ME301 Final Project Technical Memorandum

Project: 1-DOF Base Excitation Response with Varying Driven Frequencies

To: Dr. Melody Baglione

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Abstract

The vibration response of a horizontal, one degree of freedom (1-DOF) mass-spring system is observed under base excitation. A forced vibration is induced using a crankshaft mechanism that is powered by a motor and data is collected using a piezo accelerometer. A theoretical model is made to compare experimental data with theory. The damping ratio of the system is estimated using the Half-Power and Log-Decrement methods.

Introduction

Understanding 1-DOF is critical when studying mechanical vibrations, as it is heavily applicable in the real-world. 1-DOF is commonly used in structural engineering to analyze the response of buildings, bridges, and other structures due to external loads such as earthquakes, wind, or other vibrations. It is also applicable to vehicle dynamics and suspension systems to create the smoothest and most comfortable ride possible for passengers. Overall, the significance of 1-DOF lies in creating the safest and most comfortable design for clients, whether that is building developers, automobile manufacturers, or other infrastructure clients.

Experimental

The design of the system allows for masses to be added and removed without losing the integrity of the model, however, 1-DOF was the primary focus of the study.

The apparatus consists of two sliders that act as masses along a horizontal rod connected by 2 springs. Four 15 N/m springs are used in parallel, with two springs connected to each side of a slider. A motor is mounted to a sheet of metal with a crankshaft mechanism that attaches the motor to the masses. Ball bearings are used to allow for the rotational motion of the motor to translate to a linear motion of the masses. The masses slide on a rod that is mounted to two wooden planks to improve stability. The first mass has an accelerometer attached to it to measure its acceleration, which is then converted to displacement. The second mass is fixed and is immobile, allowing for the system to function as 1-DOF. Figure 1 shows the schematic of the system, while Figure 2 shows the fully assembled rig.

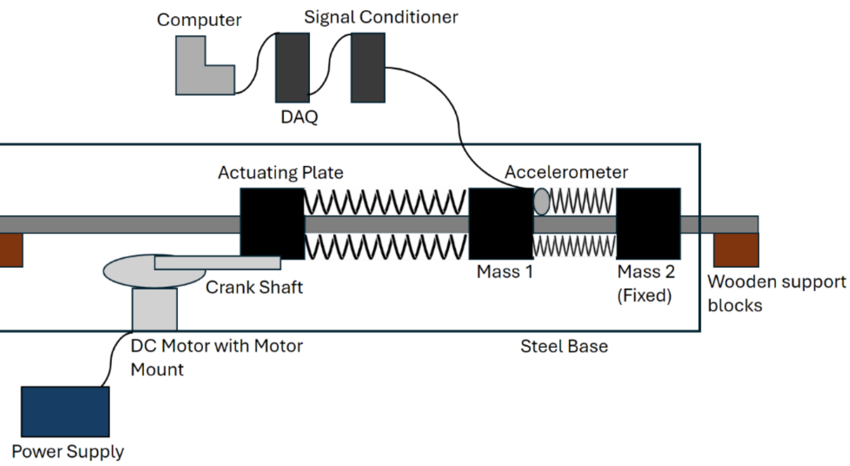


Figure 1: Schematic of Base Excitation System

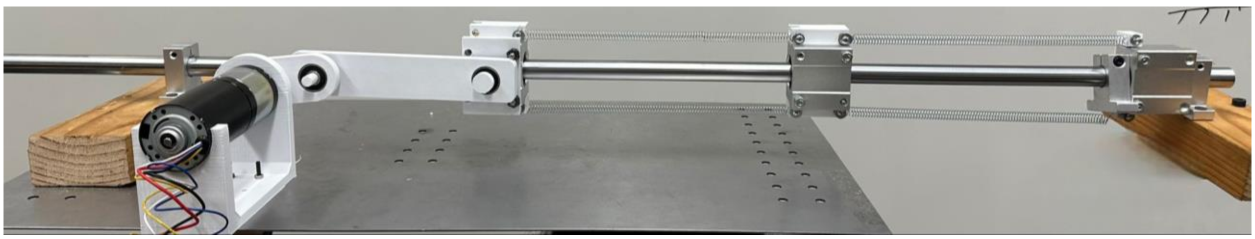


Figure 2: Image of Base Excitation System

The data collection for this system relied on a Piezo Electric Accelerometers, Model 352C65, which has a broadband resolution = 0.0015 m/s^2. When connected to LabVIEW software, the acceleration of the mass was recorded. The accelerometer was mounted to the slider through a 3d-printed slot (Figure 3), which allowed the accelerometer to be secure and aligned with the motion of the mass. Wires connected the accelerometer to a signal conditioner, National Instruments USB-4431, which has an uncertainty of 0.15%, and further to a computer with Data Acquisition System (DAQ), LabVIEW, which has an acceleration uncertainty of 0.05 g's, or 0.4905 m/s^2 and the period has an uncertainty of 0.02s. On LabVIEW, the DAQ Assistant was set to Acquisition Mode with 10000 samples to read at a rate of 1000 Hz.

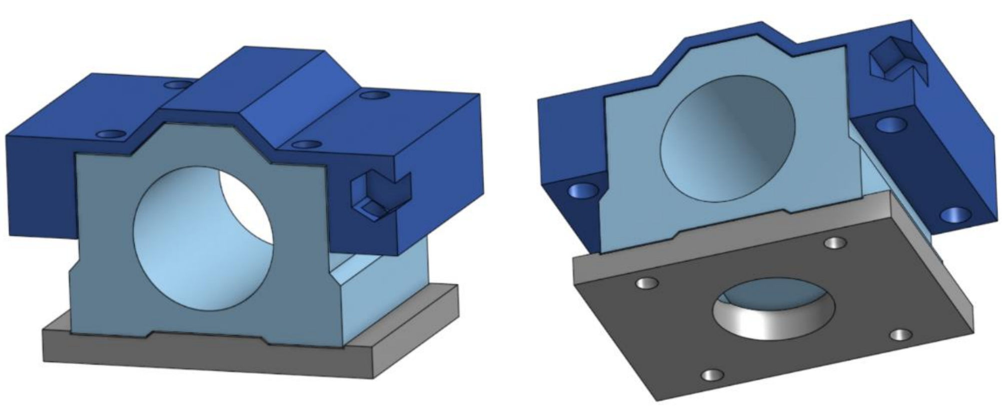


Figure 3: Mass Mount Design Including Accelerometer Mount

Results

The following data is extracted from LabVIEW, which was acquired from the accelerometer. The mass was excited at varying frequencies by changing the input voltage to the motor. Appendix 1 shows the acceleration response of the mass as the input voltage to the motor varied from 14V to 24V. The amplitude unit is in terms of g's (9.81 m/s^2) and time is in seconds.
Additionally, data was collected of the system with an initial displacement to see the free response (Figure 4). The amplitude is in g's and time is in seconds. This data was later used to determine the damping ratio of the system using log decrement.

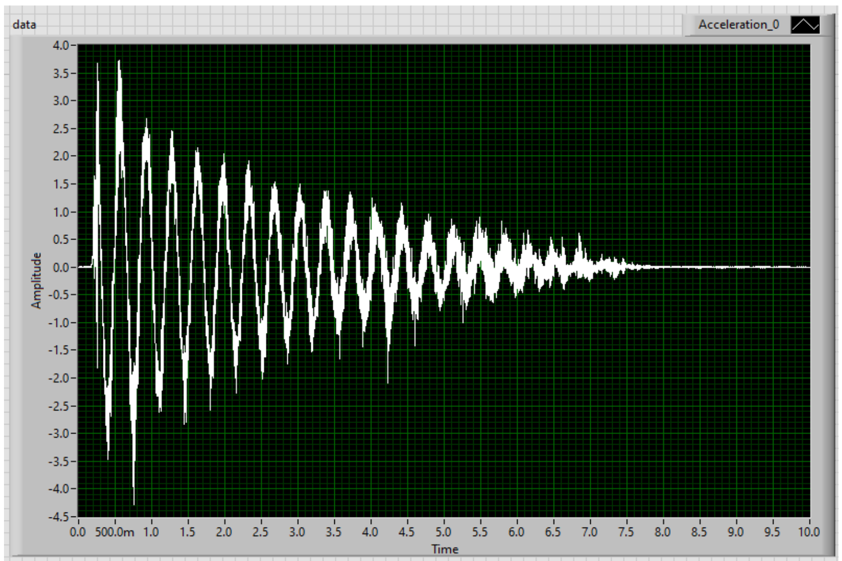


Figure 4: Free Response of 1-DOF mass-spring system

Next, a spectrum graph was created to see the Fast Fourier transform (FFT) graph of the system (Figure 5). This was used to determine the damping ratio using the Half-Power Method.

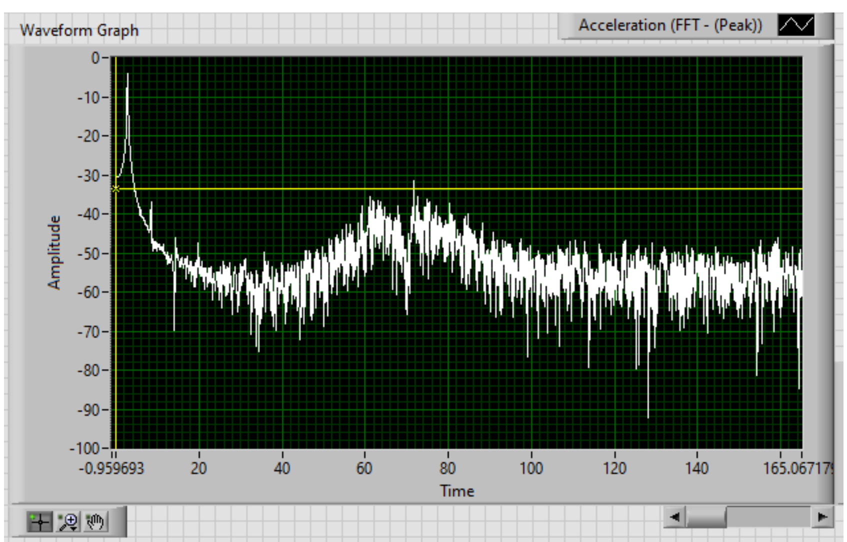


Figure 5: FFT of 1-DOF mass-spring system

Analysis

Theoretical Model

The displacement of a 1-DOF mass-spring system under base excitation is as follows:

$$X = A\sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

A is the amplitude of the base excitation, zeta (ζ) is the damping coefficient, and r is the frequency ratio of the driven frequency and natural frequency. A variety of damping coefficients were tested to account for real-life constraints and because the true damping ratio of the system is unknown.

Using python, a plot of the theoretical model was formulated to see the effect of the frequency ratio on the amplitude. Based on this model, attenuation occurs at a frequency ratio of approximately $\sqrt{2}$. Given a mass of 0.176 kg and spring constant of 60 N/m, the natural frequency of the system is $\omega_n = \sqrt{\frac{k}{m}} = 18.4\,rad/s$. The minimum driven frequency required to achieve attenuation is 26.1 rad/s, or 250 RPM. As seen in Figure 6, the displacement of the system is plotted with various damping ratios.

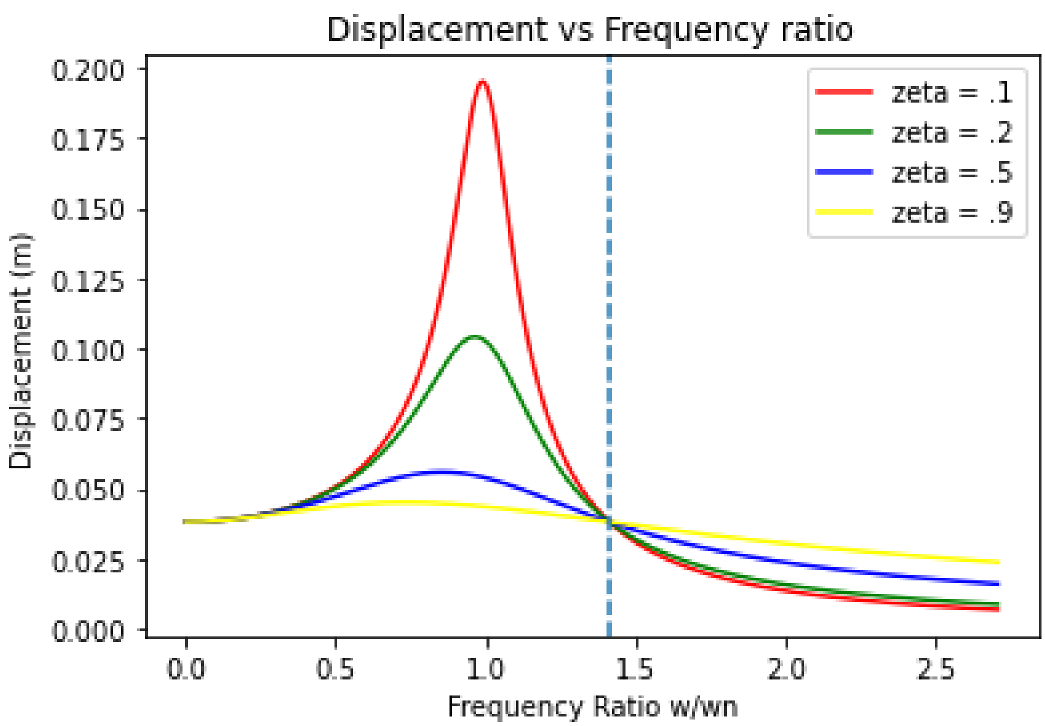


Figure 6: Theoretical Plot of Displacement at various zeta values

To find the displacement of the mass from the data acquired from the accelerometer, it is assumed that at steady state, the acceleration of the mass has a sinusoidal solution of

$a = Asin(\omega t + \phi)$. After double integrating, the displacement was found to have a solution of $x = -\frac{A}{\omega^2} \sin(\omega t + \phi)$, which a magnitude of $|x| = \frac{A}{\omega^2}$.

The following equation was used to solve the error associated with the displacement of the mass:

$$\text{Error: } \Delta x = \sqrt{\left(\frac{\partial x}{\partial A_a}\Delta A_a\right)^2 + \left(\frac{\partial x}{\partial T}\Delta T\right)^2} = \sqrt{\left(\frac{T^2}{4\pi^2}\Delta A_a\right)^2 + \left(\frac{A_a T}{2\pi^2}\Delta T\right)^2}$$

Here, $\Delta A_a = 0.003 * A_a + 0.4905$ and $\Delta T = 0.02$

Damping Ratio

After the initial data was collected, the next step was to solve for the damping ratio of the system. This was done by setting up the system for initial condition testing, the mass was given an initial condition and allowed to propagate naturally. For this test, the system motion was started prior to running the data collection software, which collected acceleration data for 10 seconds. After collecting this data, log decrement was used to solve for the damping ratio.

$$\delta = \frac{1}{N}\ln\left(\frac{X_1}{X_{N+1}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

When using this equation, it was found that the damping ratio was small, under 0.1. Because of this, the damping ratio was further verified using the alternative equation for small ratios.

$$\zeta = \frac{\delta}{2\pi} \cong \frac{1}{2\pi N}\ln\left(\frac{X_1}{X_{N+1}}\right)$$

Half-Power Method

In this analysis, the half-power method was implemented using LabVIEW to estimate the damping ratio of the Modeler system based on peak measurements from spectrum analysis. This technique involves determining the bandwidth at which the power of the frequency response is 3dB below its peak value. The numerator is the bandwidth frequencies over which the displacement response is greater than 3dB below the peak amplitude. The denominator is double the frequency of the peak.

$$\zeta = \frac{\omega_2 - \omega_1}{2 * \omega_{peak}}$$

By measuring the system's response in LabVIEW, the peaks of the frequency spectrum were identified, and the bandwidth was calculated where the power fell to 3dB below its peak value. The results from the half-power method demonstrated a calculated damping ratio of 0.032, contrasting with the damping ratio obtained via the log decrement method, which was 0.019.

Experimental Data Analysis

The Log Decrement Method gave a damping ratio of 0.019, while the Half Power Method gave a damping ratio of 0.032. As seen in Figures 7 and 8, the dotted lines represent the theoretical model for the displacement of the mass as the frequency ratio increases. The x points represent the experimental data at various motor frequencies. It can be assumed that there is a gap between resonance and attenuation due to limitations in the system, where the mass is constrained by the walls. For this reason, the amplitude of the mass can never be as high as theoretically predicted. However, both of these methods yield small damping ratios.

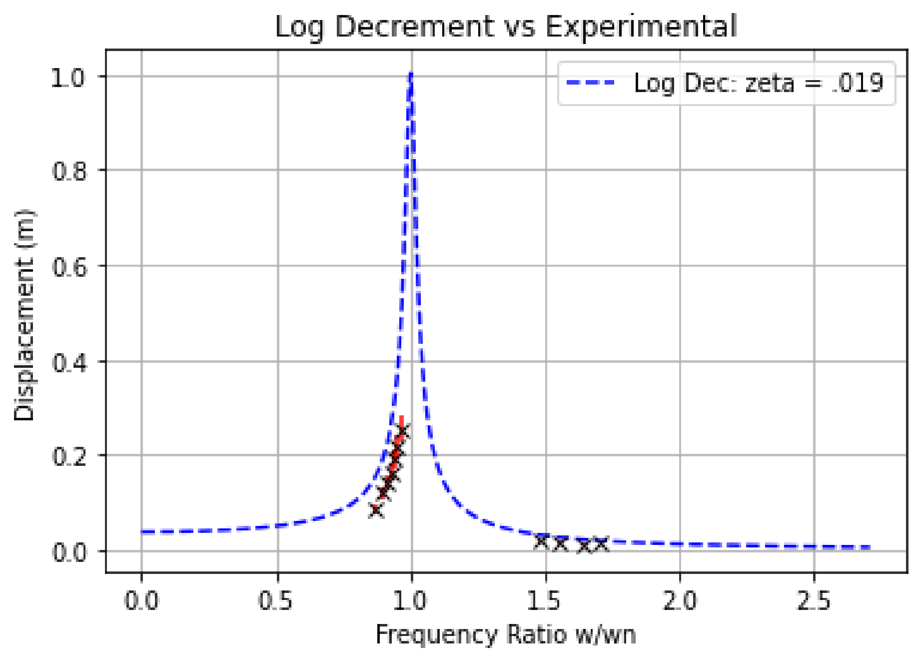


Figure 7: Experimental vs. Theoretical Results using Log Decremement Method

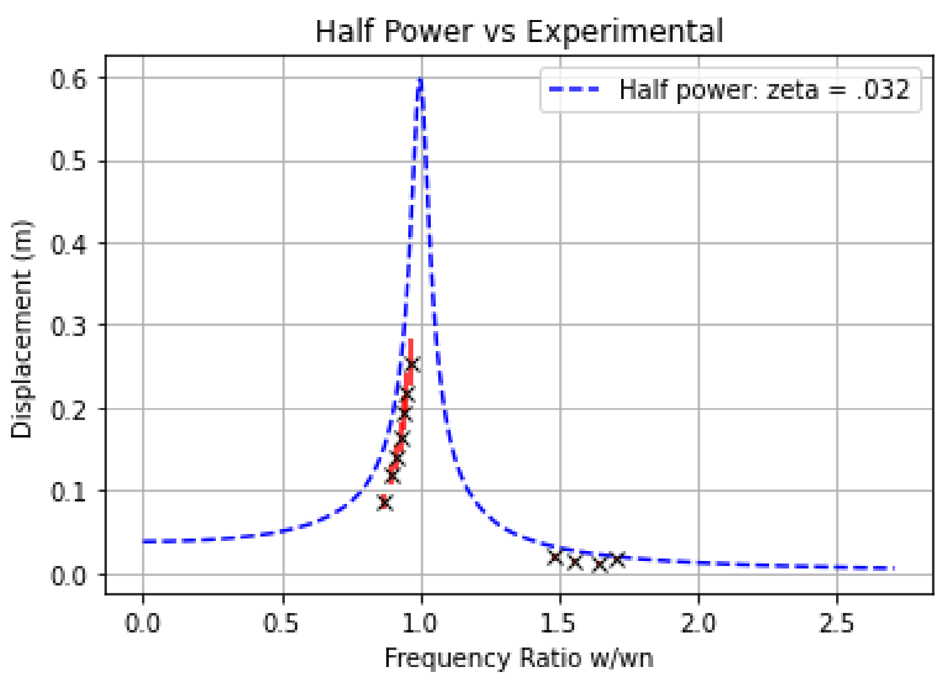


Figure 8: Experimental vs. Theoretical Results using Half Power Method

Future Work

Future work regarding the system itself includes strengthening the pins within the bearings that connect the crankshaft to the actuating plate. This can be done using a material other than PLA, specifically a metal such as steel. A stronger motor has been attached but has not been used for data collection. The stronger motor reduces stall and allows for a larger range of actuating frequencies to be tested. A sturdier and denser base can be used to allow the system to function more smoothly. With these adjustments, cleaner data can be collected.

Additionally, data can be collected from the actuating plate and Mass 1 simultaneously. The second mass can be unfixed, and an accelerometer additionally mounted to it as well. This would allow for 2-DOF motion to also be analyzed while using the same system. Different linear arm lengths within the crankshaft and spring stiffnesses can be used in conjunction with one another to test the response with different stiffnesses, while also preventing the displacement from being larger than the length of the rail. Lastly, data can be plotted in the frequency domain to analyze the modes of the system.

Acknowledgements

Thank you to Kevin Luo for his support throughout this project as the ME301 Consultant.

Thank you to Professors Kamau Wright and David Wootton for advising this project from an ME360 perspective through each design iteration.

Lastly, thank you to Professor Melody Baglione for advising this project from an ME301 perspective and supporting the group with resources such as the Vibrations Lab, as well as providing guidance regarding the design of the system and its applications at The Cooper Union.

Division of Work and Skills Gained

Akil—Half-Power Method, Electronics, and Data Collection. Analysis section.

Skills: Data collection, Team management, Problem solving. Theoretical model creation.

Francisco—Log Decrement Method, Amplitude Analysis, and Data Collection. Experimental and Analysis sections.

Skills: Log Decrement, Data collection with LabVIEW.

Angelica—Log Decrement Method, Amplitude Analysis, Design and Construction of System, Data Collection. Abstract, Introduction, Experimental, Analysis, and Future Work sections.

Skills: Data collection and using LabVIEW, using Logarithmic Decrement to solve for the damping coefficient, converting acceleration data to displacement values.

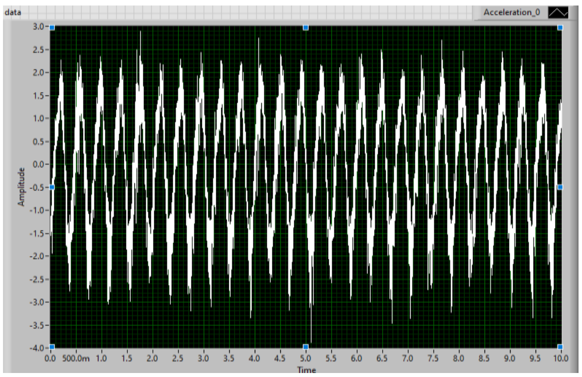
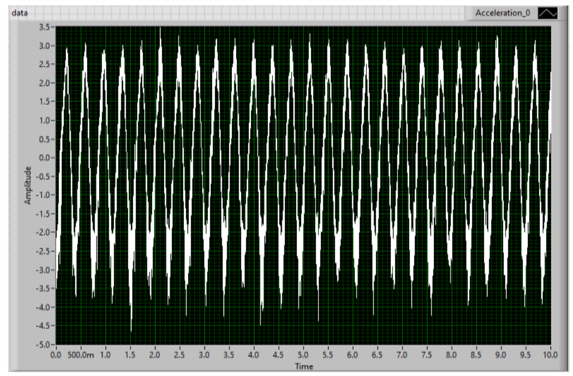
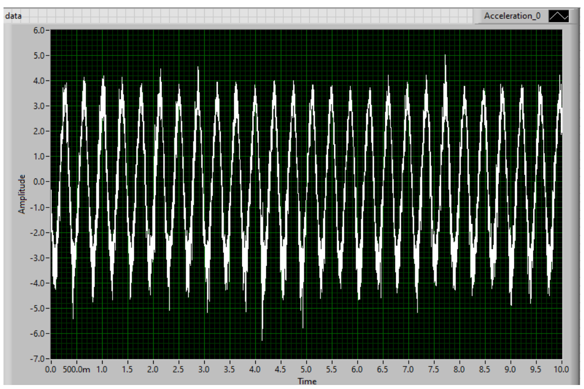
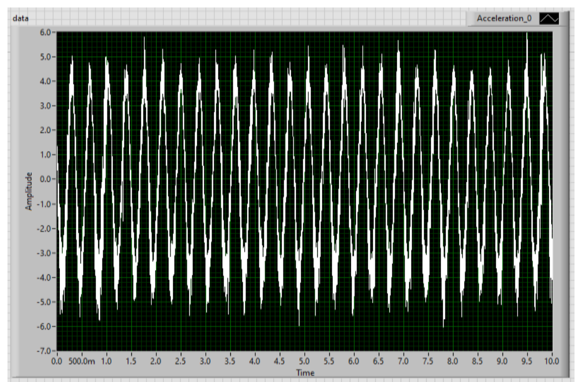
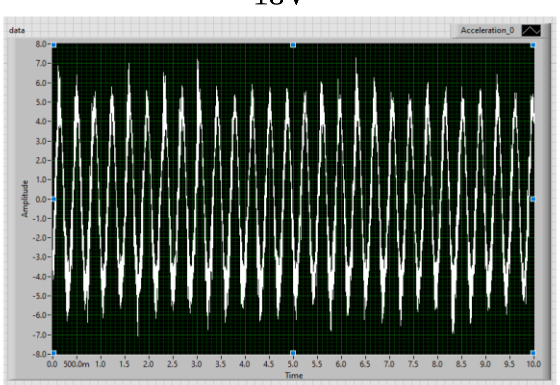
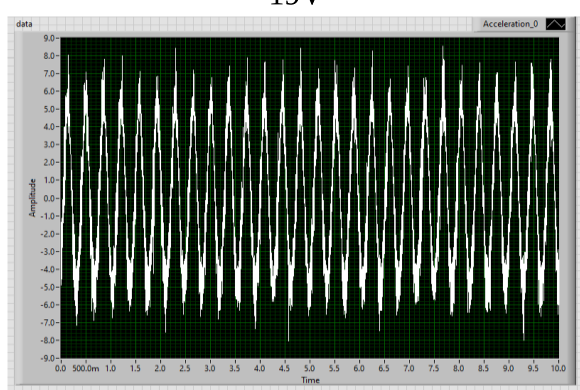
Saira—Theoretical Model, Amplitude Analysis, Design and Construction of System, Data Collection. Experimental, Results, and Analysis sections.

Skills: Using LabVIEW and setting it up to take various kinds of measurements.

References

"Theory of Vibrations with Applications," Thomson and Dahleh, Prentice Hall, Fifth Edition, 1998.

Appendix

14V	15V
	
16V	17V
	
18V	19V
	

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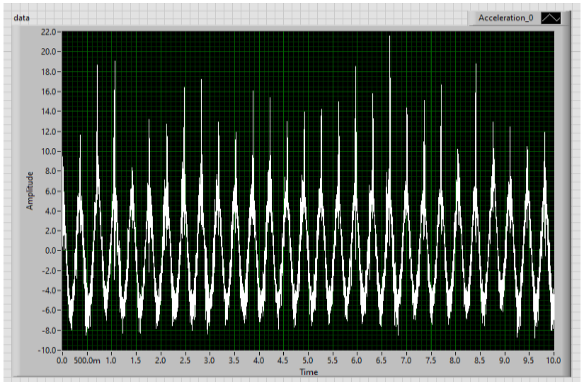
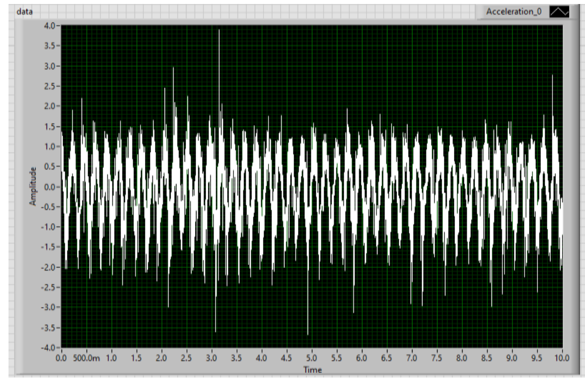
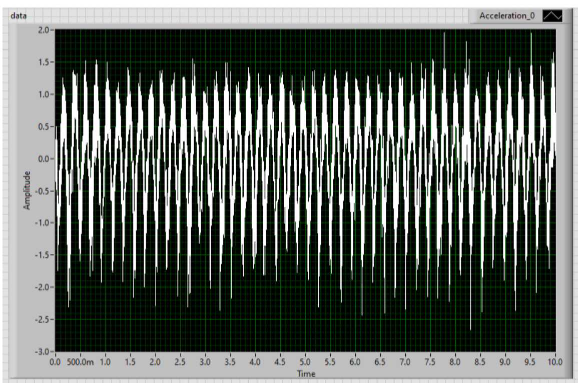


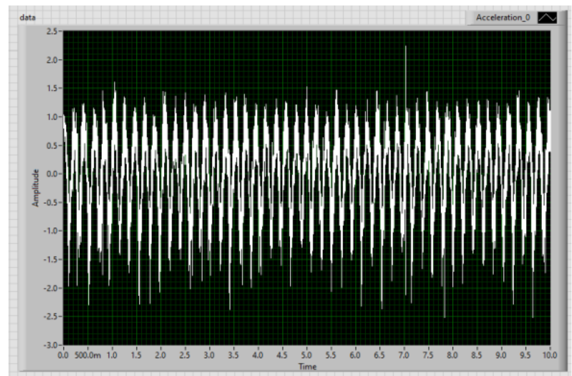
21V



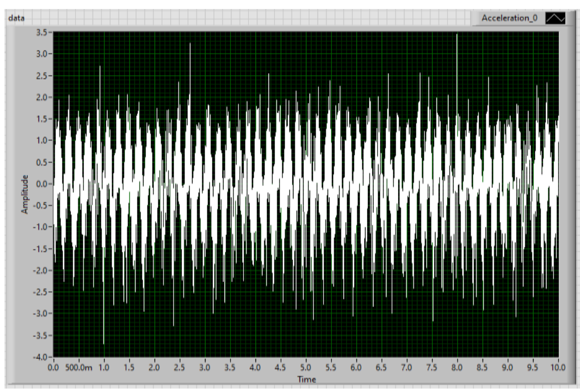
22V



23V



24V



Appendix 1: Raw Data Plots of Acceleration with Varying Voltages