



SAIR

Spatial AI & Robotics Lab

CSE 473/573-A

L10: HOUGH TRANSFORM

Chen Wang

Spatial AI & Robotics Lab

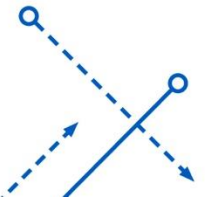
Department of Computer Science and Engineering



University at Buffalo The State University of New York

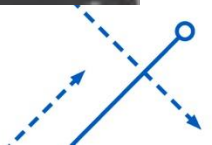
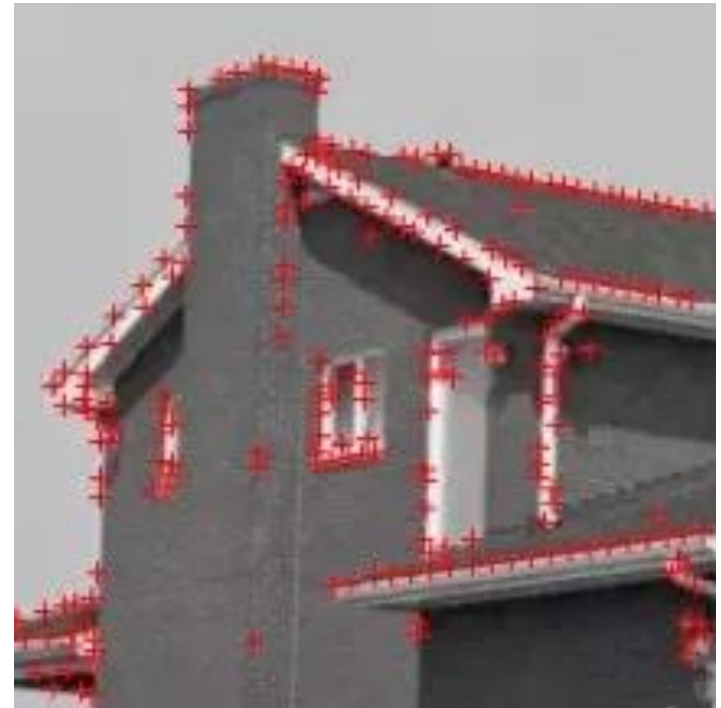
Content

- Hough Transform
 - Line Parameterization
 - Slope Intercept Form
 - Double Intercept Form
 - Normal Form
 - Line Detection
 - Image Space
 - Parameter Space
 - Hough Voting
 - Circles and Others



Hough Transform

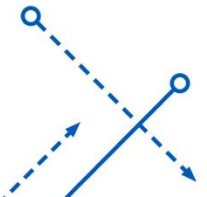
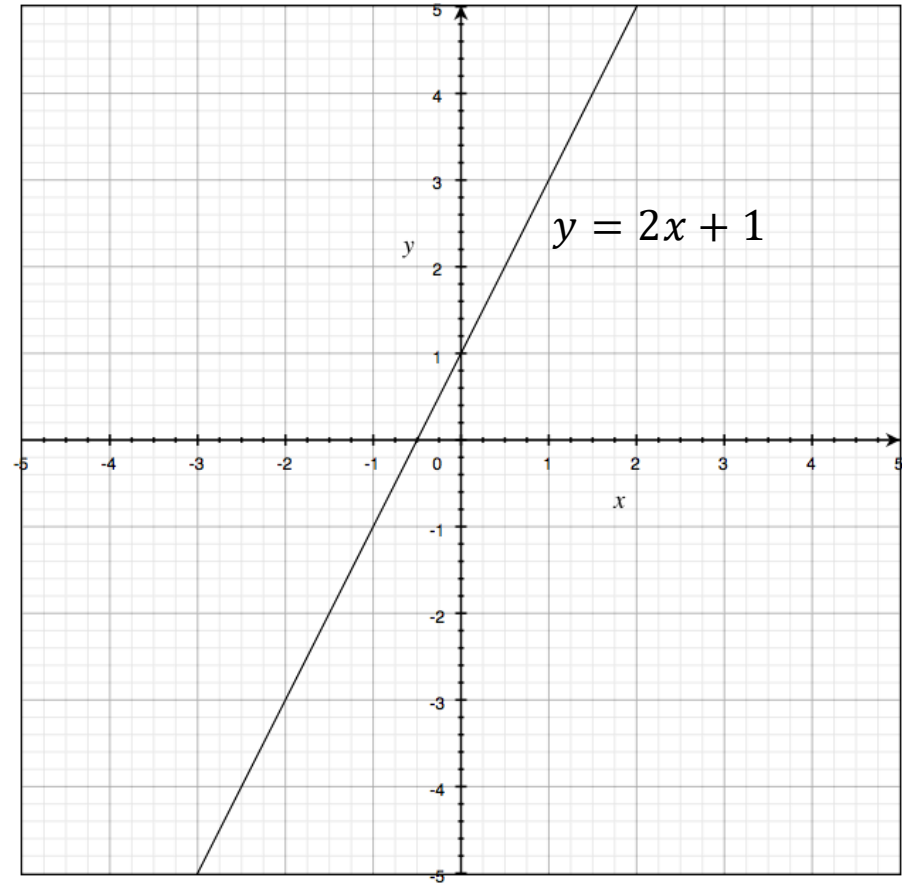
- Hough Transform can detect basic shapes
 - Detect points/edges \rightarrow Find shapes.
 - Lines, Circles, etc.
- Line parameterizations
 - Slope intercept form
 - Double intercept form
 - Normal Form



Slope intercept form

$$y = mx + b$$

↑ ↑
slope y-intercept



Double intercept form

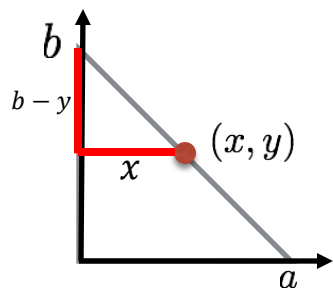
$$\frac{x}{a} + \frac{y}{b} = 1$$

x-intercept

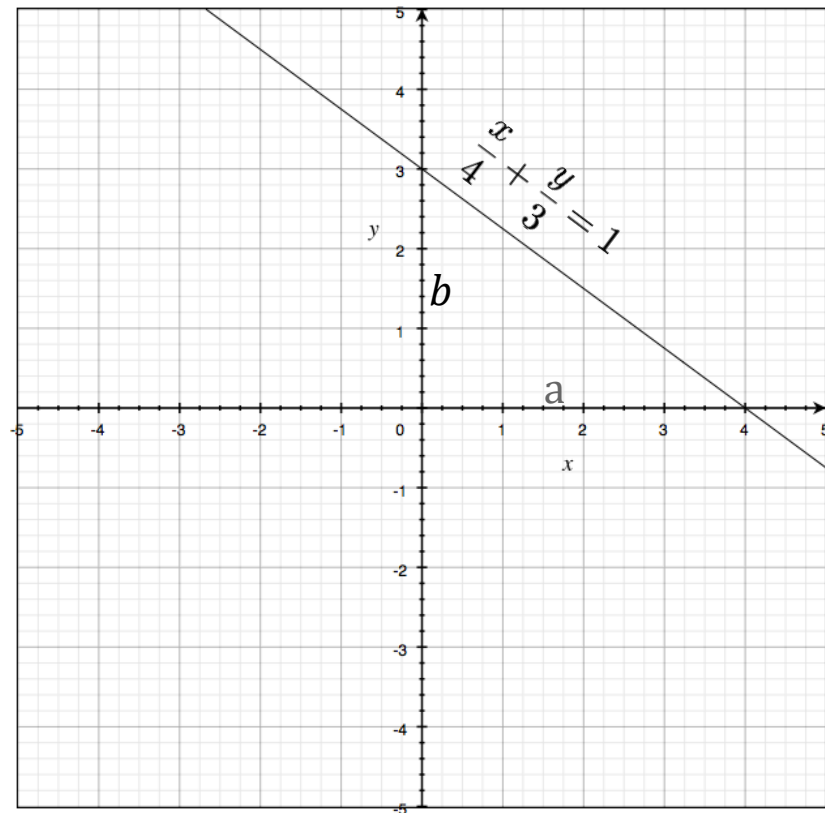
y-intercept

Derivation:

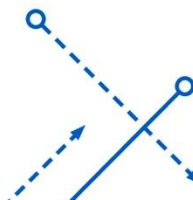
(Similar Triangles)



$$\frac{x}{a} = \frac{b - y}{b}$$



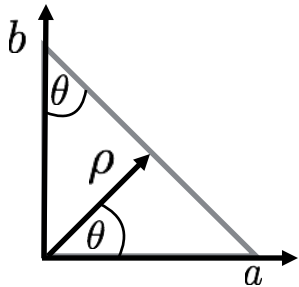
What are a and b?



Normal Form

$$x \cos \theta + y \sin \theta = \rho$$

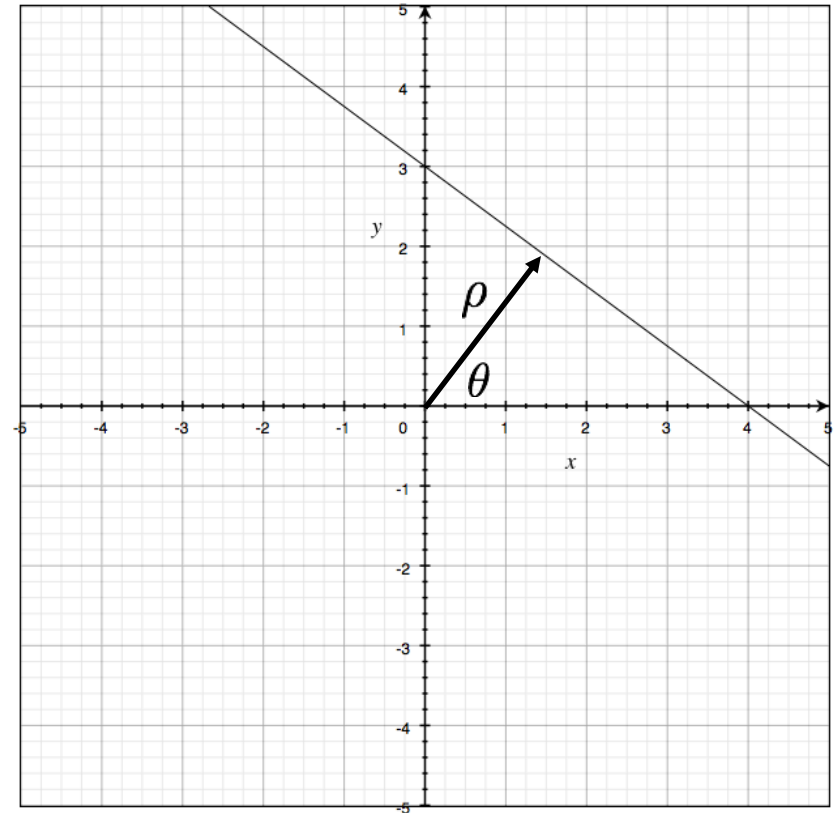
Derivation:



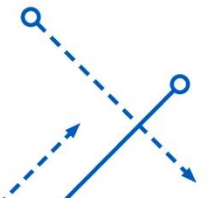
$$a = \frac{\rho}{\cos \theta}$$
$$b = \frac{\rho}{\sin \theta}$$

plug into:

$$\frac{x}{a} + \frac{y}{b} = 1$$



What are ρ and θ ?



Hough Transform

- Slope intercept form
- Normal Form

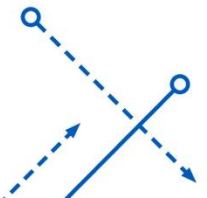


Image and parameter space

variables

$$y = mx + b$$

parameters

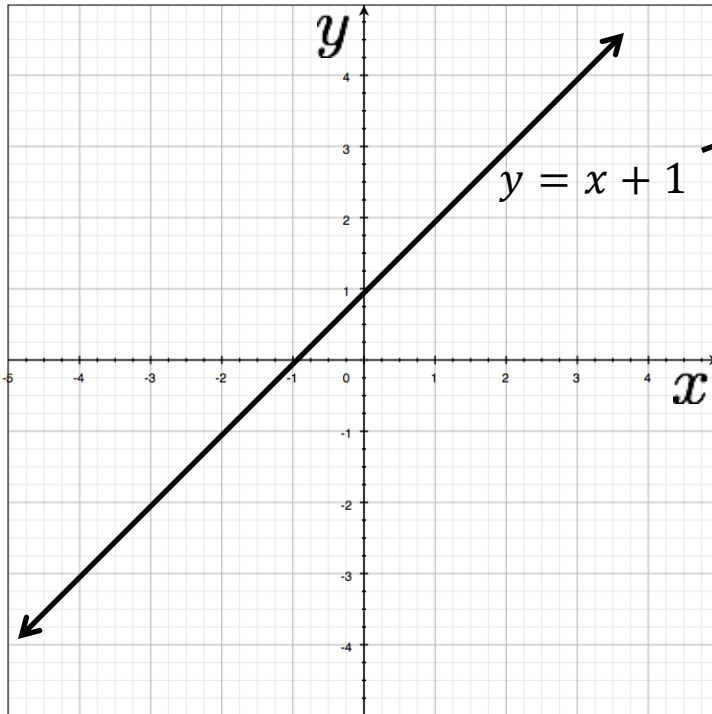
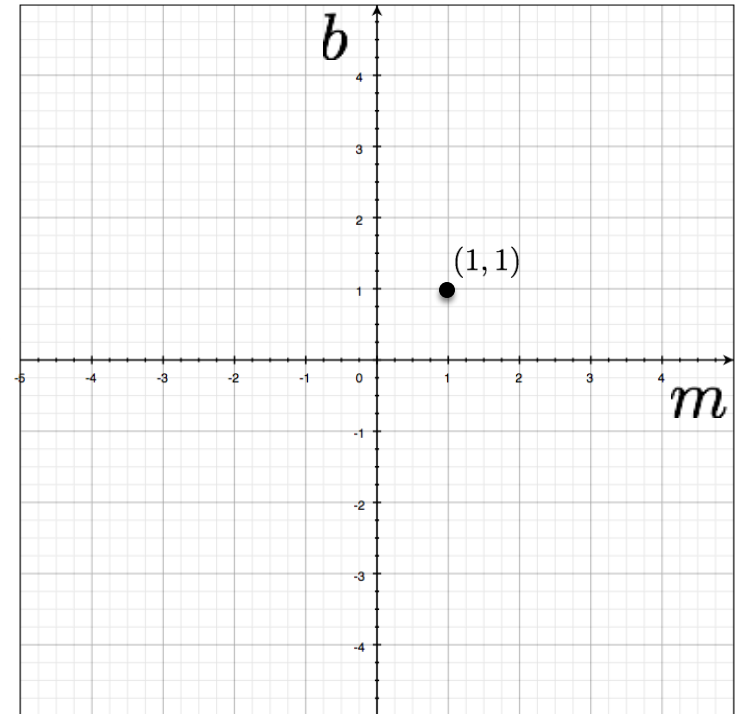


Image space

variables

$$b = -xm + y$$

parameters



Parameter space

$m = 1$
 $b = 1$

a line
becomes
a point

Image and parameter space

variables

$$y = mx + b$$

parameters

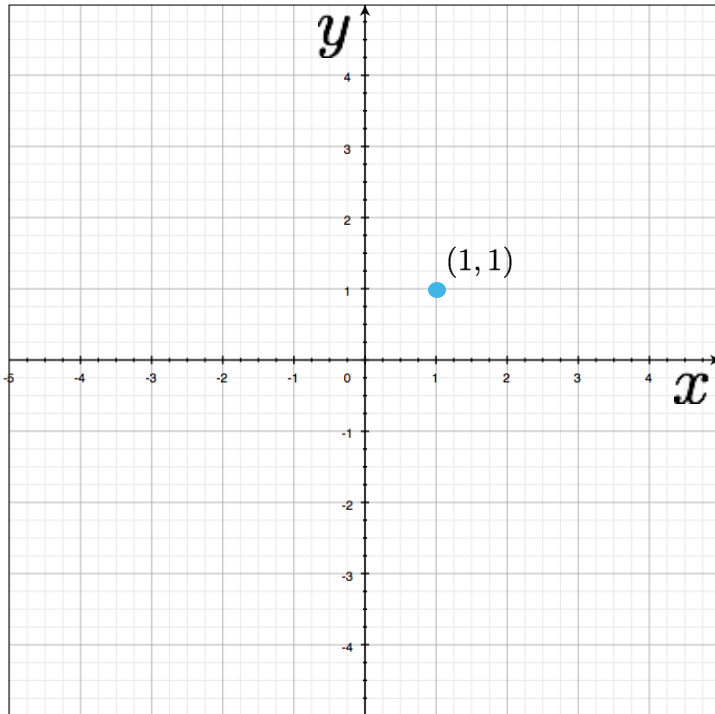


Image space

What would a point in image space become in parameter space?

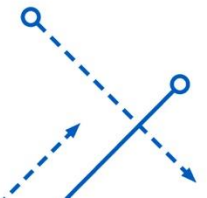


Image and parameter space

variables

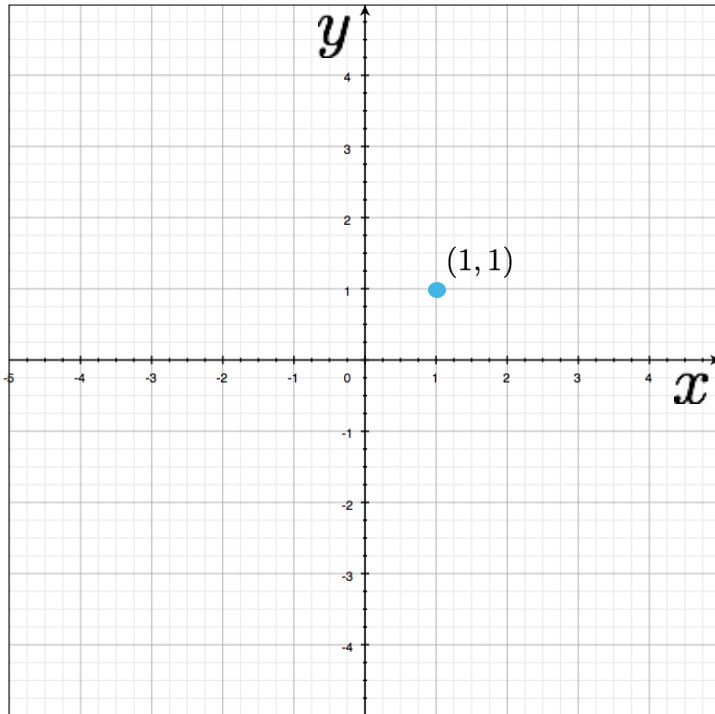
$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters



a point
becomes
a line

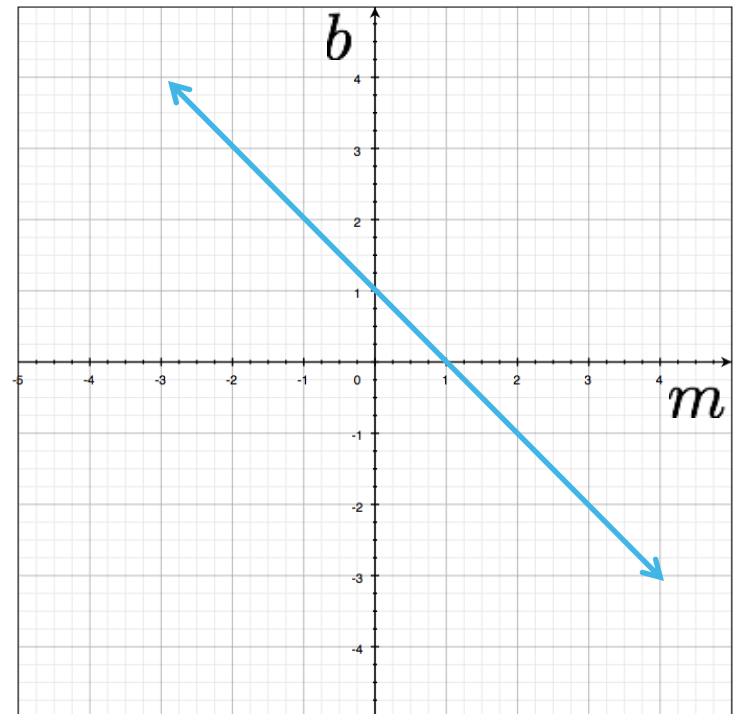


Image space

Parameter space

Image and parameter space

variables

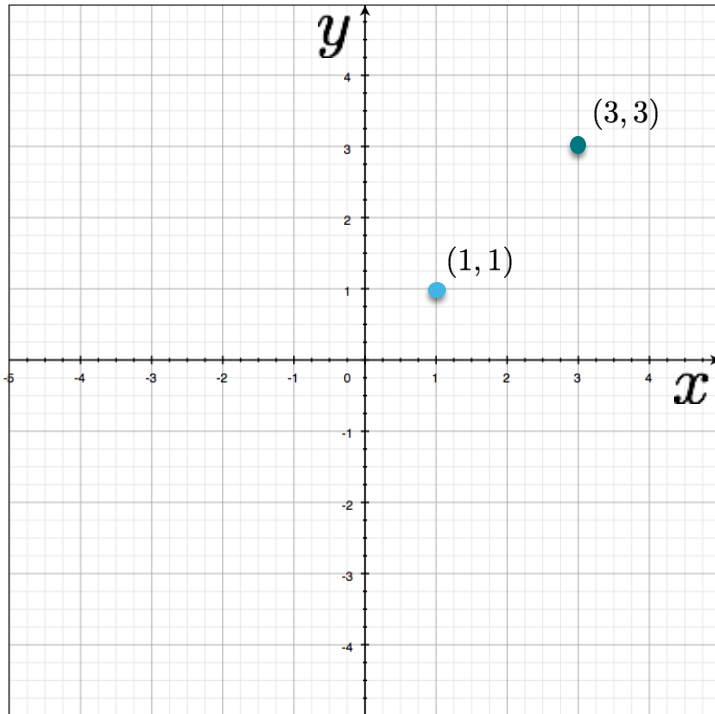
$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters



two points
become
?

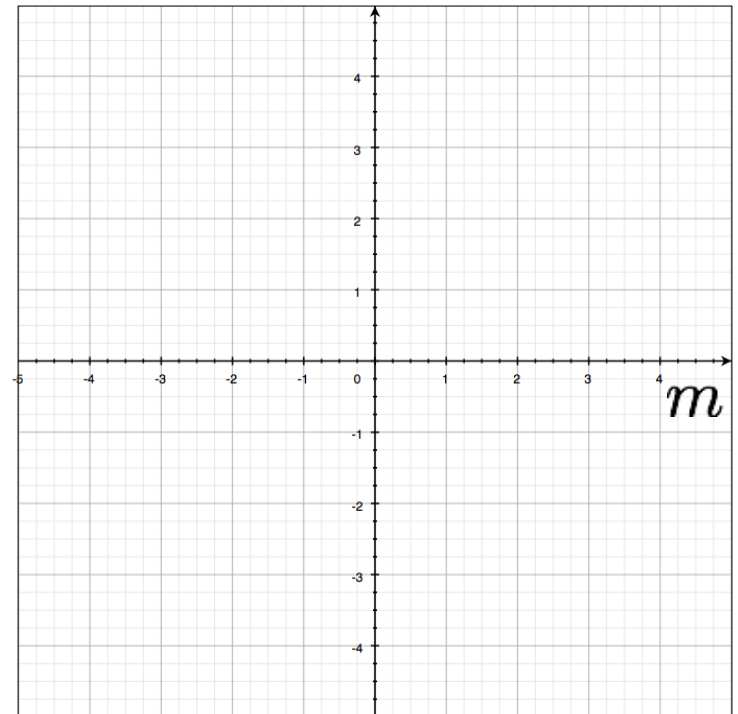


Image space

Parameter space

Image and parameter space

variables

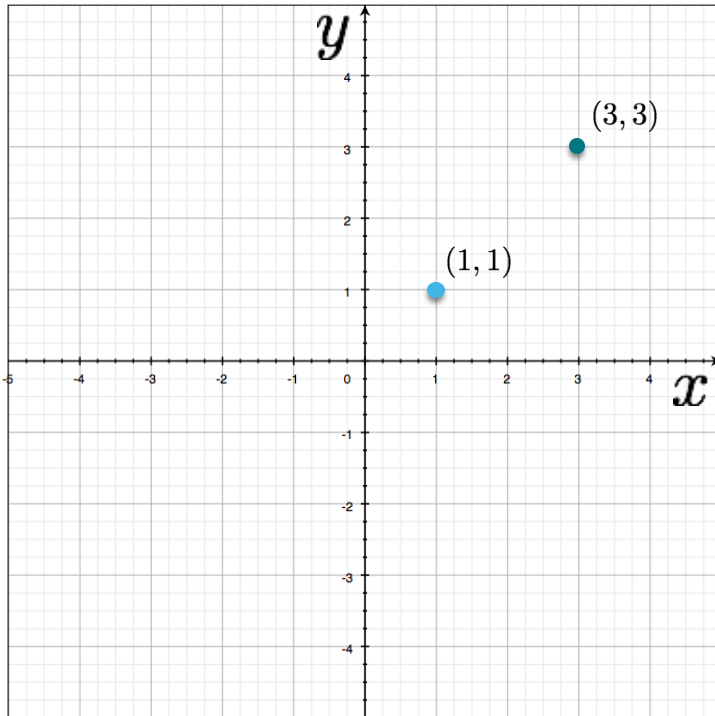
$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters



two points
become
?

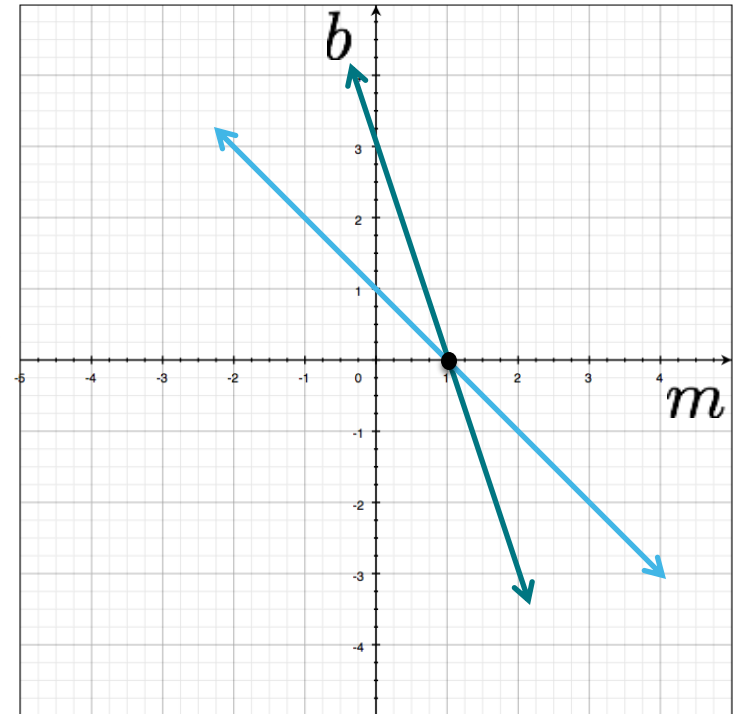


Image space

Parameter space

Image and parameter space

variables

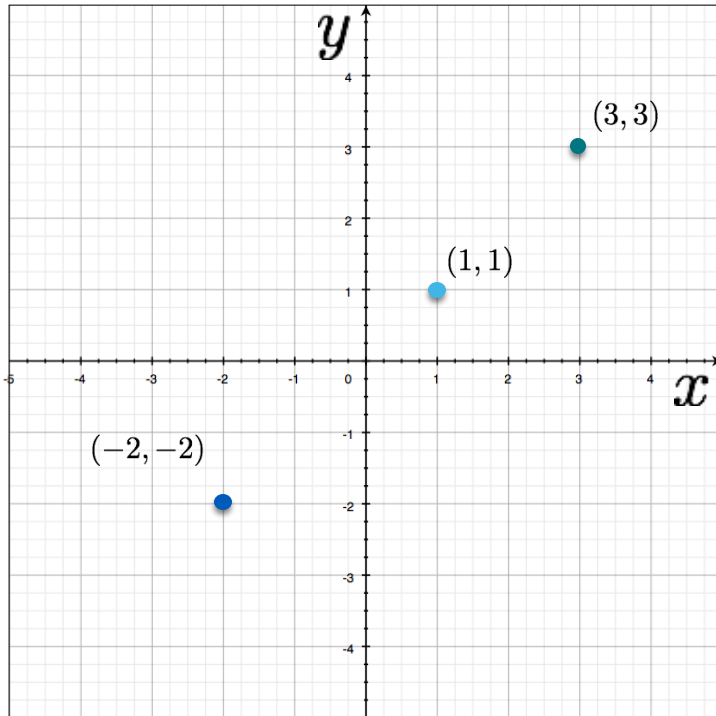
$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters



three points
become
?

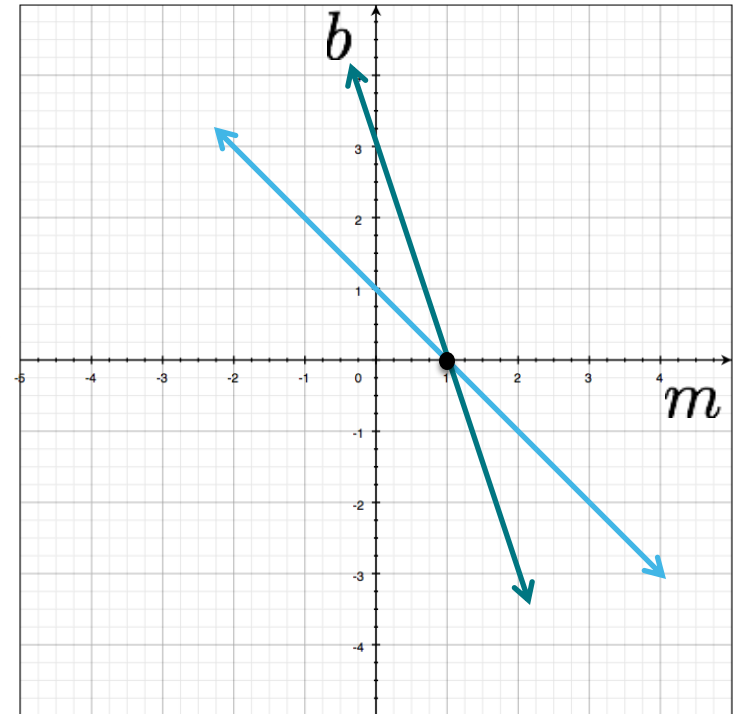


Image space

Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters

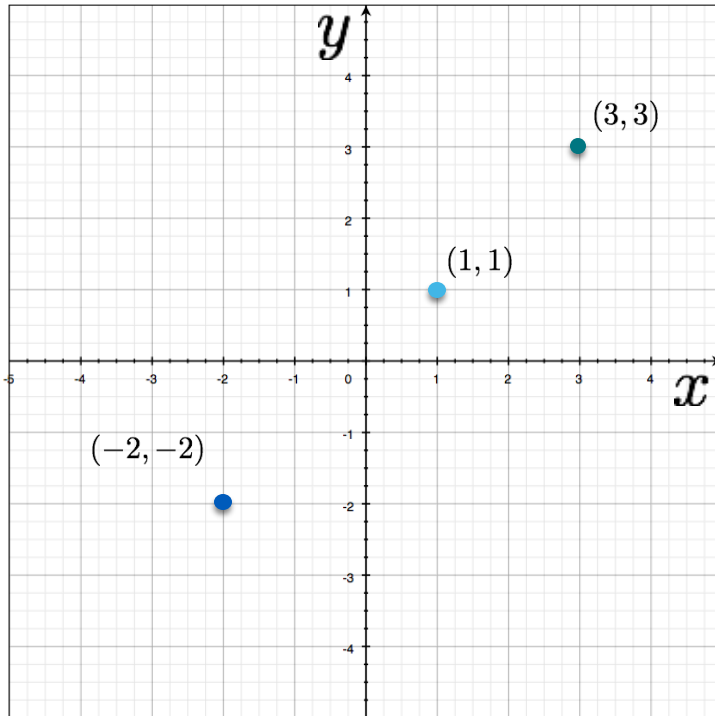
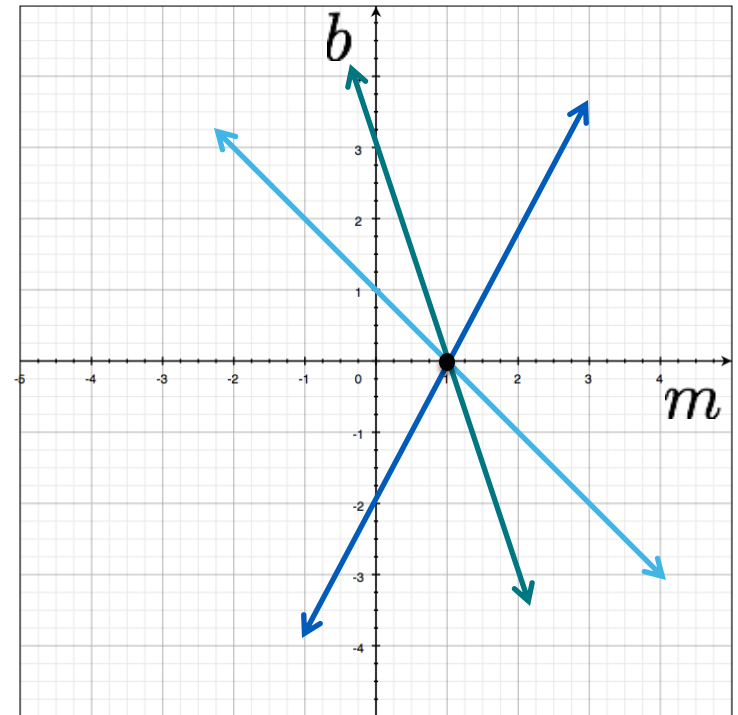


Image space

three points
become
?



Parameter space

Image and parameter space

variables

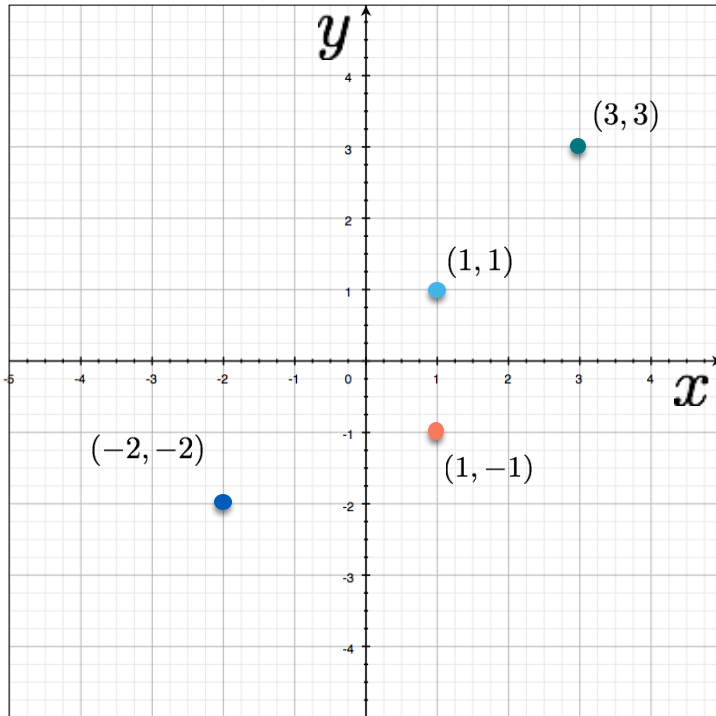
$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters



four points
become
?

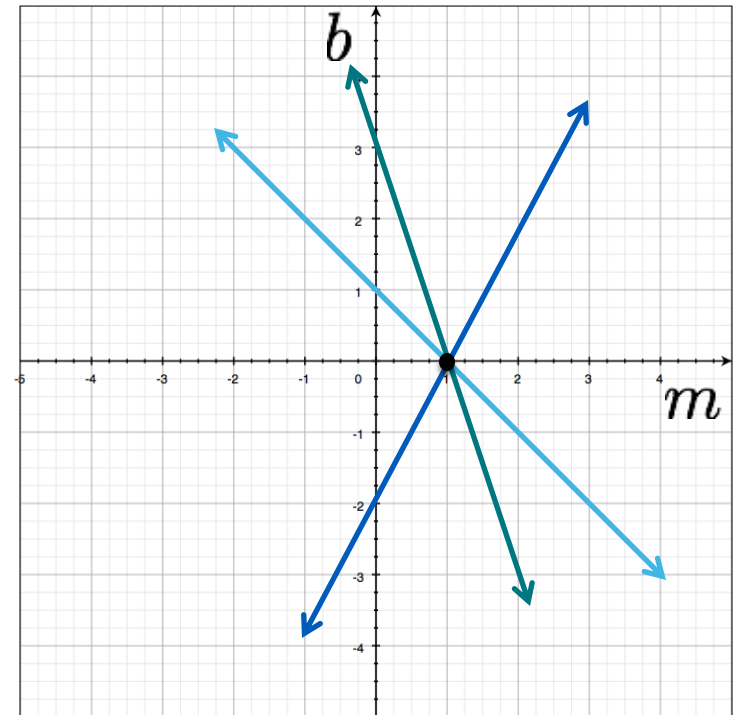


Image space

Parameter space

Image and parameter space

variables

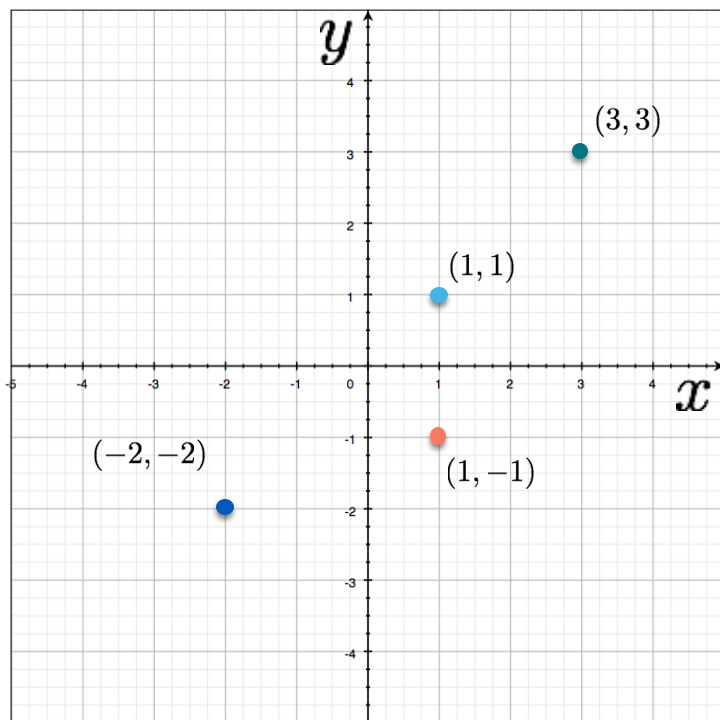
$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters



four points
become
?

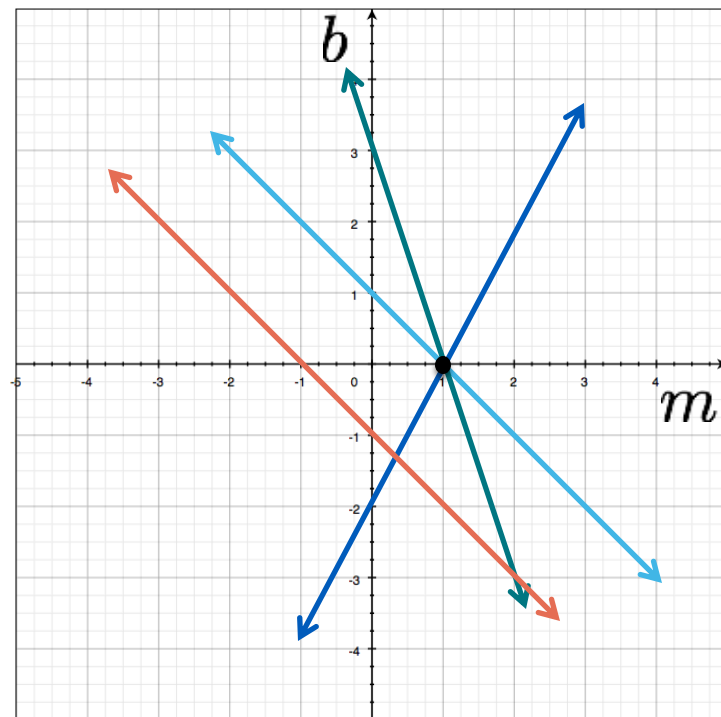


Image space

Parameter space

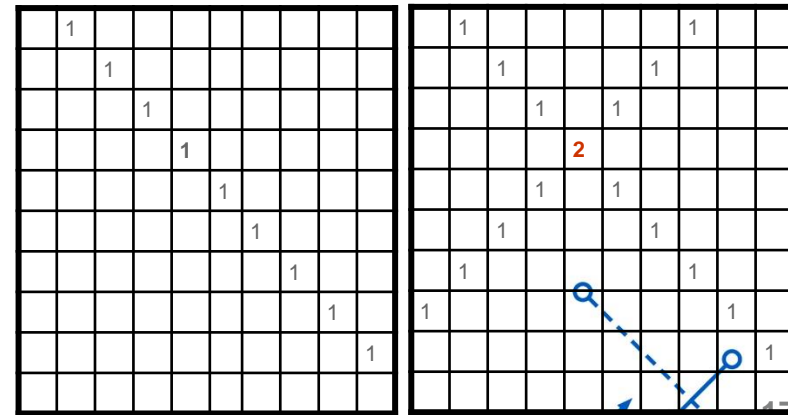
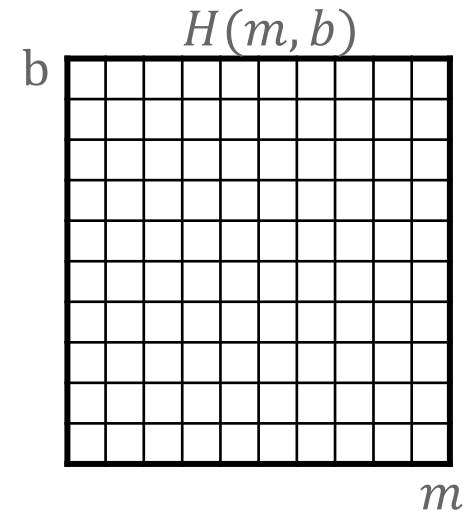
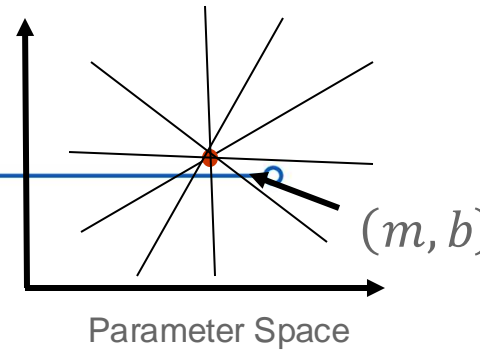
Hough Voting

Line Detection Algorithm:

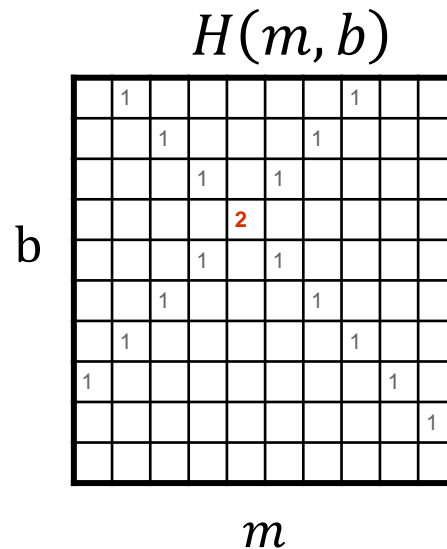
1. Quantize Parameter Space (m, b) .
2. Create Hough Space Array $H(m, b) = 0$.
3. For each image point (x_i, y_i) :
For all points (m, b) on $b = -x_i m + y_i$:
 $H(m, b) = H(m, b) + 1$
4. Find local maxima in $H(m_m, b_m)$.
5. The detected line: $y = m_m x + b_m$.

Is it able to detect multiple lines?

Is this solution good enough?



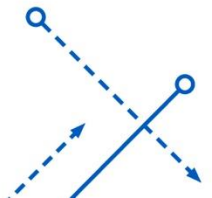
How big does the Hough array have to be?



The space of m is huge! The space of b is huge!

$$-\infty \leq m \leq \infty$$

$$-\infty \leq b \leq \infty$$



Hough Transform with Normal Form

Use normal form:

$$x \cos \theta + y \sin \theta = \rho$$

The Hough space become $H(\rho, \theta)$

Hough Space

$$0 \leq \theta \leq \pi$$

$$0 \leq \rho \leq \rho_{max}$$

(Finite Hough Array Size)

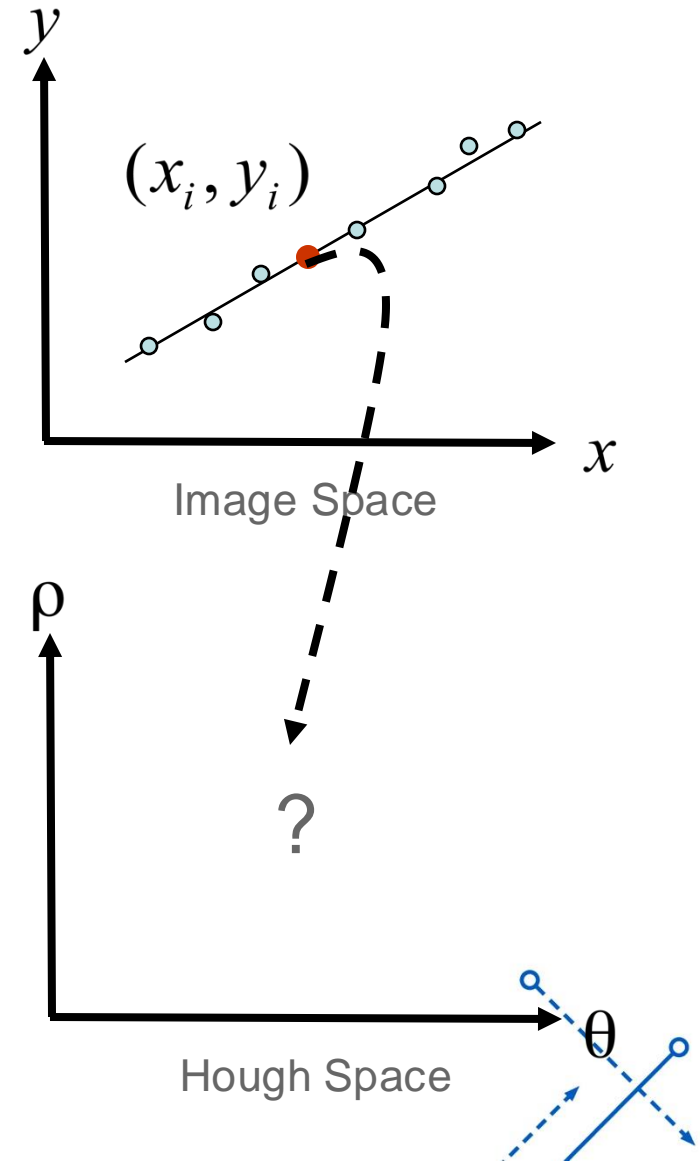
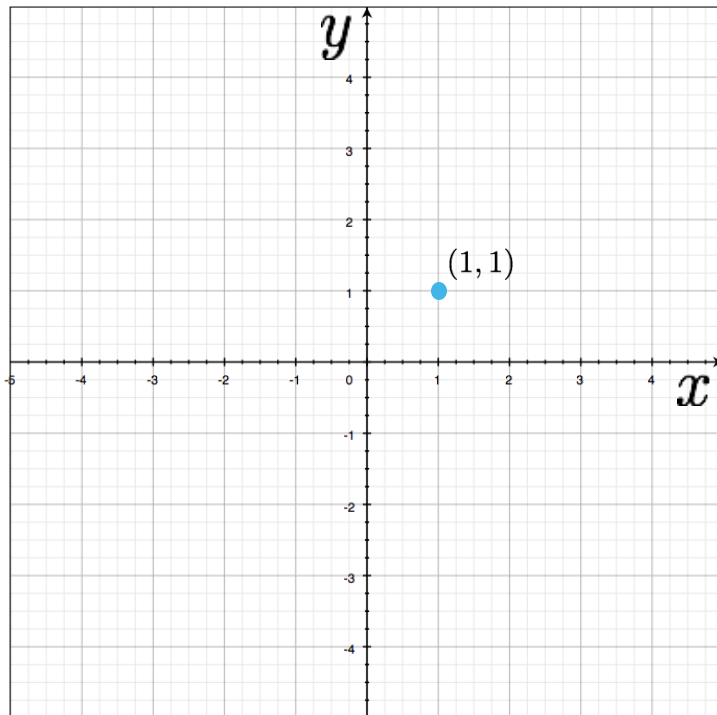


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a point
becomes?

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

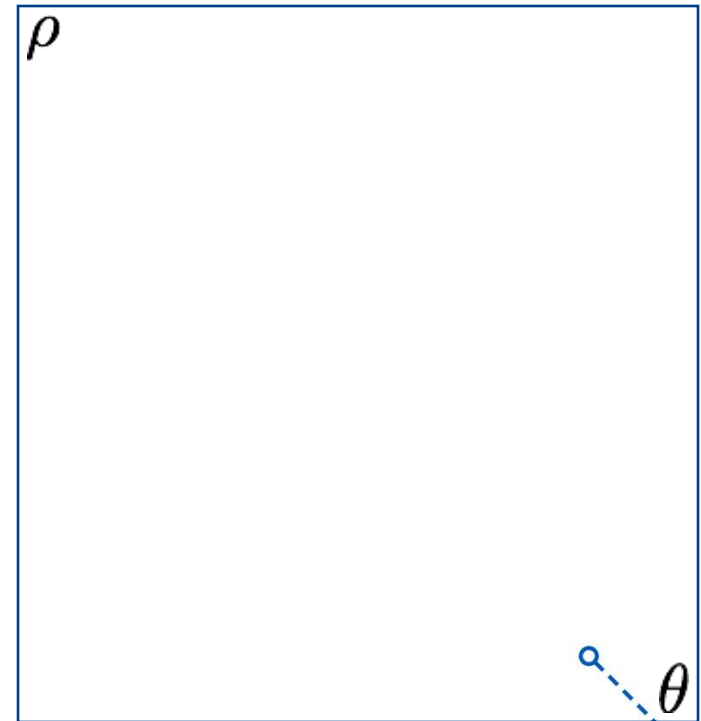
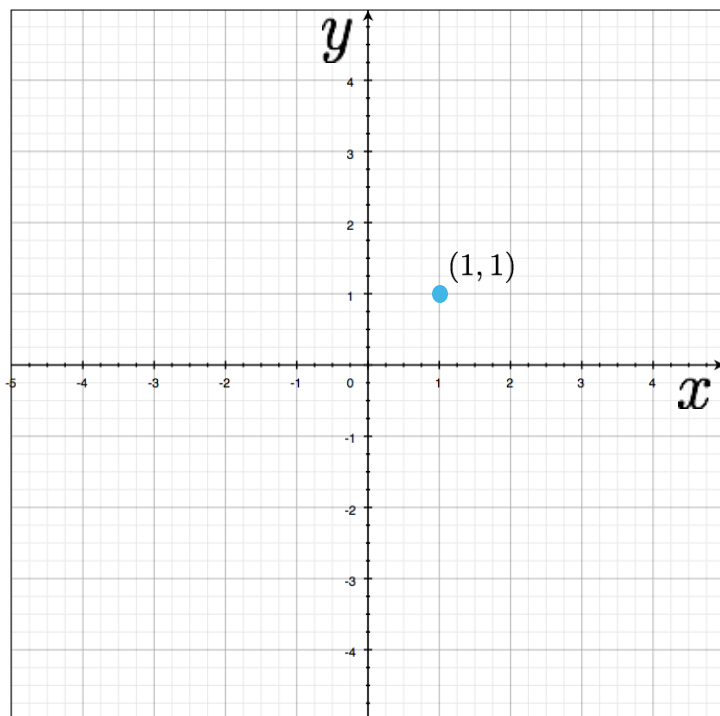


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a point
becomes
a wave

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

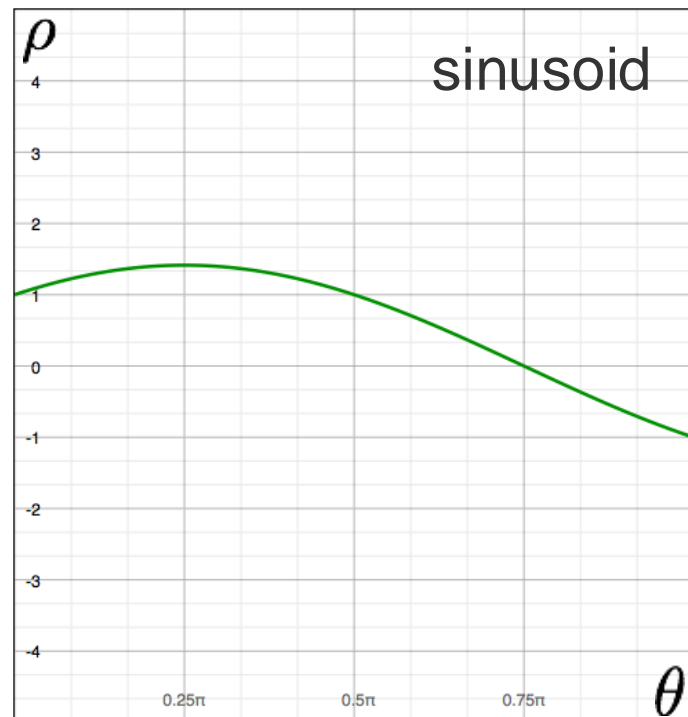
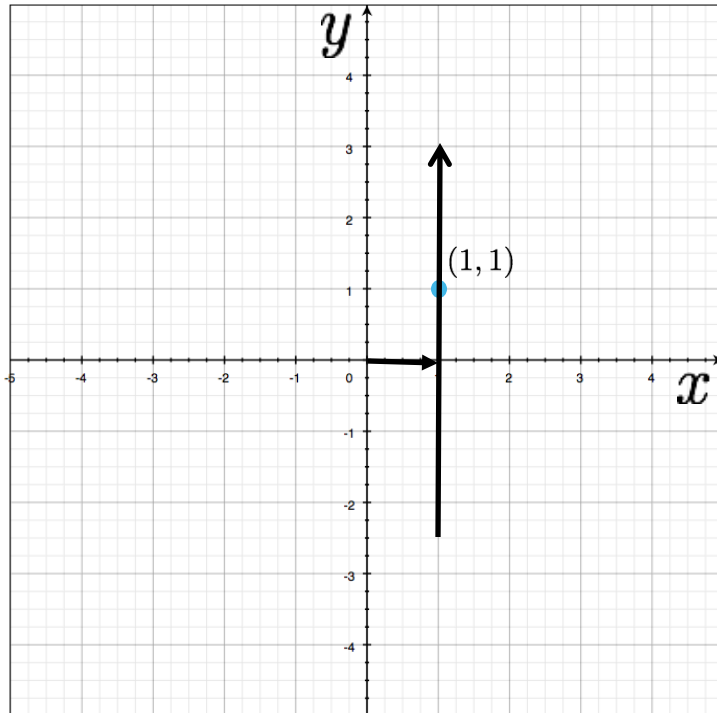


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a line
becomes?

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

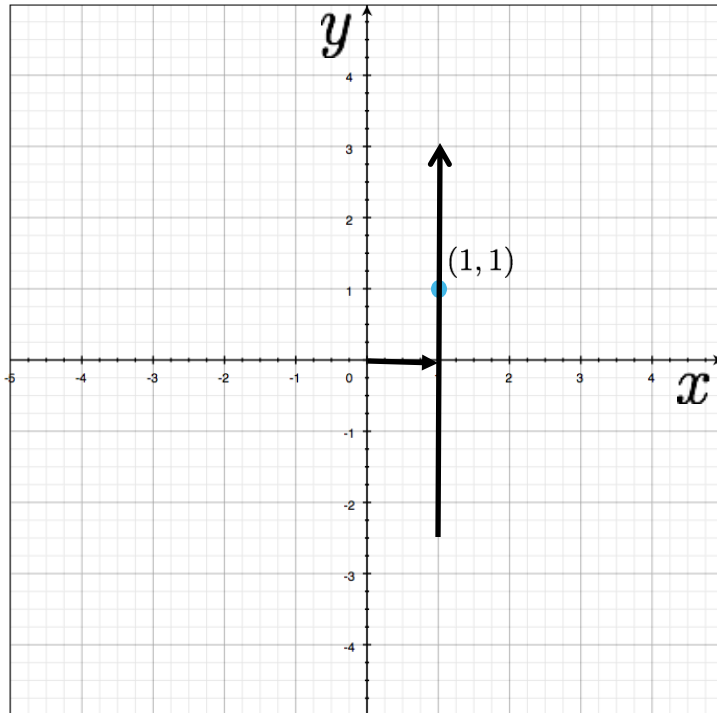


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a line
becomes
a point

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

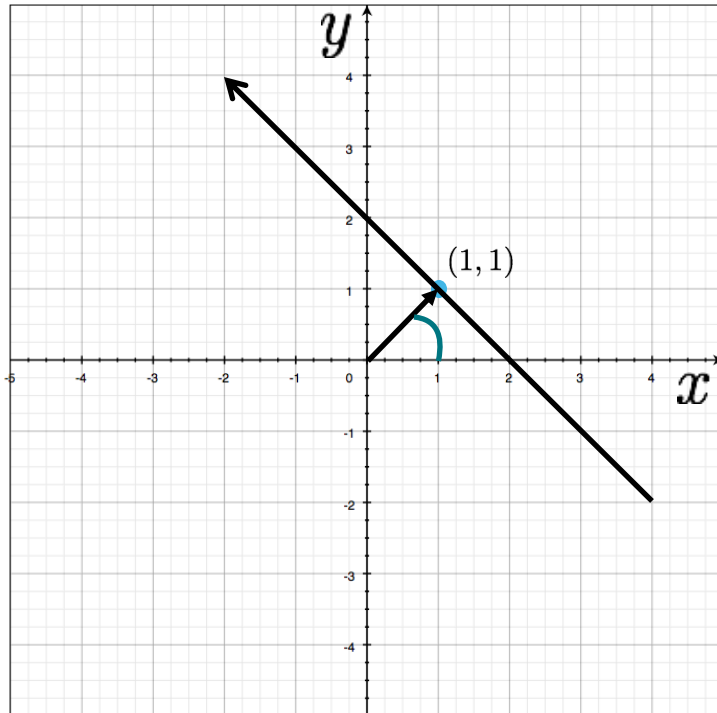


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a line
becomes?

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

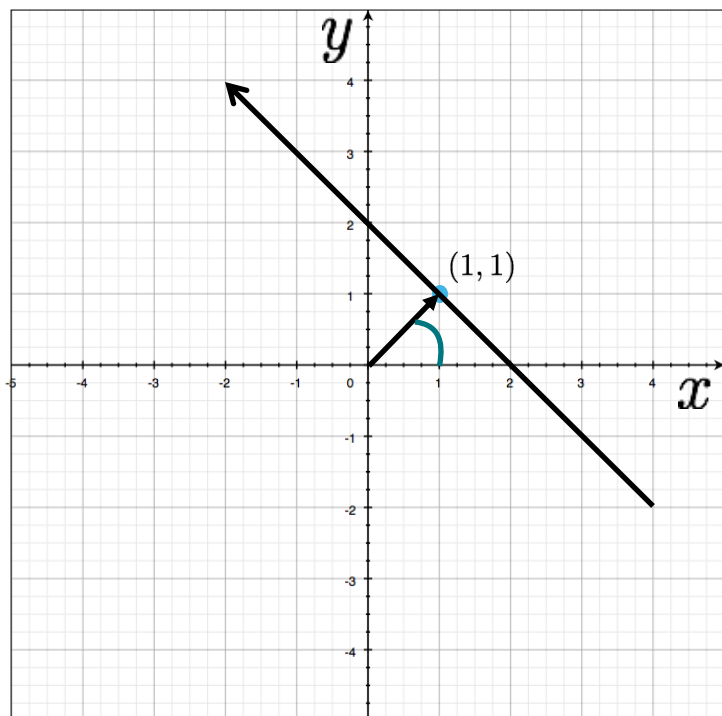


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a line
becomes
a point

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

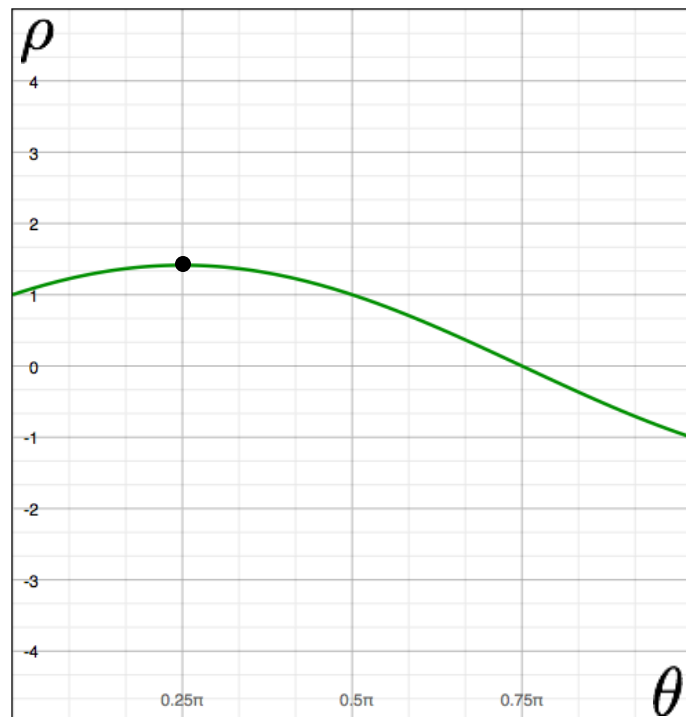
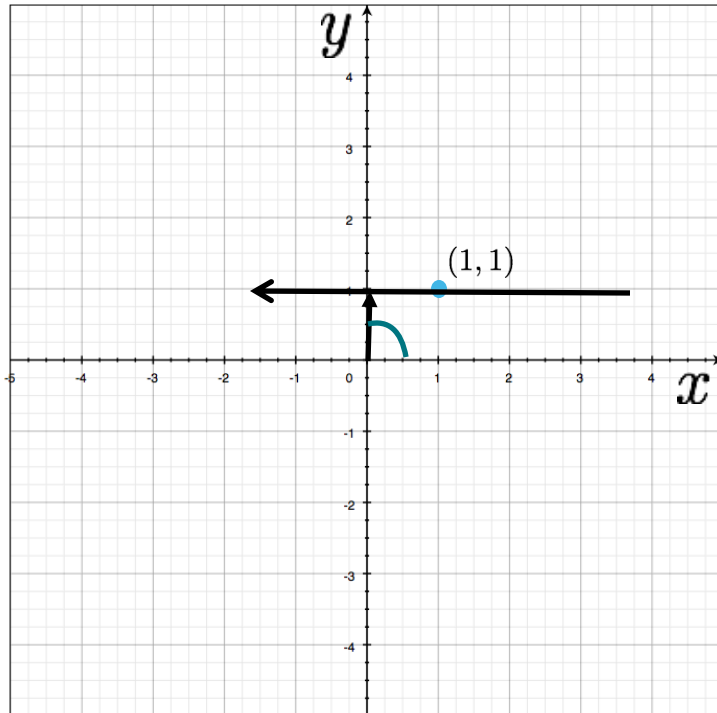


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a line
becomes
a point

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

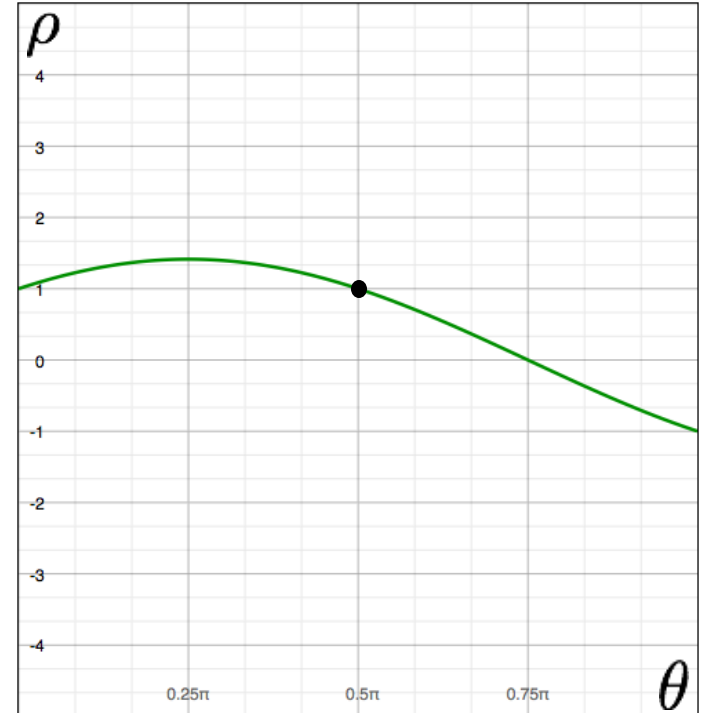
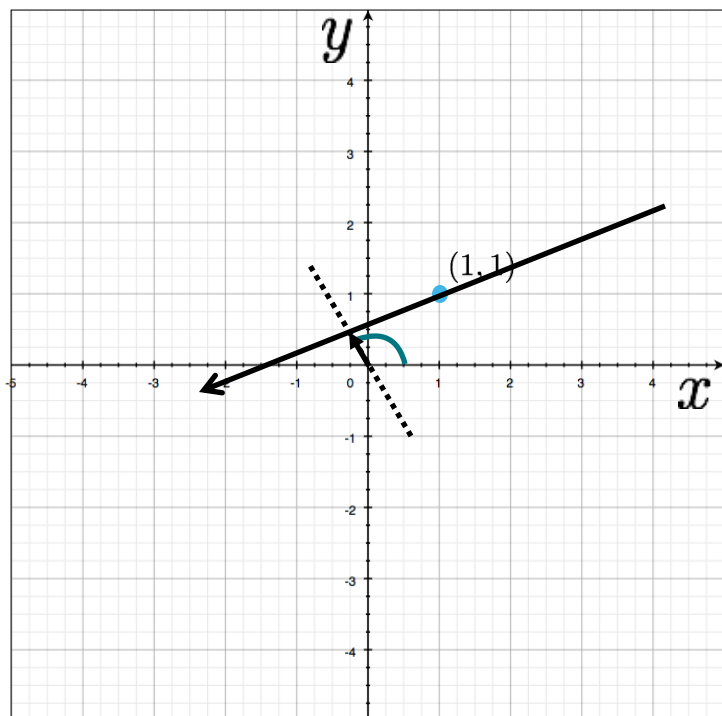


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a line
becomes
a point

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

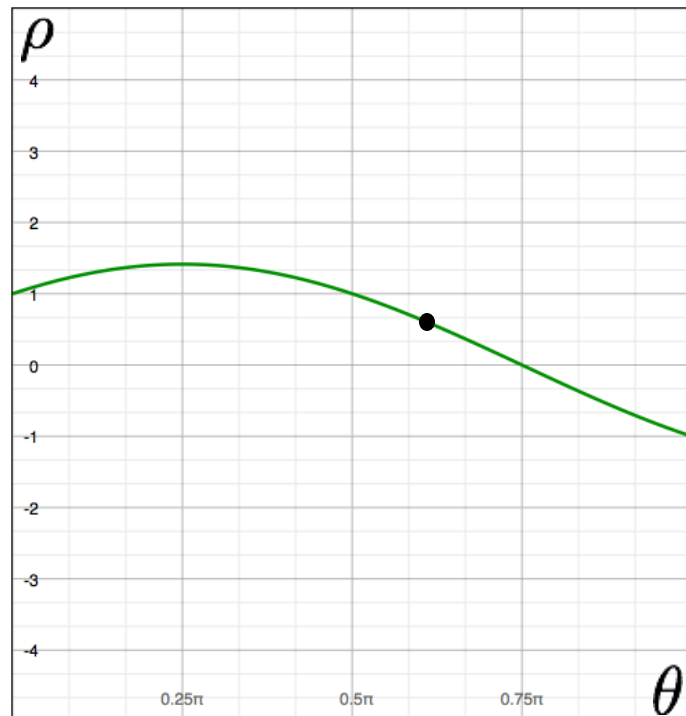
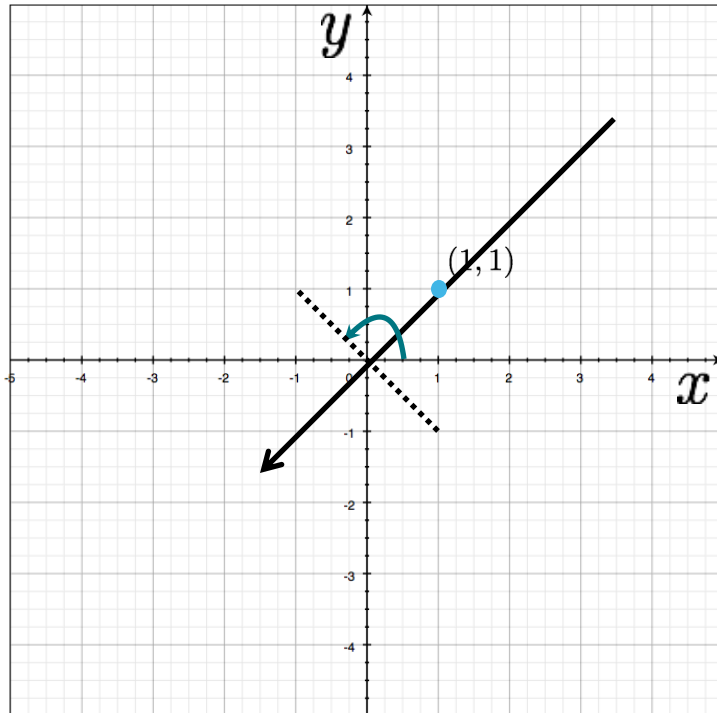


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a line
becomes
a point

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

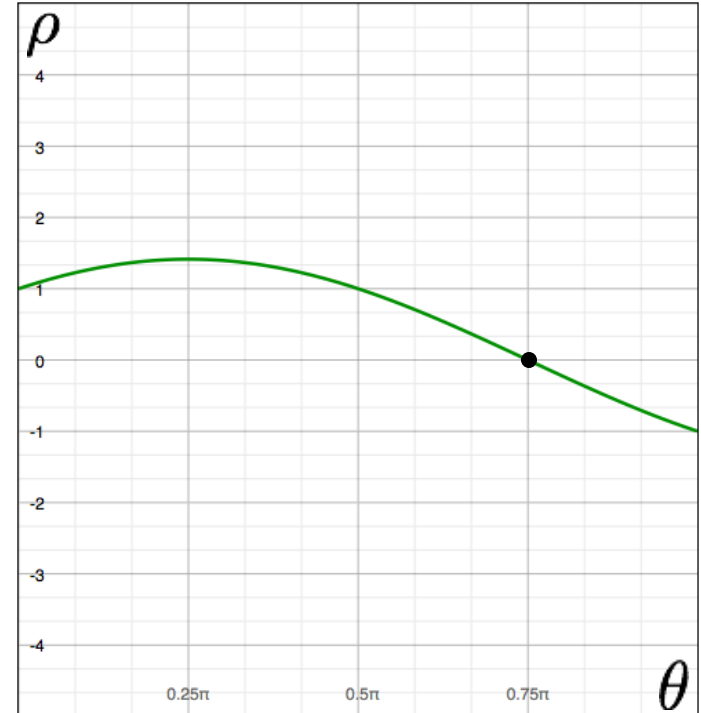
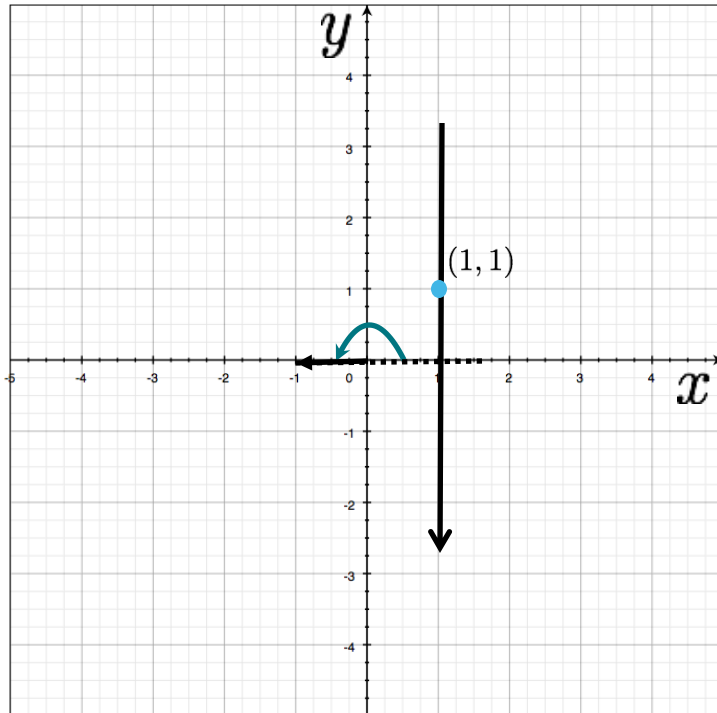


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a line
becomes
a point

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables



Image and parameter space

variables

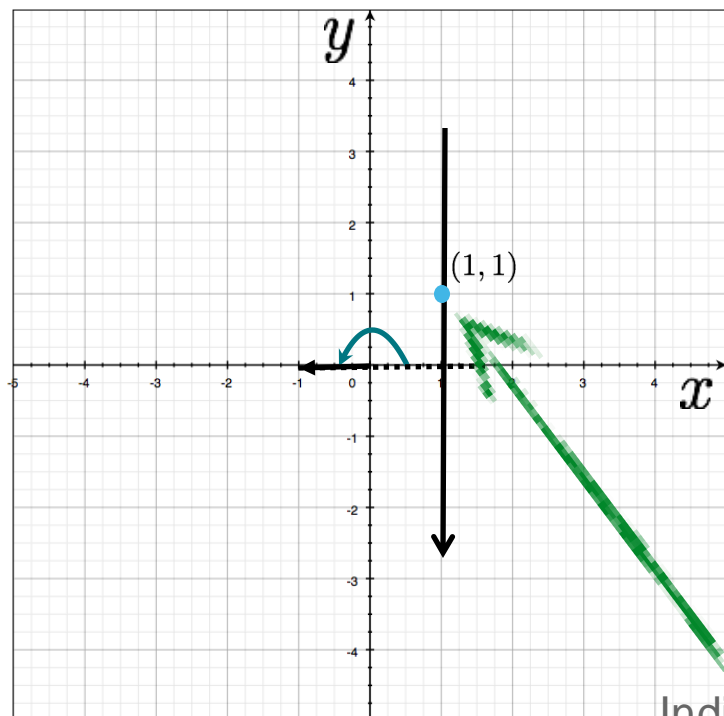
$$x \cos \theta + y \sin \theta = \rho$$

parameters

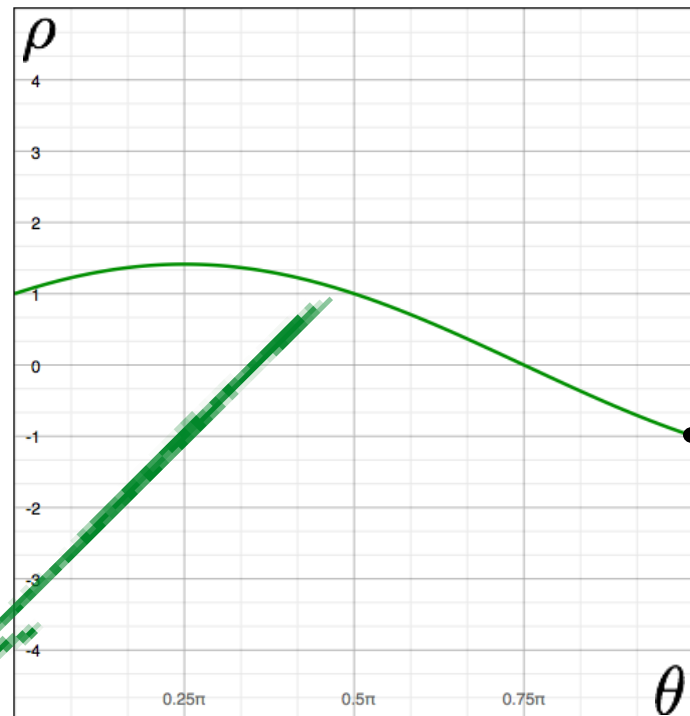
parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables



a line
becomes
a point



Indicate all lines
pass through (1,1)

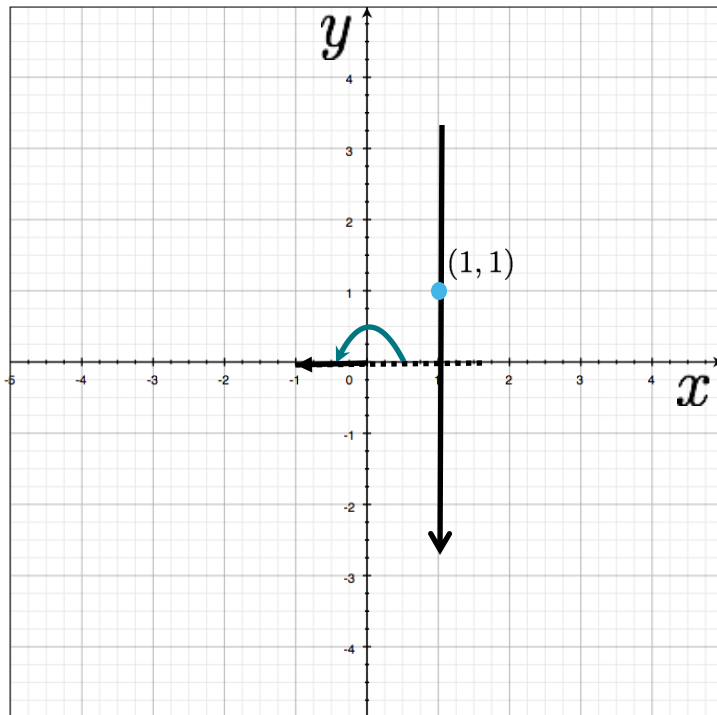
Parameter space

Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



a line
becomes
a point

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

Wait ... why is ρ negative?

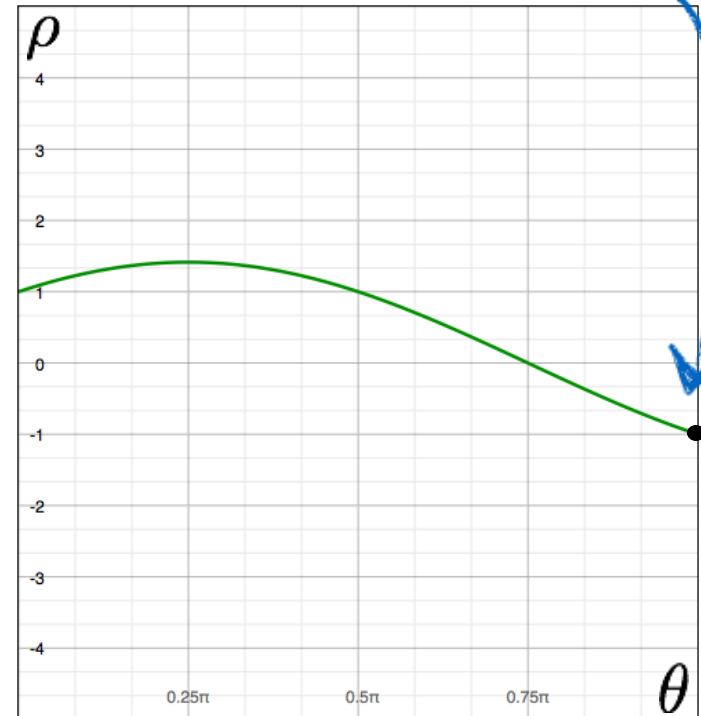
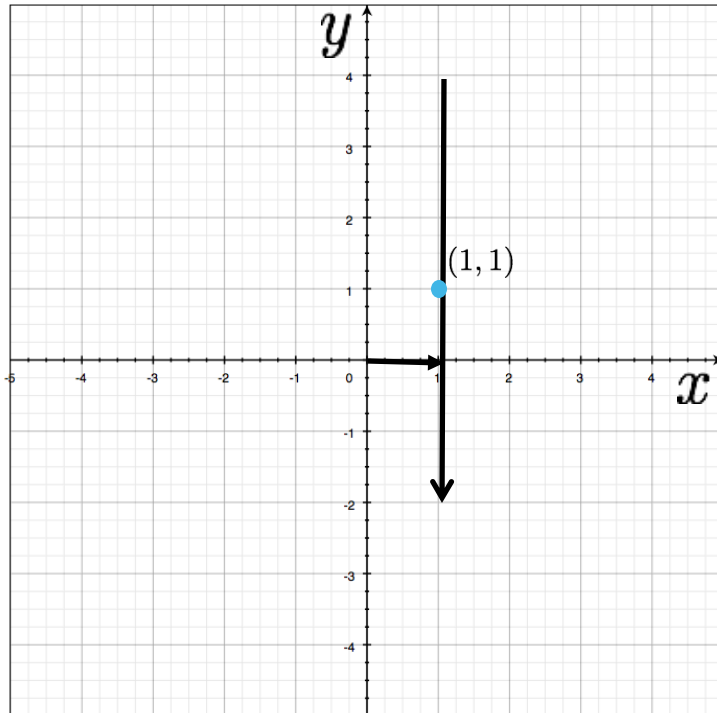


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters

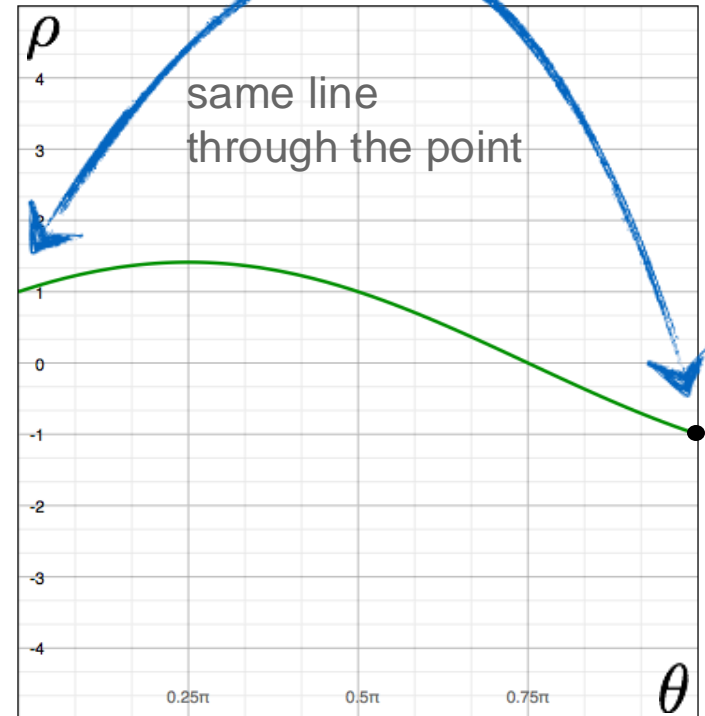


a line
becomes
a point

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables



There are two ways to write the same line

Positive ρ version:

$$x \cos \theta + y \sin \theta = \rho$$

Negative ρ version:

$$x \cos(\theta + \pi) + y \sin(\theta + \pi) = -\rho$$

Recall:

$$\sin(\theta) = -\sin(\theta + \pi)$$

$$\cos(\theta) = -\cos(\theta + \pi)$$

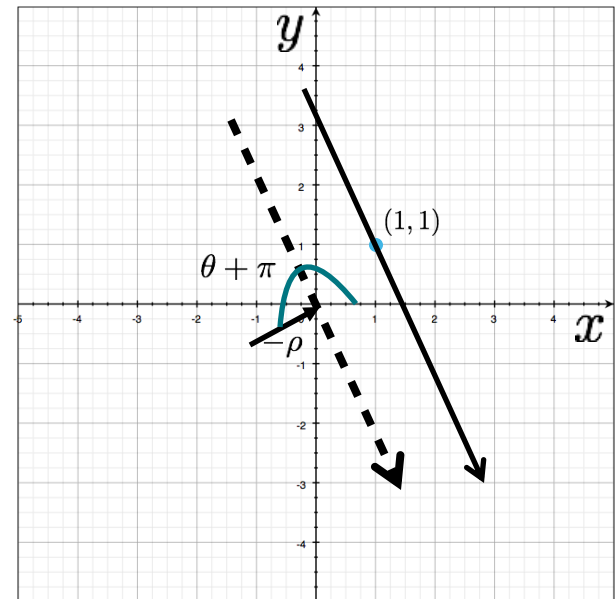
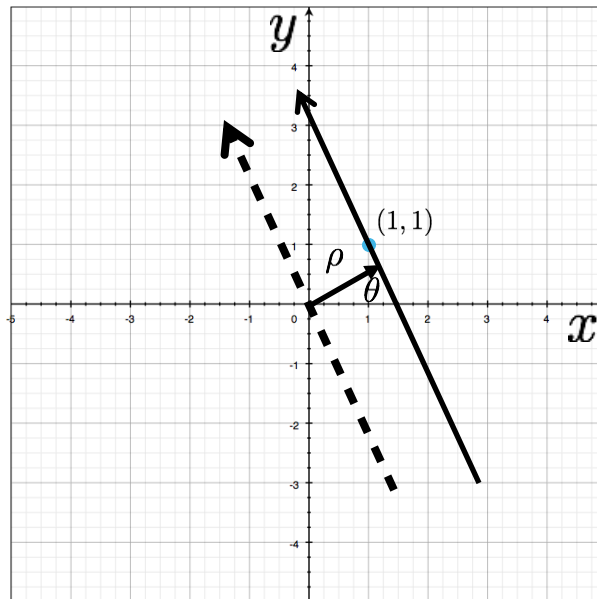
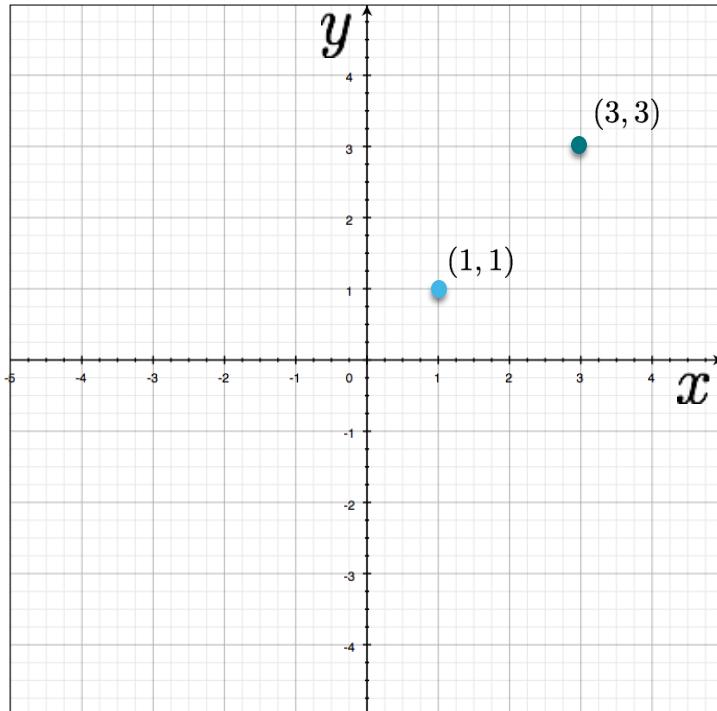


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



two points
become
?

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

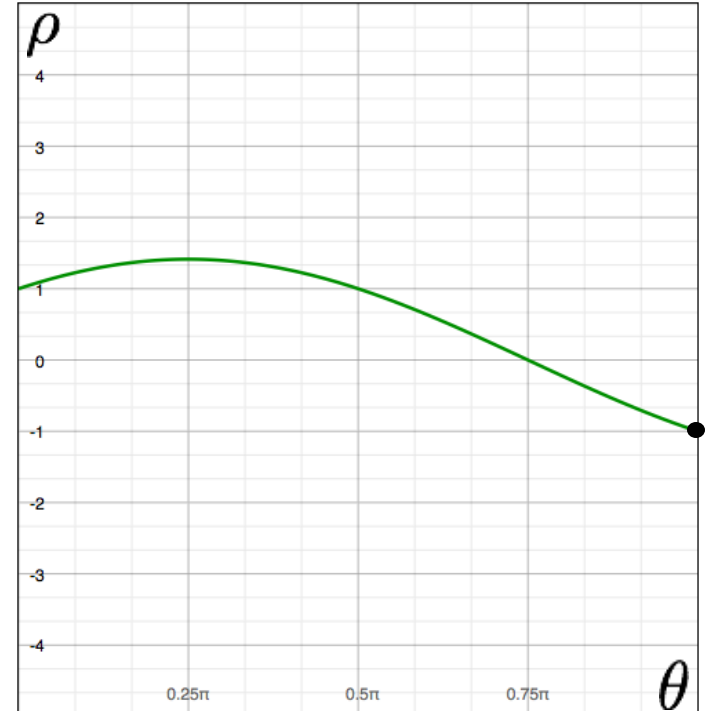
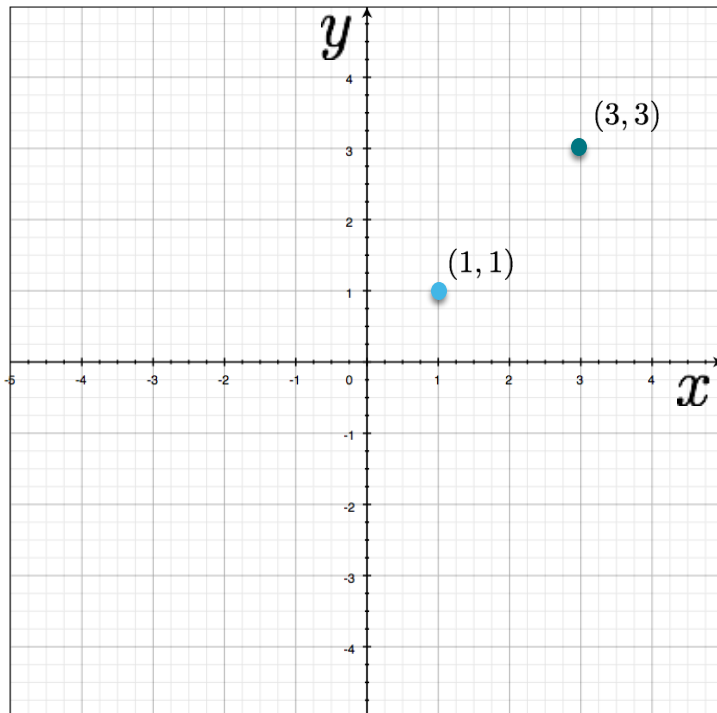


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters

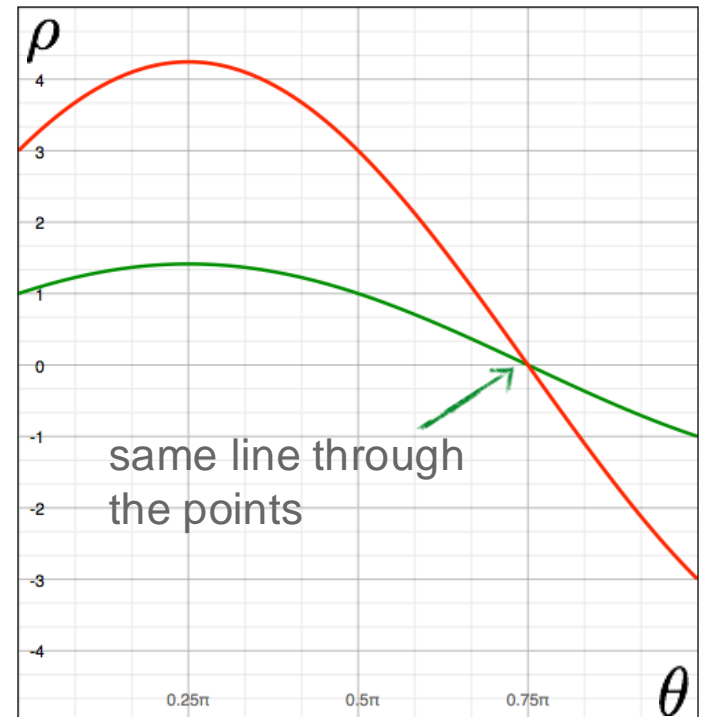


two points
become
?

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables



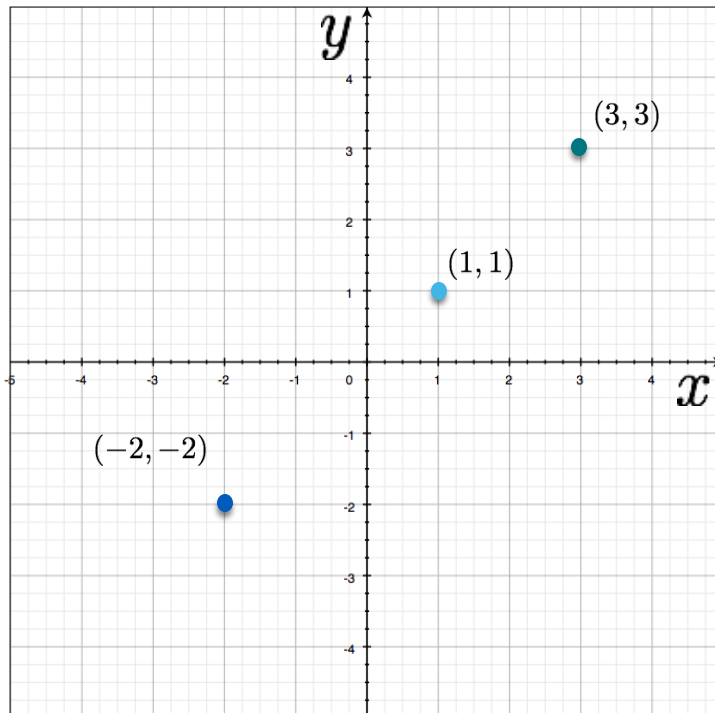
Parameter space

Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters



three points
become
?

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables

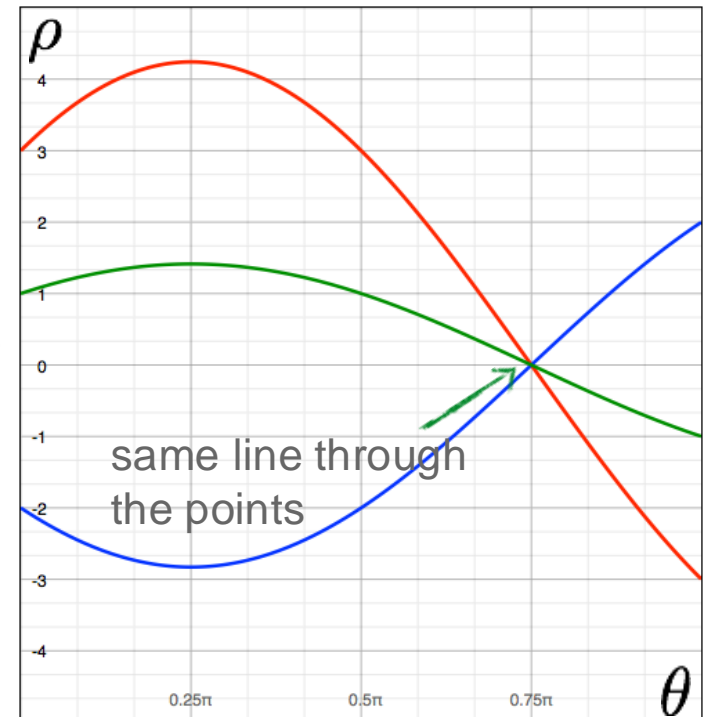
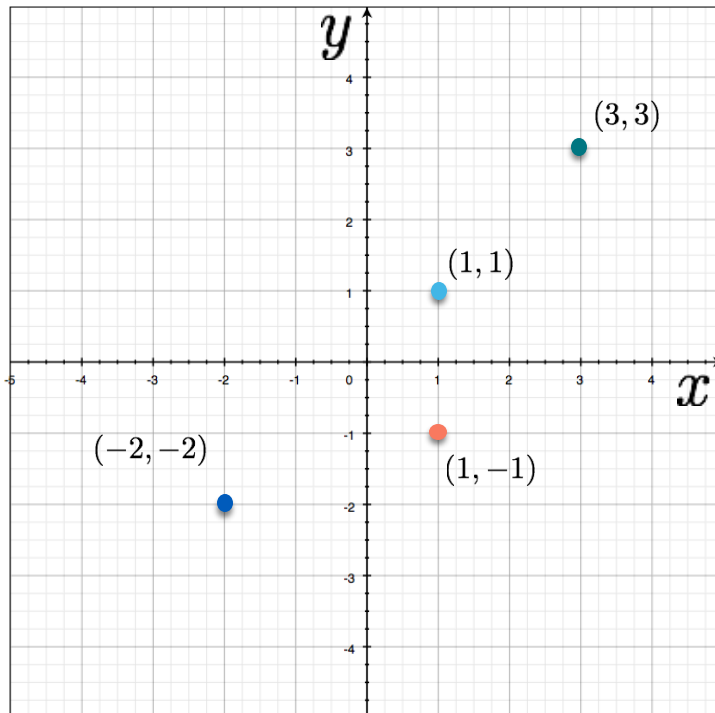


Image and parameter space

variables

$$x \cos \theta + y \sin \theta = \rho$$

parameters

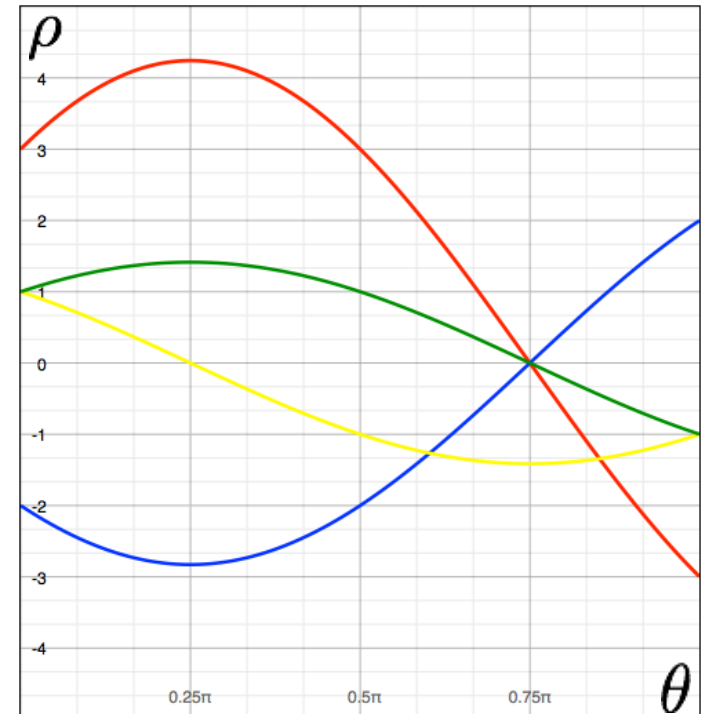


four points
become
?

parameters

$$x \cos \theta + y \sin \theta = \rho$$

variables



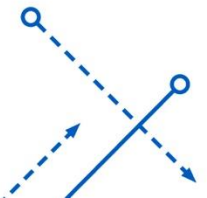
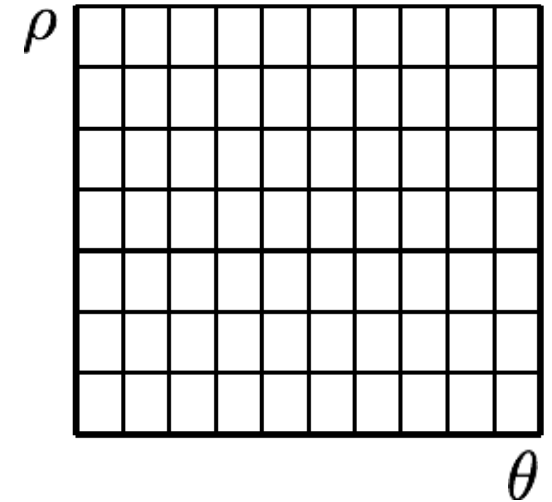
Parameter space

Line Detection by Hough Voting

Algorithm:

1. Quantize Parameter Space (θ, ρ) .
2. Create Hough Space Array $H(\theta, \rho) = 0$.
3. For each image point (x_i, y_i) :
For all points (θ, ρ) on $\rho = x_i \cos \theta + y_i \sin \theta$:
 $H(\theta, \rho) = H(\theta, \rho) + 1$
4. Find local maxima $H(\theta_m, \rho_m)$.
5. The detected line: $x \cos \theta_m + y \sin \theta_m = \rho_m$

H: accumulator array (votes)



Line Detection by Hough Voting

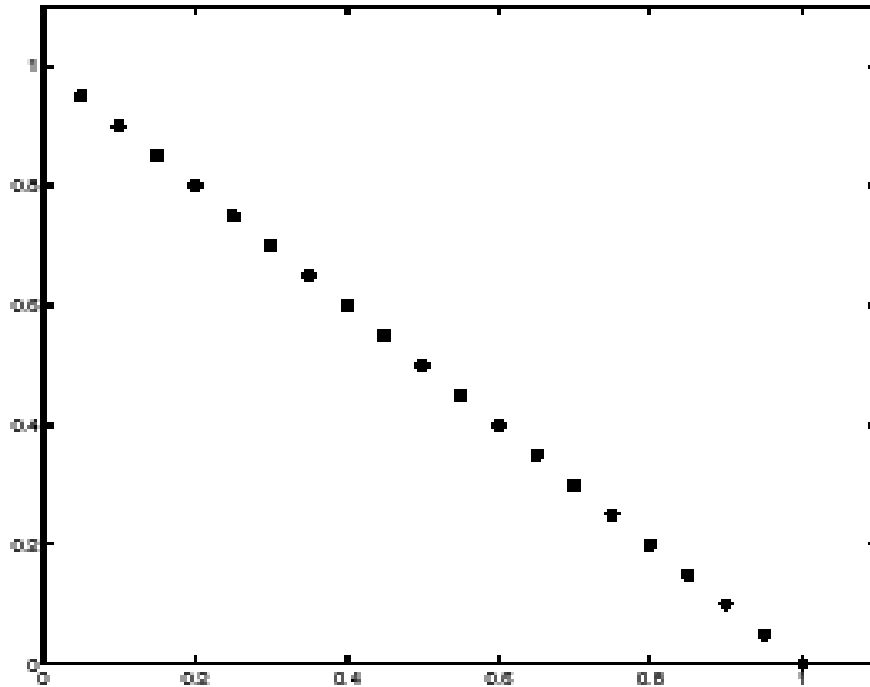
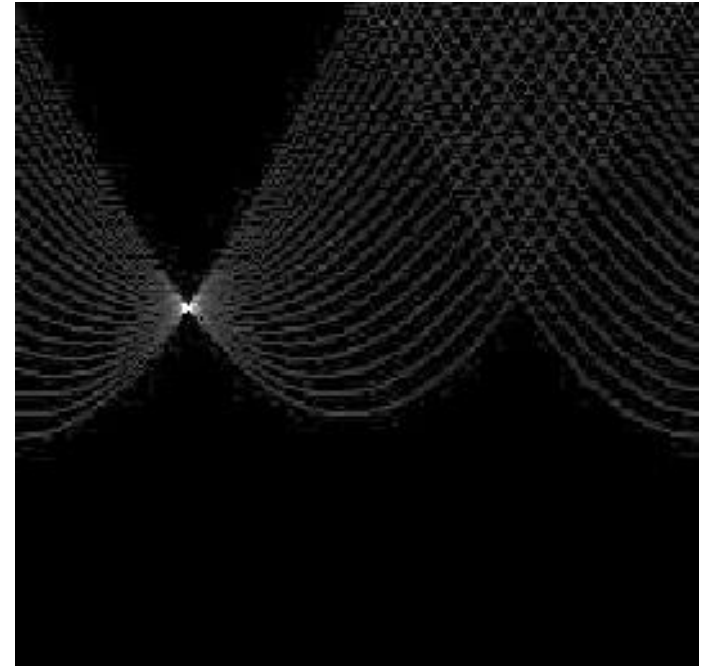


Image space



Votes

If images are noisy...

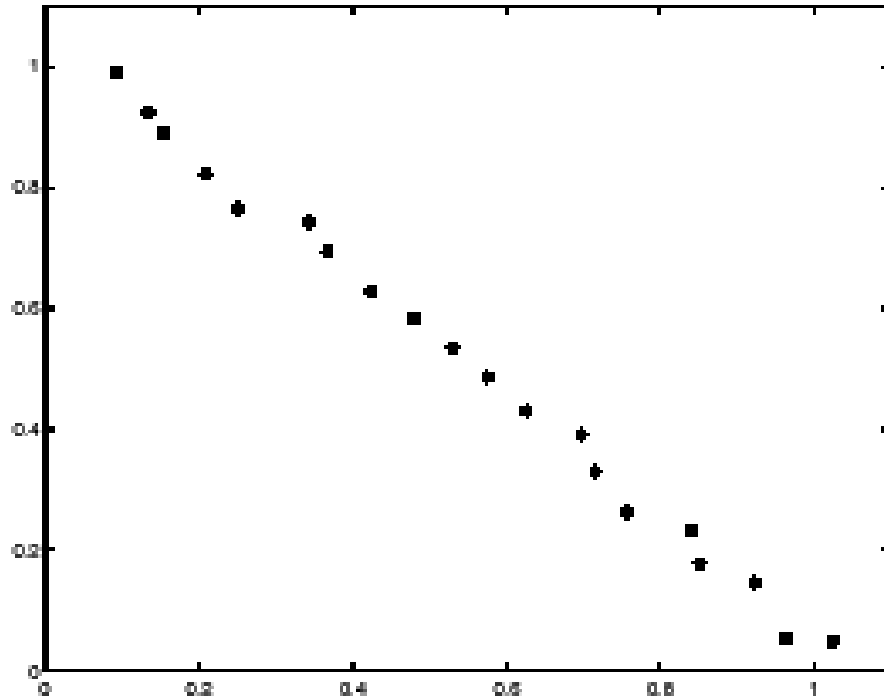
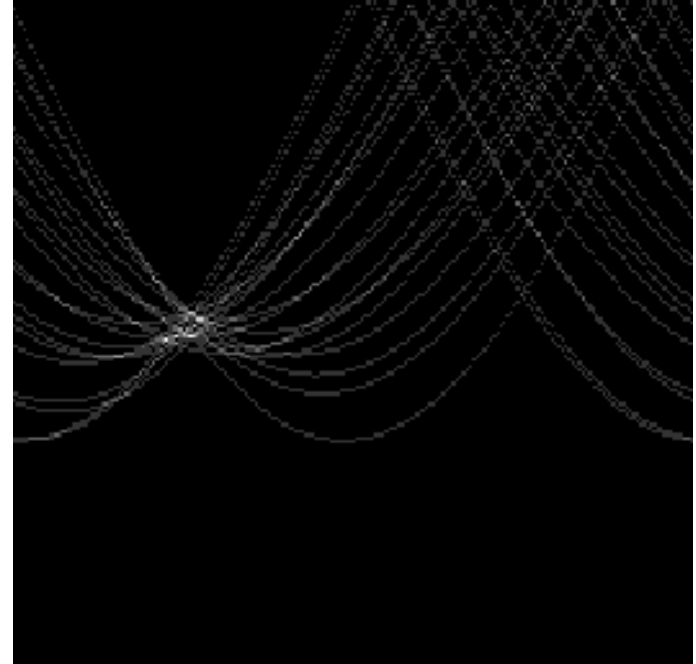


Image space



Votes

Too much noise

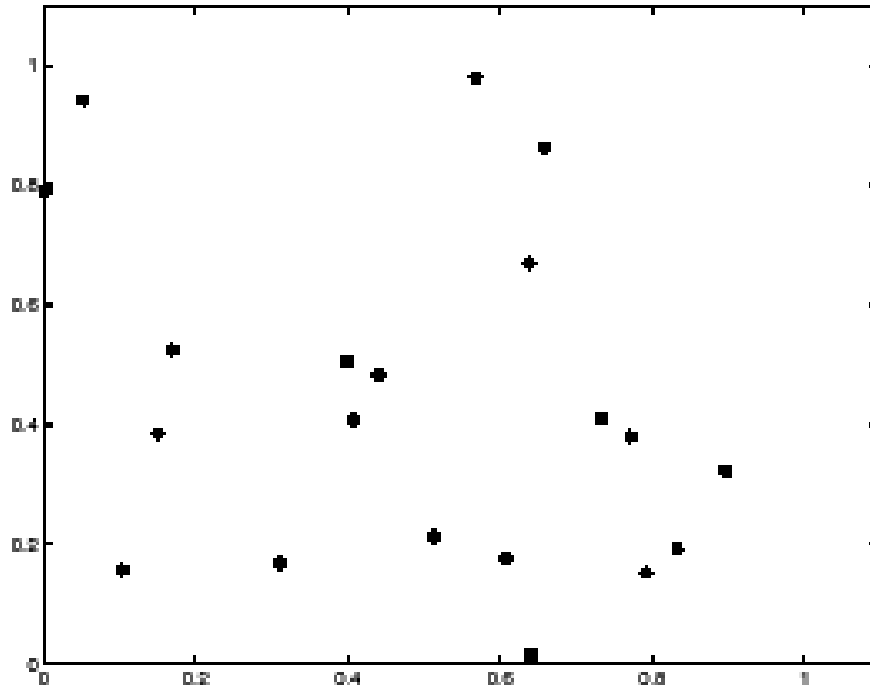
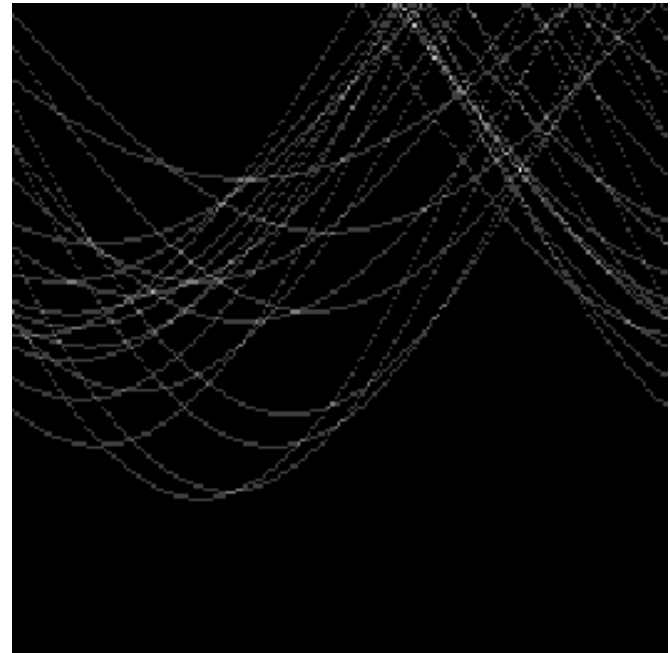


Image space

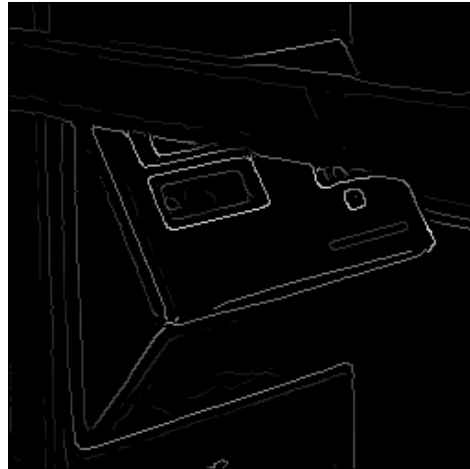


Votes

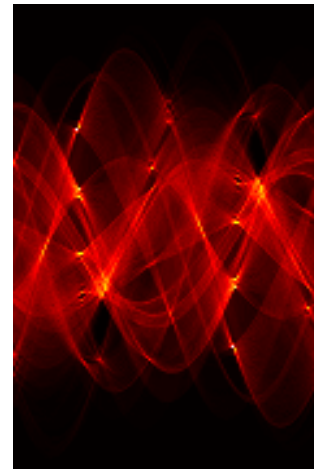
Real-world example



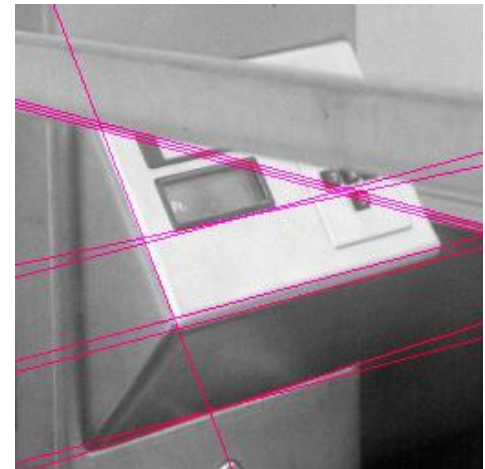
Original



Edges

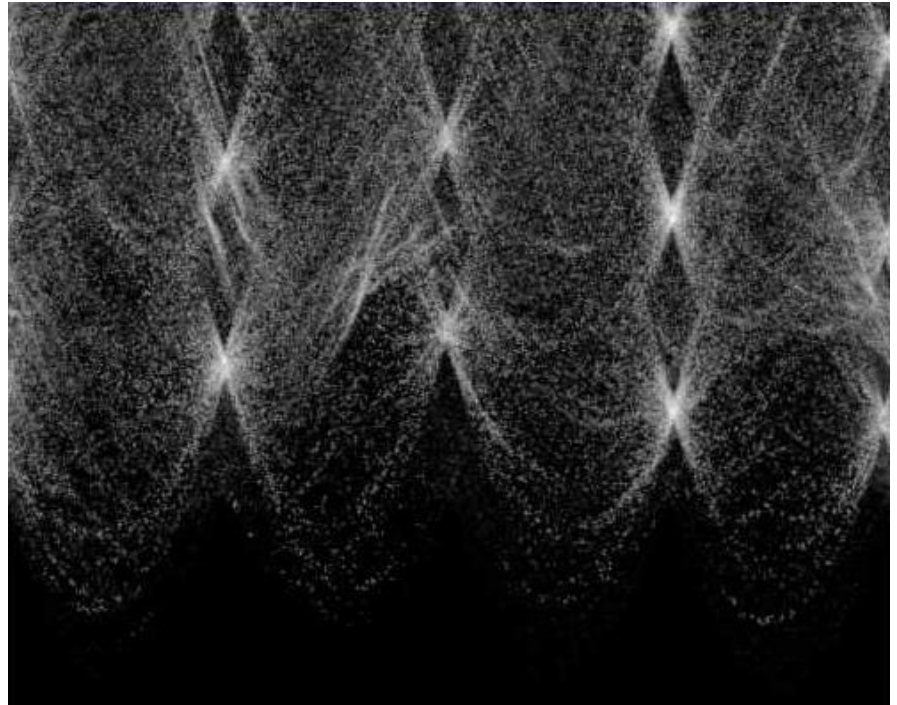


Parameter
Space



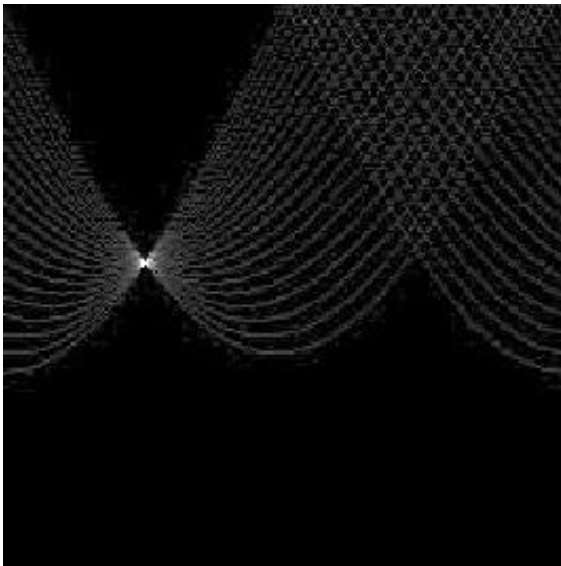
Hough Lines

More complex image

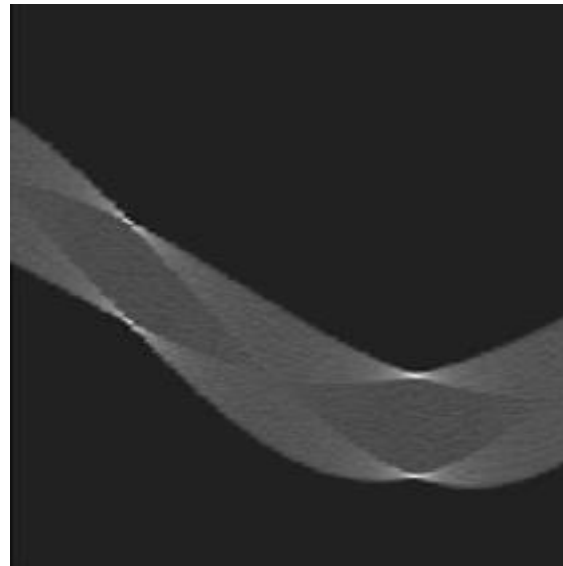


Basic Shapes

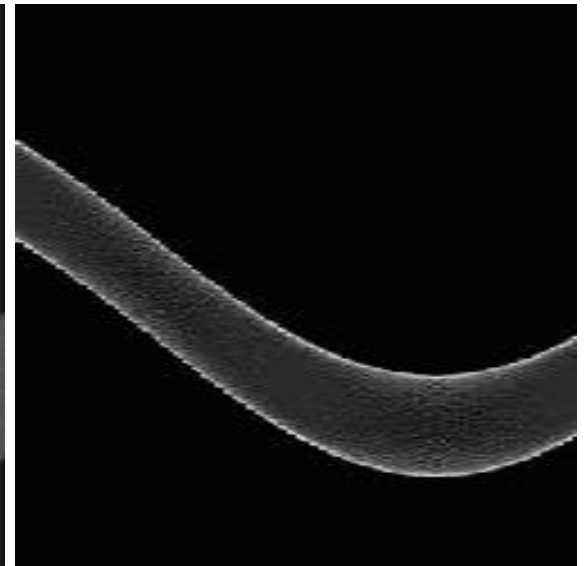
Parameter space



Line



rectangle
(parallelogram)



Circle

Can you guess the shape in image space?

Hough Circles

Let's assume **known** radius

$$(x - a)^2 + (y - b)^2 = r^2$$

Diagram illustrating the Hough Circle transform equation with annotations:

- parameters** (red arrows pointing to a and b)
- variables** (green arrows pointing to x and y)
- Fixed** (green arrow pointing to r^2)

$$(x - a)^2 + (y - b)^2 = r^2$$

Diagram illustrating the Hough Circle transform equation with annotations:

- parameters** (red arrows pointing to a and b)
- variables** (green arrows pointing to x and y)
- Fixed** (green arrow pointing to r^2)

What is the dimension of the parameter space?

Hough Circles

parameters

$$(x - a)^2 + (y - b)^2 = r^2$$

variables

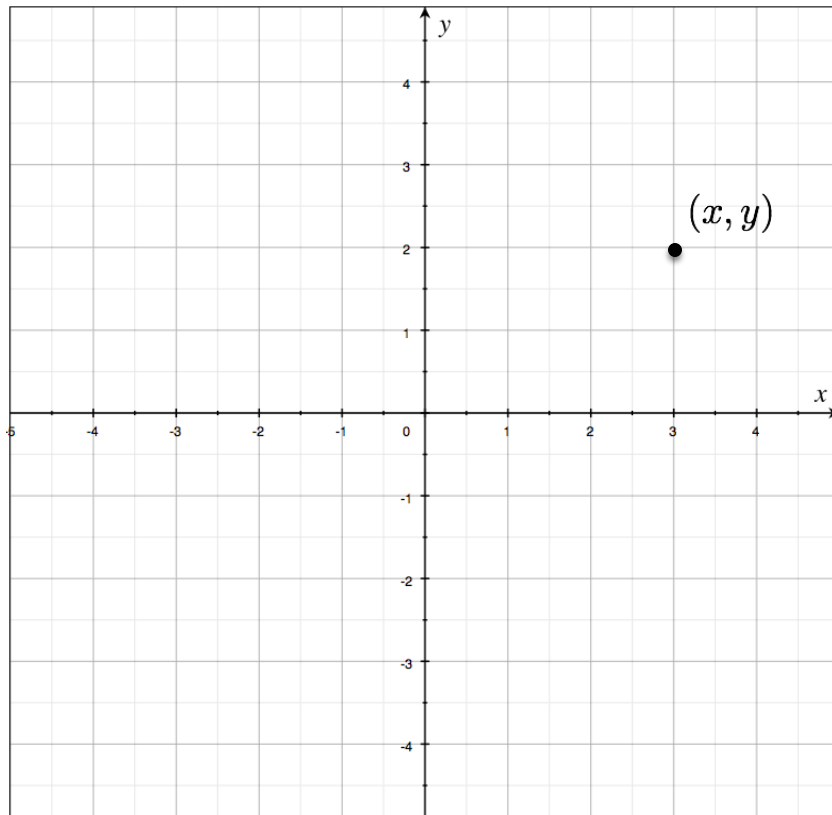
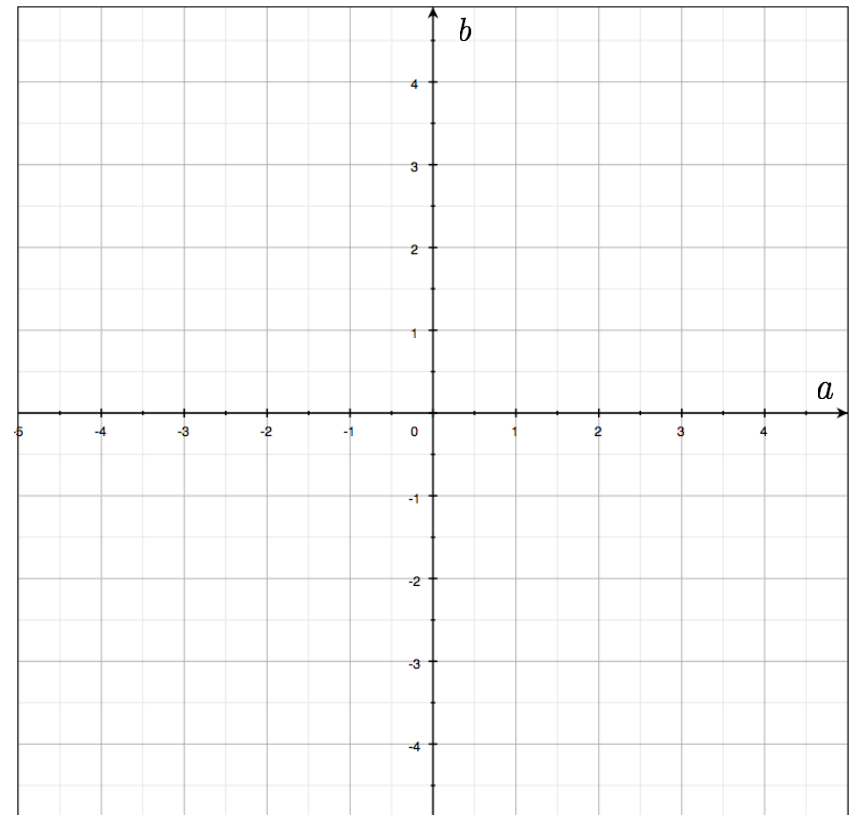


Image space

parameters

$$(x - a)^2 + (y - b)^2 = r^2$$

variables



Parameter space

What does a point in image space correspond to in parameter space?

Hough Circles

parameters

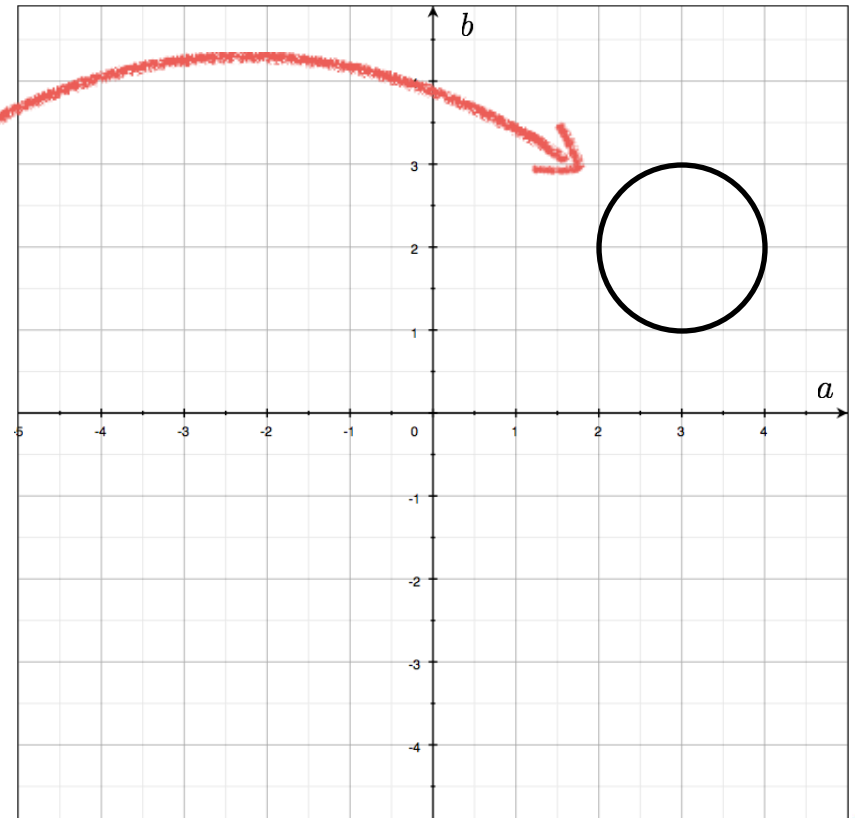
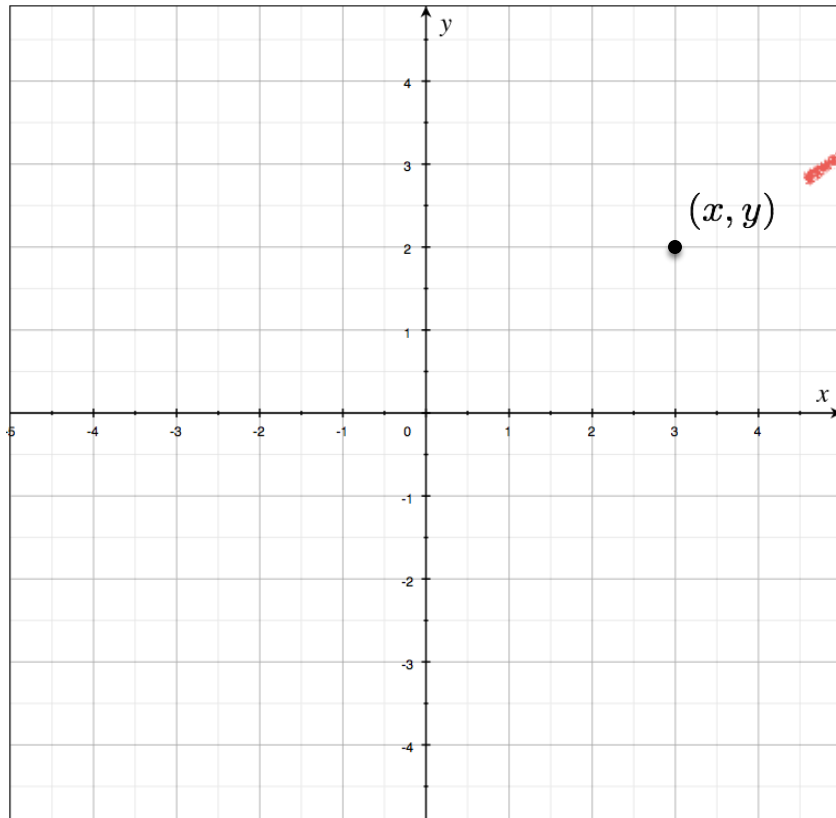
$$(x - a)^2 + (y - b)^2 = r^2$$

variables

parameters

$$(x - a)^2 + (y - b)^2 = r^2$$

variables

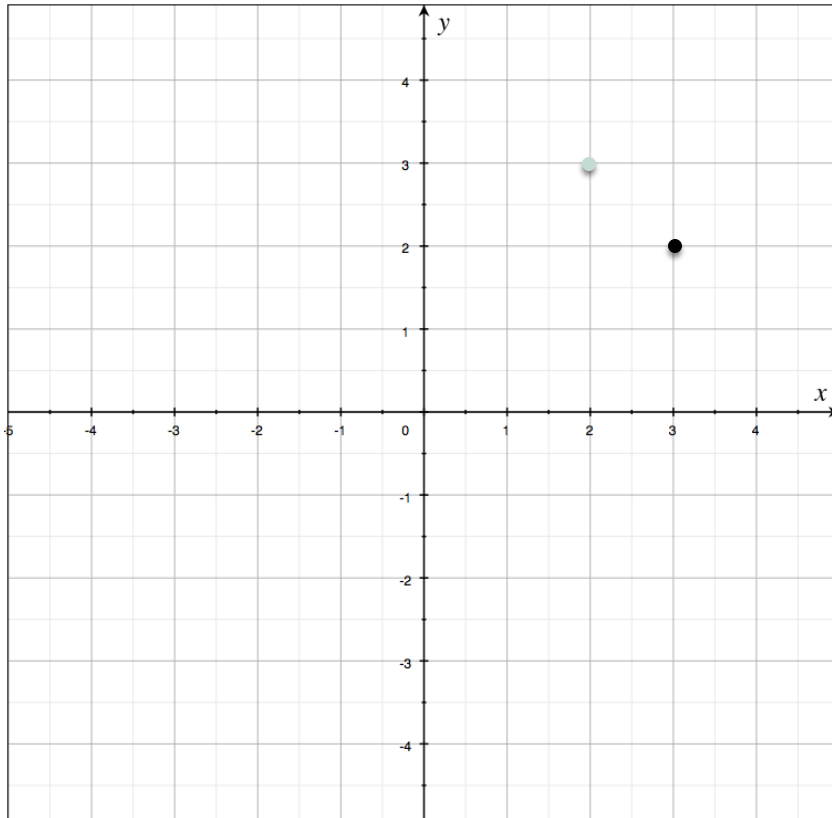


Hough Circles

parameters

$$(x - a)^2 + (y - b)^2 = r^2$$

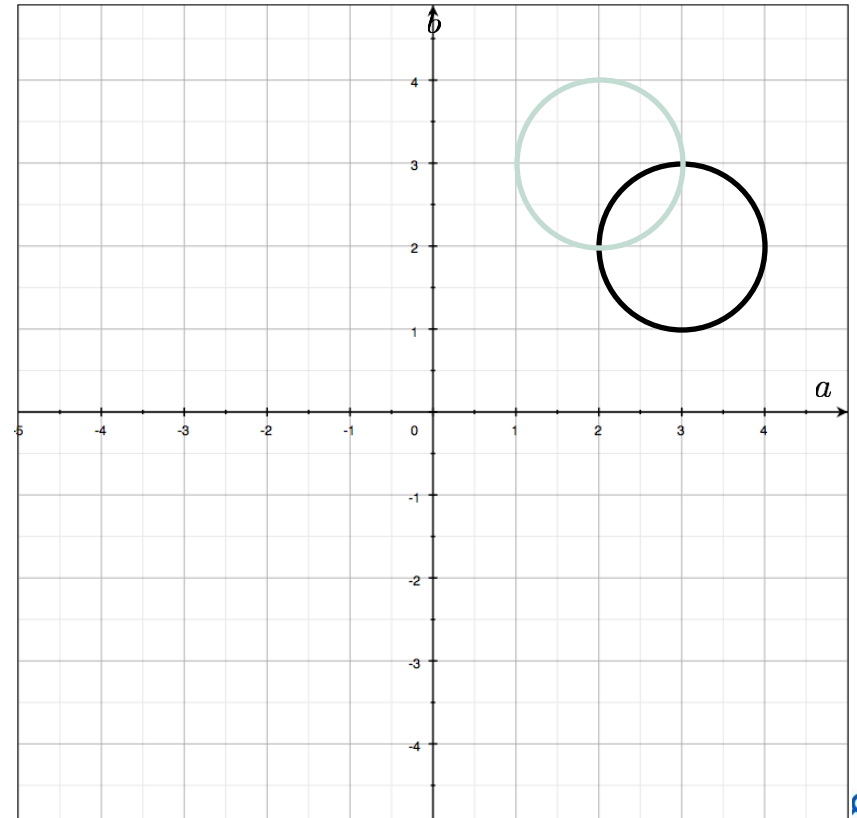
variables



parameters

$$(x - a)^2 + (y - b)^2 = r^2$$

variables

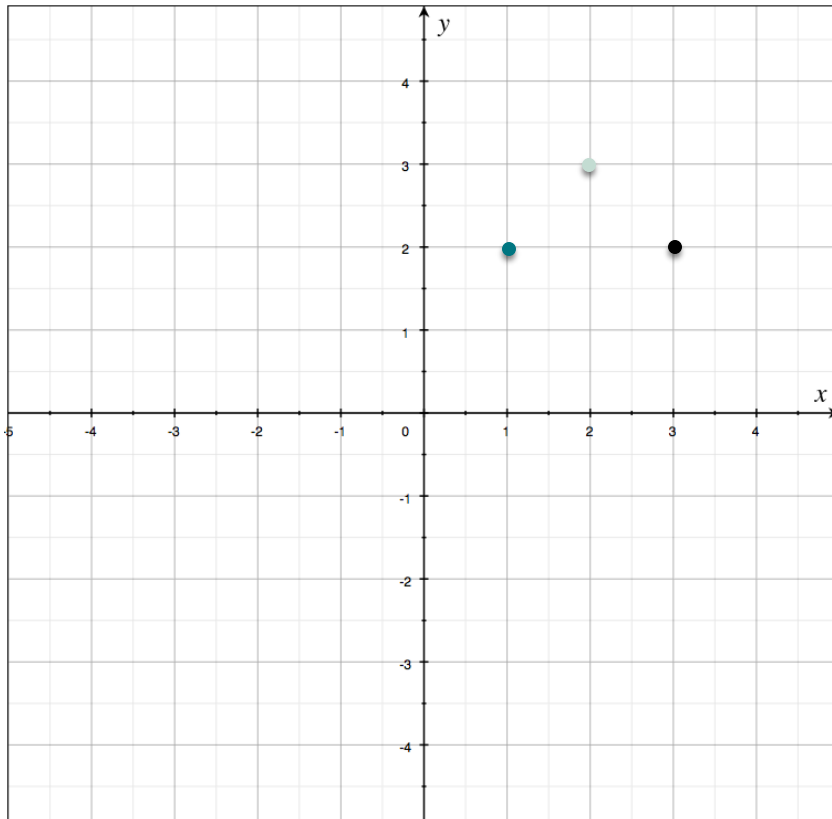


Hough Circles

parameters

$$(x - a)^2 + (y - b)^2 = r^2$$

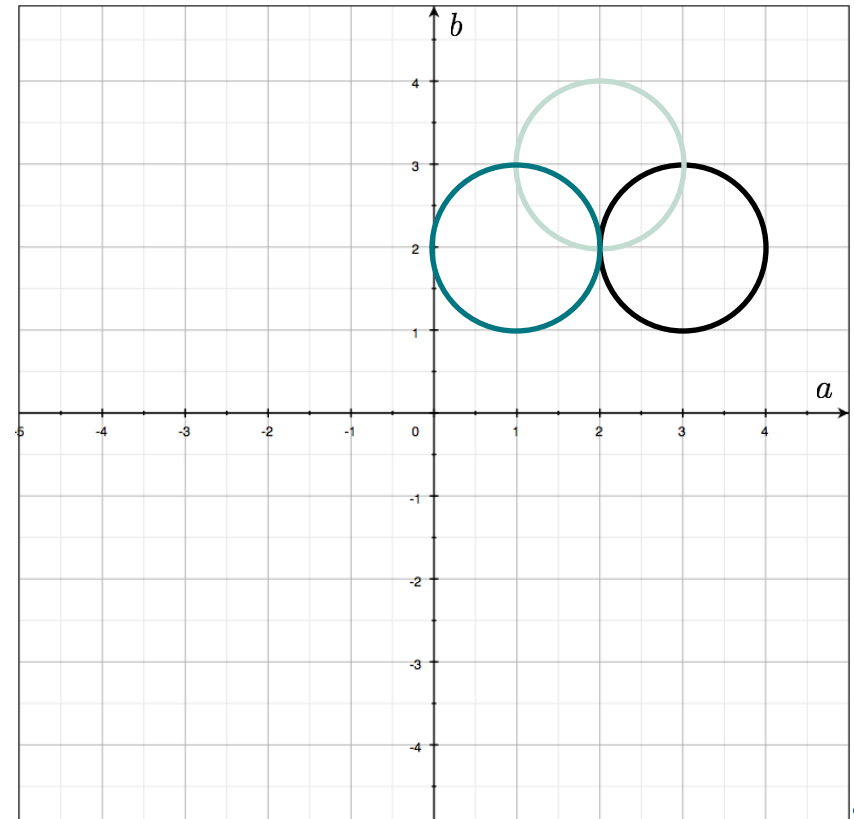
variables



parameters

$$(x - a)^2 + (y - b)^2 = r^2$$

variables

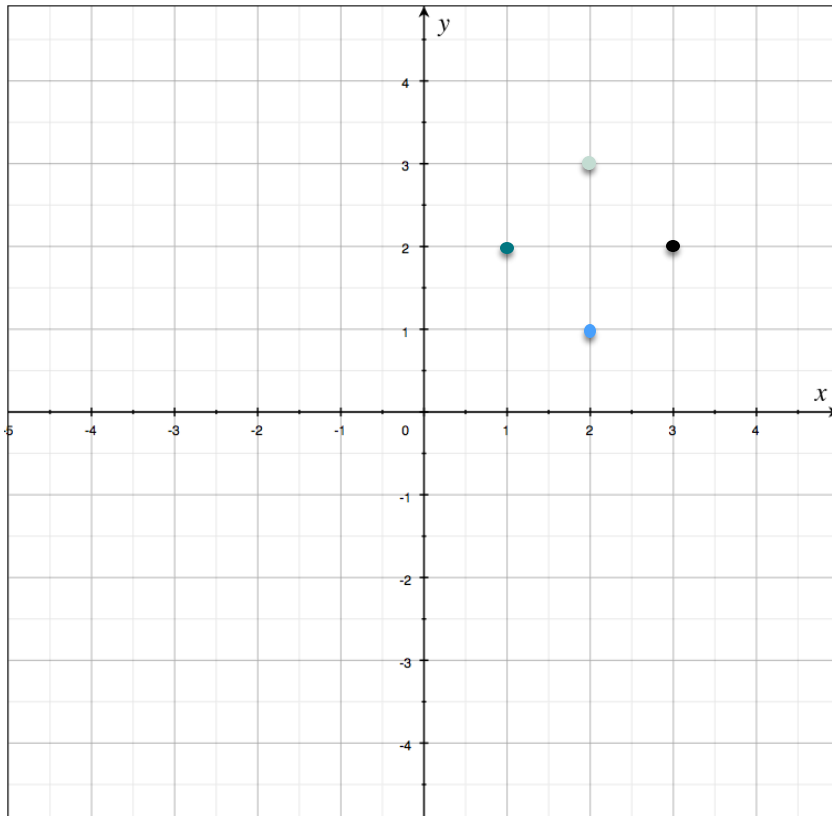


Hough Circles

parameters

$$(x - a)^2 + (y - b)^2 = r^2$$

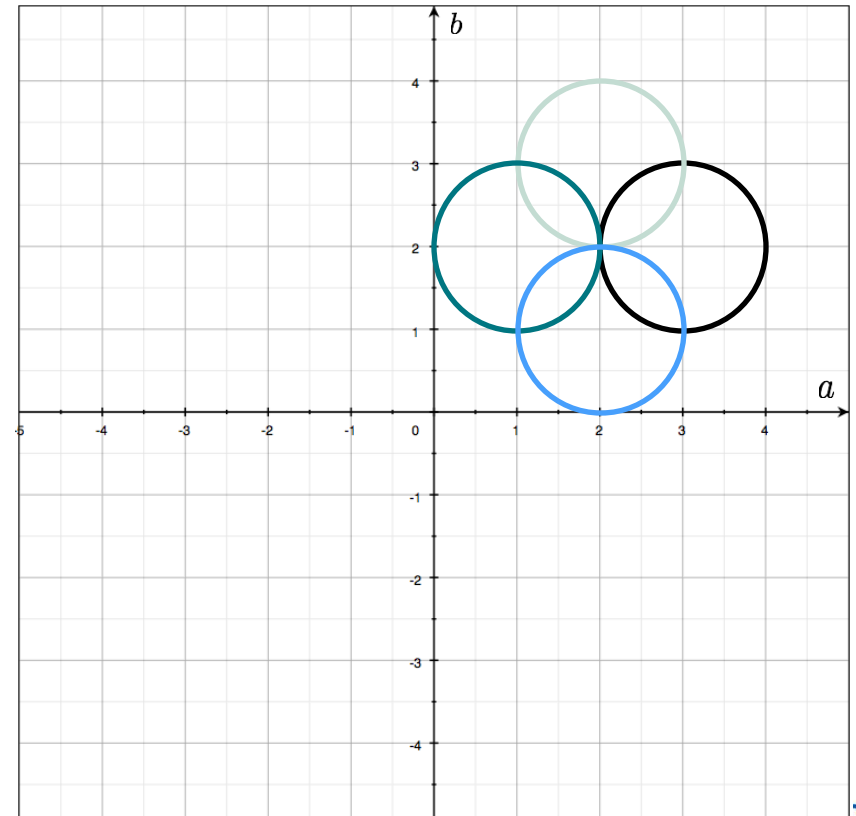
variables



parameters

$$(x - a)^2 + (y - b)^2 = r^2$$

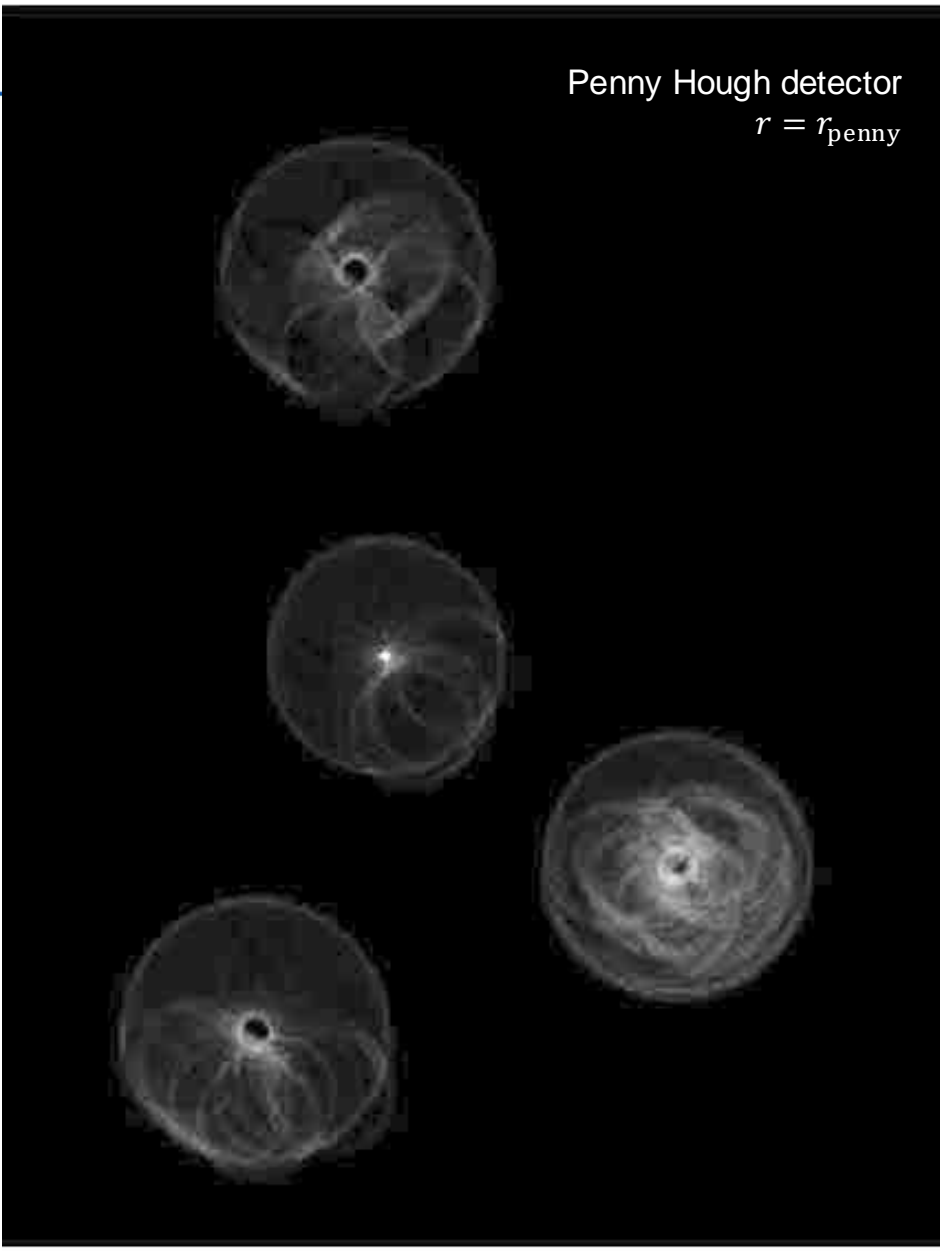
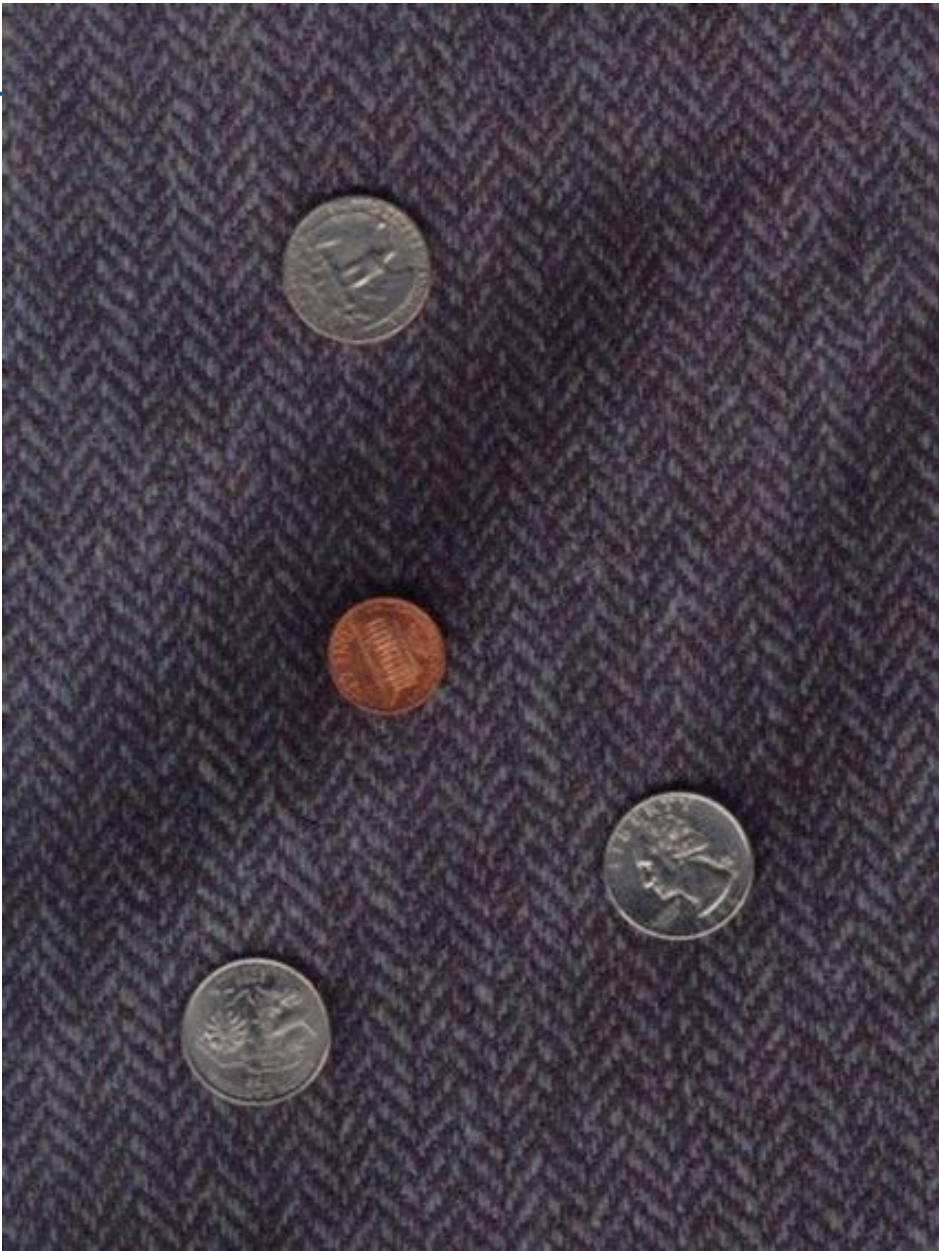
variables

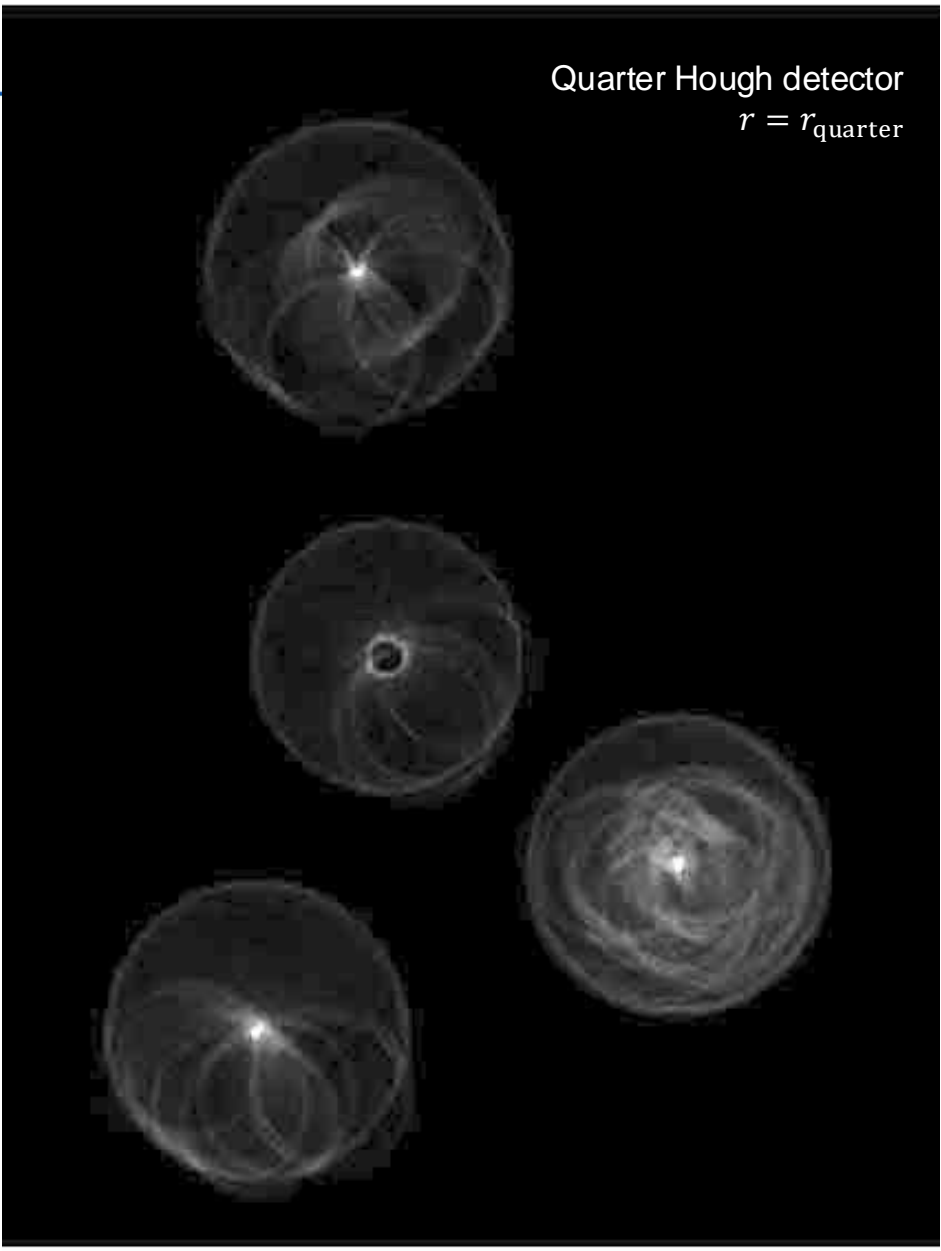
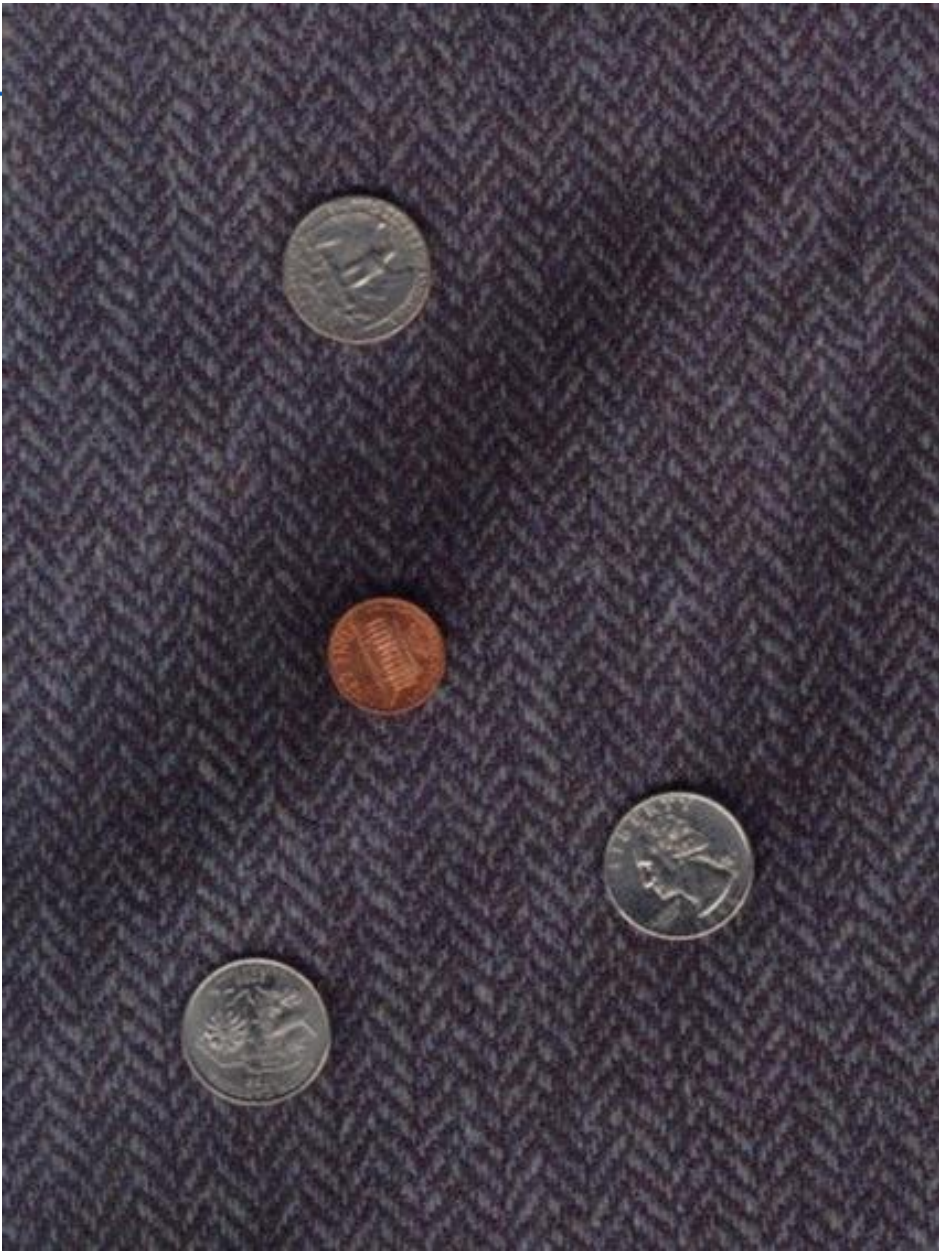


Quarter

Penny





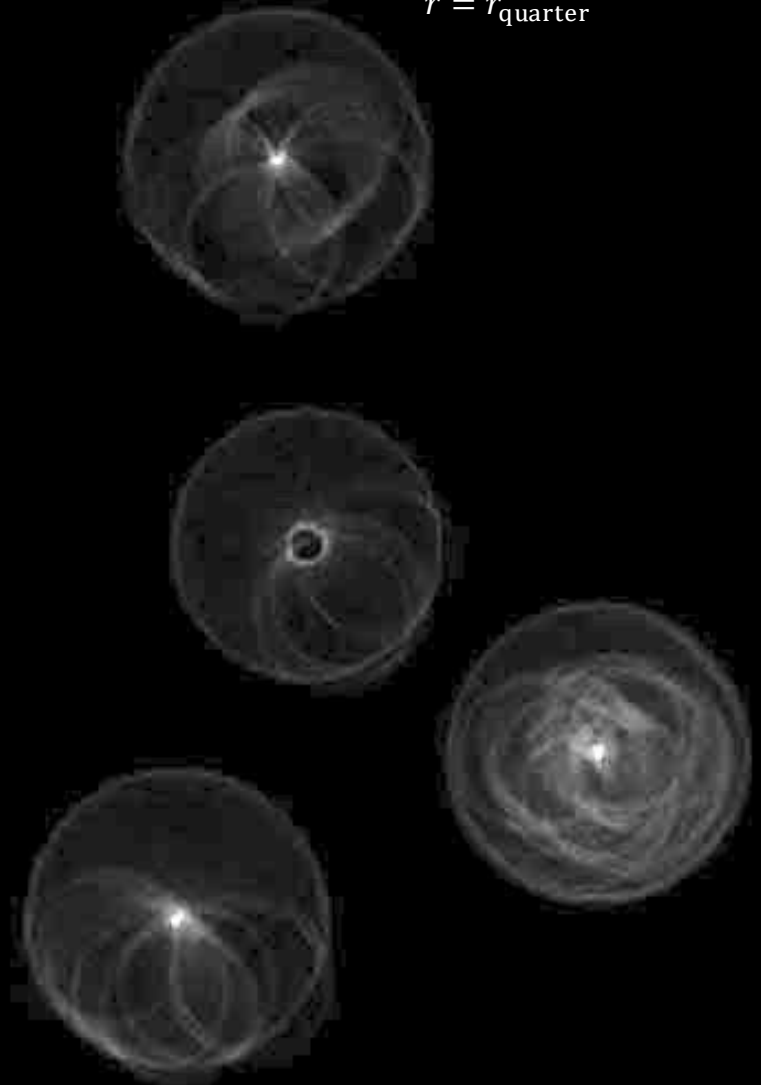
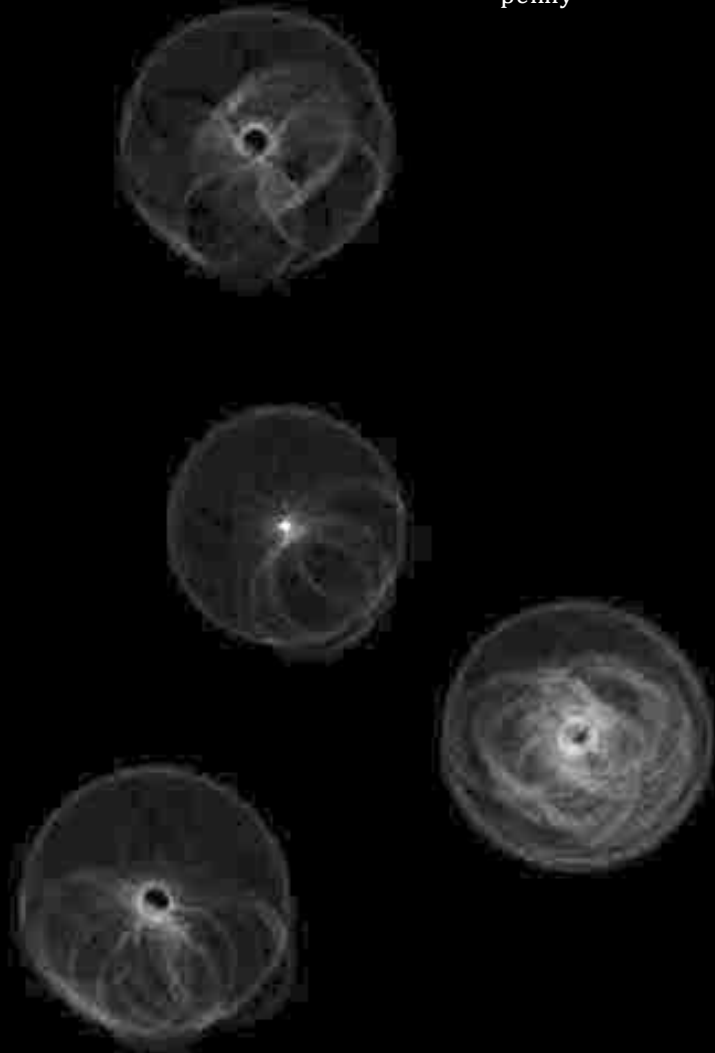


Penny Hough detector

$$r = r_{\text{penny}}$$

Quarter Hough detector

$$r = r_{\text{quarter}}$$



Penny Hough detector

$$r = r_{\text{penny}}$$



Quarter Hough detector

$$r = r_{\text{quarter}}$$



What if radius is unknown?

$$(x - a)^2 + (y - b)^2 = r^2$$

parameters

variables

$$(x - a)^2 + (y - b)^2 = r^2$$

parameters

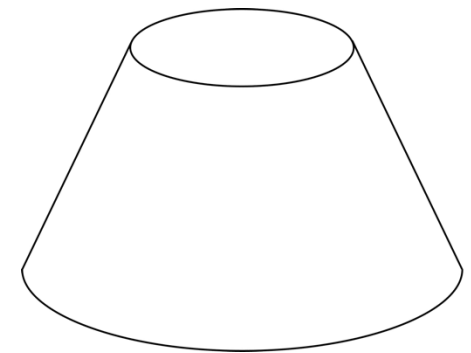
variables

If radius is unknown:

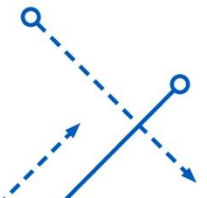
3D Hough Space!

Use Hough array $H(a, b, r)$.

Surface shape in Hough space is complicated.



Frustum of cone

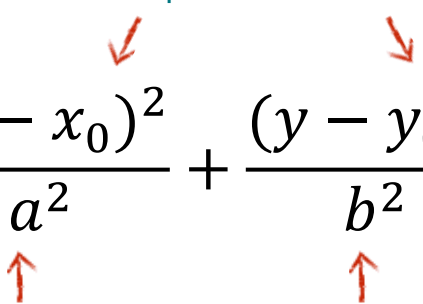


Other Shapes?

Vertical Ellipse:

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

parameters



$H(x_0, y_0, a, b)$

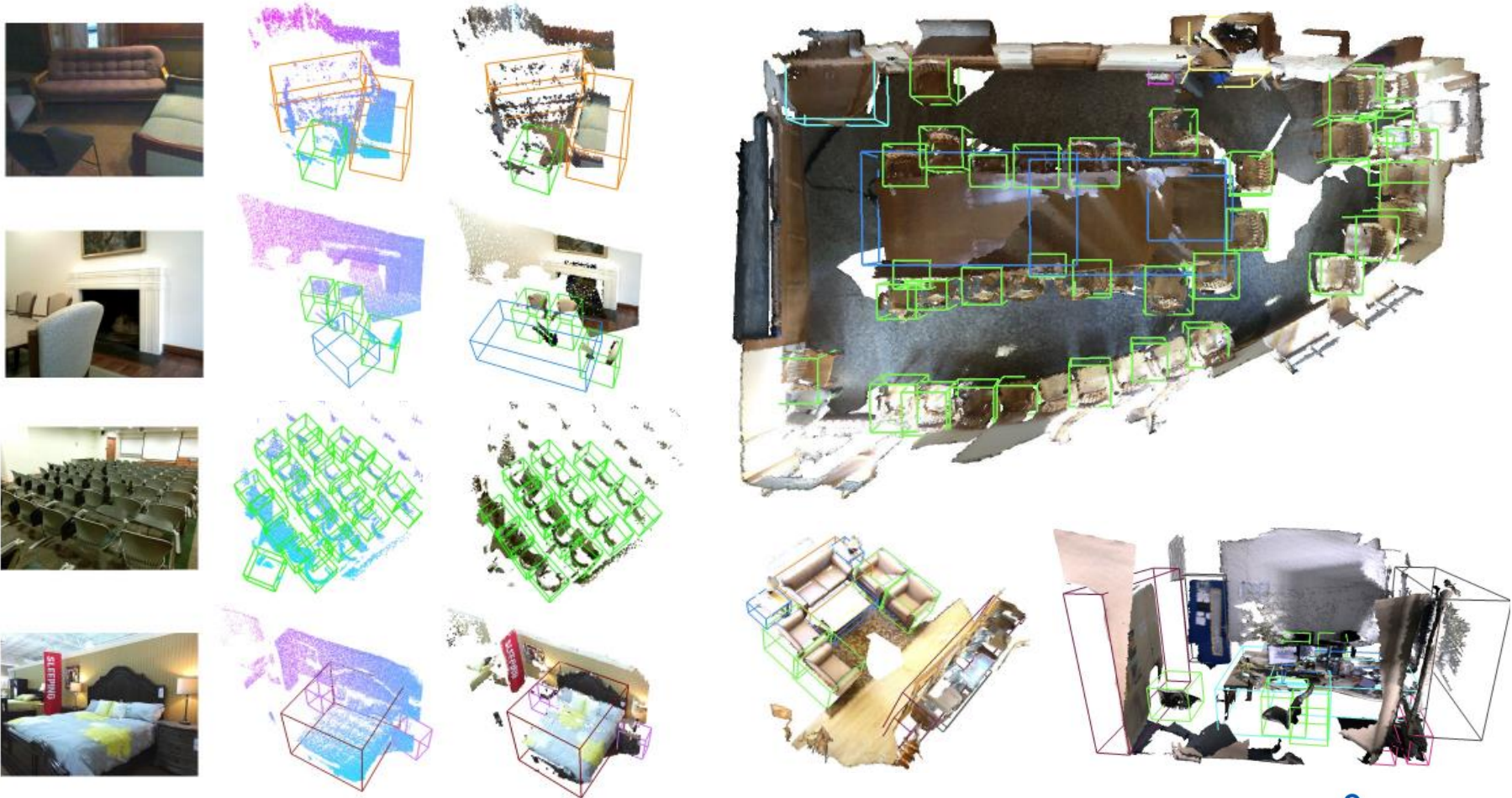
Ellipse:

$$\frac{[(x - x_0) \cos \theta + (y - y_0) \sin \theta]^2}{a^2} + \frac{[-(x - x_0) \sin \theta + (y - y_0) \cos \theta]^2}{b^2} = 1$$

$H(x_0, y_0, a, b, \theta)$

Applications of Hough Voting

Scenes Prediction Ground Truth



Conclusion

Is the following correct about Hough transform ...



- Detects multiple instances (lines/circles)?



- Robust to noise?



- Can be used for other shapes beyond lines/circles?



- Good computational complexity?



- Deals with occlusion well?