Numbers

Dr. Liam Naughton

1 Numbers

We are familiar with many sets of numbers:

- The **natural numbers** denoted $\mathbb{N}$ are the positive whole numbers.

$$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}.$$

  Sometimes in computer science we include zero in the set of natural numbers. This is a matter of convention. Mathematicians don't usually include zero as an element of $\mathbb{N}$ but if they do they write

$$\mathbb{N}_0 = \{0, 1, 2, 3, 4, 5, \ldots\}.$$

- The **integers** denoted $\mathbb{Z}$ are all the natural numbers and their negatives and zero

$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots, \}$$

- The **rational numbers** denoted $\mathbb{Q}$ are all the numbers of the form

$$\mathbb{Q} = \left\{ \frac{a}{b} : a, b \in \mathbb{Z} \right\}.$$

  We think of the rational numbers as being all numbers which can be written as a **fraction** where the numerator and the denominator are integers. In fact it's quite acceptable to think of the rational numbers as being all fractions as long as you don't forget that whole numbers are fractions! So, for example all of the following are elements of $\mathbb{Q}$.

$$7, -\frac{2}{5}, 0, \frac{5}{3}, \frac{91}{2}.$$

- The set of **real numbers** denoted $\mathbb{R}$ includes all numbers, in particular it includes all of the above as subsets. The real numbers are not so easy to define so for the purposes of this course we will say that $\mathbb{R}$ is a complete ordered field and leave it at that. Just remember that there are numbers in $\mathbb{R}$ which **are not** in $\mathbb{Q}$. For example, each of the following are irrational numbers:

$$\sqrt{2}, \pi, e, \phi.$$

  Even though we often approximate π as

$$\pi \approxeq \frac{22}{7}$$

this is only an approximation to the true value of π. The number e is known as **Euler's number** and its decimal expansion begins

$$e = 2.718281828459045235 36\ldots$$

the last number in the list is ϕ which is often used to represent the **Golden Mean** and it pops up in lots of places. Its value to a few decimal places is

$$\phi = \frac{1+\sqrt{5}}{2} = 1.61803\ldots$$

When you actually think about it, numbers, and number systems, are much more complicated than they look. Our brains have adjusted themselves to seamlessly carry out arithmetic with fractions, decimals etc. but how can we ensure that a computer can mimic these operations in the same way? The number systems above are well defined in a rigorous mathematical fashion, and if we want to be able to carry out simple numerical operations on a computer we need to make sure that we define things in a rigorous way. That's why we use **data types**. We need to ensure that if a computer is asked to add numbers of different types it has some logical set of rules to follow.

1.1 Numbers & Data Types

Data type specifies the size and type of values that can be stored in an identifier. Different data types allow you to select the type appropriate to the needs of the application.

1.1.1 Integers

Integer types can hold whole numbers which are integers (unsurprisingly). The size of the values that can be stored depends on the integer type that we choose.

Type	Size	Range
byte	1 byte	-128 to 127
short	2 bytes	-32768 to 32767
int	4 bytes	$-2,147,483,648$ to $2,147,483,647$
long	8 bytes	$9,223,372,036,854,775,808$ to $9,223,372,036,854,755,807$

1.1.2 Floating Point

Floating point data types are used to represent numbers rational and real numbers other than integers. The degree of precision depends on the type that we choose

Type	Size	Range
float	4 bytes	$3.4 \times e^{-38}$ to $3.4 \times e^{38}$
double	8 bytes	$1.7 \times e^{-308}$ to $1.7 \times e^{308}$

Java also includes the data types **character** (`char`) and **boolean** (`boolean`) to store characters and values with true/false states respectively.