# Prime Number

•) A number is said to be prime if it's only divisors are 1 and itself.

| n = 13 | n = 4 | n = 101 |
|---|---|---|
| ↓ | ↓ | ↓ |
| 1 × 13 | 2 × 2 | 1 × 101 |
| Prime | 4 × 1 Not prime | Prime |

Note: 2 is the only even prime number

1 is not prime it is a composit number

## Naive solution

```
boolean isPrime(int n)
{
    if (n==1)
        return false;

  for (int i=2; i<n; i++)
    {
        if(n % i == 0)
            return false;
    }
  return true;
}
```

n = 7

i = 2  7 % 2 ×

i = 3  7 % 3 ×

i = 4  7 % 4 ×

i = 5  7 % 5 ×

i = 6  7 % 6 ×

i = 7  7 % 7 ✓

Time complexity : O(n)

## Effective Solution

Divisors always appear in Pairs.

30 : (1, 30) (2, 15) (3, 10) (5, 6)

65 : (1, 65) (5, 13)

25 : (1, 25) (5, 5)

if $(x, y)$ is a pair

$$x * y = n$$

And if $x \leq y$

$$x * x \leq n$$

$$x \leq \sqrt{n}$$

## Effective Solution

```
boolean isPrime(int n)
{
   if(n==1)
      return false;
   for (int i=2; i*i<=n ; i++)
      {
         if(n%i==0)
            return false;
      }
   return true;
}
```

n = 65

i = 2  65%2

i = 3  65%3

i = 4  65%4

i = 5  65%5

## Another Efficient Solution

By checking $n \% 2 == 0$ and $n \% 3 == 0$, we can save many iterations for large 'n'.

```
boolean isPrime(int n)
{
   if (n==1)
       return false;
   if( n==2 || n==3)
       return true;

   for(int i=5; i*i <=n ; i=i+6)
     {
       if( n%i==0 || n%(i+2)==0)
             return false;
     }
   return true;
}
```

n = 121

i = 5

i = 11