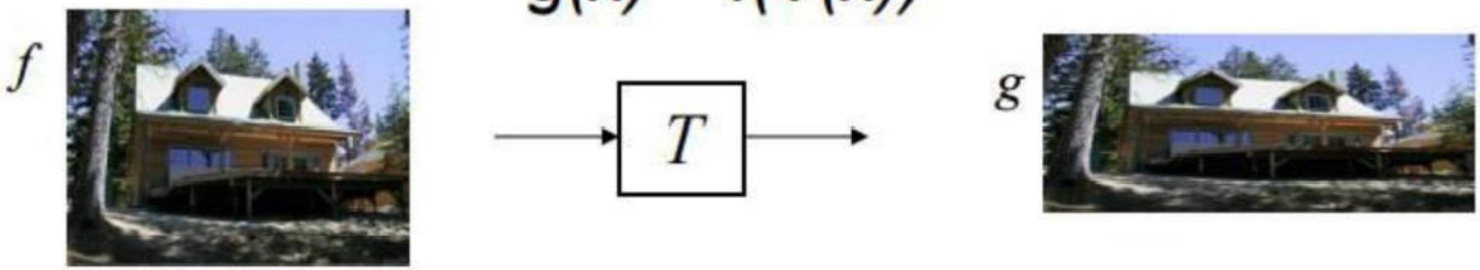


image filtering: change ***range*** of image (Next Week)

$$g(x) = T(f(x))$$

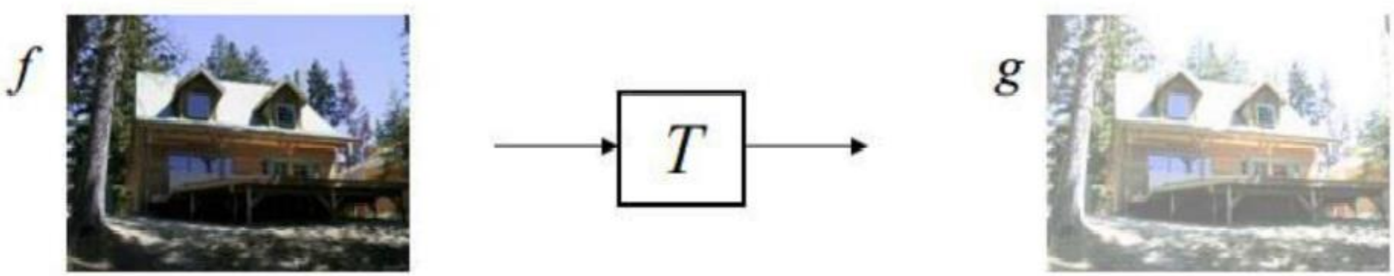
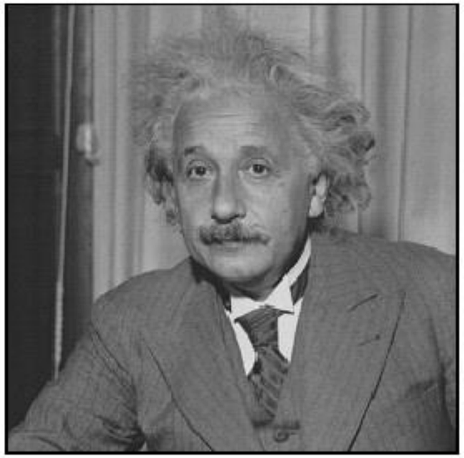


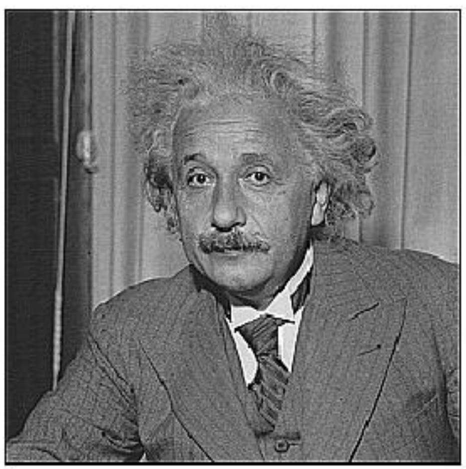
Image filtering

- Image filtering: compute a function of the local neighborhood at each position

- Really important!
  - Enhance images
    - Denoising, resize, increase contrast, etc.
  - Extract information from images
    - Texture, edges, distinctive points, etc.
  - Detect patterns
    - Template matching
    - Deep Convolutional Networks

Image Filtering





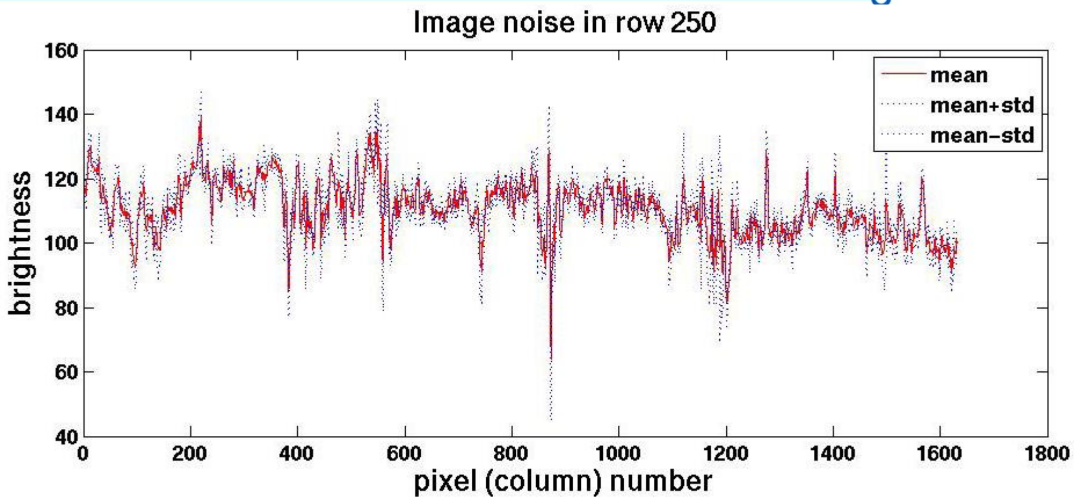
Smooth/Sharpen Images…

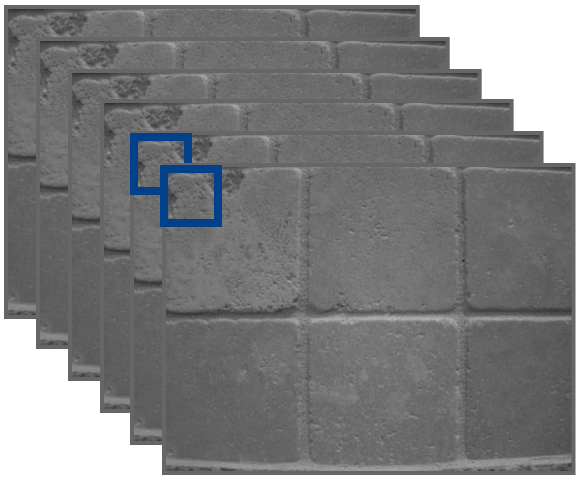
Find edges…

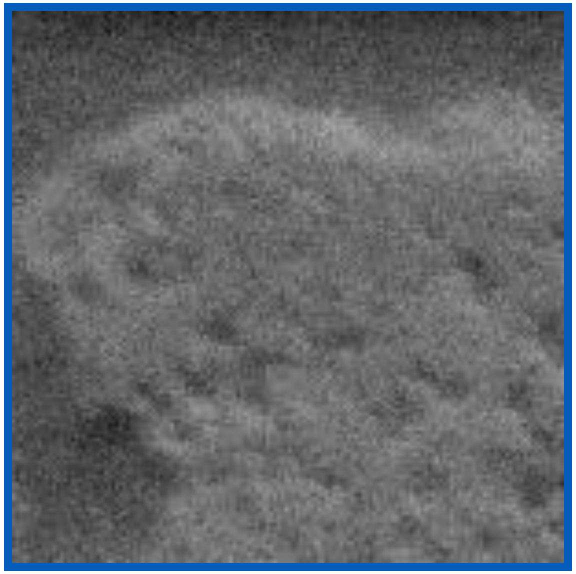
Find Waldo…

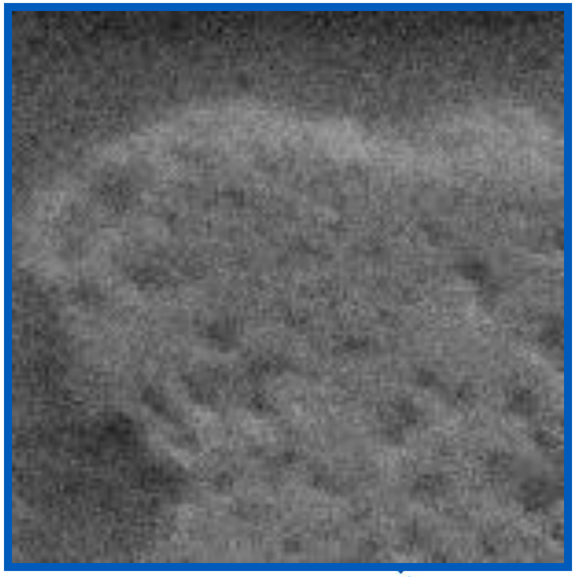
How can we do noise reduction?

- We can measure **noise** in multiple images of the same static scene.
- How could we reduce the noise, i.e., give an estimate of the true intensities?



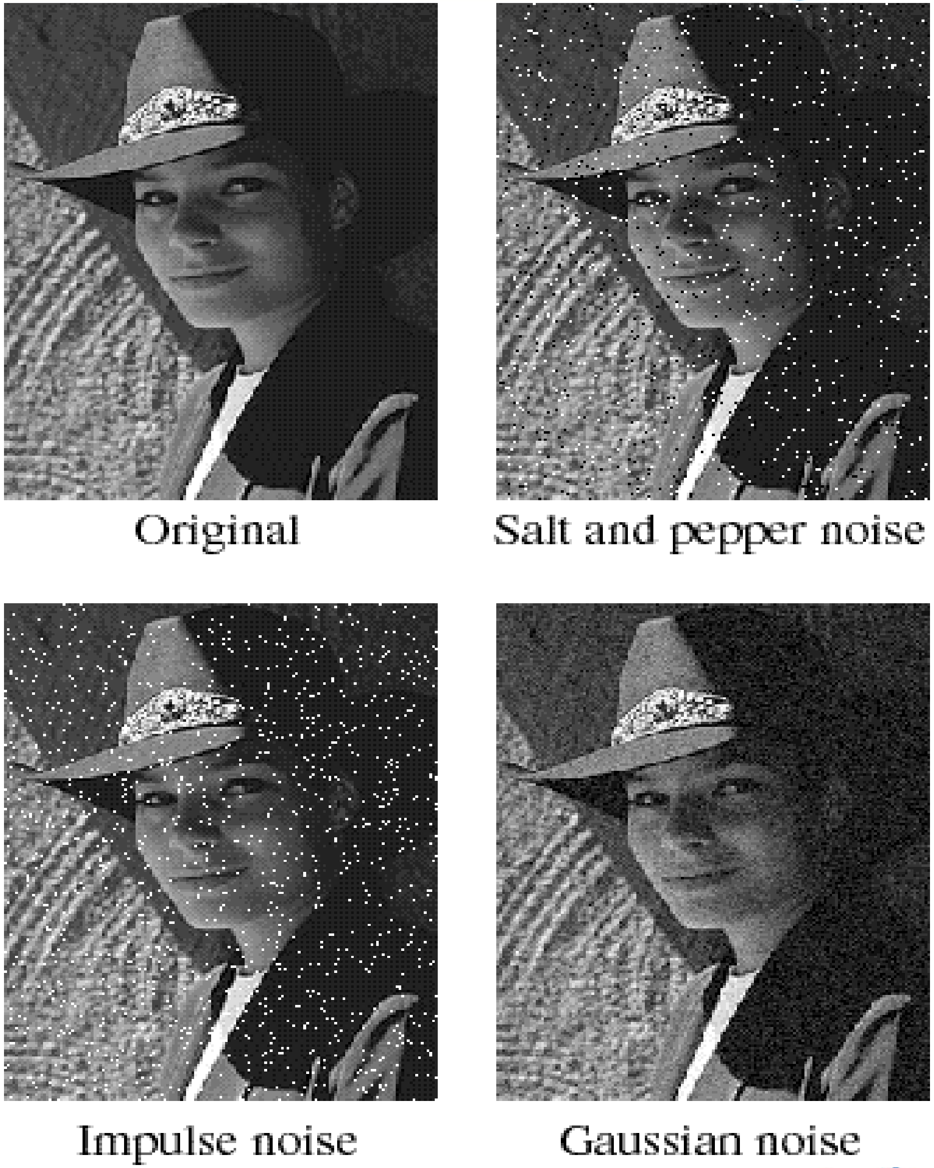






Common types of noise

- **Impulse noise:** random occurrences of white pixels
- **Salt and pepper noise**: random occurrences of black and white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution



Source: S. Seitz

Gaussian noise

- Additive Noise

$$f(x,y) = \overbrace{\bar{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. ("white") noise:
$\eta(x,y) \sim \mathcal{N}(\mu,\sigma)$

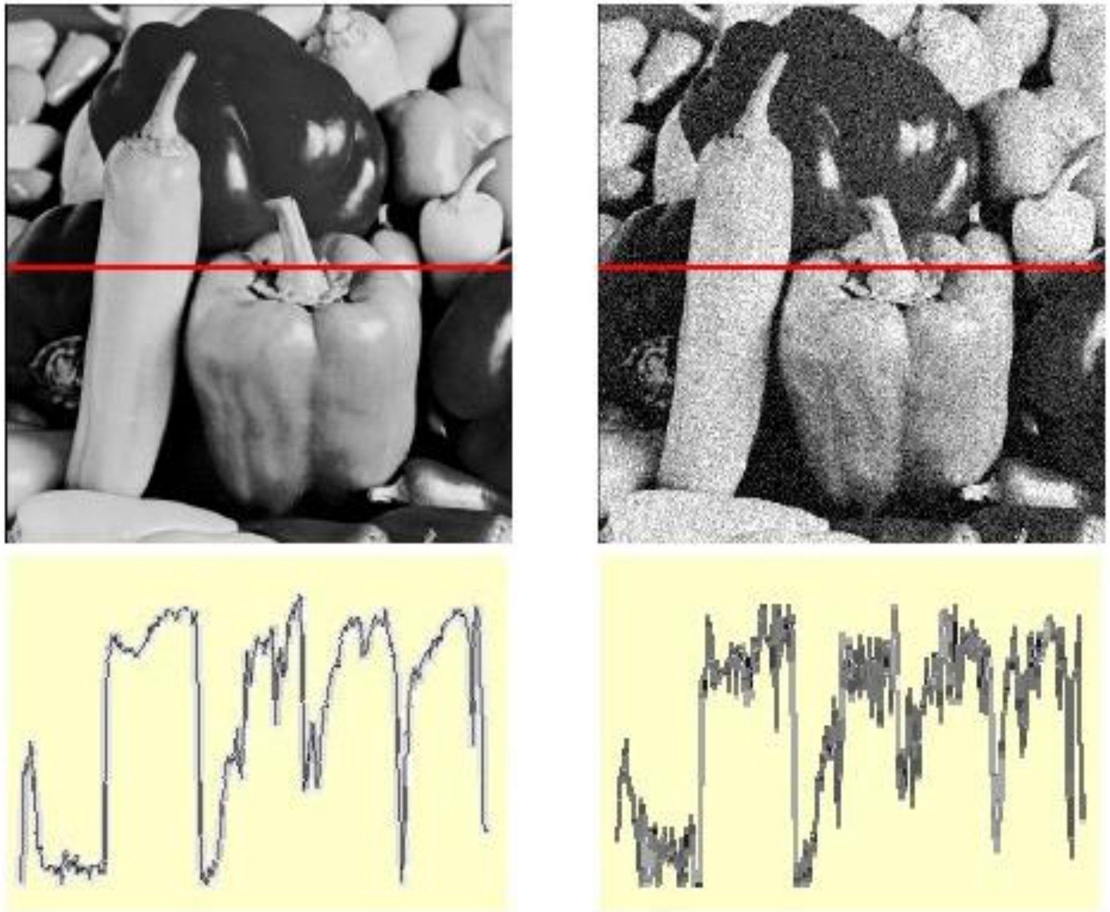
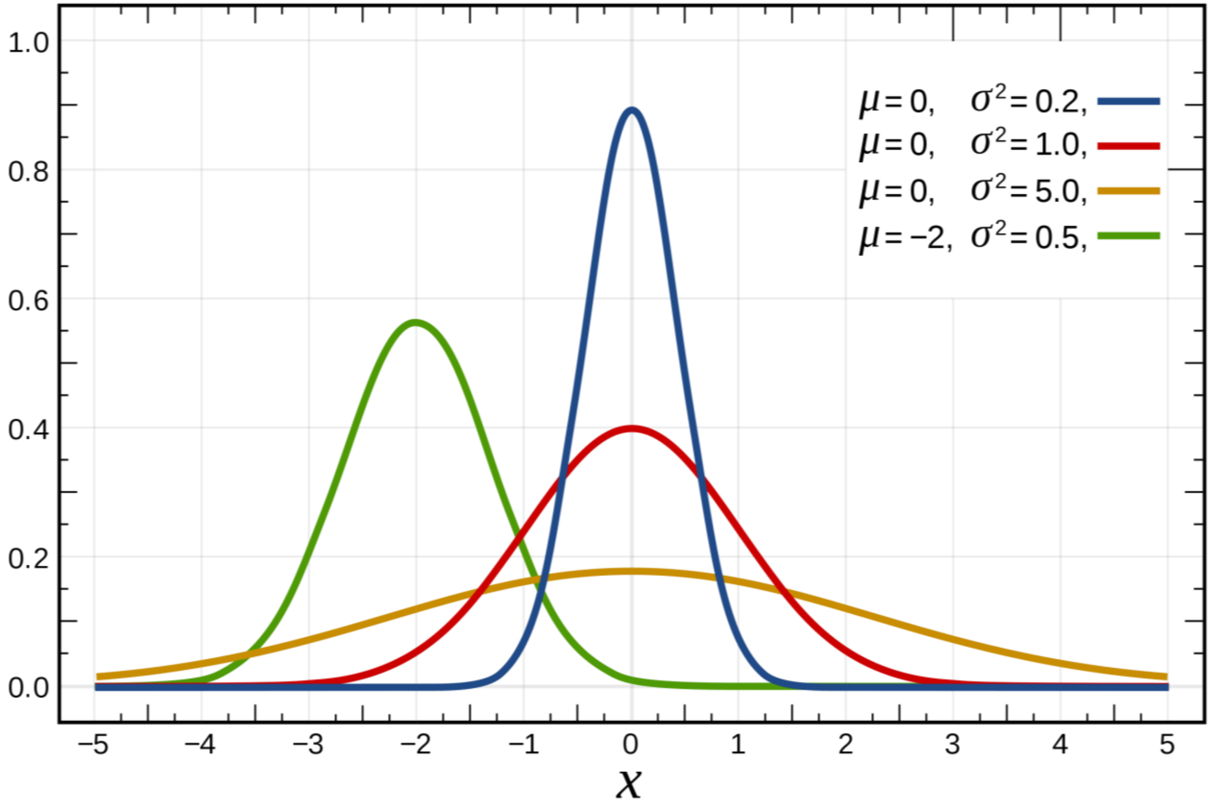


Fig: M. Hebert

PDF of Gaussian distribution

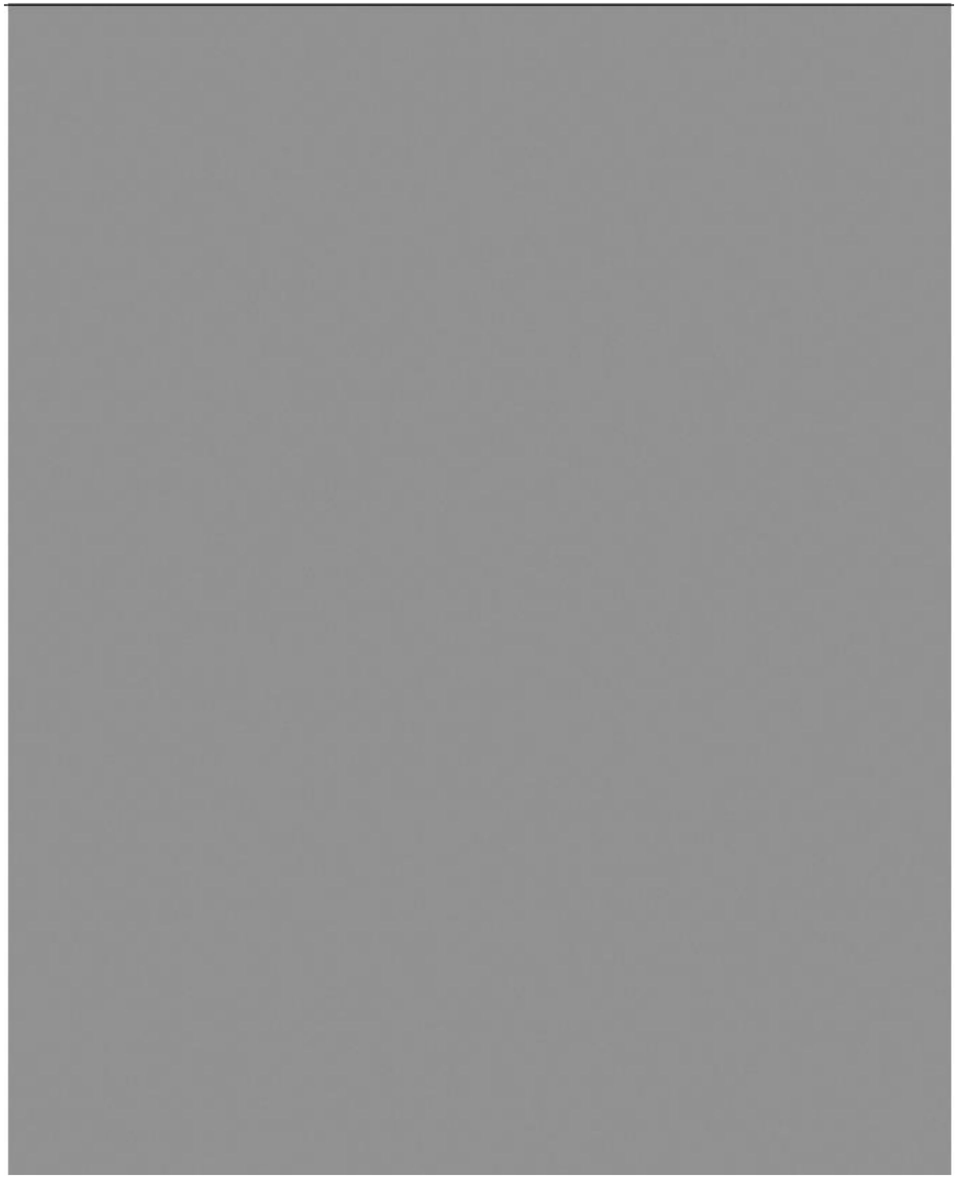
- **Probability density function**



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Gaussian noise

sigma=1

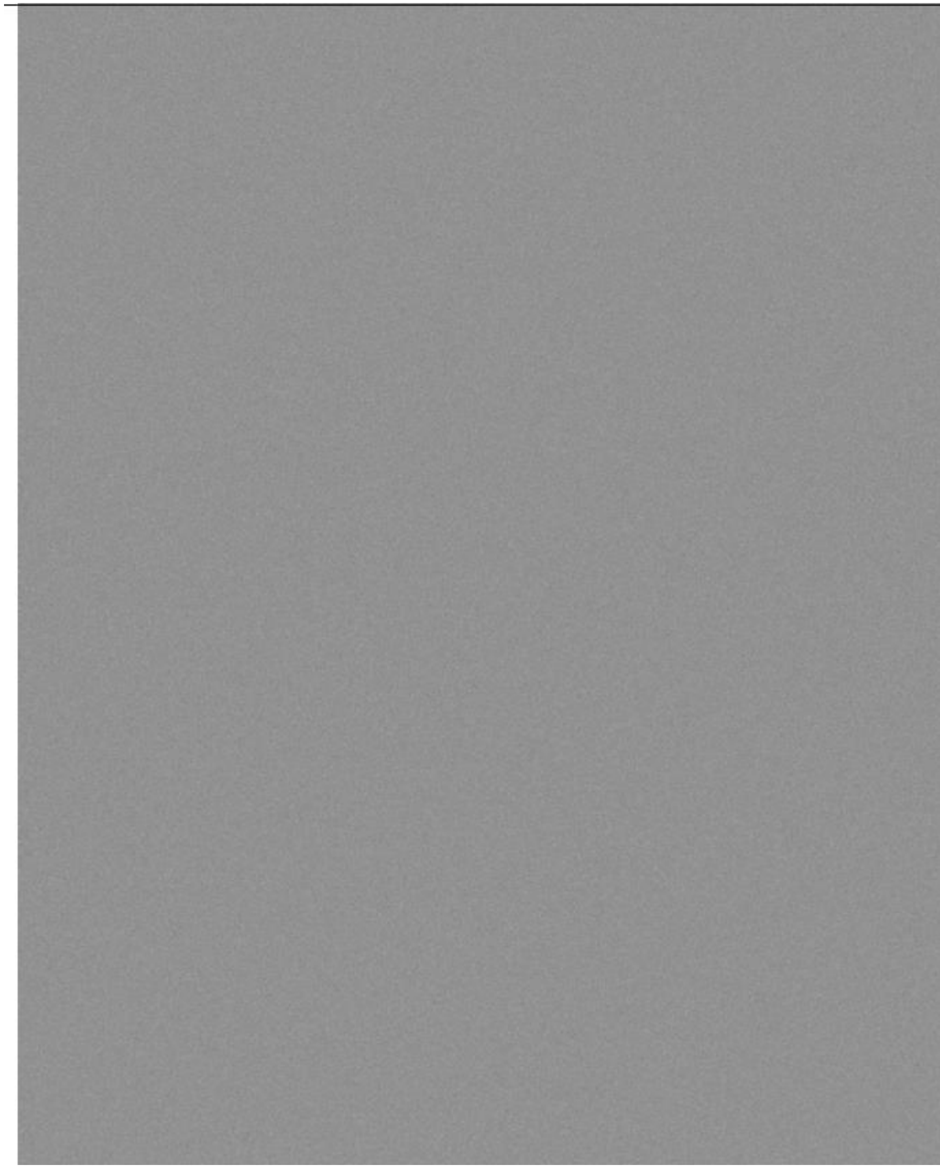


Effect of sigma on Gaussian noise:

Image shows the noise values themselves.

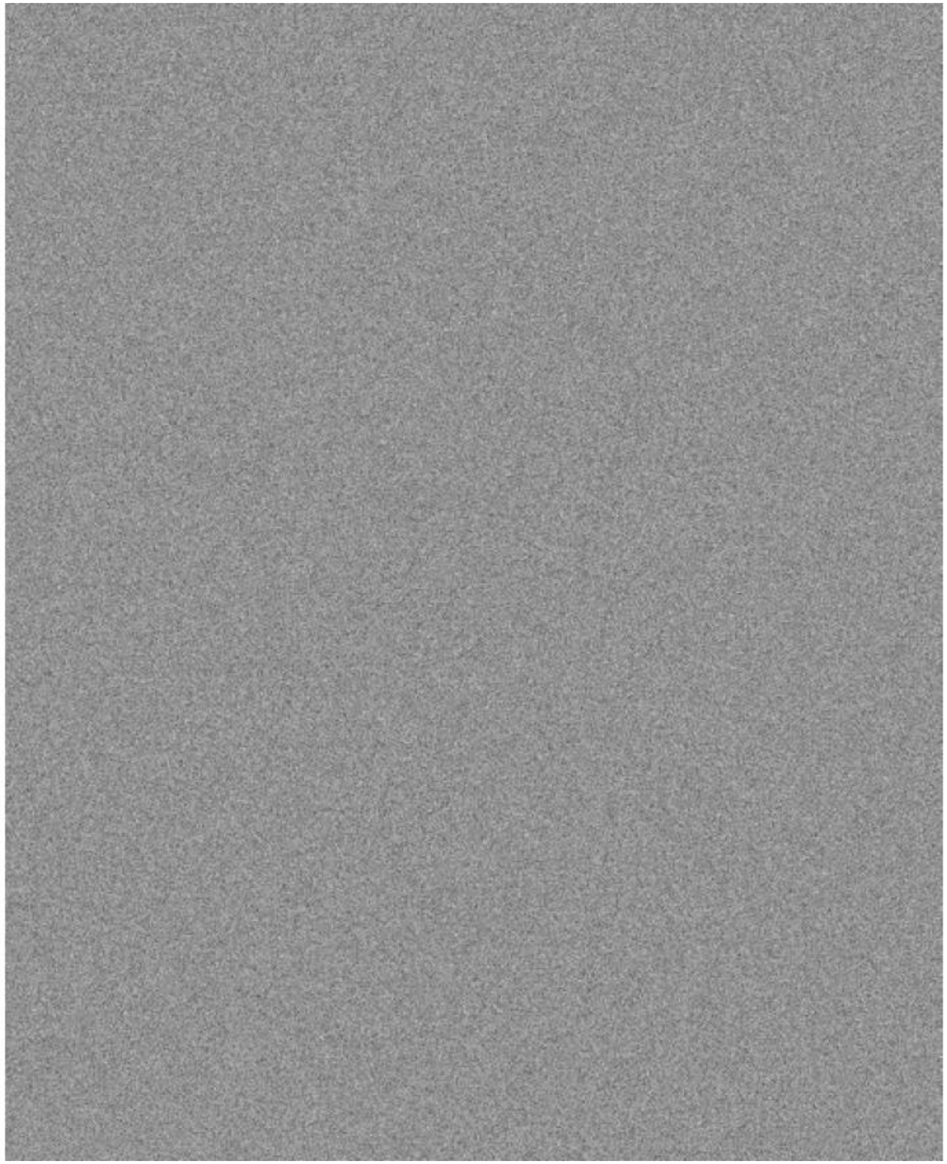
Gaussian noise

sigma=4



Gaussian noise

sigma=16



Gaussian noise

sigma=1

Effect of sigma on Gaussian noise:

This shows the noise values added to the raw intensities of an image.

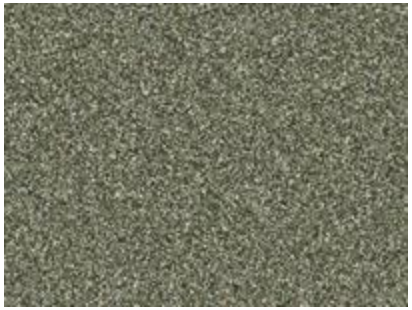
Gaussian noise

sigma=16

Pixel neighborhoods are important.

- Q: What happens if we reshuffle all pixels?



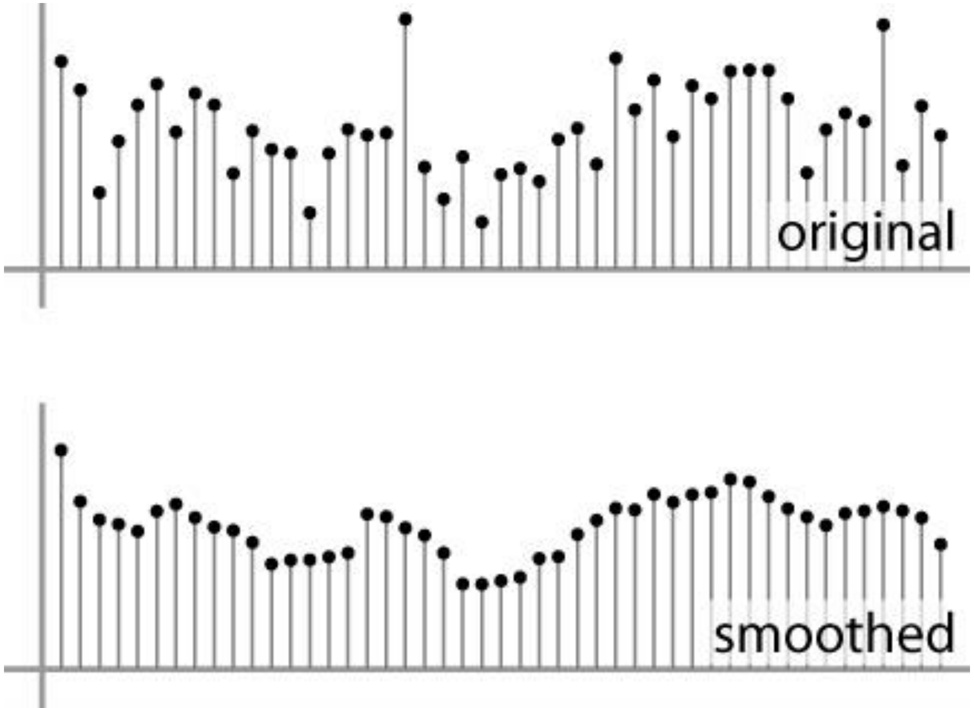



- A: Its histogram won’t change.
  Point-wise processing unaffected.

- Can we use neighborhoods to remove image noise?

First attempt at a solution

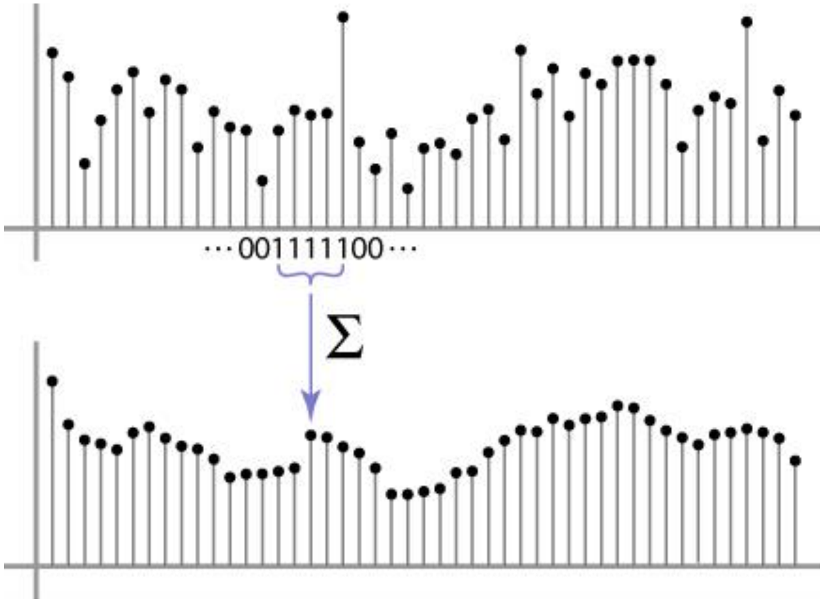
- Let's replace each pixel with an average of all the values in its neighborhood
  - Expect pixels to be like their neighbors
  - Expect noise processes to be independent from pixel to pixel.
  - Moving average in 1D:



Source: S. Marschner

Weighted Moving Average

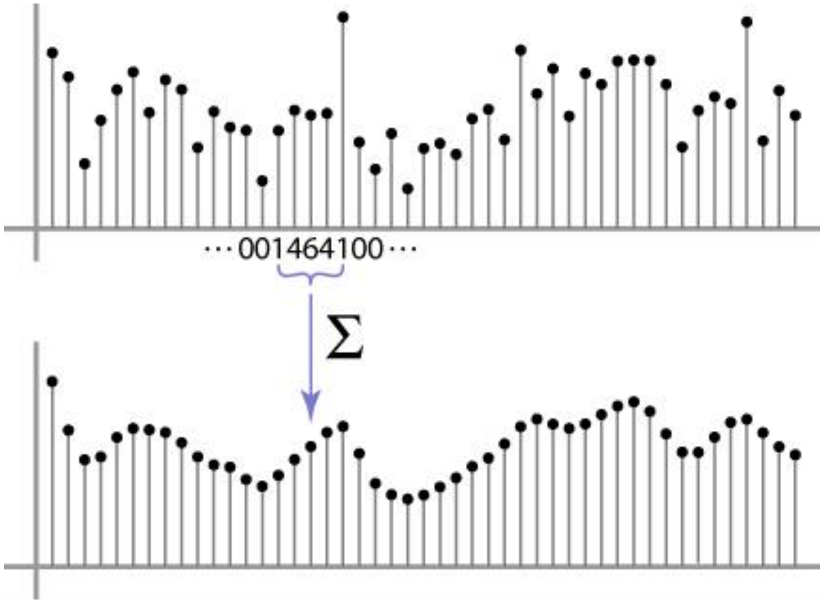
- We can add weights to moving average
- *Weights* [1, 1, 1, 1, 1] / 5



Source: S. Marschner

Weighted Moving Average

- Non-uniform weights [1, 4, 6, 4, 1] / 16



Source: S. Marschner

Example: Box Filter

$$f[\cdot,\cdot]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$h[\cdot\,,\cdot\,] \qquad \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Box Filter

$h[\cdot\,,\cdot\,]$ $\frac{1}{9}$ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$f[.\,,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.\,,.]$

$$g[m,n] = \sum_{k,l} h[k,l]\ f[m+k, n+l]$$

SAIR
Spatial AI & Robotics Lab

Credit: S. Seitz

Box Filter

$h[\cdot,\cdot]$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10							

$$g[m,n] = \sum_{k,l} h[k,l]\ f[m+k, n+l]$$

Credit: S. Seitz

Box Filter

$h[\cdot\,,\cdot\,]$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20						

$$\mathrm{g}[m,n] = \sum_{k,l} h[k,l]\;\; f[m+k, n+l]$$

Credit: S. Seitz

Box Filter

$h[\cdot\,,\cdot\,]$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30					

$$g[m,n] = \sum_{k,l} h[k,l]\; f[m+k,n+l]$$

Box Filter

$h[\cdot\,,\cdot\,]$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				

$$g[m,n] = \sum_{k,l} h[k,l]\ f[m+k, n+l]$$

SAIR
Spatial AI & Robotics Lab

Box Filter

$h[\cdot\,,\cdot\,]$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				
				?					

$$g[m,n] = \sum_{k,l} h[k,l]\ f[m+k, n+l]$$

Box Filter

$h[\cdot\,,\cdot\,]$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				
						?			
				50					

$$g[m,n] = \sum_{k,l} h[k,l]\ f[m+k,n+l]$$

Credit: S. Seitz

Box Filter

$h[\cdot\,,\cdot\,]$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

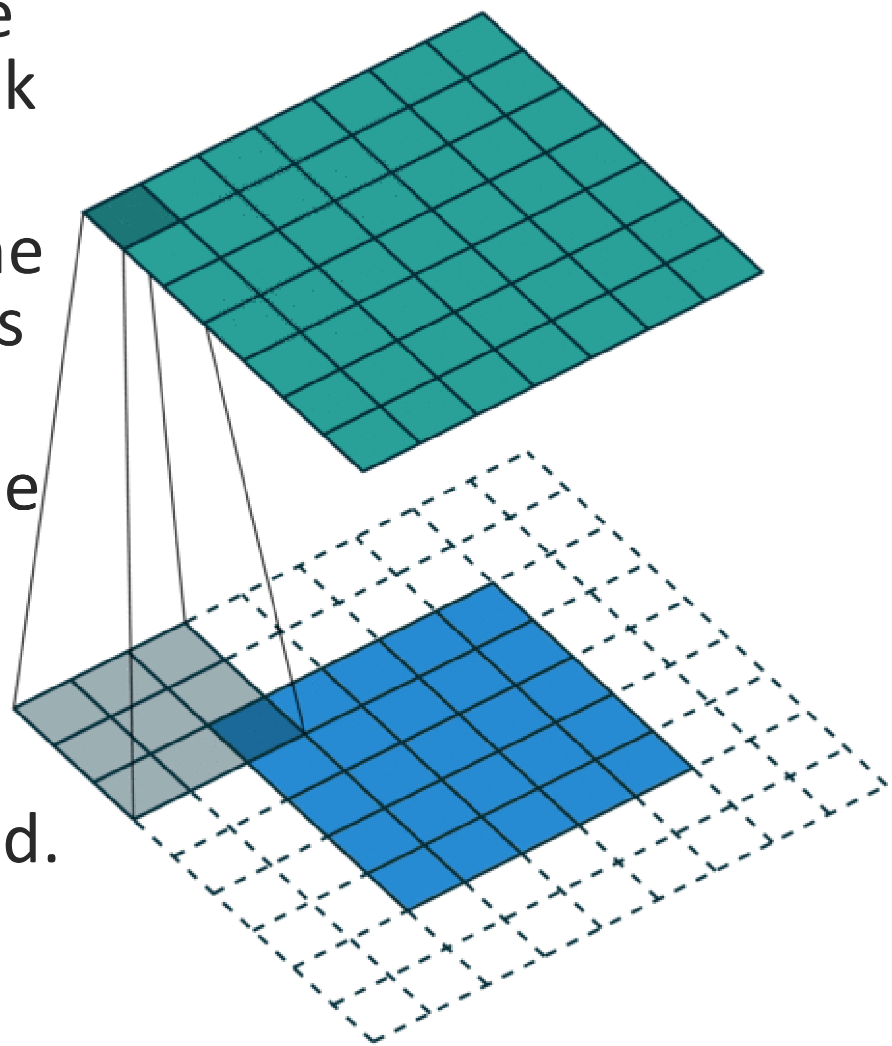
$g[.,.]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$\mathrm{g}[m,n] = \sum_{k,l} h[k,l]\ \ f[m+k,n+l]$$

Convolution & Correlation

- Convolution/Correlation is the process of moving a filter mask over an image
- At each point in the image, one computes the sum of products at each location
- Filter is often referred to as the **Kernel or Mask.**
- A function of displacement
- **Convolution**: filter is flipped.
- **Correlation**: filter is not flipped.



Box Filter

What does it do?

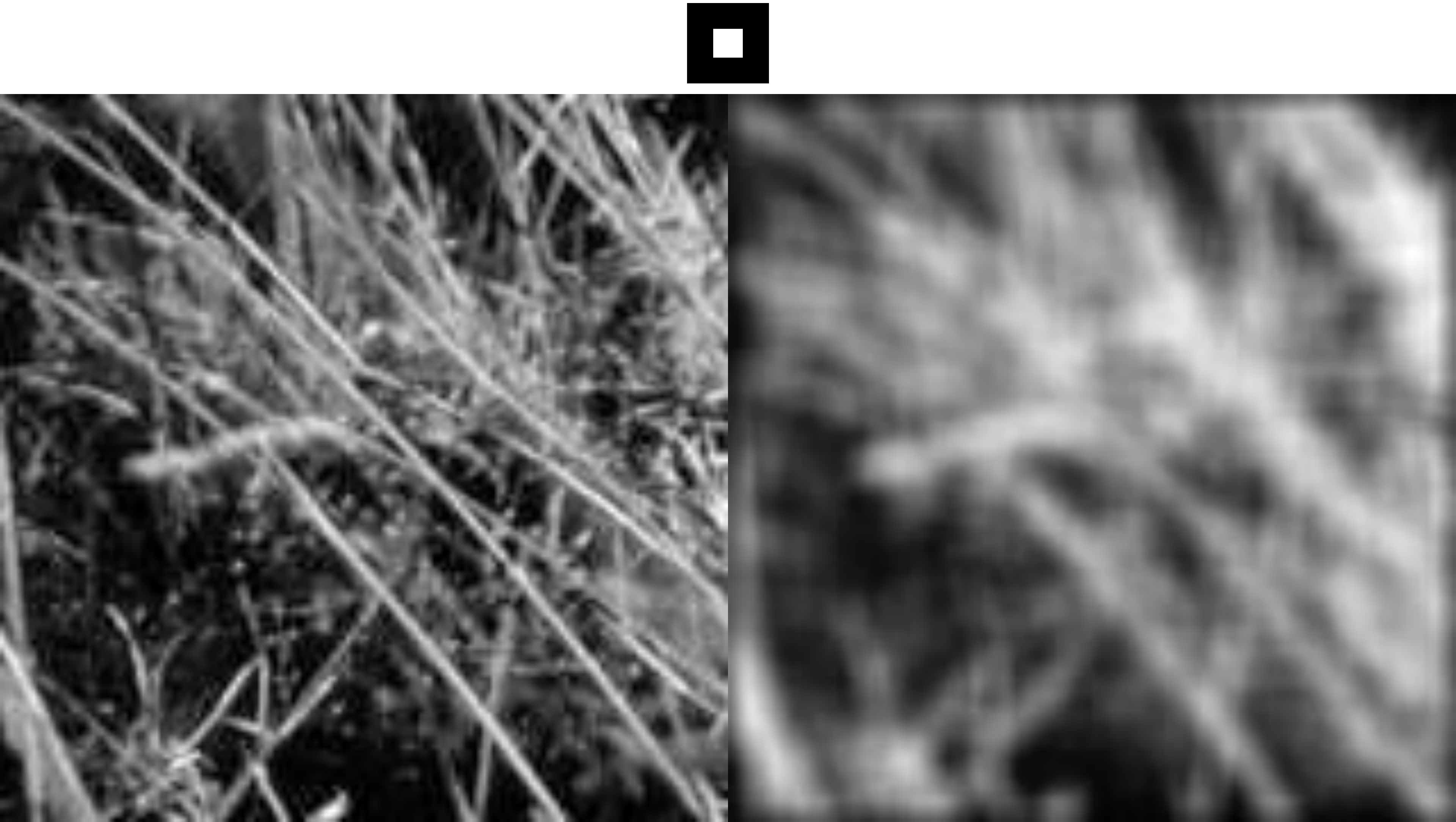
- Replaces each pixel with an average of its neighborhood

- Achieve smoothing effect (remove sharp features)

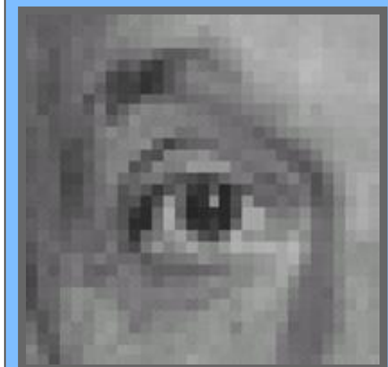
h[· ,·]

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Smoothing with box filter



Predict the filtered outputs



$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = ?$$

$$* \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = ?$$

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = ?$$

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

Practice with linear filters

0	0	0
0	1	0
0	0	0

Original

Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
1	0	0
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
1	0	0
0	0	0

Shifted left
By 1 pixel

Assume using convolution (filter flipped)

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ?$$

(Note that filter sums to 1)

Source: D. Lowe

Practice with linear filters

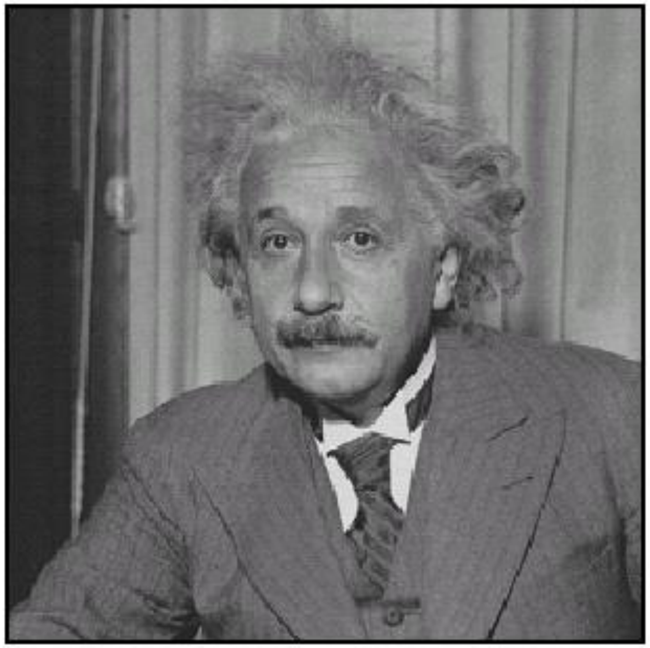
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

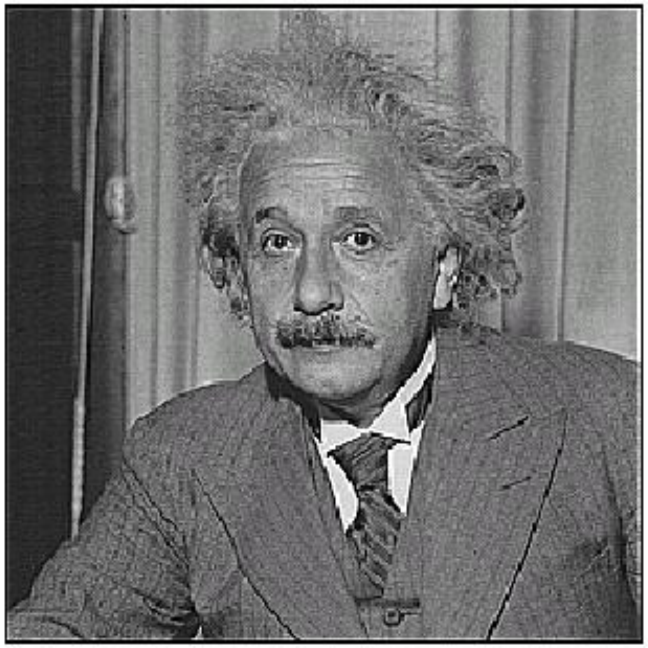
Sharpening filter
Accentuates differences with local average

Source: D. Lowe

Sharpening



before



after

Source: D. Lowe

Correlation (another name for filtering)

H=Filter F=Signal or Image

$$G[m,n] = \sum_{k,l} H[k,l] \; F[m+k, n+l]$$

- 2D **Convolution**
  - PyTorch implements correlation as convolution
  - import torch.nn.functional as F
  - `G = F.conv2d(I, f)`

I=image f=filter

$$G[m,n] = \sum_{k,l} H[k,l] \; F[m-k, n-l]$$

Correlation vs. Convolution

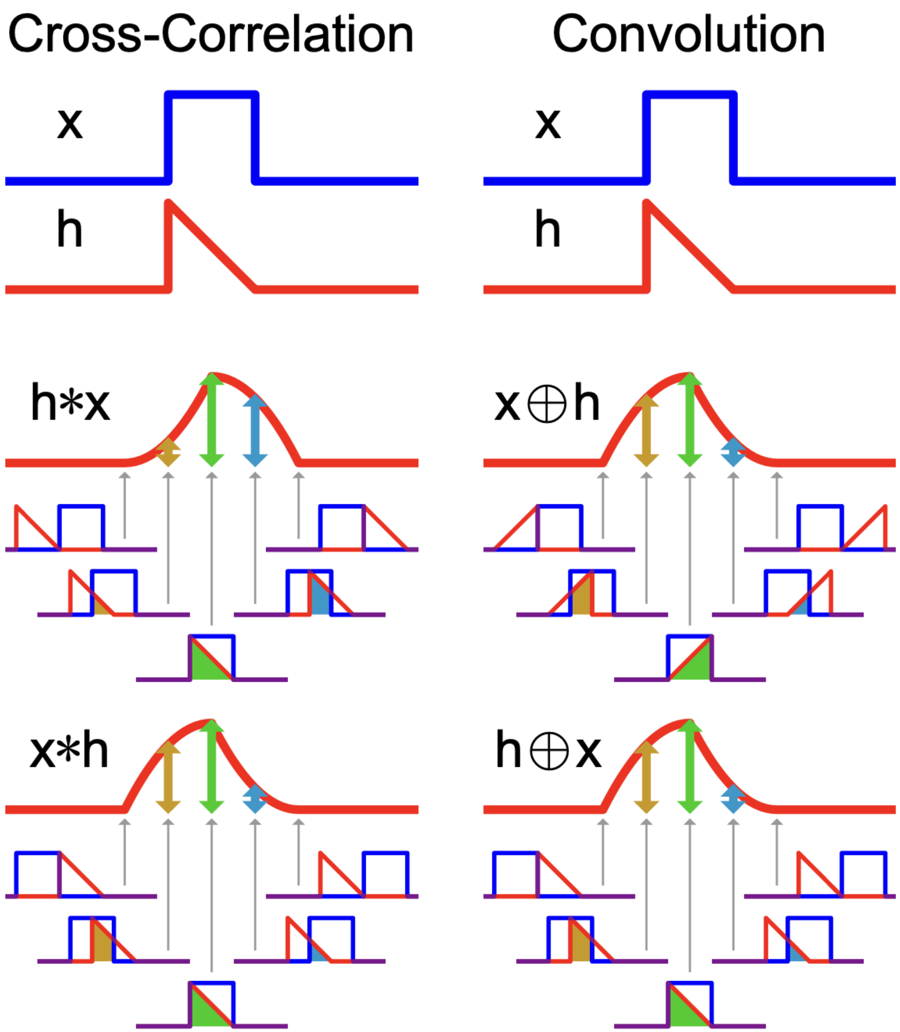
- 2D Correlation

$$h[m,n] = \sum_{k,l} f[k,l]\ I[m+k, n+l]$$

- 2D Convolution

$$h[m,n] = \sum_{k,l} f[k,l]\ I[m-k, n-l]$$

- 1D Case



Effect on the filter

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

When k = l = -1 → (m-1, n-1)

1	2	3
4	5	6
7	8	9

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m-k, n-l]$$

When k = l = -1 → (m+1, n+1)

1	2	3
4	5	6
7	8	9

EXERCISE

Convolution Filters are rotated by 180 degrees

44

What Properties do they have?

- Both extract information from the image
- Both are shift or translation Invariant
- Both are linear (a linear combination of neighbors)
- Only the **Convolution** is Associative
  - F * (G * I) = (F * G) * I

When to use which?

- Correlation
  - Applying a template or filter to an image
  - Measuring Similarity
  - When we don't care of it is associative

- Convolution
  - Applying an operation to an image (filtering)
  - When we want association

What to keep in mind

- If we say we are applying a **correlation** and we provide a filter, assume it is a correlation filter.
- If we say we are applying a **convolution** and we provide a filter, assume it is a convolution filter. No rotation is needed.
- You can get the same point, with either approach

Correlation filtering

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Say the averaging window size is (2k+1) x (2k+1):

$$G[i,j] = \underbrace{\frac{1}{(2k+1)^2}}_{\substack{\textit{Attribute uniform} \\ \textit{weight to each pixel}}} \underbrace{\sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]}_{\substack{\textit{Loop over all pixels in neighborhood} \\ \textit{around image pixel F[i,j]}}}$$

Now generalize to allow different weights depending on neighboring pixel's relative position:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} \underbrace{H[u,v]}_{\textit{Non-uniform weights}} F[i+u, j+v]$$

Correlation filtering

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

More formally, it is called cross-correlation, i.e.,

$$G = H \otimes F$$

Filtering an image: replace each pixel with a linear combination of its neighbors.

The filter is also called “kernel”, “mask”, or a “window”.

More Formal/Strict Terminology

- Cross-correlation:
  - Often referred as **correlation** in computer vision.

$$G[m,n] = \sum_{k,l} H[k,l] \; F[m+k, n+l]$$

- Correlation (in statistics):

$$\mathrm{corr}(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\mathrm{E}[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}.$$

- Reading more:
  - Chen Wang, et al. "Kernel Cross-correlator." In Proceedings of the AAAI Conference on Artificial Intelligence, 2018.
  - Convolution vs Cross-correlation
    - Section 2.2 in "Chen Wang, Kernel learning for visual perception". PhD thesis.

Properties

- Commutative:

$$\boldsymbol{x} \otimes \boldsymbol{y} = \boldsymbol{y} \otimes \boldsymbol{x}$$

  - Conceptually no difference between filter and signal
  - But particular filtering implementations might break this equality
- Associative:

$$\boldsymbol{x} \otimes (\boldsymbol{y} \otimes \boldsymbol{z}) = (\boldsymbol{x} \otimes \boldsymbol{y}) \otimes \boldsymbol{z}$$

  - Often apply several filters one after another: $\boldsymbol{x} \otimes \boldsymbol{y_1} \otimes \boldsymbol{y_2} \otimes \boldsymbol{y_3}$
  - This is equivalent to applying one filter: $\boldsymbol{x} \otimes (\boldsymbol{y_1} \otimes \boldsymbol{y_2} \otimes \boldsymbol{y_3})$
- Distributes over addition:

$$\boldsymbol{x} \otimes (\boldsymbol{y} + \boldsymbol{z}) = (\boldsymbol{x} \otimes \boldsymbol{y}) + (\boldsymbol{x} \otimes \boldsymbol{z})$$

- Scalars factor out:

$$\mathrm{a}\boldsymbol{x} \otimes \boldsymbol{y} = \boldsymbol{x} \otimes a\boldsymbol{y} = a(\boldsymbol{x} \otimes \boldsymbol{y})$$

- Identity (unit impulse), e.g., *e* = [0, 0, 1, 0, 0],

$$\boldsymbol{x} \otimes \boldsymbol{e} = \boldsymbol{x}$$

Source: S. Lazebnik

Key properties of Linear Filters

- Assume $f(\boldsymbol{x})$ is image filtering: $f(\boldsymbol{x}) = \boldsymbol{x} \otimes \boldsymbol{w}$.
- Linearity (superposition property):

$$f(a \cdot \boldsymbol{x} + b \cdot \boldsymbol{y}) = af(\boldsymbol{x}) + bf(\boldsymbol{y})$$

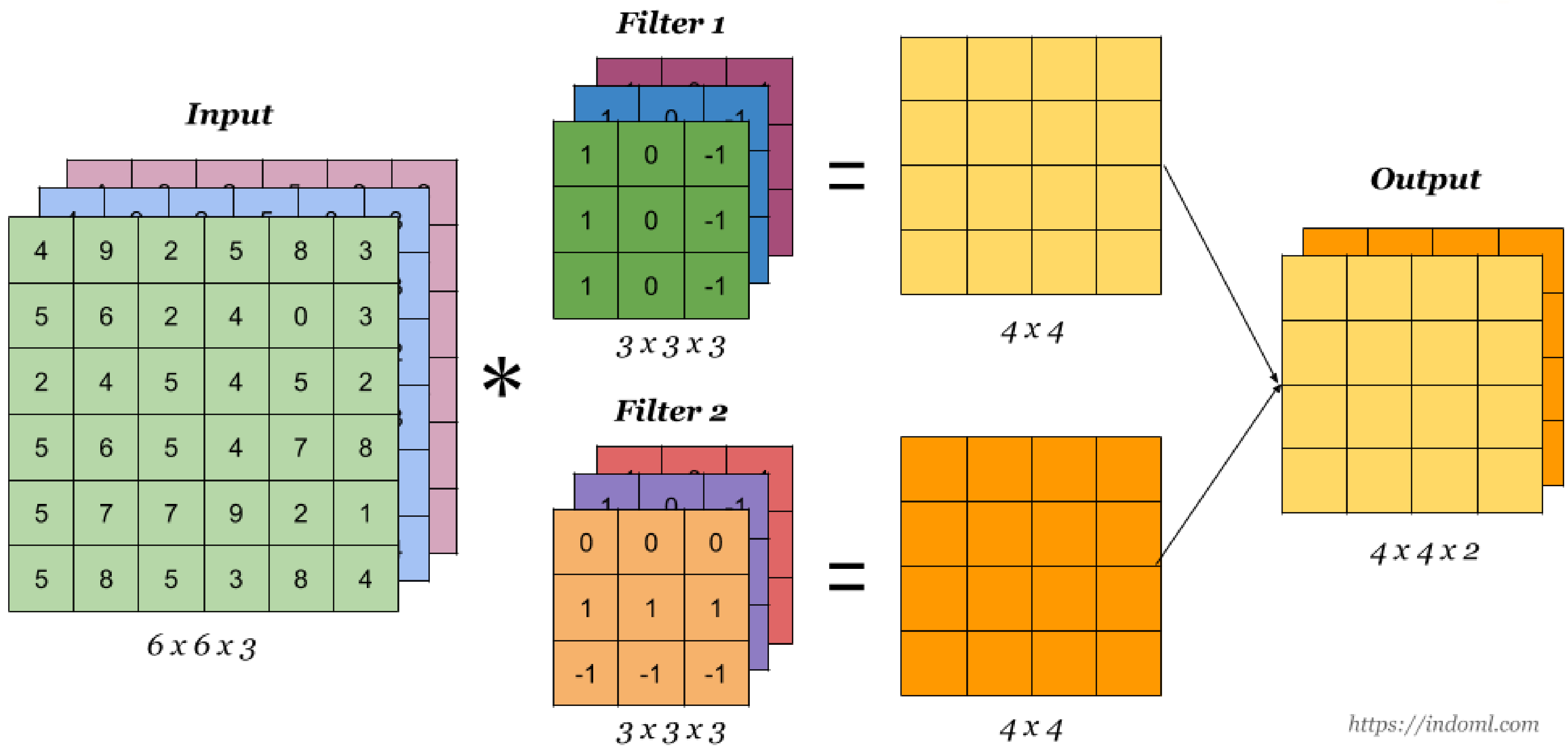
- Linear filter is **equivariant** (not **invariant)** to translation.
- Assume $T(\boldsymbol{x})$ is image shift (translation):
  - **Equivariance**:

$$f(T(X)) = T\big(f(x)\big)$$

  - **Invariance**:

$$f(T(X)) = f(x)$$

Multi-channel correlation



Examples:

```
>>> # With square kernels and equal stride
>>> filters = torch.randn(8, 4, 3, 3)
>>> inputs = torch.randn(1, 4, 5, 5)
>>> F.conv2d(inputs, filters, padding=1)
```

What is shape of the output in the left example?

(1, 8, 5, 5)

Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

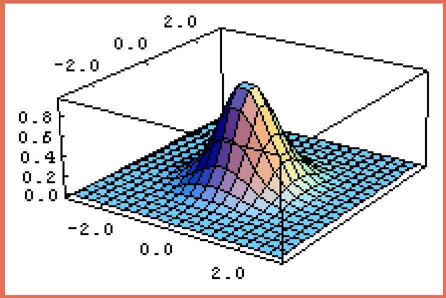
$F[x, y]$

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$H[u, v]$

This kernel is an approximation of a Gaussian function:

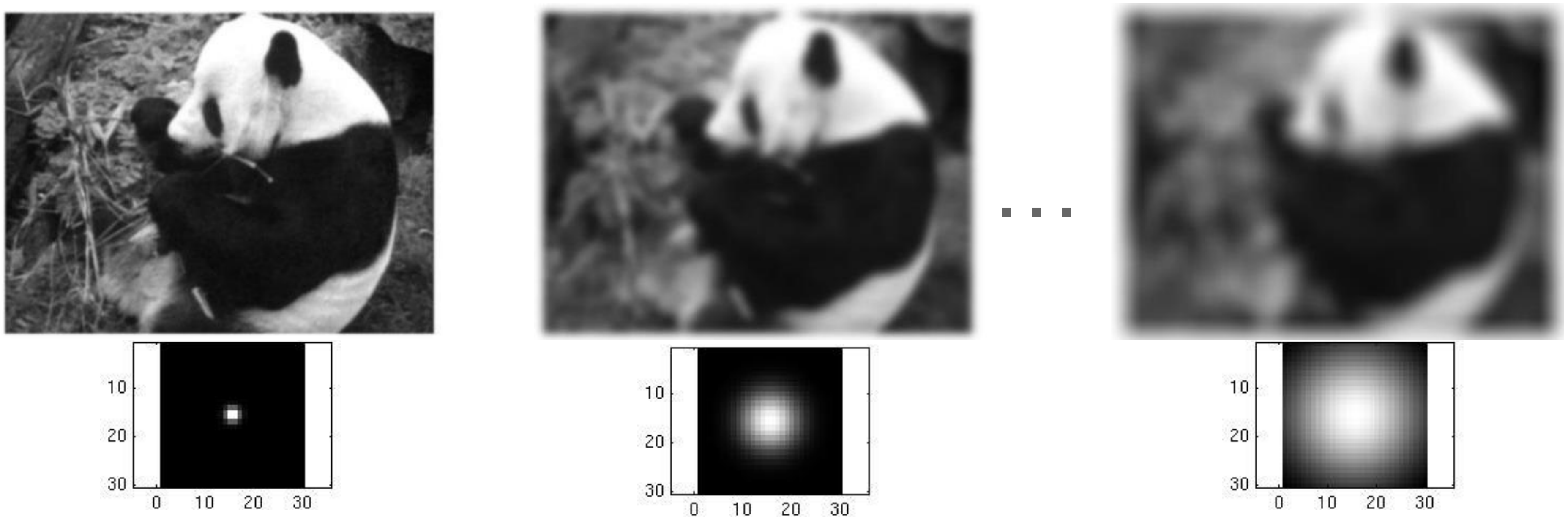
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



Source: S. Seitz

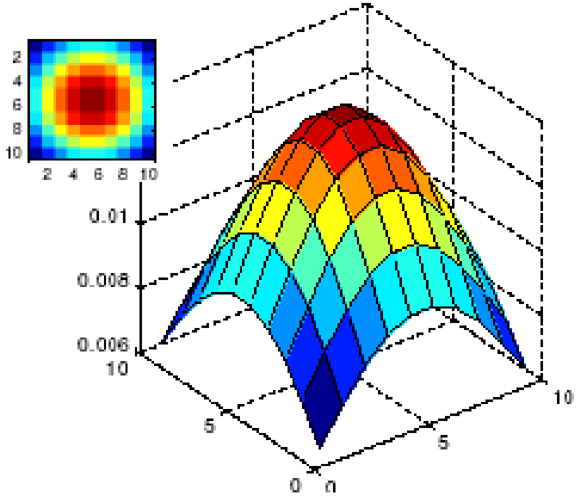
Smoothing with a Gaussian

- Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.

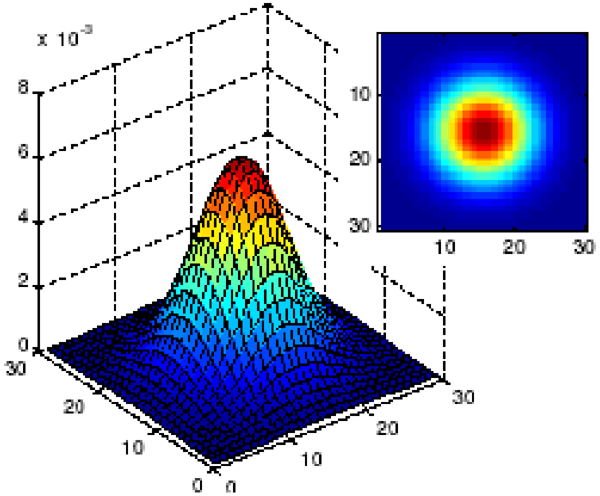


Gaussian filters

- What parameters matter here?
- **Size** of kernel / mask
  - Gaussian function has infinite support, but discrete filters use finite kernels



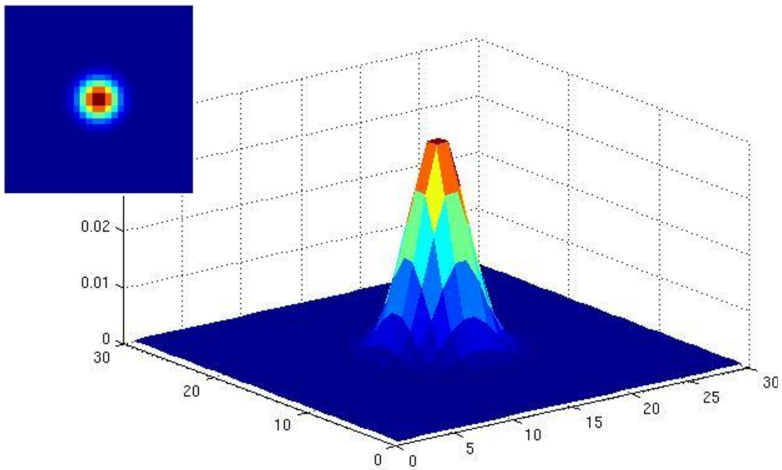
σ = 5 with 10 x 10 kernel



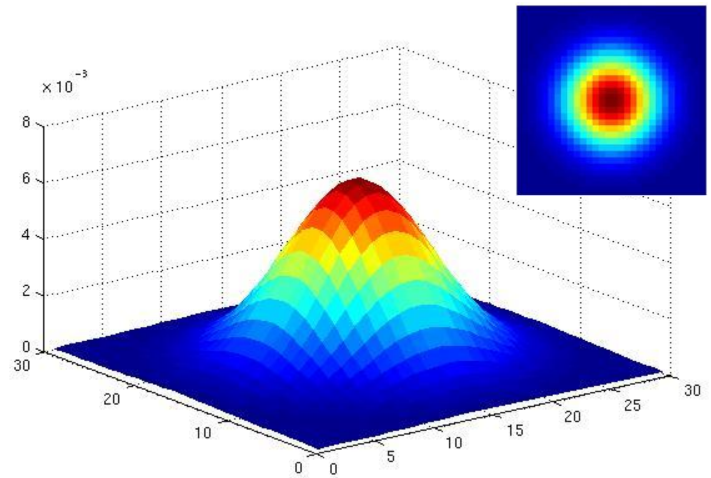
σ = 5 with 30 x 30 kernel

Gaussian filters

- **Variance**: determines extent of smoothing



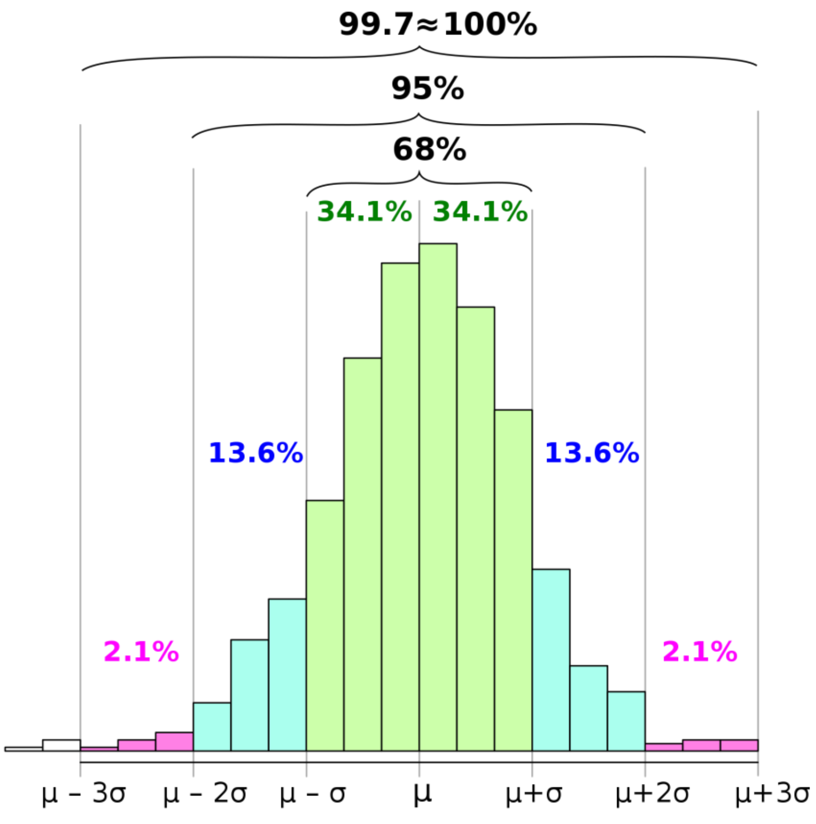
σ = 2 with
30 x 30
kernel



σ = 5 with
30 x 30
kernel

How big should the filter be?

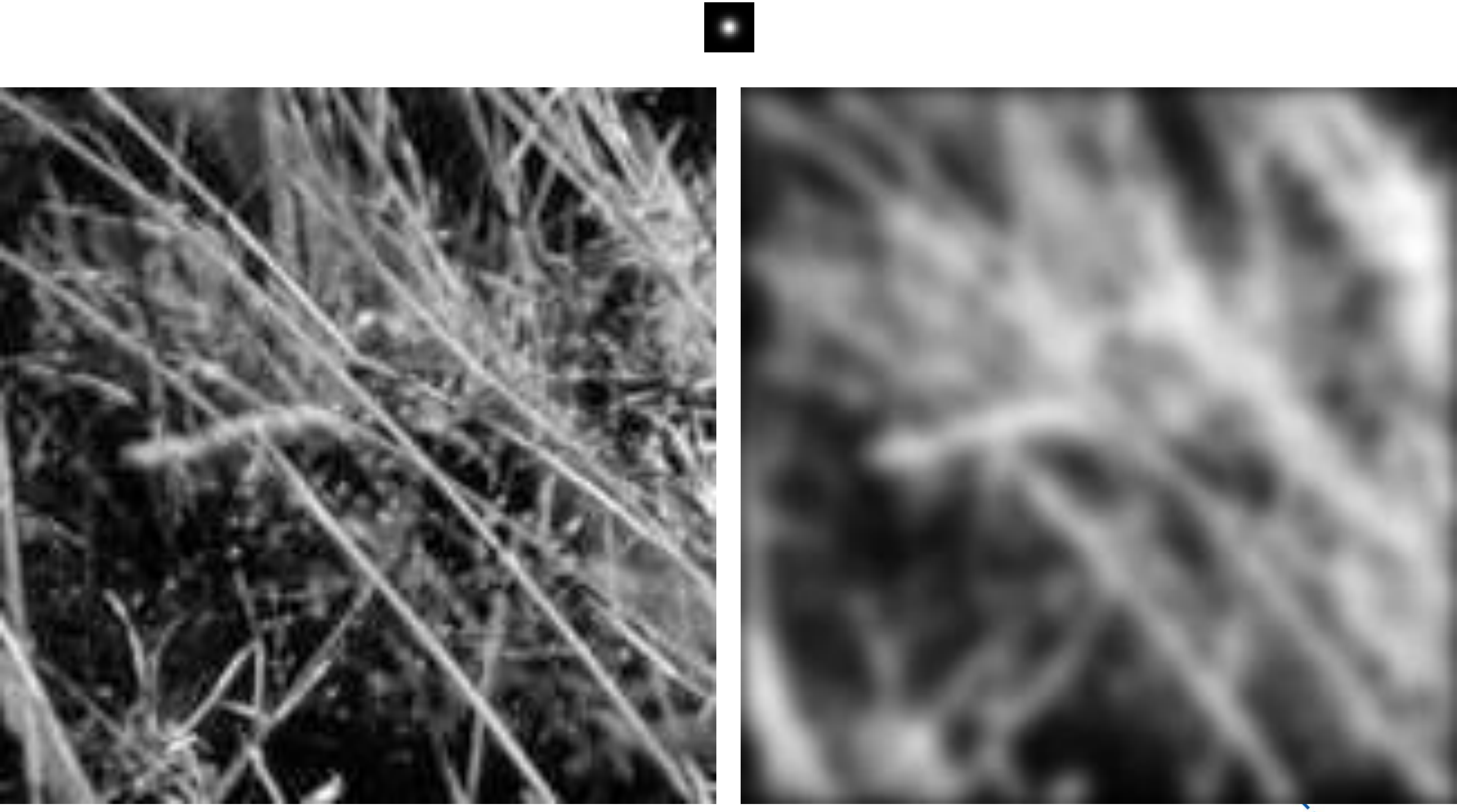
- Values at edges should be near zero
- Rule of thumb for Gaussian filter:
  - set filter half-width to about 3σ



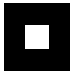
Gaussian filters

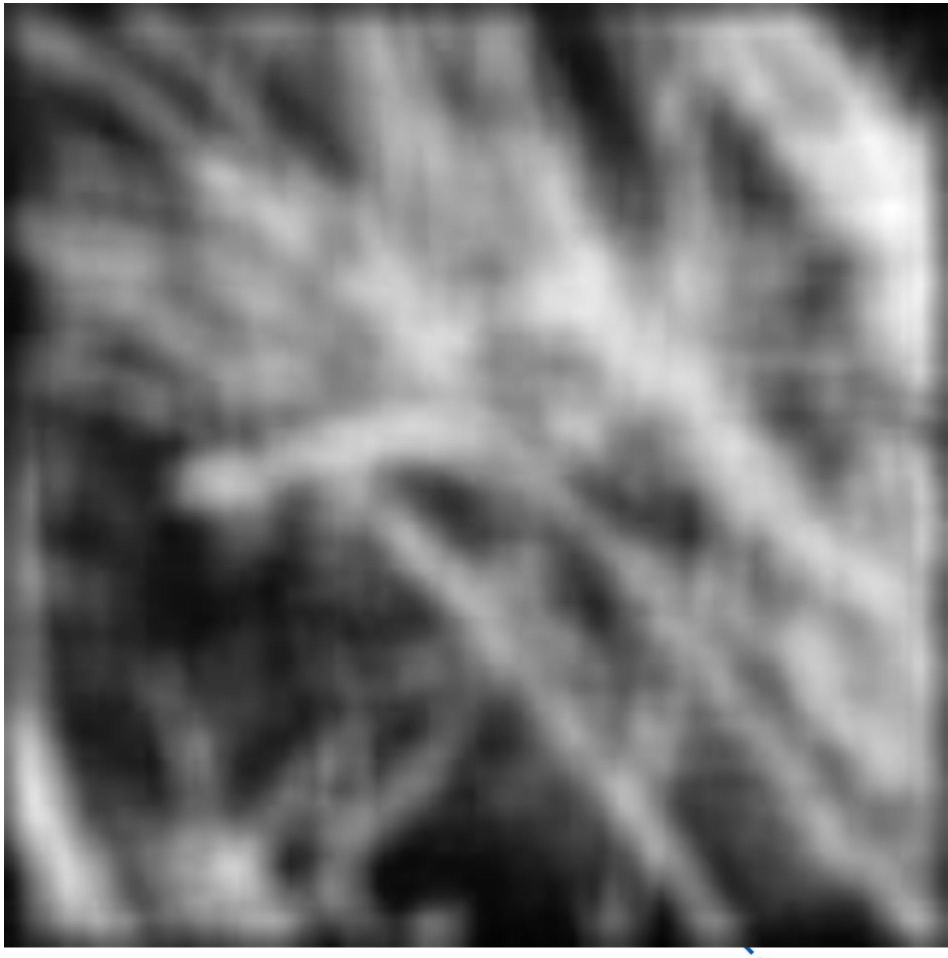
- Remove “high-frequency” components from image
  - Low-pass filter: Images become smoother.
- Recap of frequency in a signal (image)
  - High-frequency components
    - Fine details and edges
  - Low-frequency components
    - Large-scale structures and smooth regions

Smoothing with Gaussian filter



Smoothing with box filter





Gaussian filters

- Convolution with self is another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving two times with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$
\begin{aligned}
G_\sigma(x,y) &= \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}} \\
&= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}}\right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}}\right)
\end{aligned}
$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

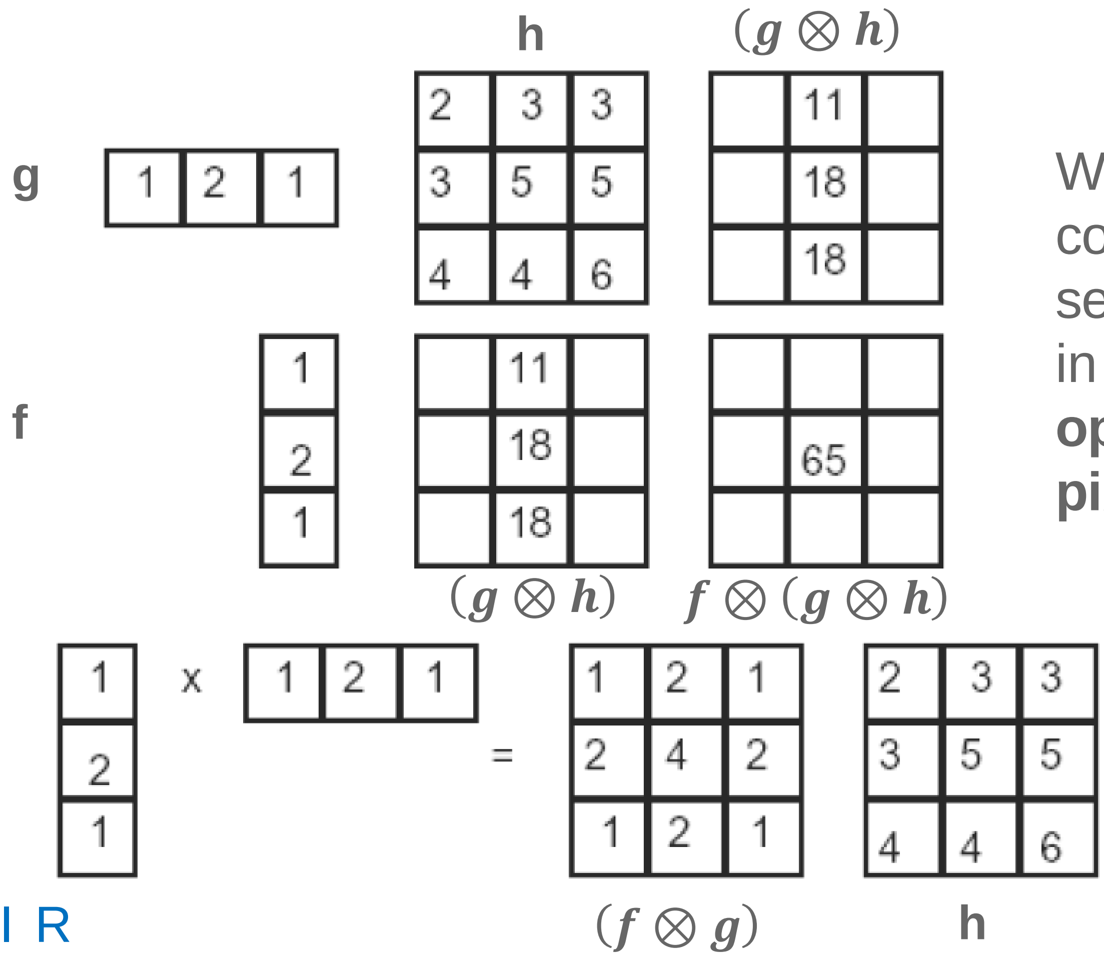
SAIR
Spatial AI & Robotics Lab

Source: D. Lowe

Separability

- In some cases, filter is separable, (factor into two steps):

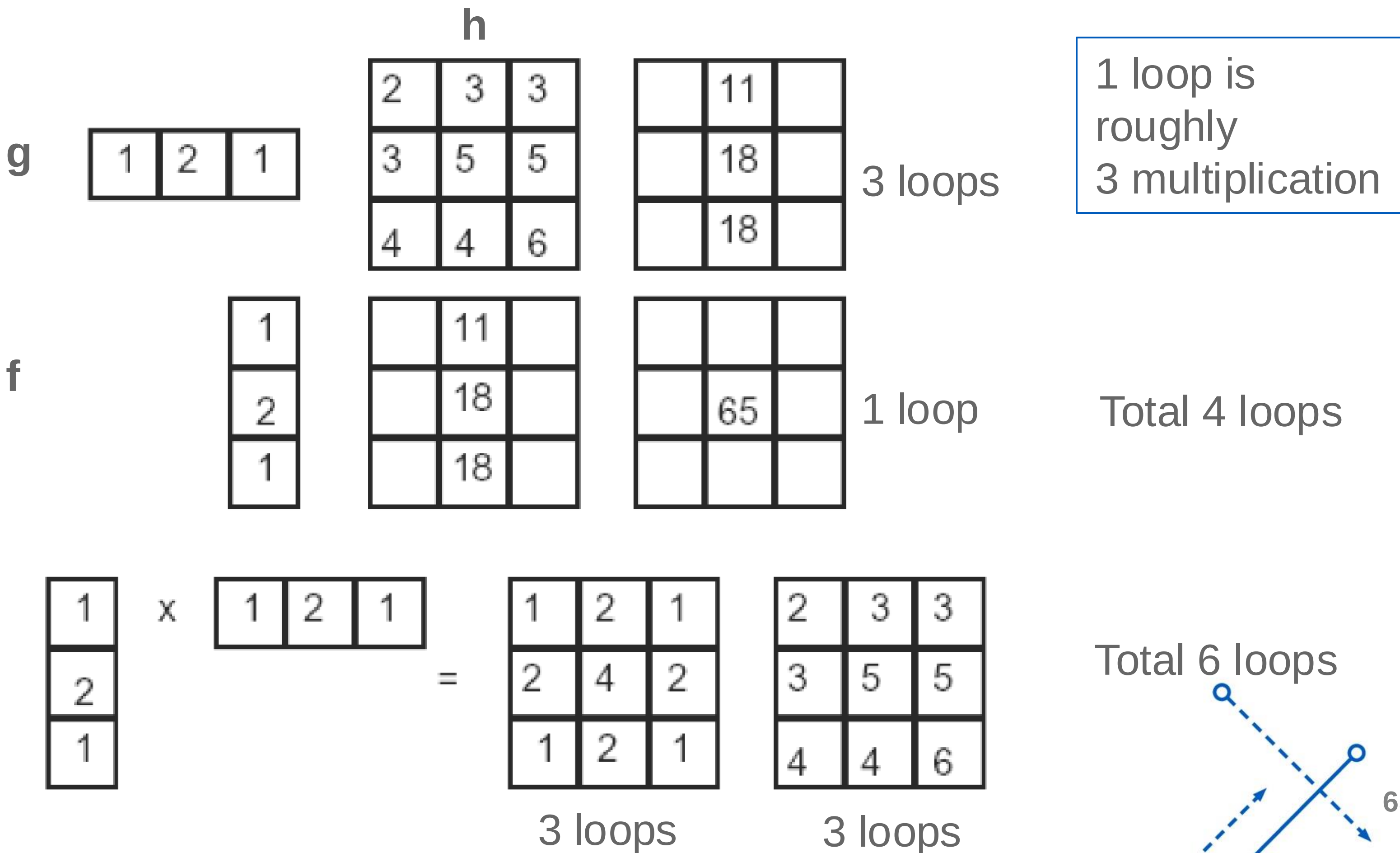
$$\boldsymbol{f} \otimes (\boldsymbol{g} \otimes \boldsymbol{h}) = (\boldsymbol{f} \otimes \boldsymbol{g}) \otimes \boldsymbol{h}$$



What is the computational complexity advantage for a separable filter of size k x k, in terms of **number of operations per output pixel**?

Separability

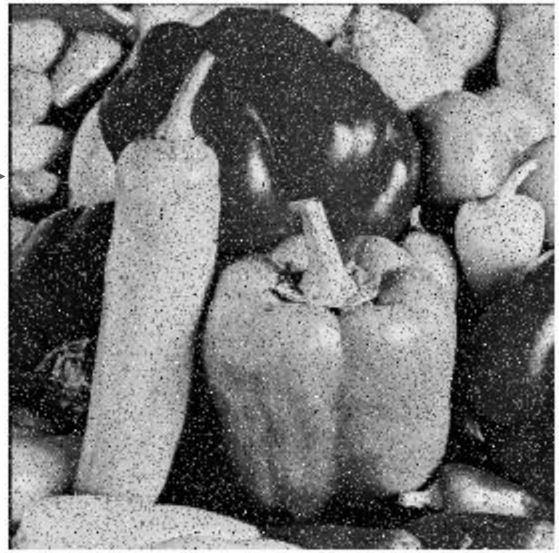
- **Advantage:** much reduced computational cost.
- **Disadvantage:** requires an extra ram **memory** to store the intermediate image, problematic in certain applications.



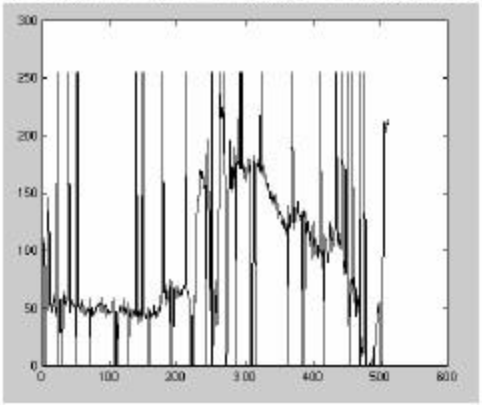
SAIR
Spatial AI & Robotics Lab

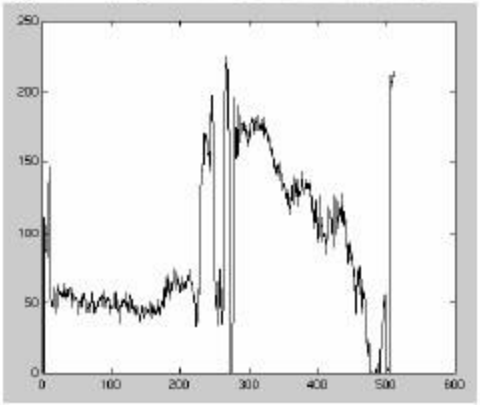
Median filter

Salt and pepper noise



Median filtered



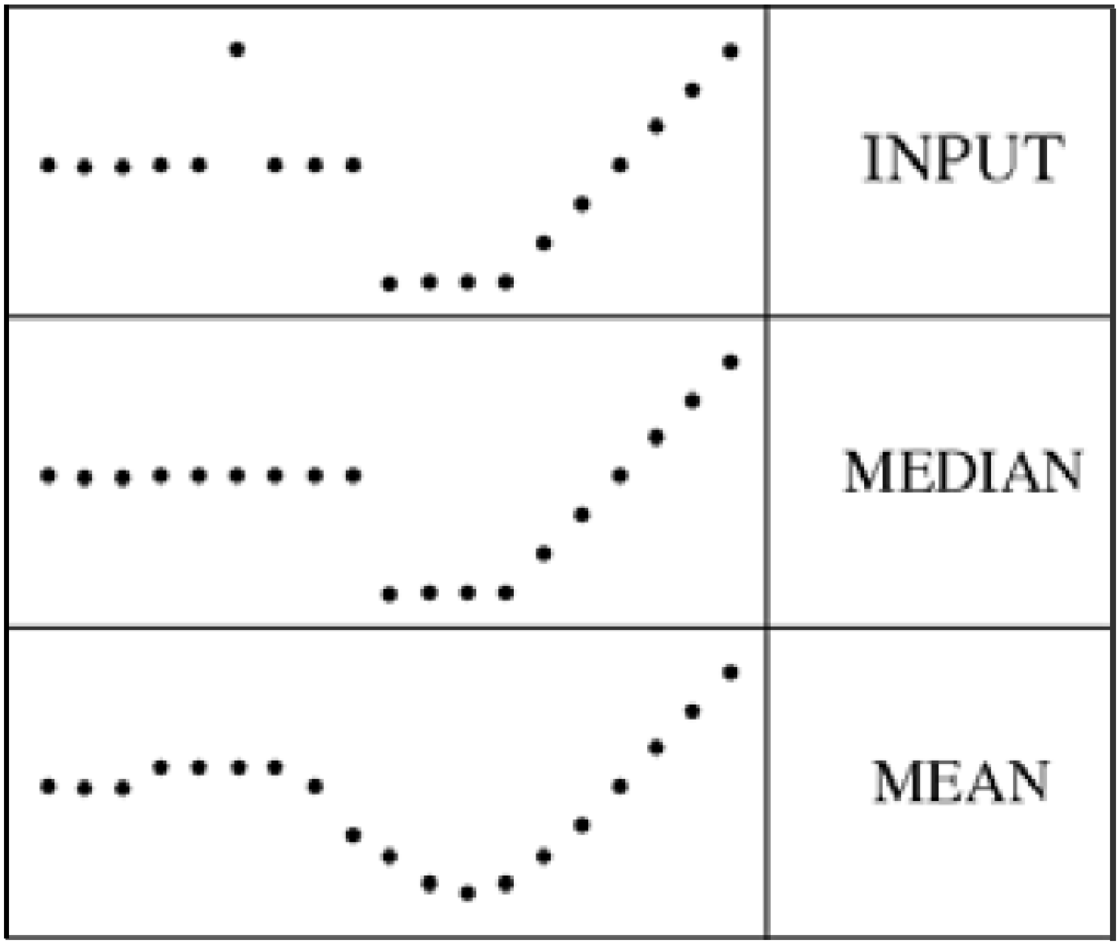


Plots of a row of the image

Source: M. Hebert

Median filter

- Median filter is edge preserving
- It doesn't introduce new intensities, which is often expected.



Content

- Filtering
  - Linear filters
  - Correlation and Convolution
  - Equivariance, Invariance
  - Smoothing, Gaussian Filter, Median filter