Turducken!

Saira Billah, Isabella Flynn, Josh Ashvil, Abigail Berg

Table of Contents

Introduction

The goal of this challenge was to design a system to fully cook a turducken with the fastest cooking time and least amount burnt. The turducken is made up of an outer layer of turkey, homogenous layer of duck and chicken, "ducken," and a center of stuffing. The turducken would be cooked in a conventional oven with heated skewers through it. Many different combinations of oven temperature and skewer heat flow were tested to get the best time and least amount of burnt turducken.

Requirements

The method to cook the turducken had specific requirements. The entire turkey begins the cooking process at 42 degrees Fahrenheit and must be fully cooked, meaning that it had to be greater than 165 °F. The turkey is burnt when it reaches 280 °F. The skewers reject a constant rate of heat and must be capable of inserting into the turkey. They also could be no bigger than 0.5 inches in any one dimension, with a cross sectional area less than 0.2 in^2. There was also a maximum of 2 skewers. The oven has a temperature range of 350 °F – 550 °F.

Design Assumptions and Skewer Design

In our design, we are using two skewers to cook our turducken, as well as a conventional oven. Each skewer is placed 3.5 inches away from the center of the turkey at a 40-degree angle. The skewer is only heating the ducken (through resistance heating), since the stuffing does not need to be cooked and the turkey burns easily due to the oven (see Figure 1). Each skewer has a hollow square cross-section (with thickness 0.125 in and side length 0.5 in), and we chose this because it is a very simple design that can emit heat evenly and be meshed easily (see Figure 2). We made the skewers hollow because it does not affect heat transfer into the turducken, and allowed us to meet the cross-section area limit of 0.2 in^2, while maximizing our side length to be 0.5 in. The ends of the skewers are pointed to allow users to insert them into the turducken with ease. Lastly, we decided to make our skewers out of aluminum due to its high thermal conductivity, which allows it to transfer heat efficiently (which is why it is commonly used in a lot of cookware).

Engineering Drawings

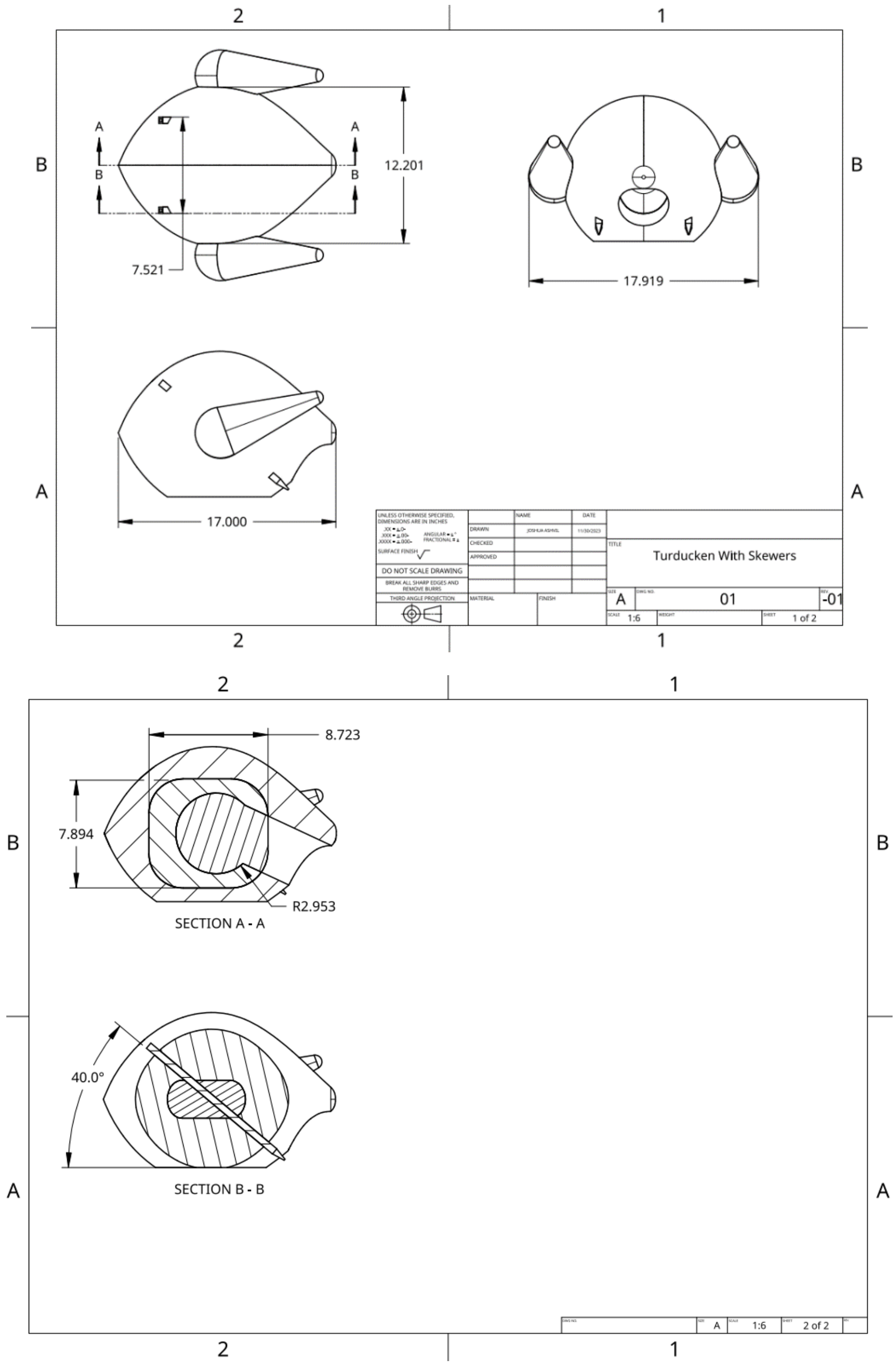


Figure 1: Engineering Drawing of Turducken with Skewers

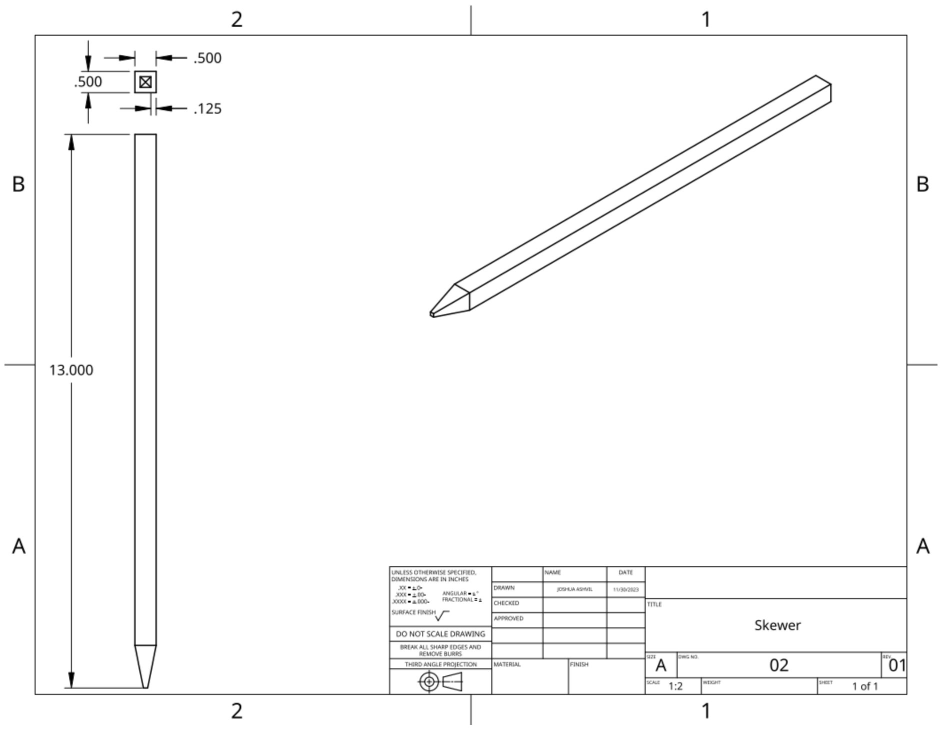


Figure 2: Engineering Drawing of Skewer

Material Properties of Turducken

In order to model the cooking process of the turducken, some thermal properties must be known, including the density, thermal conductivity, and specific heat. These thermal properties are dependent on the composition of the food, and vary depending on the ratios of protein, fat, carbohydrates, and ash. To determine these properties, the turducken was analyzed in 3 components: the turkey, ducken, and stuffing.

Composition of Turducken		
	Component	Composition
Turkey	Turkey	100%
Ducken	Duck	50%
	Chicken	50%
Stuffing	Pureed Carrots	33%
	Lean Beef	33%
	Mashed Potatoes	33%

Table 1: Composition of Turducken

The composition of foods varies as a function of time, so the composition of moisture, protein, fat, ash, and carbohydrates were analyzed at 3 points: before boiling, during boiling, and at burnt, where there is only ash. The following information was used from the 2006 ASHRAE handbook, Chapter 9 of Thermal Properties of Foods.

Food Composition at 45-212 (before boiling)					
	Moisture	Protein	Fat	Ash	Carbohydrates
Turkey	0.704	0.204	0.080	0.009	0.000
Chicken	0.660	0.186	0.151	0.008	0.000
Duck	0.485	0.115	0.393	0.007	0.000
Ducken	0.572	0.150	0.272	0.007	0.000
Pureed Carrots	0.879	0.010	0.002	0.009	0.101
Lean Beef	0.708	0.220	0.049	0.011	0.000
Mashed Potatoes	0.790	0.021	0.001	0.009	0.016
Stuffing	0.792	0.084	0.017	0.009	0.039

Food Composition at 212 (boiling)					
	Moisture	Protein	Fat	Ash	Carbohydrates
Turkey	0	0.696	0.274	0.030	0.000
Chicken	0	0.540	0.437	0.023	0.000
Duck	0	0.223	0.764	0.013	0.000
Ducken	0	0.350	0.633	0.017	0.000
Pureed Carrots	0	0.084	0.016	0.071	0.829
Lean Beef	0	0.787	0.175	0.038	0.000
Mashed Potatoes	0	0.444	0.021	0.191	0.343
Stuffing	0	0.560	0.115	0.063	0.262

Food Composition at 280+ (burnt)					
	Moisture	Protein	Fat	Ash	Carbohydrates
Turkey	0	0	0	1	0
Chicken	0	0	0	1	0
Duck	0	0	0	1	0
Ducken	0	0	0	1	0
Pureed Carrots	0	0	0	1	0
Lean Beef	0	0	0	1	0
Mashed Potatoes	0	0	0	1	0
Stuffing	0	0	0	1	0

Table 2: Composition of Each Element at Various Temperatures

To solve for the thermal conductivity, density, and specific heat of each element, the following equations were used from the ASHRAE Handbook. As seen in Table 3, the thermal properties were calculated for moisture, protein, fat, carbohydrates, and ash at 42, 165, 212, and 280 °F respectively.

Table 1 Thermal Property Models for Food Components ($-40 \le t \le 300$°F)

Thermal Property	Food Component	Thermal Property Model
Thermal conductivity, Btu/(h·ft·°F)	Protein	$k = 9.0535 \times 10^{-2} + 4.1486 \times 10^{-4}t - 4.8467 \times 10^{-7}t^2$
	Fat	$k = 1.0722 \times 10^{-1} - 8.6581 \times 10^{-5}t - 3.1652 \times 10^{-8}t^2$
	Carbohydrate	$k = 1.0133 \times 10^{-1} + 4.9478 \times 10^{-4}t - 7.7238 \times 10^{-7}t^2$
	Fiber	$k = 9.2499 \times 10^{-2} + 4.3731 \times 10^{-4}t - 5.6500 \times 10^{-7}t^2$
	Ash	$k = 1.7553 \times 10^{-1} + 4.8292 \times 10^{-4}t - 5.1839 \times 10^{-7}t^2$
Thermal diffusivity, ft^2/h	Protein	$\alpha = 2.3170 \times 10^{-3} + 1.1364 \times 10^{-5}t - 1.7516 \times 10^{-8}t^2$
	Fat	$\alpha = 3.8358 \times 10^{-3} - 2.4128 \times 10^{-7}t - 4.5790 \times 10^{-10}t^2$
	Carbohydrate	$\alpha = 2.7387 \times 10^{-3} + 1.3198 \times 10^{-5}t - 2.7769 \times 10^{-8}t^2$
	Fiber	$\alpha = 2.4818 \times 10^{-3} + 1.2873 \times 10^{-5}t - 2.6553 \times 10^{-8}t^2$
	Ash	$\alpha = 4.5565 \times 10^{-3} + 8.9716 \times 10^{-6}t - 1.4644 \times 10^{-8}t^2$
Density, lb/ft^3	Protein	$\rho = 8.3599 \times 10^{1} - 1.7979 \times 10^{-2}t$
	Fat	$\rho = 5.8246 \times 10^{1} - 1.4482 \times 10^{-2}t$
	Carbohydrate	$\rho = 1.0017 \times 10^{2} - 1.0767 \times 10^{-2}t$
	Fiber	$\rho = 8.2280 \times 10^{1} - 1.2690 \times 10^{-2}t$
	Ash	$\rho = 1.5162 \times 10^{2} - 9.7329 \times 10^{-3}t$
Specific heat, Btu/(lb·°F)	Protein	$c_p = 4.7442 \times 10^{-1} + 1.6661 \times 10^{-4}t - 9.6784 \times 10^{-8}t^2$
	Fat	$c_p = 4.6730 \times 10^{-1} + 2.1815 \times 10^{-4}t - 3.5391 \times 10^{-7}t^2$
	Carbohydrate	$c_p = 3.6114 \times 10^{-1} + 2.8843 \times 10^{-4}t - 4.3788 \times 10^{-7}t^2$
	Fiber	$c_p = 4.3276 \times 10^{-1} + 2.6485 \times 10^{-4}t - 3.4285 \times 10^{-7}t^2$
	Ash	$c_p = 2.5266 \times 10^{-1} + 2.6810 \times 10^{-4}t - 2.7141 \times 10^{-7}t^2$

Source: Choi and Okos (1986)

Table 2 Thermal Property Models for Water and Ice ($-40 \le t \le 300$°F)

	Thermal Property	Thermal Property Model
Water	Thermal conductivity, Btu/(h·ft·°F)	$k_w = 3.1064 \times 10^{-1} + 6.4226 \times 10^{-4}t - 1.1955 \times 10^{-6}t^2$
	Thermal diffusivity, ft^2/h	$\alpha_w = 4.6428 \times 10^{-3} + 1.5289 \times 10^{-5}t - 2.8730 \times 10^{-8}t^2$
	Density, lb/ft^3	$\rho_w = 6.2174 \times 10^{1} + 4.7425 \times 10^{-3}t - 7.2397 \times 10^{-8}t^2$
	Specific heat, Btu/(lb·°F) (For temperature range of −40 to 32°F)	$c_w = 1.0725 - 5.3992 \times 10^{-3}t + 7.3361 \times 10^{-5}t^2$
	Specific heat, Btu/(lb·°F) (For temperature range of 32 to 300°F)	$c_w = 9.9827 \times 10^{-1} - 3.7879 \times 10^{-5}t + 4.0347 \times 10^{-7}t^2$

Figure 3 – equations for thermal properties from ASHRAE handbook

Material Properties of Components at 42 °F			
Component	Thermal Conductivity	Specific Heat	Density
Protein	0.107	0.481	82.848
Fat	0.103	0.476	57.591
Ash	0.195	0.264	150.591
Carbohydrates	0.120	0.372	99.546
Water	0.336	0.997	62.399

Material Properties of Components at 165 °F			
Component	Thermal Conductivity	Specific Heat	Density
Protein	0.146	0.499	80.647
Fat	0.092	0.493	55.808
Ash	0.242	0.290	149.395

Carbohydrates	0.162	0.397	98.218
Water	0.385	1.003	62.980

Material Properties of Components at 212 °F			
Component	Thermal Conductivity	Specific Heat	Density
Protein	0.157	0.505	79.805
Fat	0.087	0.497	55.126
Ash	0.255	0.298	148.937
Carbohydrates	0.171	0.402	97.710
Water	0.394	1.008	63.202

Material Properties of Components at 280 °F			
Component	Thermal Conductivity	Specific Heat	Density
Protein	0.169	0.513	78.588
Fat	0.080	0.500	54.140
Ash	0.271	0.307	148.276
Carbohydrates	0.179	0.407	96.976
Water	0.397	1.019	63.522

Table 3: Material Properties at Various Temperatures

After calculating these thermal properties at various temperatures, an assumption was made that the thermal property of an item is proportional to the composition of its elements. The thermal properties may be determined by the following equation, $A = \sum A_i\, x_i$, where A represents the thermal property (density, thermal conductivity, or specific heat), and x is the mass fraction of that component. Table 4 shows the estimated thermal properties of the turkey, ducken, and stuffing. These values were put into Ansys under engineering data for each component.

Thermal Conductivity of Turducken at varying temperatures (Btu/hrft°F)				
Component	42 °F	165 °F	212 °F	280 °F
Turkey	0.268	0.310	0.141	0.271
Ducken	0.238	0.269	0.114	0.271
Stuffing	0.283	0.327	0.168	0.271

Specific Heat (Btu/lb°F)				
Component	42 °F	165 °F	212 °F	280 °F
Turkey	0.841	0.850	0.497	0.307
Ducken	0.774	0.785	0.497	0.307
Stuffing	0.856	0.863	0.471	0.307

Density of Turducken at varying temperatures (lb/ft^3)				
Component	42 °F	165 °F	212 °F	280 °F
Turkey	66.803	66.609	75.130	148.276
Ducken	64.956	64.956	65.369	148.276
Stuffing	62.687	62.869	84.971	148.276

Table 4: Density of Turducken at Various Temperatures

Hand Calculations for Cook Time of Turducken

The turducken was modeled as three concentric spheres in order to simplify it for a circuit analysis.

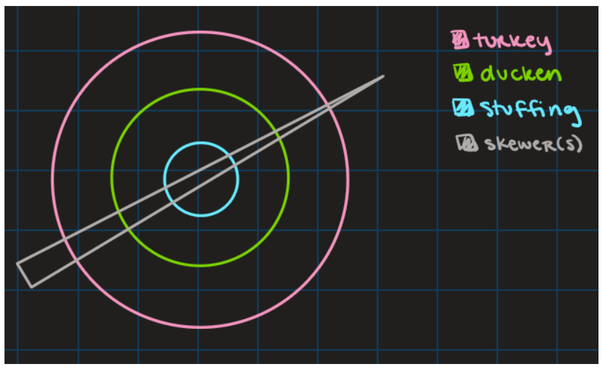


Figure 4: Simplified Geometry of Turducken for Analysis

Simple heat transfer can be modeled as circuit where the interfaces between materials are resistances, and the bodies are capacitances. Voltages in the system are representative of temperature and current is a representation of heat flow. The following figures show potential circuits for the given system, one of which takes into consideration the radiation of the oven and one that neglects this radiation. In order to further simplify the system, radiation was neglected.

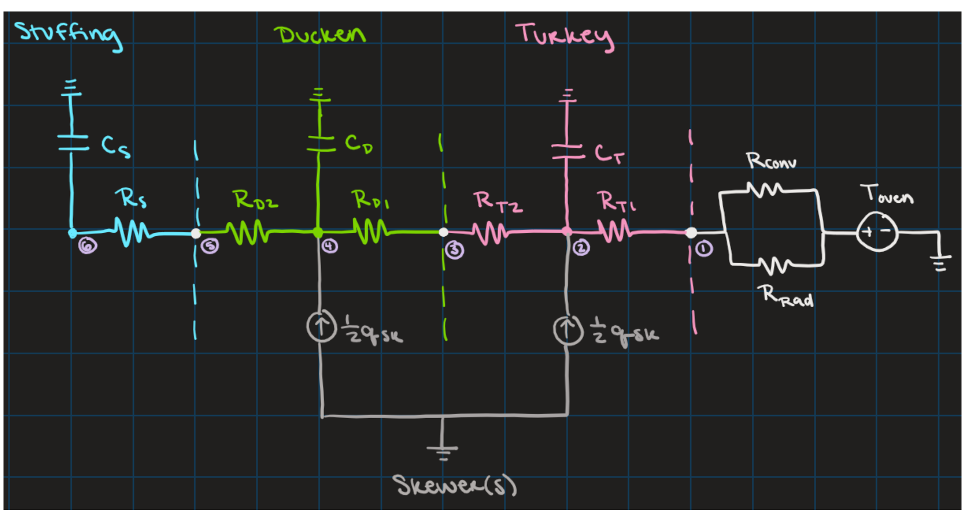


Figure 5: Circuit Model of System, Including Effects of Radiation

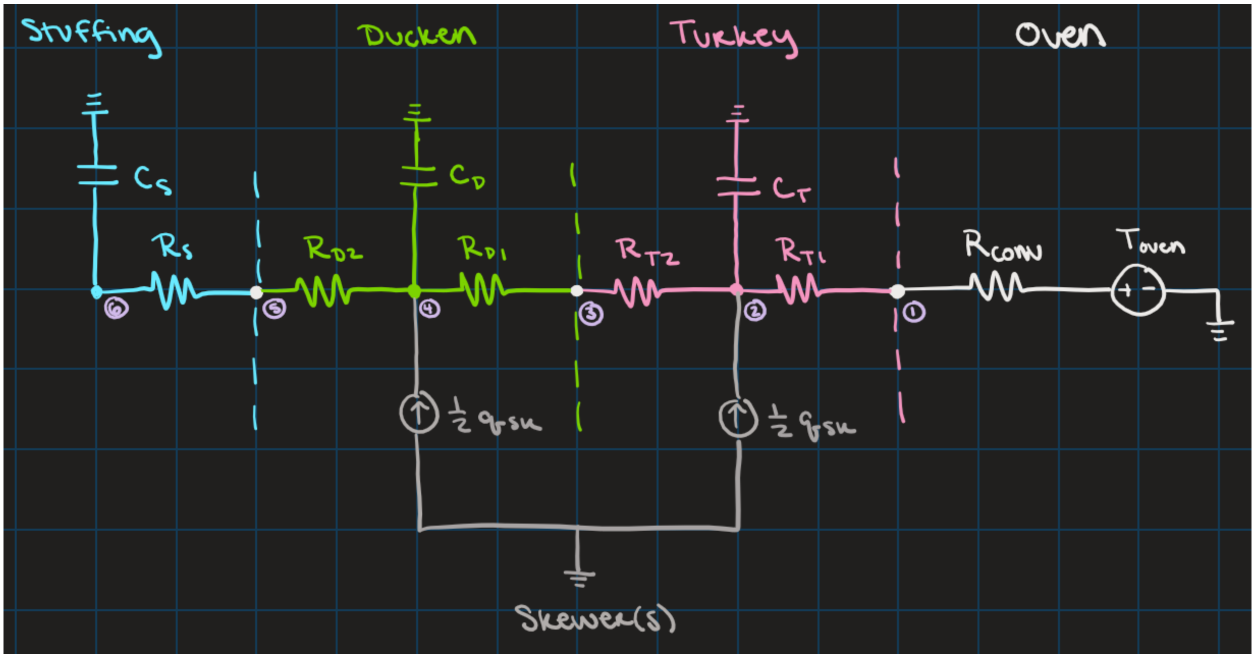


Figure 6: Circuit Model of the System, Neglecting Effects of Radiation

In order to model the system as a circuit, the system properties must be expressed in terms of circuit elements. The following sections discuss these relations.

<u>Conduction: (R_T, R_D, and R_S)</u>

The equation for conduction is $Q = \frac{kA\Delta T}{\Delta x}$, where A is area ($A = 4\pi r^2$), Δx is the distance, and k is the material property of thermal conductivity. The thermal resistance is then found to be $R_{cond} = \frac{\Delta T}{Q} = \frac{\Delta x}{kA} = \frac{\mathbf{1}}{\mathbf{4\pi k}}\left(\frac{\mathbf{1}}{\mathbf{r_1}} - \frac{\mathbf{1}}{\mathbf{r_2}}\right)$. The thermal resistance for the turkey, ducken, and stuffing are all based on conduction.

<u>Convection: (R_{Oven})</u>

The equation for convection is $Q = hA\Delta T$, where h is the heat transfer coefficient. The thermal resistance is found to be $R_{conv} = \frac{\Delta T}{Q} = \frac{1}{hA}$. The thermal resistance for the oven is based on convection.

<u>Radiation:</u>

The equation of radiation is $Q = \varepsilon\sigma A(T_{surface}^4 - T_{surroundings}^4)$, where ε is the emissivity of the surface, σ is the Stefan-Boltzmann constant. The thermal resistance is found to be $R_{rad} = \frac{1}{\varepsilon\sigma A\left(T_{surface}^2 + T_{surroundings}^2\right)\left(T_{surface} + T_{surroundings}\right)}$. The thermal resistance for the oven is also based on radiation, however, for our analysis, we neglected radiation.

<u>Capacitances</u>:

The capacitance equation is $C_x = \rho_x V_x c_x$, where ρ is density, V is volume ($V = \frac{4\pi}{3}(r_o^3 - r_i^3)$), c is specific heat. The capacitances are applied to the turkey, ducken, and stuffing.

With these relations and the fact that the skewers are modeled as heat flow, the following equations are governed by Kirchoff's Current Law and a nodal analysis of the system to solve for temperature at each interface and body. The temperature and material properties were assumed to be uniform at these interfaces and bodies to simplify the system. The geometry was also simplified to be concentric spheres to simplify the material property calculations. This, however, means that the temperatures solved for would be an overestimate of the actual system and would give better insight into the order in which each body cooked than the speed.

<u>Nodal Analysis:</u> $Q = \frac{\Delta T}{R}$, $Q = C\frac{dT}{dt}$

$$\frac{T_1 - T_{oven}}{R_{oven}} + \frac{T_1 - T_2}{R_{T1}} = 0$$

(Node 1 – outer turkey)

$$C_T\frac{dT_2}{dt} + \frac{T_2 - T_1}{R_{T1}} + \frac{T_2 - T_3}{R_{T2}} = \frac{1}{2}Q_{skewers}$$

(Node 2 – middle of turkey)

$$\frac{T_3 - T_2}{R_{T2}} + \frac{T_3 - T_4}{R_{D1}} = 0$$

(Node 3 – turkey/ducken contact)

$$C_D\frac{dT_4}{dt} + \frac{T_4 - T_3}{R_{D1}} + \frac{T_4 - T_5}{R_{D2}} = \frac{1}{2}Q_{skewers}$$

(Node 4 – middle of ducken)

$$\frac{T_5 - T_4}{R_{D2}} + \frac{T_5 - T_6}{R_S} = 0$$

(Node 5 – ducken/stuffing contact)

$$C_S\frac{dT_6}{dt} + \frac{T_6 - T_5}{R_S} = 0$$

(Node 6 – middle of stuffing)

Using these governing equations, the python script (found in Appendix A) was used to visualize these cooking times and temperatures. The following figures show these times at different ambient conditions and inputs.

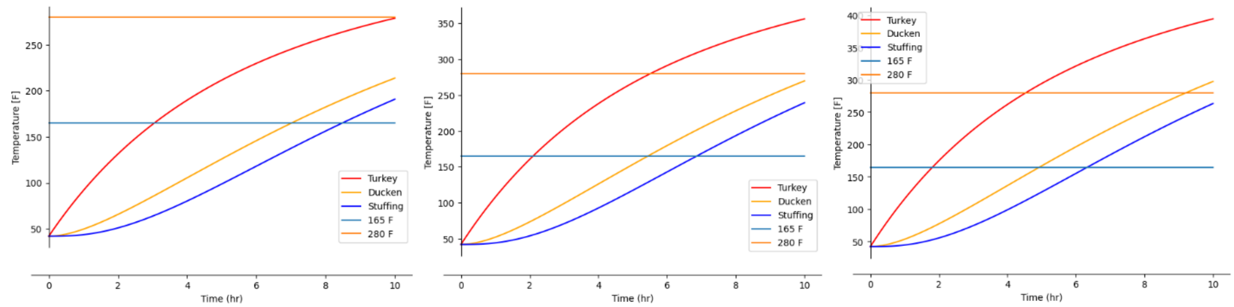


Figure 7: Hand Calc Visualization of Cooking Times with no Q_{in}-- 350°F, 450°F, 500°F

Naturally, the turkey cooks fastest as the primary source of heat transfer is from convection due to ambient conditions. The ducken follows and the stuffing takes the longest to fully cook. While the 3 conditions tested do not take an abnormally long time to cook, fully cooking within at most 7.5 hours, by the time the ducken fully cooks, the turkey burns. While the 350 °F case takes longest to cook, it has the least amount of meat burnt, so this condition was used for the simulations. In order to speed up the internal heat transfer, cooking the inside faster, a skewer was added to the turducken. The following graphs show the cooking times at 350 °F with various input power.

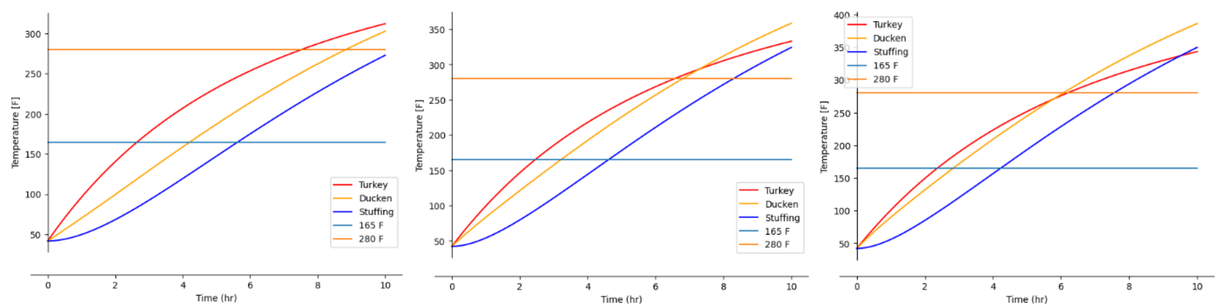


Figure 8: Hand Calc Visualization of Cooking Times at 350F with Q_{in} -- 32W, 52W, 62W

Here, while the cooking time is still a bit slow, there is significantly less turkey burnt by the time the rest of the turducken cooks. It appears that the best ratio of speed to percent burnt would fall between an input power of 32W and 62W for this system at 350 °F.

Meshing of Turducken

For this task, it was important to get a good mesh, so we achieve accurate results. Thus, we made use of Altair HyperMesh to meet this goal. We meshed a model including our turkey, ducken, stuffing, and wings. All were imported as separate bodies, but before doing so we had to fix the given turducken geometry. There was some overlapping geometry between the turkey and the ducken, and we had to add stuffing. We fixed these issues in Onshape using a boolean feature, and also added in the holes for the skewers (we did not import the skewers themselves into the model).

Once we solved the geometry issues, we began the meshing process. We started by trimming a quadrilateral around each skewer hole with the surface edit tool (offset 0.197 in). We did this because when the geometry changes from a large turkey to a smaller hole, a transition mesh is needed since we want finer elements around the smaller region for a more accurate mesh. Next, we created a surface mesh (using the automesh tool) of the turkey and ducken together (to keep element connectivity within the skewer holes). All elements were trias of size 0.01 m (0.394 in) and 0.003 m (0.118 in) for the trimmed quadrilaterals around the holes (see Figure 10). Next, we surface meshed the stuffing with trias elements of size 0.01 m (0.394 in), and finally the wings with trias of size 0.005 m (0.197 in) (the wings had some smaller geometry, so we wanted the mesh to be finer).

Next, we began our 3D mesh of the bodies. For this, we used the CFD Tetramesh tool. For all components, we created 3 boundary layers, with the first being 0.0005 m (0.0197 in) thick, with a growth rate of 1.2. We used the standard tetrameshing algorithm in hypermesh and had a growth rate of 1.3 for our pyramids. As seen in Figures 11 and 12, the boundary layers between the turkey and the ducken, as well as the ducken and the stuffing, all maintained connectivity, which is important especially in the regions where the skewers pass through multiple bodies. Thus, we were confident that this mesh would provide accurate results when imported into ANSYS Workbench for analysis.

Legend:

Component	**Turkey**	**Ducken**	**Stuffing**	**Wings**
Boundary Layer	Light Blue	Yellow	Orange	Maroon
Tetramesh	Navy Blue	Red	Green	Purple

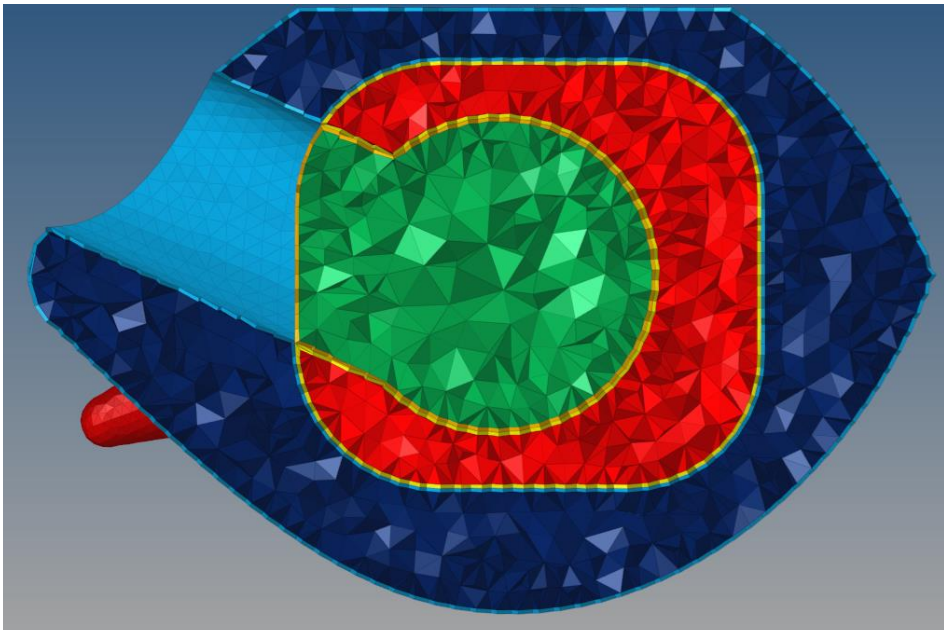


Figure 9: Section View of Turducken CFD Tetramesh

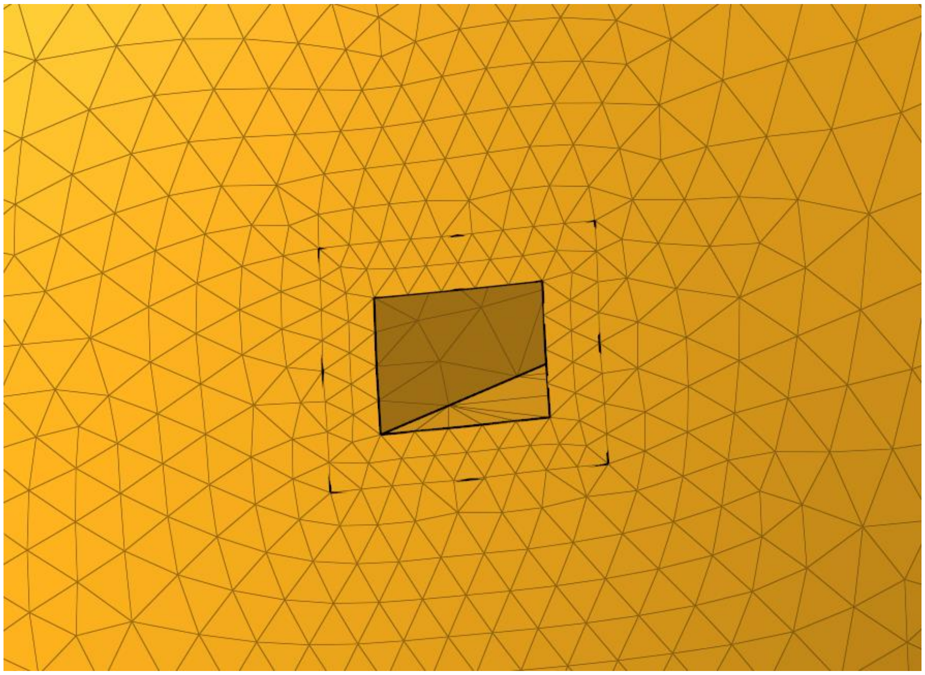


Figure 10: Close-Up View of Transition Mesh Around Skewer Holes

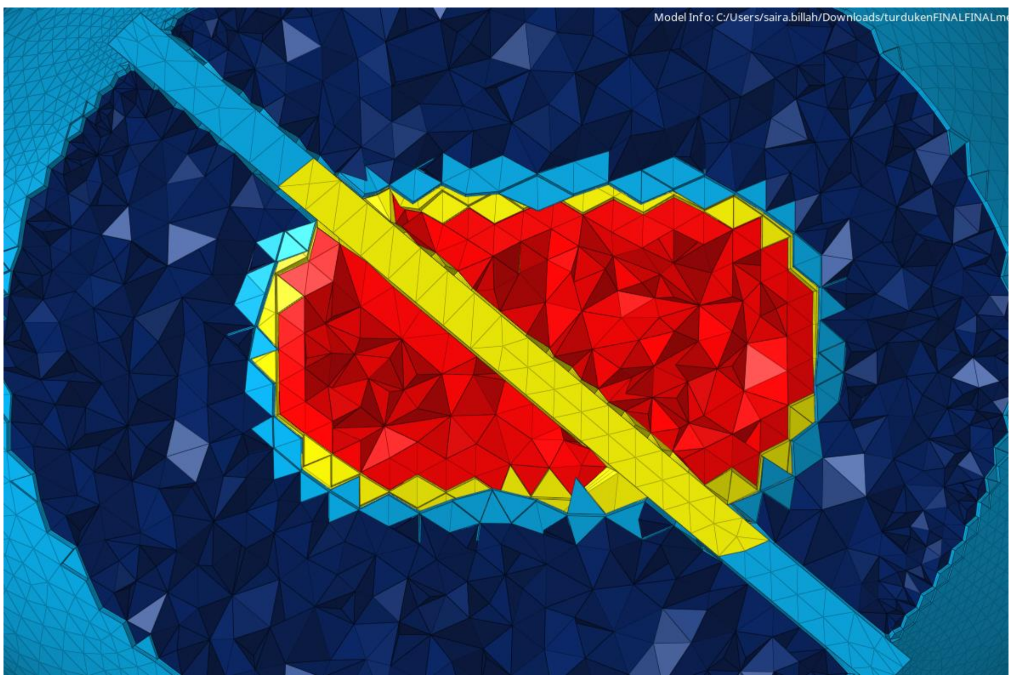


Figure 11: Close-Up View of Skewer Hole Boundary Layer, Maintaining Connectivity from the Turkey to the Ducken

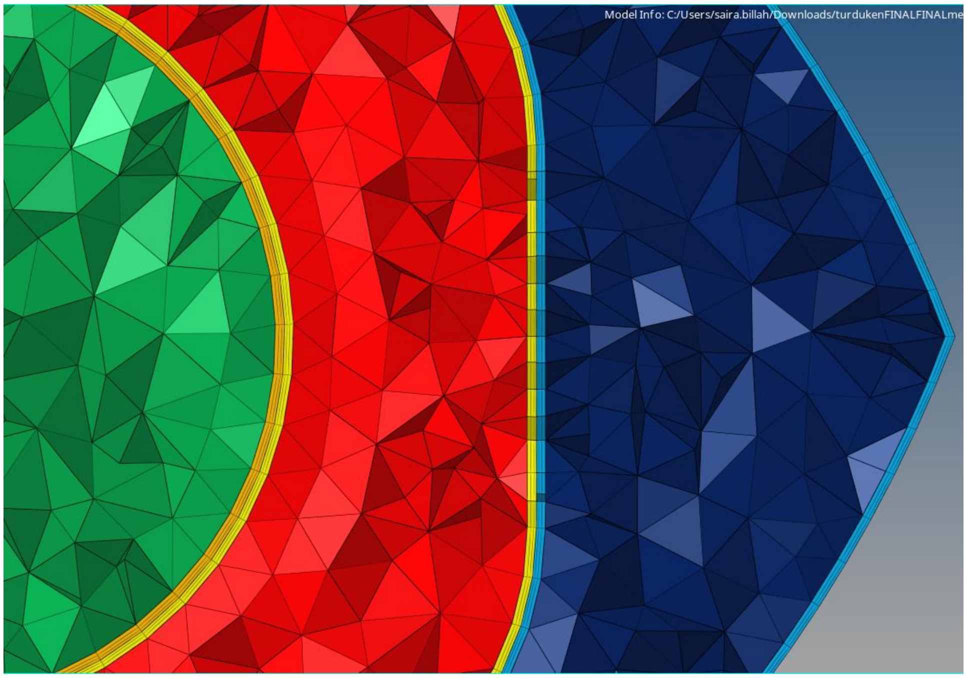


Figure 12: Second Close-Up View Showing Connectivity Between Boundary Layers

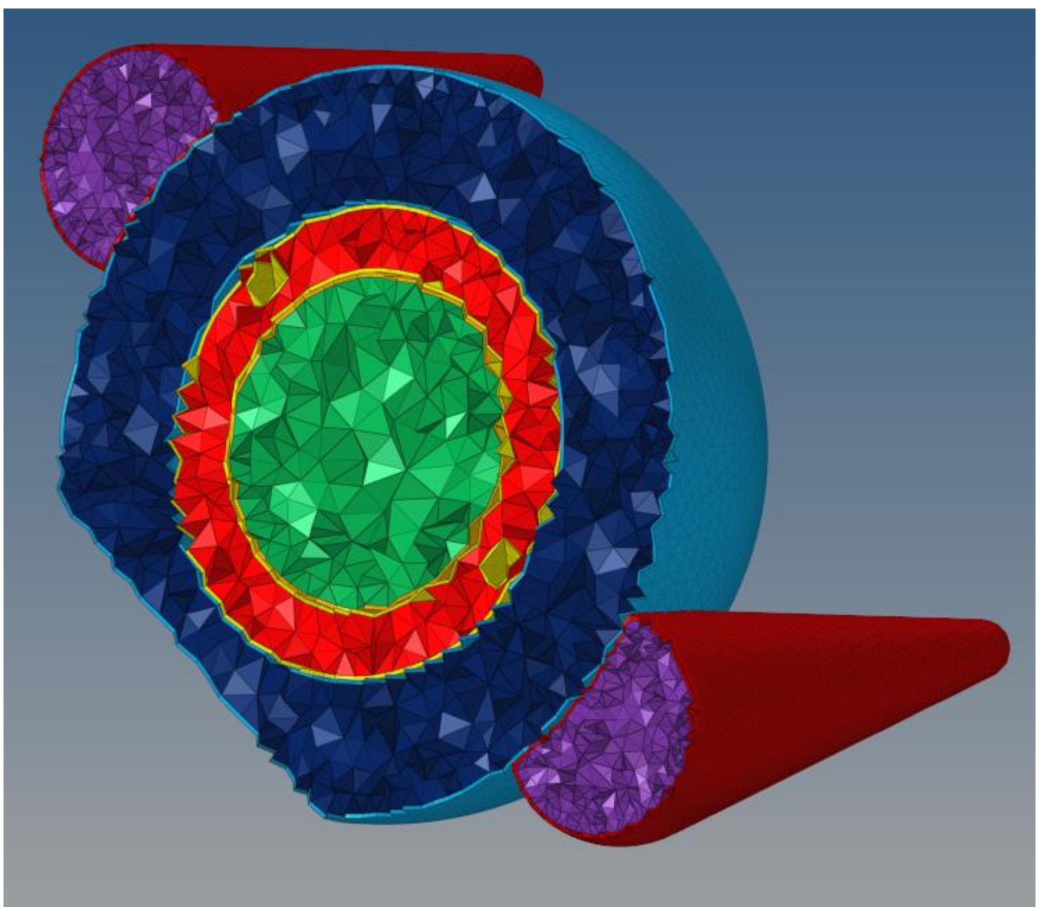


Figure 13: Section View Showcasing Tetrameshes of Every Component of the Turducken

Modeling on Ansys Workbench

We assumed that the turducken is in a conventional oven, so the majority of heat is transferred to the turducken by convection. This was modeled with convection on Ansys, where we input the film coefficient and the temperature of the oven, assuming it is preheated. With the addition of skewers, a heat flow was used to model the heat transferred from the skewers. Various values for heat flow were tested, and their cooking time and percent burned were recorded to determine the optimal method for cooking the turducken.

To determine the cooking time of the turducken, a temperature was added in the solution that looked at the turkey, ducken, and wings, because it is required that these components must be greater than 165 °F. On the other hand, the stuffing has no temperature requirement and was therefore not used to determine cooking time. To find the percent burned, the results file of the turducken at the point the minimum temperature is 165 °F was extracted and analyzed. The number of nodes above 280 °F divided by the total number of nodes was used to determine the volume burned.

Transient Thermal Results Without Skewers

In order to get a clear understanding of how much heat we need to cook the turducken, we modeled the turducken cooking in an oven with no skewers at first. To get consistent results, Figure 14 shows the analysis settings used. There were 200 steps, and the model was tested for 12 hours. The turducken was modeled at 400 °F and 350 °F to see which temperature gives the best results. Figure 15 shows the loads applied in the model of a turducken placed in a preheated 400 °F oven.

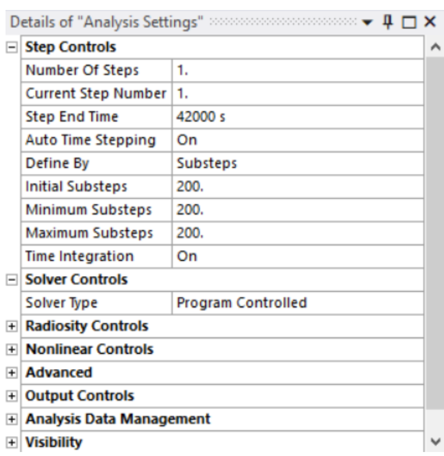


Figure 14: Analysis Settings

Loads

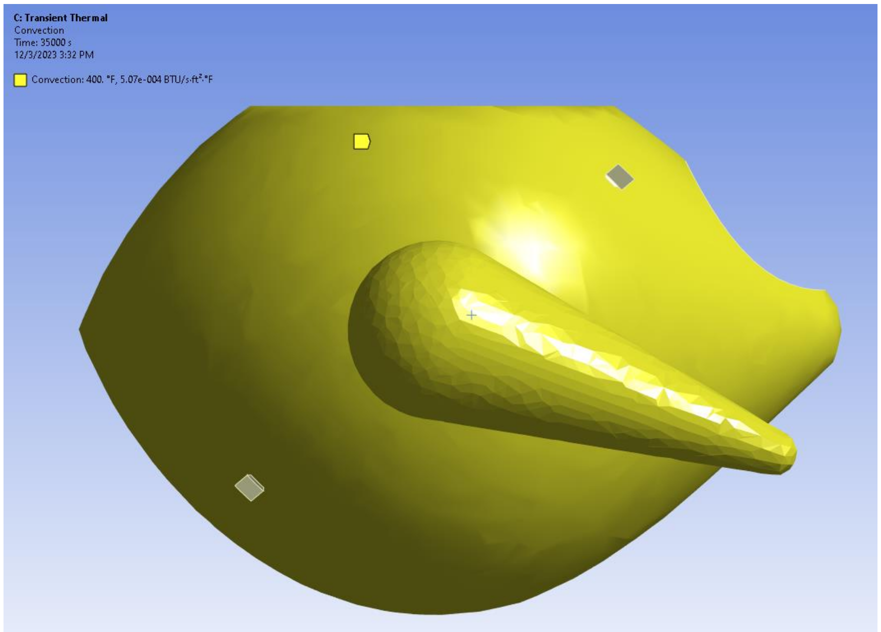


Figure 15: Boundary Conditions with No Rods

Experiment 1: 350 °F external temperature, 0 Q_{in} from rods

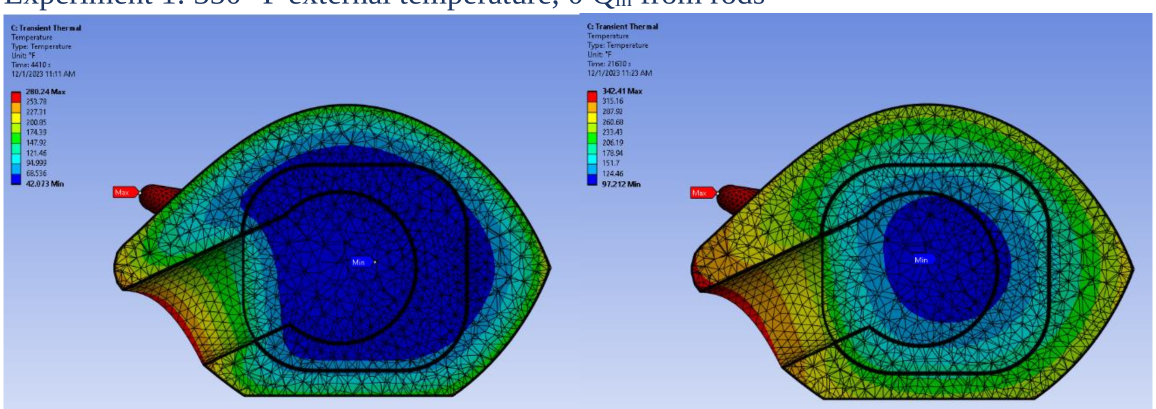
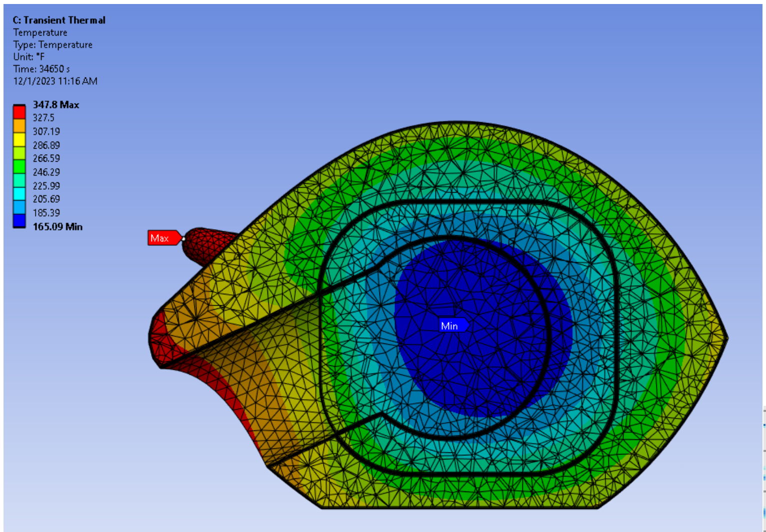


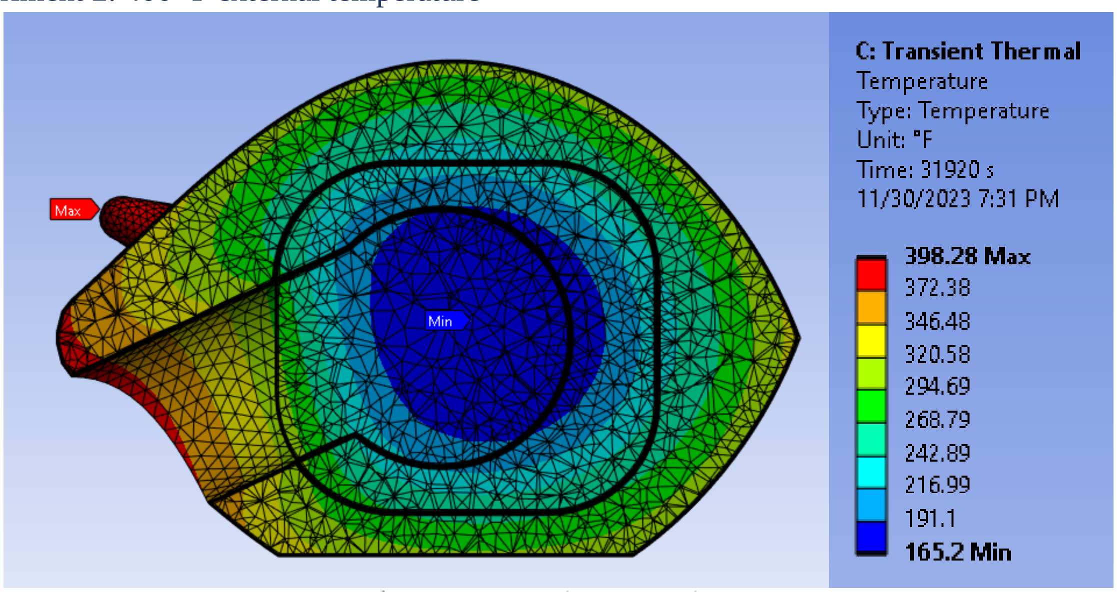
Figure 16: Experiment 1 -- Turducken at 1.2 Hours and 6 Hours Respectively



# Nodes	103693
# Nodes B	62616
% Burnt	60.38595

Figure 17: Experiment 1 --Turducken Fully Cooked at 9.6 Hours

Experiment 2: 400 °F external temperature



# Nodes	103693	
# Nodes Burnt	71401	
% Burnt	68.85807	%

Figure 18: Experiment 2--Turducken Fully Cooked at 8.9 Hours

With an oven temperature of 400 °F, the turducken fully cooked in 8.9 hours. However, around 69% of the turducken burned at this point. A lower oven temperature of 350 °F led to less of the turducken burning, approximately 60%. However, at this temperature the turducken took 9.6 hours to fully cook. Moving forward, we decided to experiment with the oven temperature of 350 °F but incorporate a heat skewer to help the internal temperature of the turducken.

Transient Thermal Results with Skewers

As mentioned before, heat flow was used to model the heat transferred from the skewers to the turducken. To do this, the faces of the where the skewers intersect the turducken were selected and a heat flow was added. To get a starting point for heat flow, 30W were tested, which is around .03 Btu/s. Figure 19 shows the boundary conditions applied for a model with the skewers.

Loads

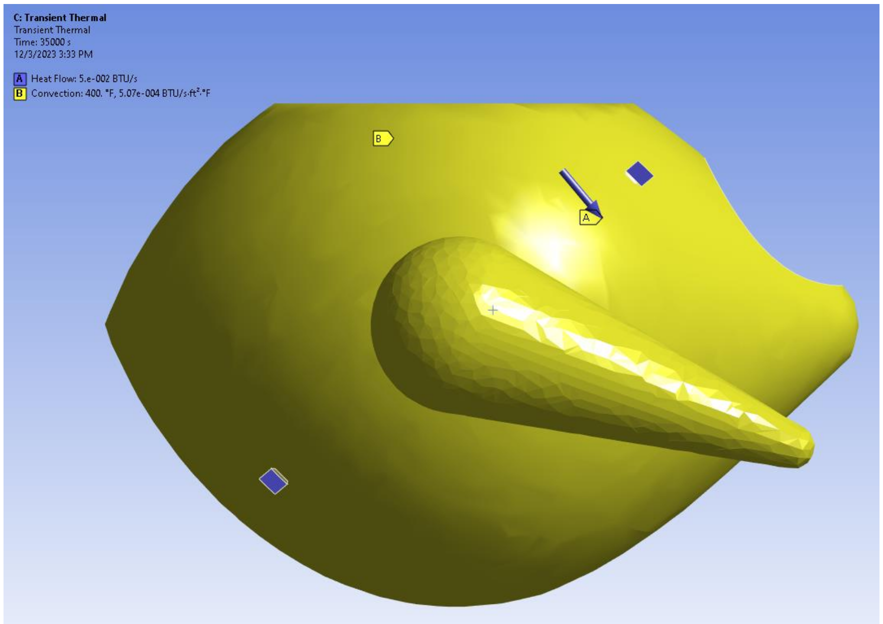


Figure 19: Boundary Conditions with Rods

Experiment #1: 350 °F and heat flow of .03 Btu/s - 49% burned

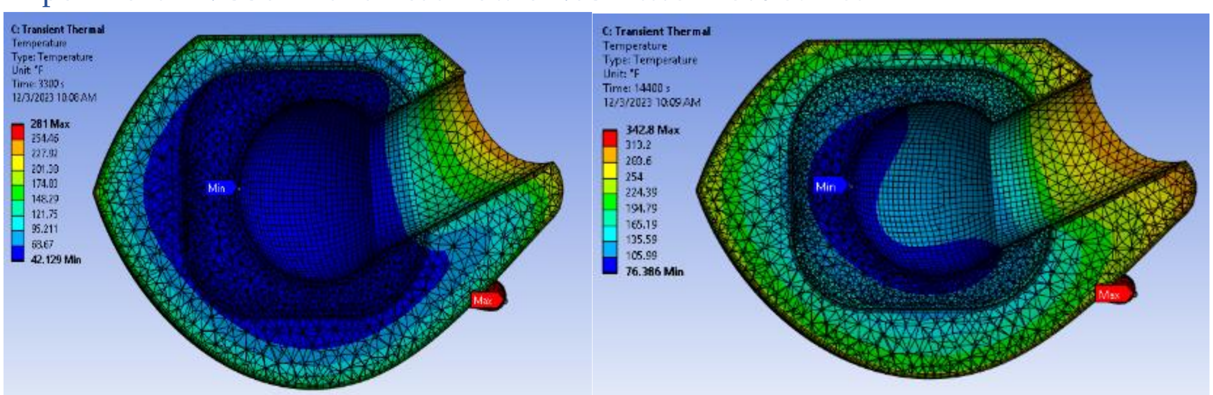


Figure 20: Experiment 1 -- Turducken at 0.9 Hours and 4 Hours Respectively

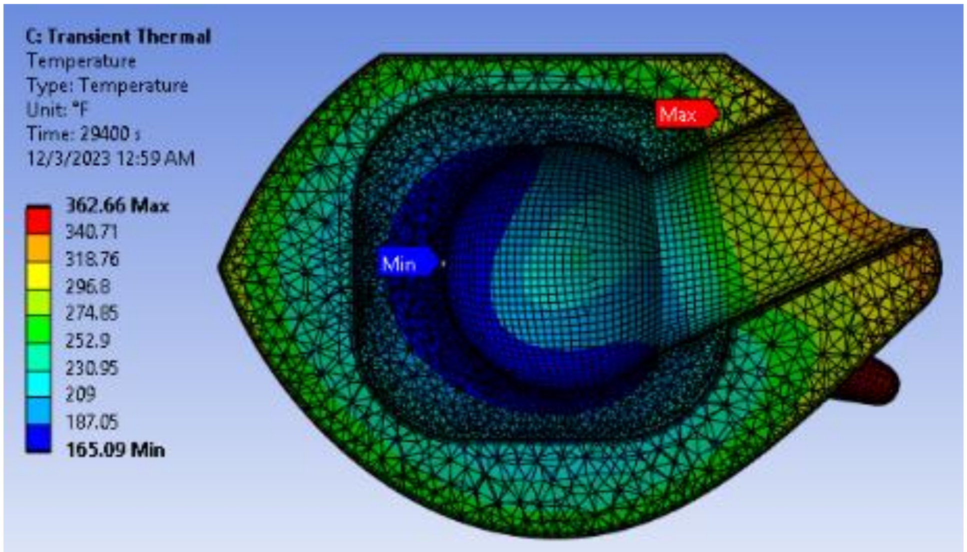


Figure 21: Experiment 1 Turducken Fully Cooked at 8.2 hours

Experiment #2: 350 °F and heat flow of .04 Btu/s - 51% burned

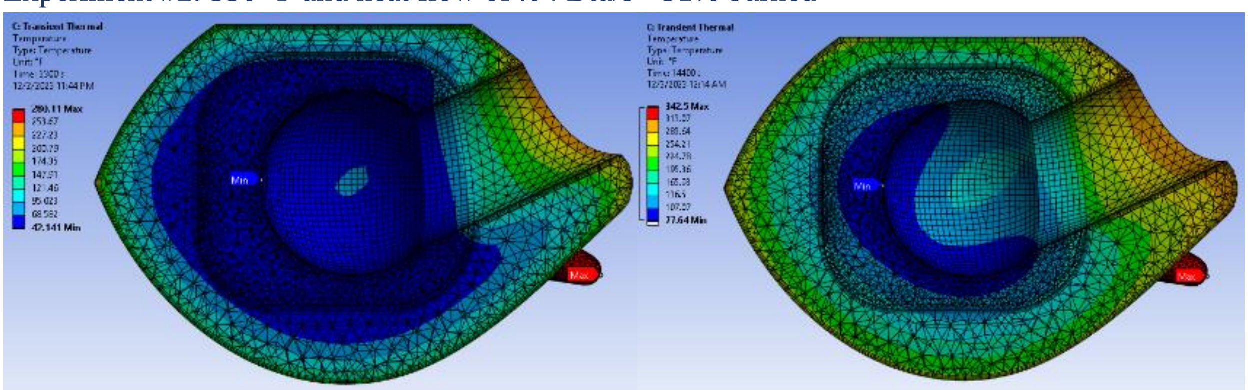


Figure 22: Experiment 2 Turducken at 0.9 hours and 4 hours Respectively

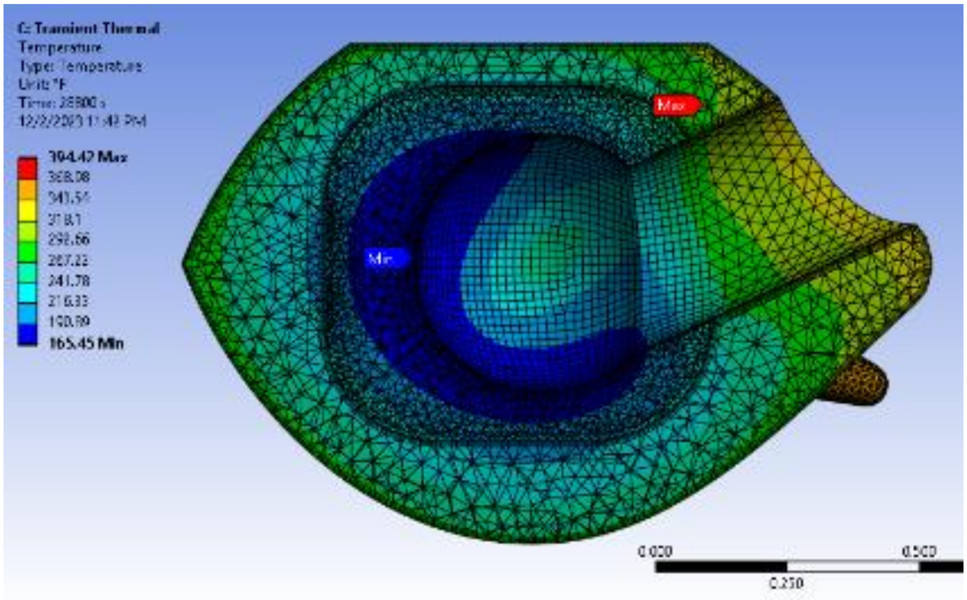


Figure 23: Experiment 2 Turducken Fully Cooked at 8 hours

Experiment #3: 350 °F oven and heat flow of .05 Btu/s - 43% burned

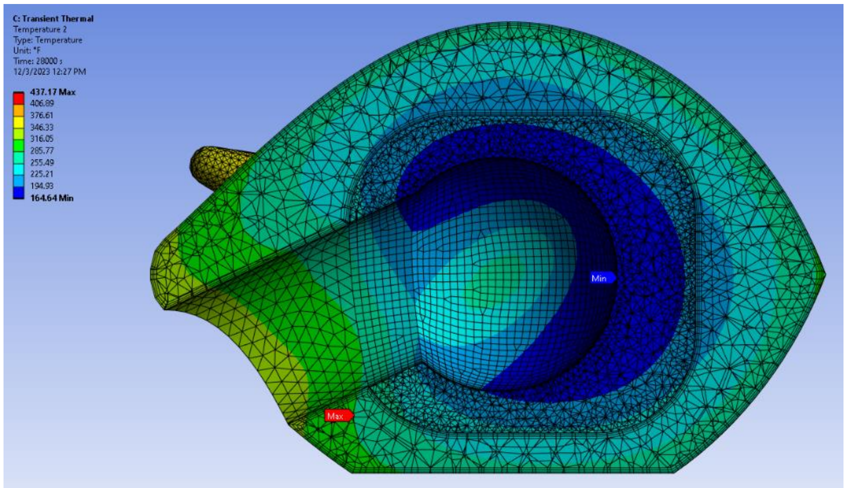


Figure 24: Experiment 3 Turducken Fully Cooked at 7.8 hours

Experiment #4: 375 °F oven and heat flow of .03 Btu/s - 58% burned

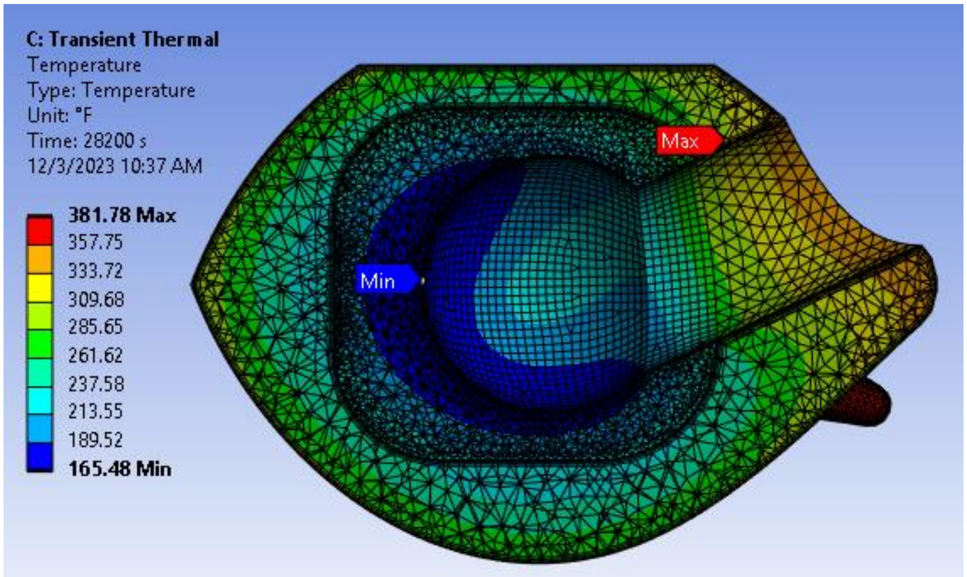


Figure 25: Experiment 4 Turducken Fully Cooked at 7.8 hours

Experiment #5: 350 °F oven and heat flow of .03 Btu/s on the middle rod – 47% burned

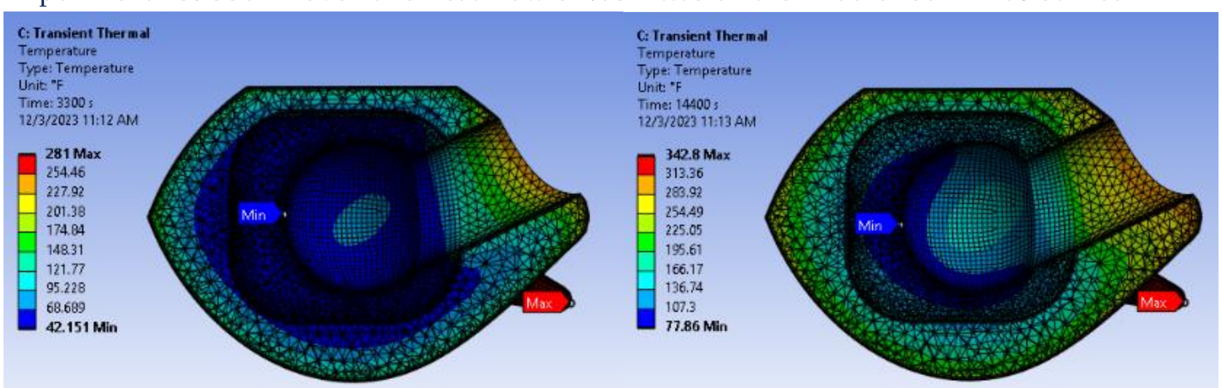


Figure 26: Experiment 5 Turducken at 0.9 hours and 4 hours Respectively

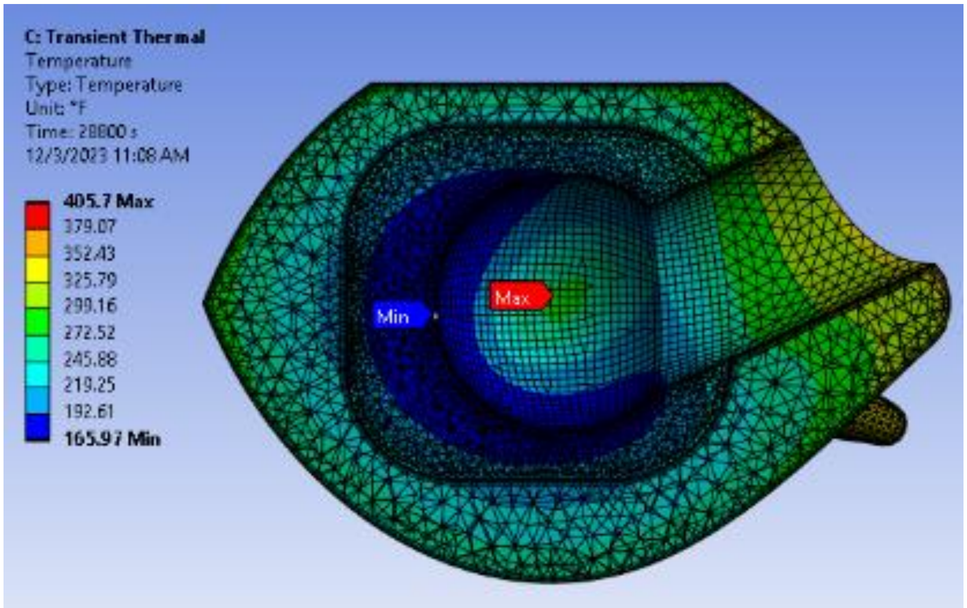


Figure 27: Experiment 5 Turducken Fully Cooked at 8 hours

Experiment #6: 350 °F oven and heat flow of .05 Btu/s on middle rod – 46% burned

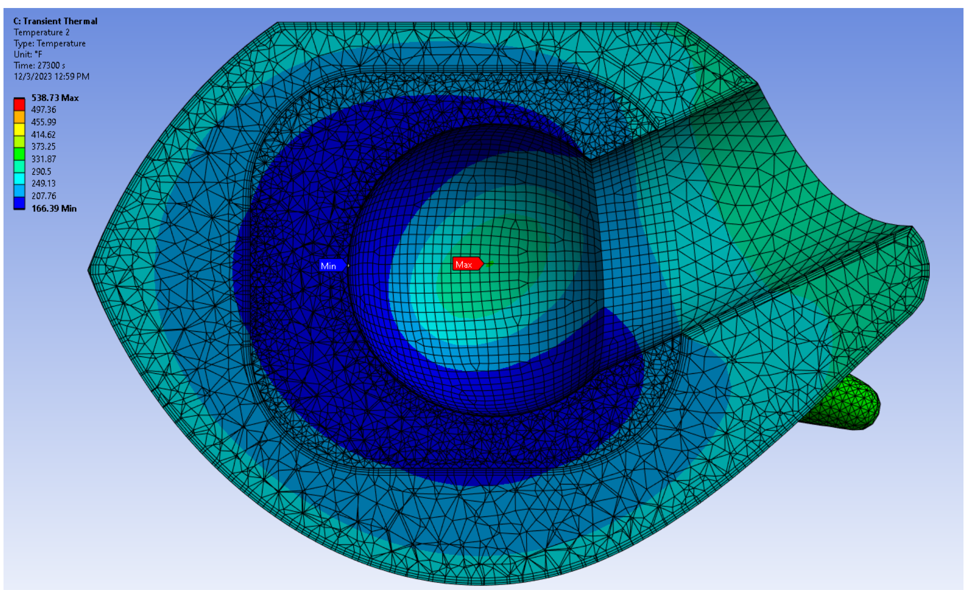


Figure 28: Experiment 6 Turducken Fully Cooked at 7.6 hours

In experiments 1-6, the magnitude and location of heat flow was varied to determine the best results to fully cook the turducken with the least amount of turducken burned. However, it was noticed that while the internal temperature of the turducken was slowly rising, the external temperature continued to increase at a fast rate, so a lot of the external turducken burned. To fix this problem, we experimented with how long the oven was turned on. As seen in Figure 29, the convection coefficient of the turducken was set to 0 at 30,000 seconds (8.33 hours). This was meant to simulate turning the oven off at 10,800 seconds, or 3 hours, so the convection slowly decreases until it reaches 0. However, the ambient temperature of 350 °F was maintained because we are assuming that no heat is lost in the oven and the door is a perfect insulator.

	Steps	Time [s]	✔ Convection Coefficient [BTU/s·ft²·°F]
1	1	0.	= 5.07e-004
2	1	10800	5.07e-004
3	1	30000	0.
*			

Figure 29: Alterations Made to Model Oven Being Turned Off

Experiment #7: 350 °F oven, turn off at 3 hours, and heat flow of .03 Btu/s on middle rod – 41% burned

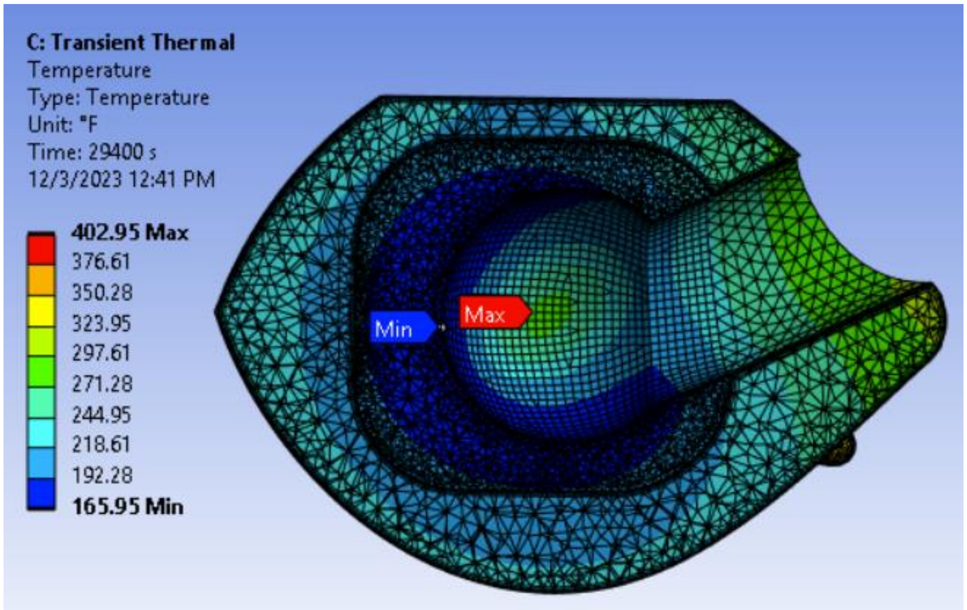


Figure 30: Experiment 7 Turducken at 8.2 hours

Experiment #8: 350 °F oven, turn off at 3 hours, and heat flow of .05 Btu/s on middle rod – 38% burned

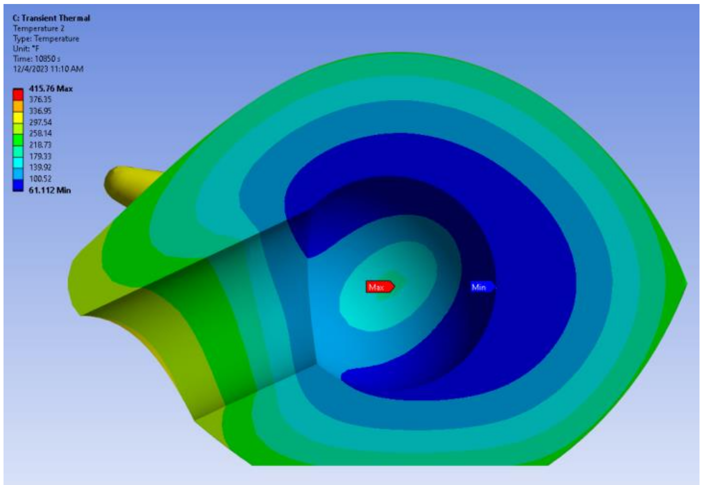
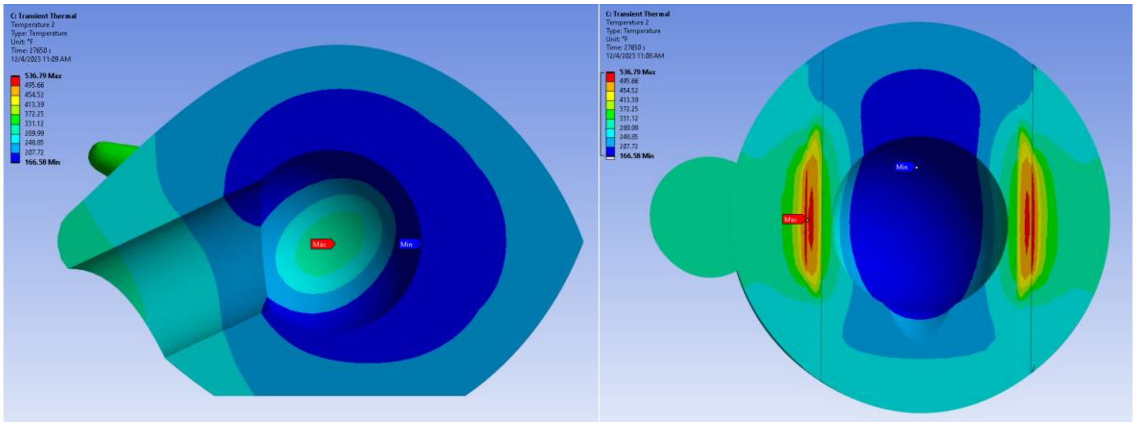


Figure 31a: Experiment 8 turducken at 3 hours



Burnt	61345
Total	159821
% burned	38.38357

Figure 32b: Experiment 8 Turducken at 7.6 hours

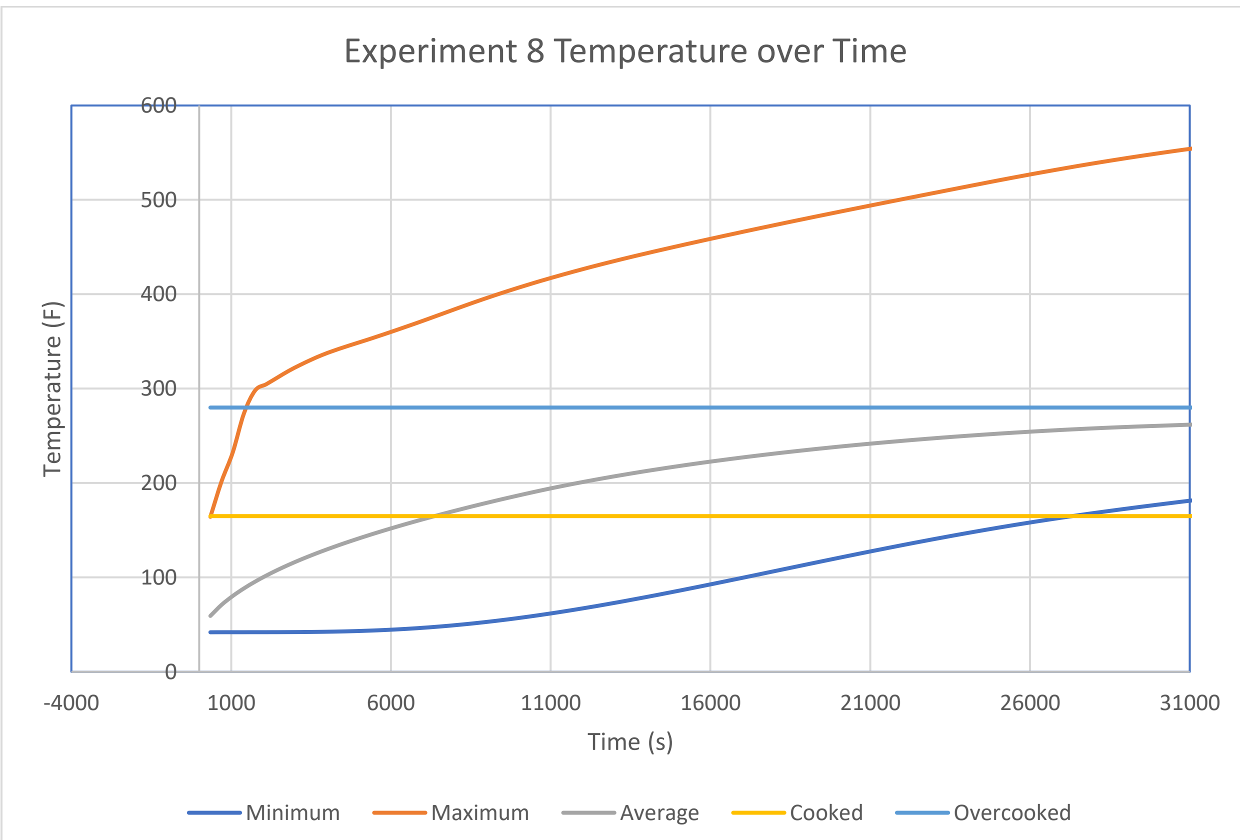


Figure 33: Experiment 8 Temperature Over Time

Analysis

Table 5 shows all the experiments and their respective total cook time and percentage burned. The experiment with the best results was found to be Experiment 8, where the turducken is cooked in an oven of 350 °F that is turned off at 3 hours, and has a skewer inserted with a heat flow of .05 Btu/s that is only activated in the ducken component. This leads to a cooking time of 7.6 hours and only 38% of the turducken burned.

Table 5 - Summary of all experiments and results

Oven Temperature (F)	Qin (Btu/s)	Rod Heat	Cook time (Hours)	Time Oven off (Hours)	Percentage Burned
400	0	N/A	8.9	N/A	69%
350	0	N/A	9.6	N/A	60%
350	.03	Total	8.2	N/A	49%
350	.04	Total	8	N/A	51%
350	.05	Total	7.8	N/A	43%
375	.03	Total	7.8	N/A	58%
350	.03	Middle	8	N/A	47%
350	.05	Middle	7.6	N/A	46%
350	.03	Middle	8.2	3	41%
350	.05	Middle	7.6	3	38%

Design Time

Task	**Time Spent (Hours)**
Thermal Properties	5
Designing skewer	3
Mesh	20
Hand Calculations	20
Simulations	30
Report and Presentation	20
Total	98

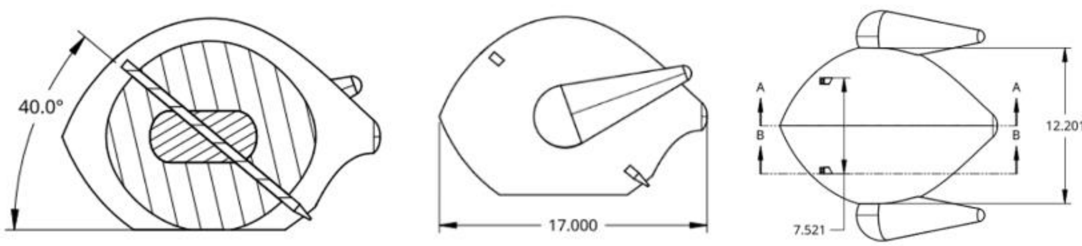
Instructions Manual for Cooking Turducken

① Create a stuffing mixture of equal parts carrots, potatoes, and lean beef.

② Stuff the turducken and place in the fridge for a few hours.

③ Preheat the oven to 350 °F.

④ Insert the skewers that are powered with 53 Watts at 40°.



⑤ Place the turducken in the oven and let cook for 3 hours.

⑥ At 3 hours, turn the oven off but keep the turducken in the oven.

⑦ After 7 hours and 36 minutes, remove the turducken from the oven and turn off the skewers.

⑧ Let the turducken cool down and enjoy!

Appendix

Python Script for Cooking Times and Temperatures

Note that in order to use the equation solver, the model was revised to combine resistors in series and use voltage division to find these intermediate temperatures. The following equations show the updated governing equations, derived from the original equations.

Voltage division (Node 1): $T_1 = (T_{oven} - T_2)\frac{R_{T1}}{R_{T1}+R_{oven}}$

Voltage division (Node 3) : $T_3 = (T_2 - T_4)\frac{R_{D1}}{R_{D1}+R_{T2}}$

Voltage division (Node 5) : $T_3 = (T_4 - T_6)\frac{R_{S1}}{R_{S1}+R_{D2}}$

(Node 2) : $\dot{T}_2 = \frac{1}{C_T}(\frac{T_{oven}-T_2}{R_{T1}+R_{T2}} - \frac{T_2-T_4}{R_{T2}+R_{D1}} + q_{in})$

(Node 4) : $\dot{T}_4 = \frac{1}{C_D}(\frac{T_2-T_4}{R_{D1}+R_{D2}} - \frac{T_4-T_6}{R_{D2}+R_{S1}} + q_{in})$

(Node 6) : $\dot{T}_6 = \frac{1}{C_S}(\frac{T_4-T_6}{R_{S1}+R_{S2}})$

```
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
#PARAMETERS
T_end = 10 #where the graph will plot until
rad = 0 #raditation status
T_oven = Tinf = 350 #deg F
q_in = 32 #W (0.03 = 32W ==194, 0.05 = 52W ==208) *2 as there are 2 rods
used
#MATERIAL PROPERTIES AT 42F
#THERMAL CONDUCTIVITY**
k_t = 0.27 #turkey (Btu/hr/ft/F)
k_d = 0.24 #ducken (Btu/hr/ft/F)
k_s = 0.28 #stuffing (Btu/hr/ft/F)
k = [k_t, k_t, k_d, k_d]
#SPECIFIC HEAT**
c_t = 0.84 #turkey (Btu/lb/F)
c_d = 0.77 #ducken (Btu/lb/F)
c_s = 0.86 #stuffing (Btu/lb/F)
#DENSITY**
rho_t = 66.80   #turkey (lbm/ft^3)
rho_d = 64.96  #ducken (lbm/ft^3)
rho_s = 62.69  #stuffing (lbm/ft^3)
#RADII (ft)
r_t = 0.7
r_d = 0.35
```

```
r_s = 0.25
r = [r_t, ((r_t + r_d)/2), r_d, (r_d + r_s)/2, r_s]
#VOLUME (ft^3)
v_t = (4/3)*(r_t**3 - r_d**3)*np.pi
v_d = (4/3)*(r_d**3 - r_s**3)*np.pi
v_s = (4/3)*(r_s**3)*np.pi
#FILM COEFFICIENT
h = 2 #(Btu/hr/ft^2/F)
#OVEN RESISTANCE
R_conv = 1/(h*4*np.pi*r_t**2) #convective resistance
R_rad = 1/((0.98*1.714*10**-9)*(350**2+350**2)*(350+350)) #radiation
(fW^3)
if rad == 0:
  R_oven = R_conv
if rad != 0:
  R_oven = (1/R_conv + 1/R_rad)^-1;
#THERMAL RESISTIVITIES OF FOOD(h*F/Btu)
R = [R_oven]
for i in range(len(r)-1):
    R.append(1/r[i+1]-1/r[i]/(4*np.pi*k[i]))
R.append(1/r_s/(4*np.pi*k_s)) #resistance of stuffing
#CAPACITANCE (Btu/F)**
C1 = C_T = 0.1*rho_t*v_t*c_t/2 #turkey capacitance
C2 = C_D = 0.1*rho_d*v_d*c_d #ducken capacitance
C3 = C_S = 0.1*rho_s*v_s*c_s #stuffing capacitance
print(C1, C2, C3)
#Solving Eqs
from sympy.solvers.ode.systems import dsolve_system
t = sp.symbols("t")
T2 = sp.Function("T2")(t)
T4 = sp.Function("T4")(t)
T6 = sp.Function("T6")(t)
eqns = []
#equations
eqns.append(sp.Eq(T2.diff(t),1/C1*((Tinf-T2)/(R[0]+R[1]) - (T2-
T4)/(R[2]+R[3])+q_in/2)))
eqns.append(sp.Eq(T4.diff(t),1/C2*((T2-T4)/(R[2]+R[3])- (T4-
T6)/(R[4]+R[5])+q_in/2)))
eqns.append(sp.Eq(T6.diff(t),1/C3*((T4-T6)/(R[4]+R[5]))))
sols = sp.dsolve(eqns,
ics={T2.subs(t,0):42,T4.subs(t,0):42,T6.subs(t,0):42})
T2s = sols[0].rhs
T4s = sols[1].args[1]
T6s = sols[2].args[1]
T1s = (Tinf - T2s)*R[0]/(R[0]+R[1])+Tinf
```

```
T3s = -(T2s-T4s)*R[2]/(R[2]+R[3])+T2s
T5s = (T4s-T6s)*R[4]/(R[4]+R[5])+T4s
cook = 165
burn = 280
p = sp.plot(T2s, T4s,T6s,cook, burn,  (t, 0, T_end), show = False,
axis_center = [0,0], xlabel = 'Time (hr)', ylabel = 'Temperature [F]',
legend = 'True')
p[0].line_color = 'r'
p[0].label = 'Turkey'
p[1].line_color = 'orange'
p[1].label = 'Ducken'
p[2].line_color = 'blue'
p[2].label = 'Stuffing'
p[3].label = '165 F'
p[4].label = '280 F'
p.show()
```

References

2006 ASHRAE Handbook - Refrigeration (2006). American Society of Heating Refrigerating & Air Conditioning Engineers Incorporated.

Fundamentals of Thermal Resistance (2020) *Celsia*. Available at: https://celsiainc.com/heat-sink-blog/fundamentals-of-thermal-resistance/ (Accessed: 20 November 2023).

Thermal Resistance (2012) *Neutrium*. Available at: https://neutrium.net/heat-transfer/thermal-resistance/ (Accessed: 20 November 2023).