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Spatial AI & Robotics Lab

CSE 473/573
L2: CAMERA MODEL

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Content

- Pinhole Camera
  - Perspective Projection
  - Projective Geometry
  - Vanishing Points and Lines
  - Homogeneous Coordinates
  - Matrix form of Perspective Projection
  - Intrinsic and Extrinsic Parameters
- Other type of projection
  - Weak Perspective Projection
  - Photo Finish Photography
  - 360 Imaging
  - Tilt-shift Imaging
- Camera Calibration

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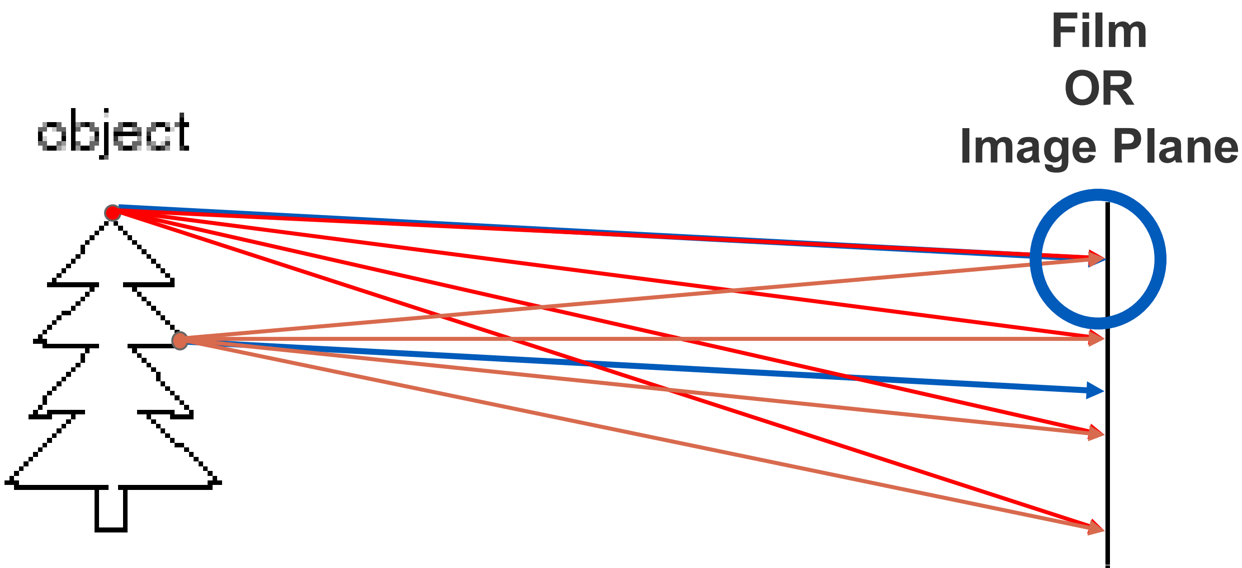
IMAGE FORMATION I

Pinhole Camera

Image formation

Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

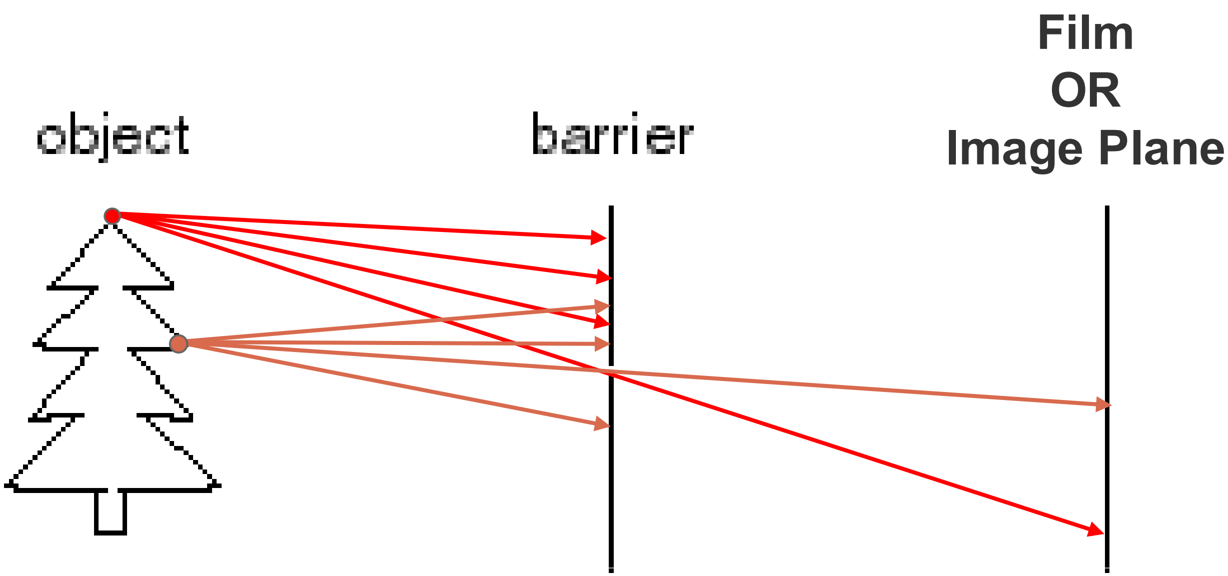


Answer is No….

Pinhole camera

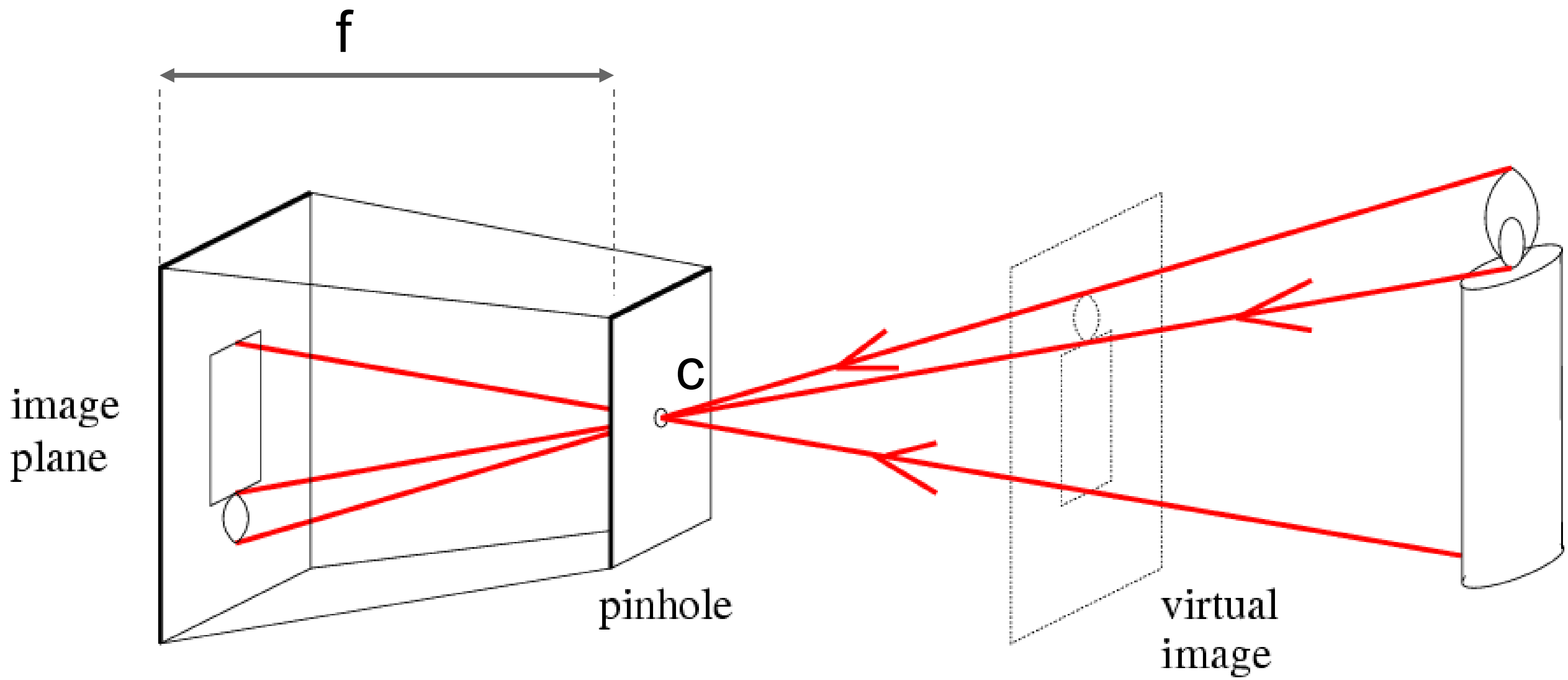
Idea 2: add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**



Slide source: Seitz

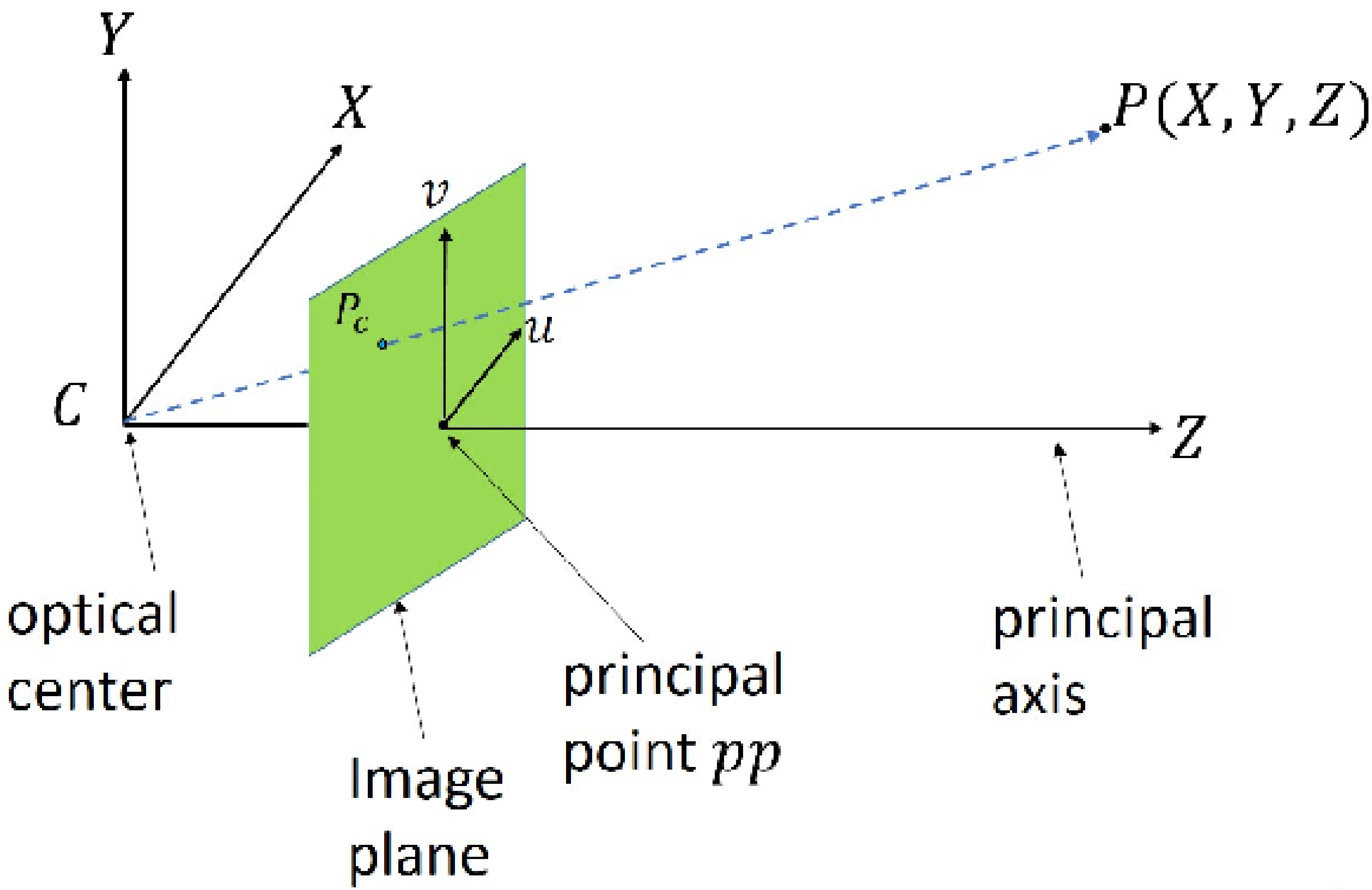
Pinhole camera



f = focal length
c = center of the camera

Figure from Forsyth

Perspective Projection



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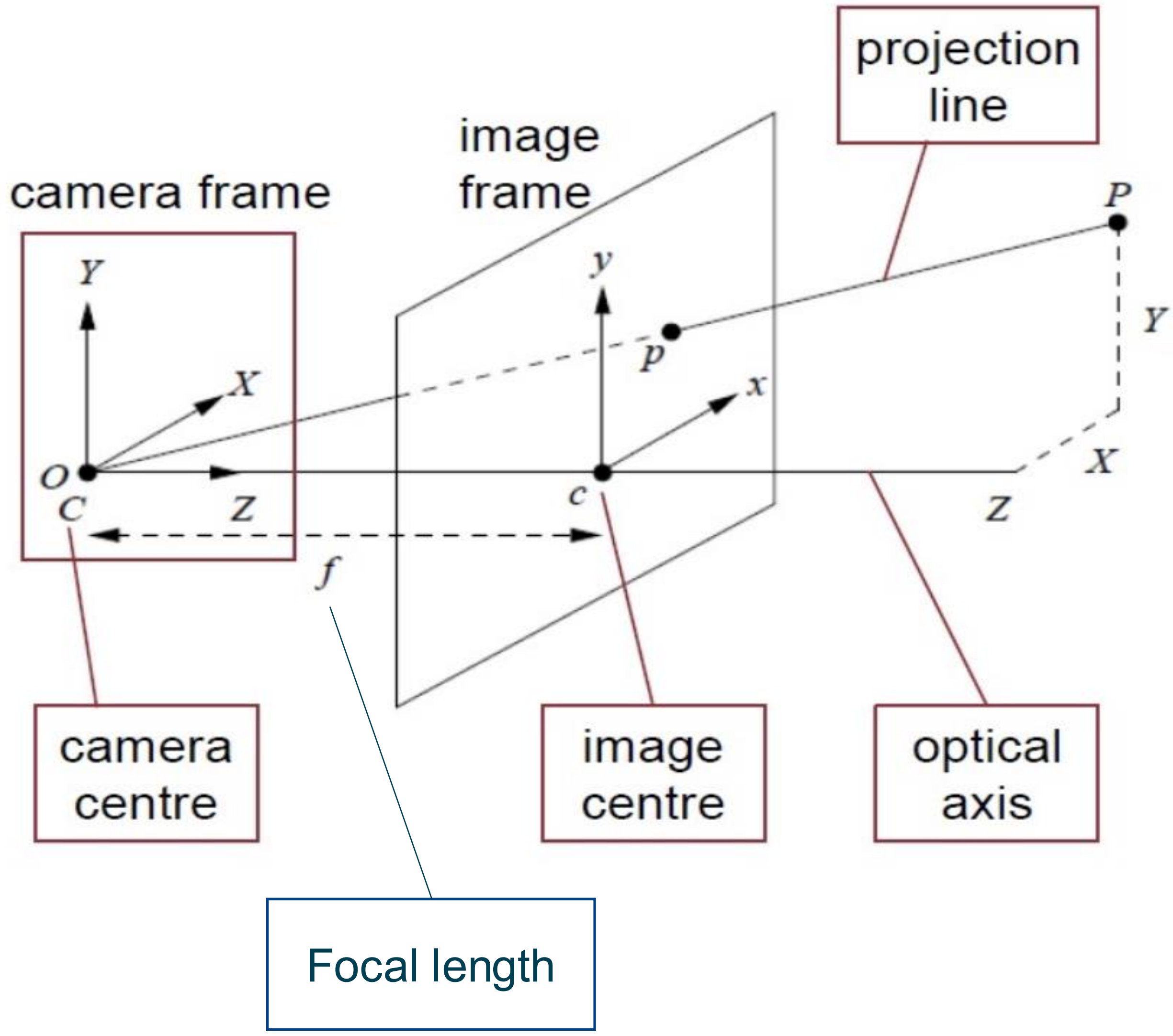
Other names

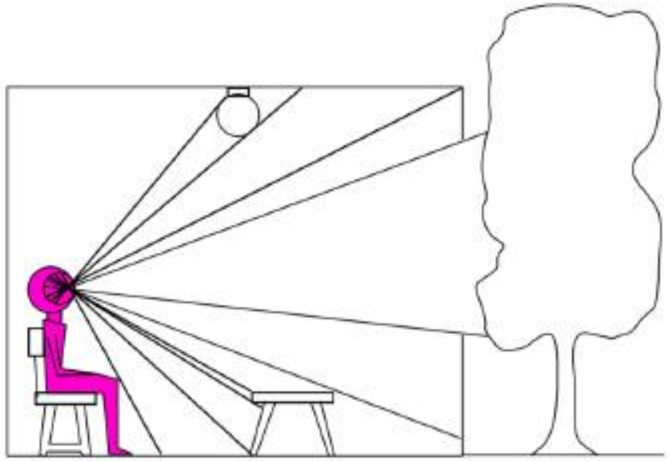


camera frame
image
frame
projection
line
Y
X
O
C
Z
y
p
x
c
P
Y
X
Z
f
camera
centre
image
centre
optical
axis
Focal length

What do we lose?

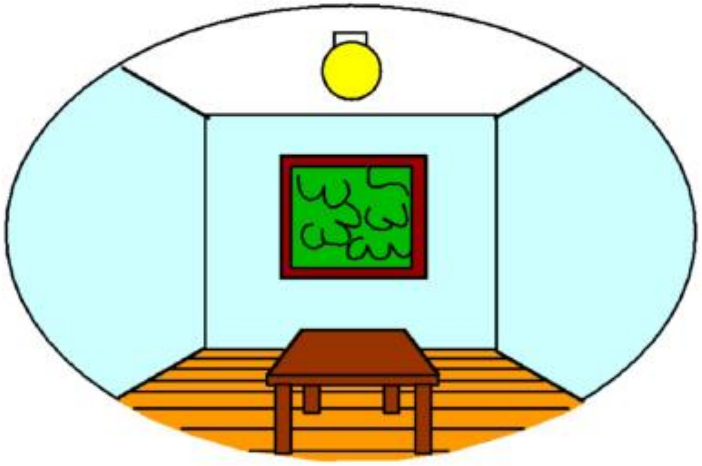
- Projection from 3D world to 2D Image

3D world



Point of observation

2D image



Long range objects appear to be small.

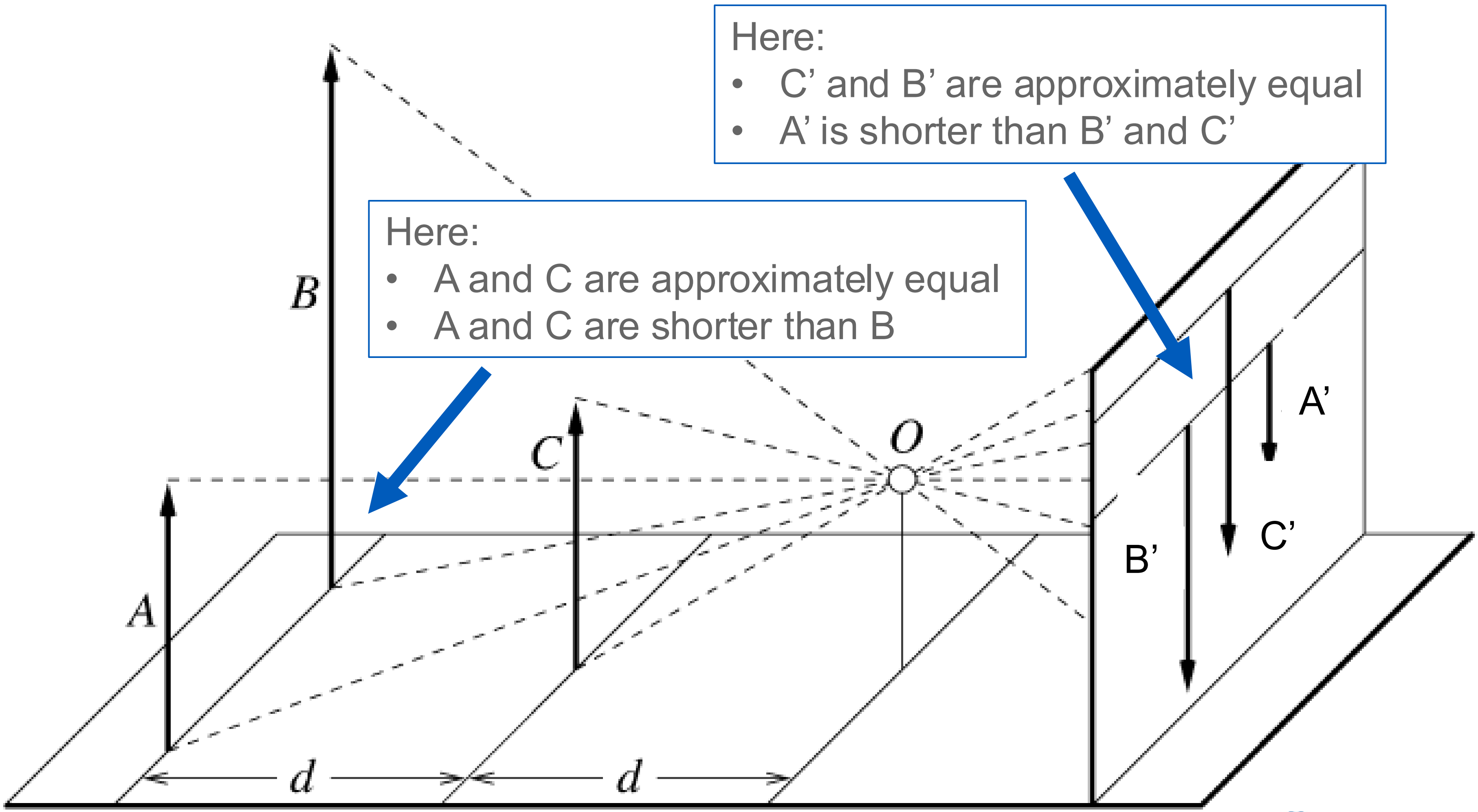


Figure by David Forsyth

Perspective Projection - Example

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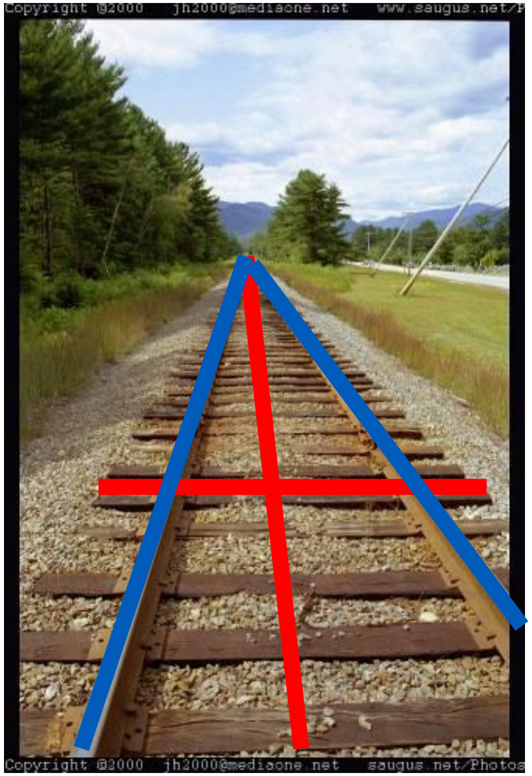
Perspective Projection - Example

What do we lose geometrically?

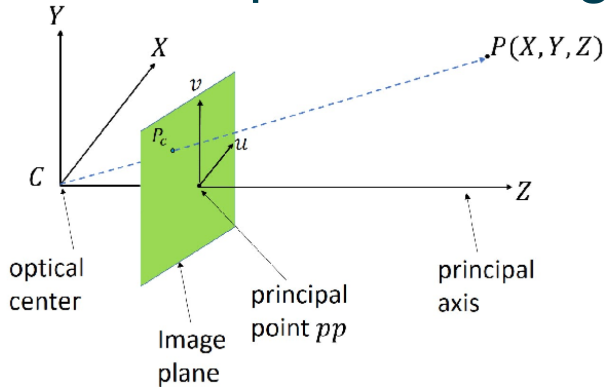


- **Angles**
- **Distances**
- **and therefore Area**

Geometrical Properties

Many-to-one:

Points along same ray map to same point in image.



Points map to → ?

- **points**

Lines map to → ?

- **lines**

Distances and angles (are / are not **?)** preserved

- **are not**

Degenerate cases:

– Line through focal point projects to a point.

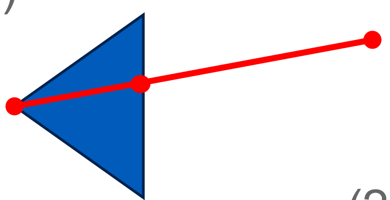
– Plane through focal point projects to line

– Plane (not parallel to image plane) projects to part of the image.

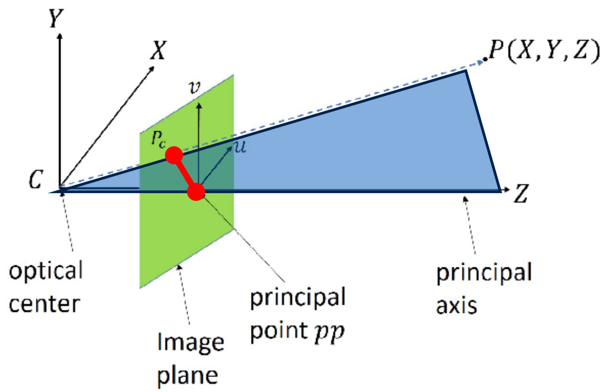
Degenerate cases

- (1) Line through focal point projects to a point.
- (2) Plane through focal point projects to line
- (3) Plane (not parallel to image plane) projects to part of the image.

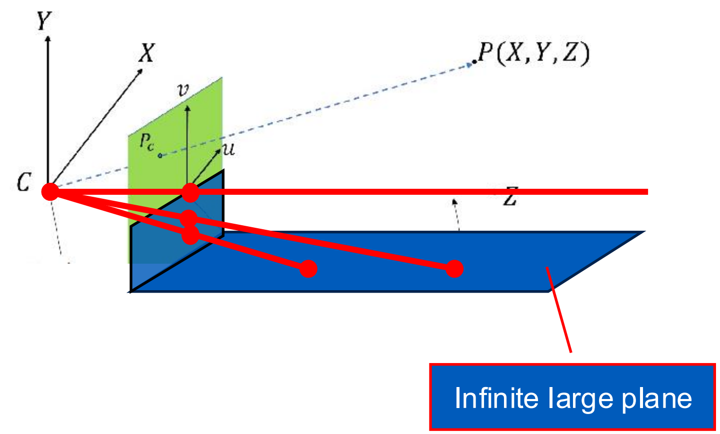
(1)



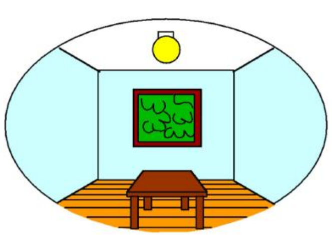
(2)



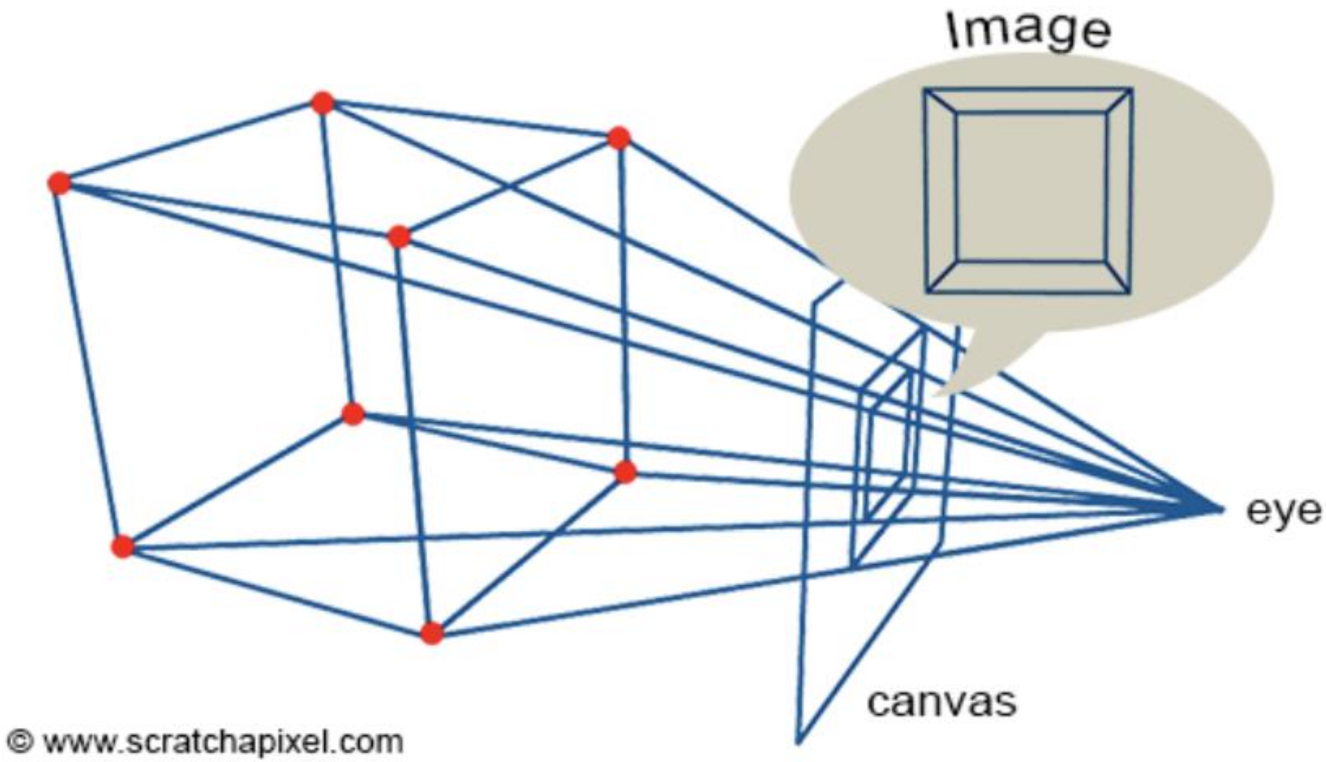
(3)



2D image



Perspective Projection - Example

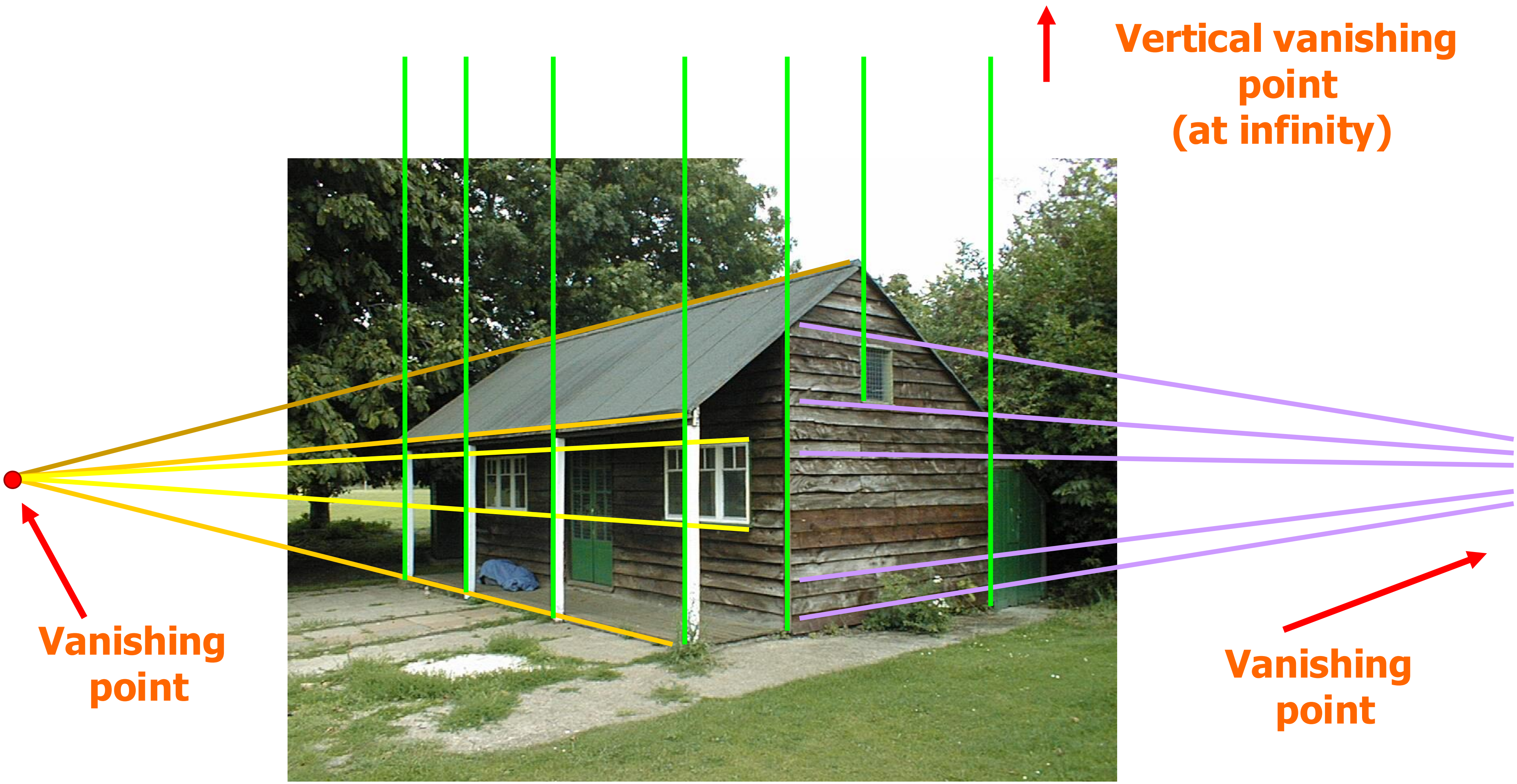


Vanishing points and lines

Parallel lines in the world intersect in the image at a “vanishing point”

- Angles are not preserved
- Parallel lines meet!

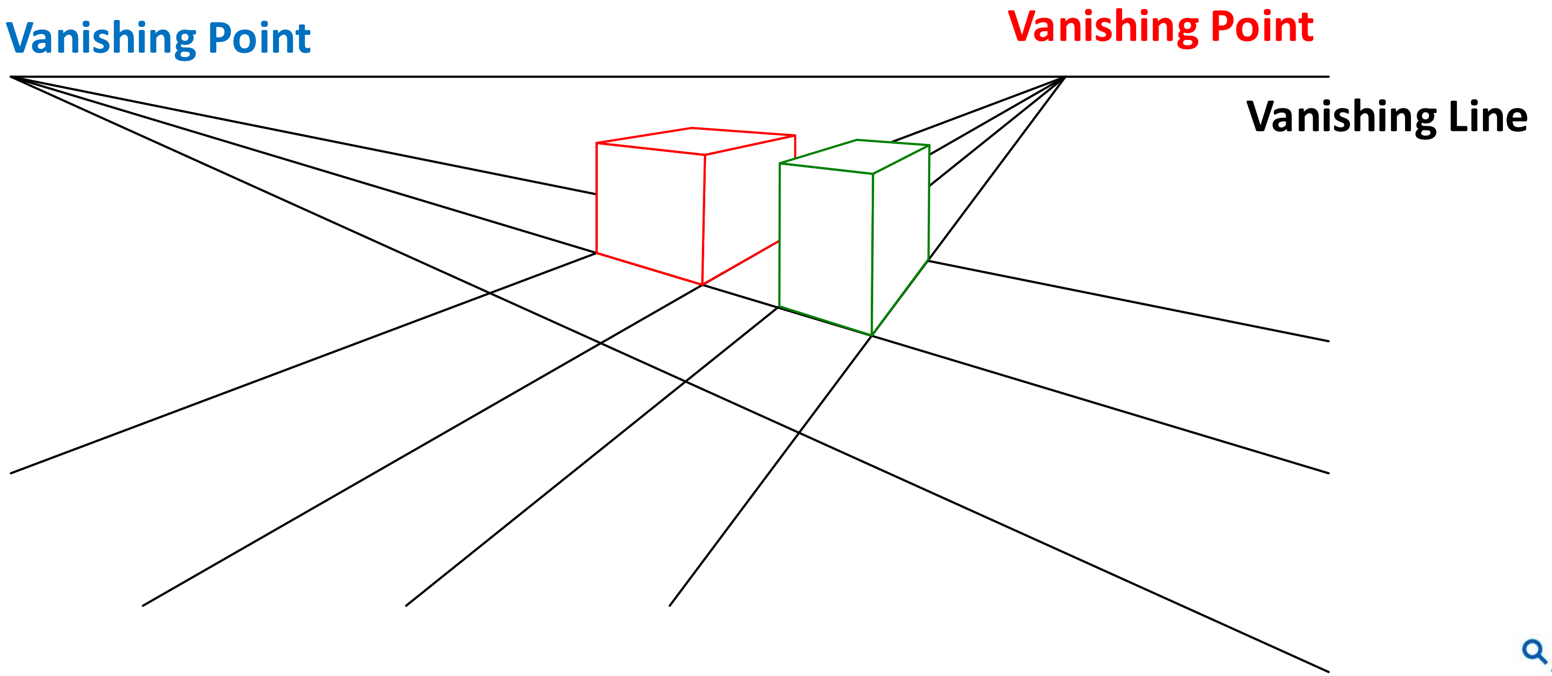
Vanishing points and lines



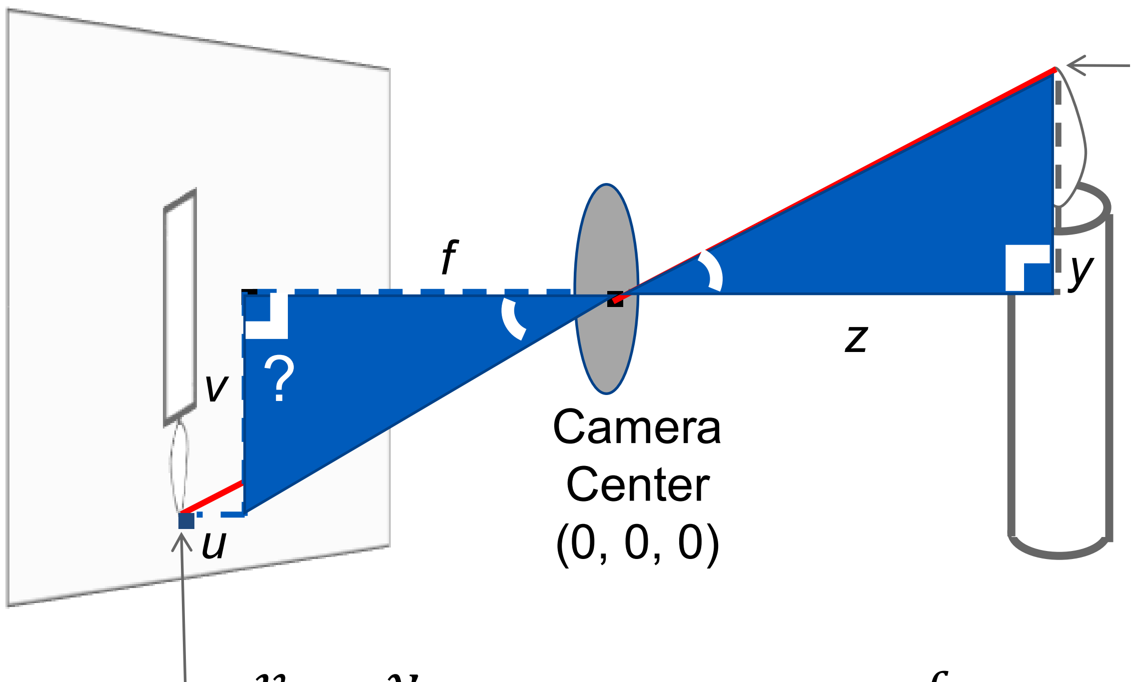
Slide from Efros, Photo from Criminisi

Vanishing points and lines

- Any two lines, parallel in 3D will meet at a unique vanishing point in image plane.
- All pairs of parallel lines on the same plane in 3D will have vanishing points on a unique vanishing line.



Perspective Projection



$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x = \begin{bmatrix} u \\ v \end{bmatrix}$$

Sides are Proportional

$$\frac{v}{-f} = \frac{y}{z} \qquad v = -y * \frac{f}{z}$$

$$\frac{u}{-f} = \frac{x}{z} \qquad u = -x * \frac{f}{z}$$

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Perspective Projection Equations

- 3D point $\mathbf{P} = (X, Y, Z)^{\mathsf{T}}$ projects to 2D image point $\mathbf{p} = (x, y)^{\mathsf{T}}$.
- By symmetry,

$$\frac{X}{Z} = \frac{x}{f}, \quad \frac{Y}{Z} = \frac{y}{f}$$

i.e.,

$$x = f\frac{X}{Z}, \quad y = f\frac{Y}{Z}$$

- Simplest form of perspective projection.

Preliminary: Homogeneous coordinates

Converting to *homogeneous* coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

- A way of representing N-dimensional coordinates with N+1 numbers.
- It allows perspective projection representing as matrix-vector multiplication.

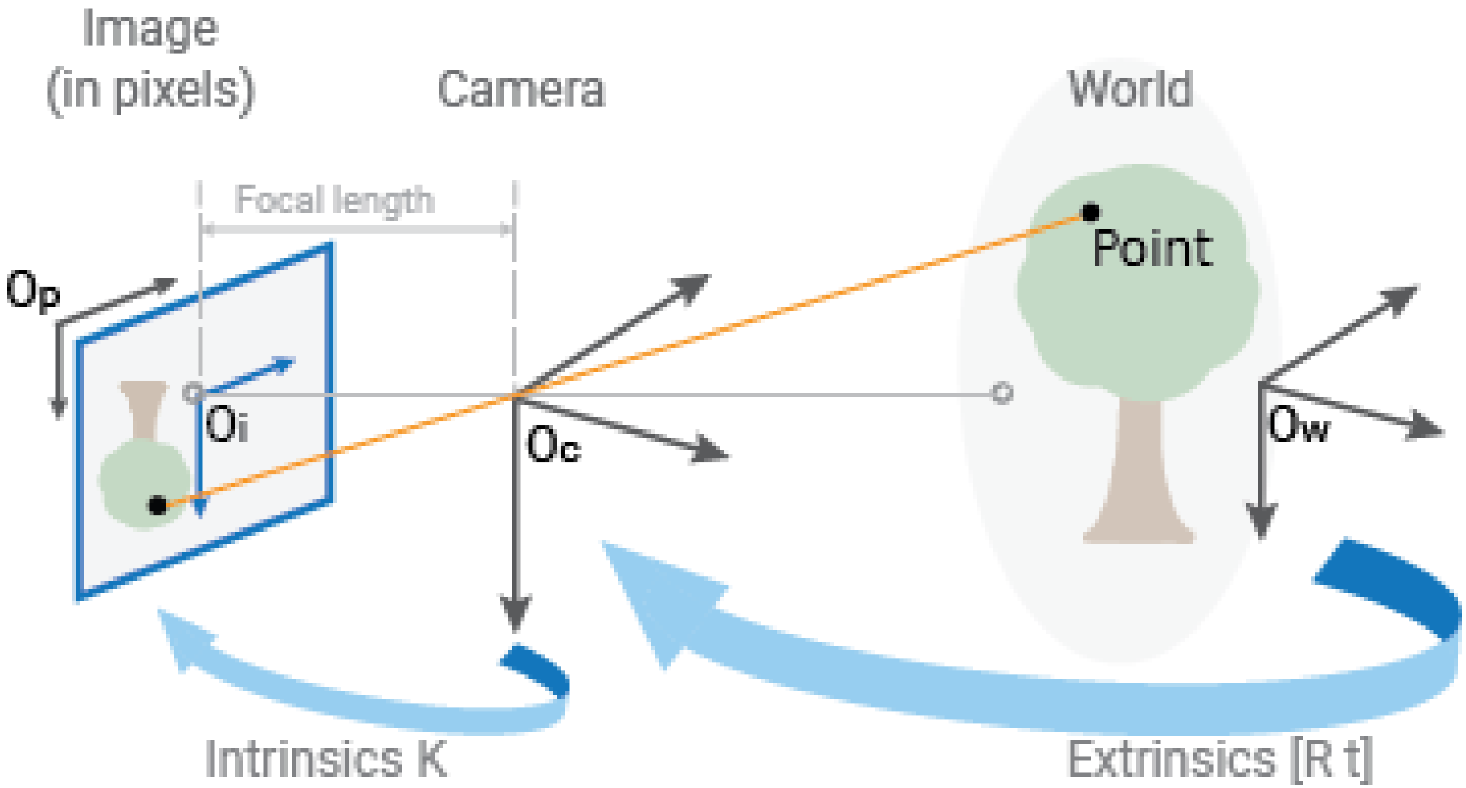
Perspective Projection

- Matrix Form

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

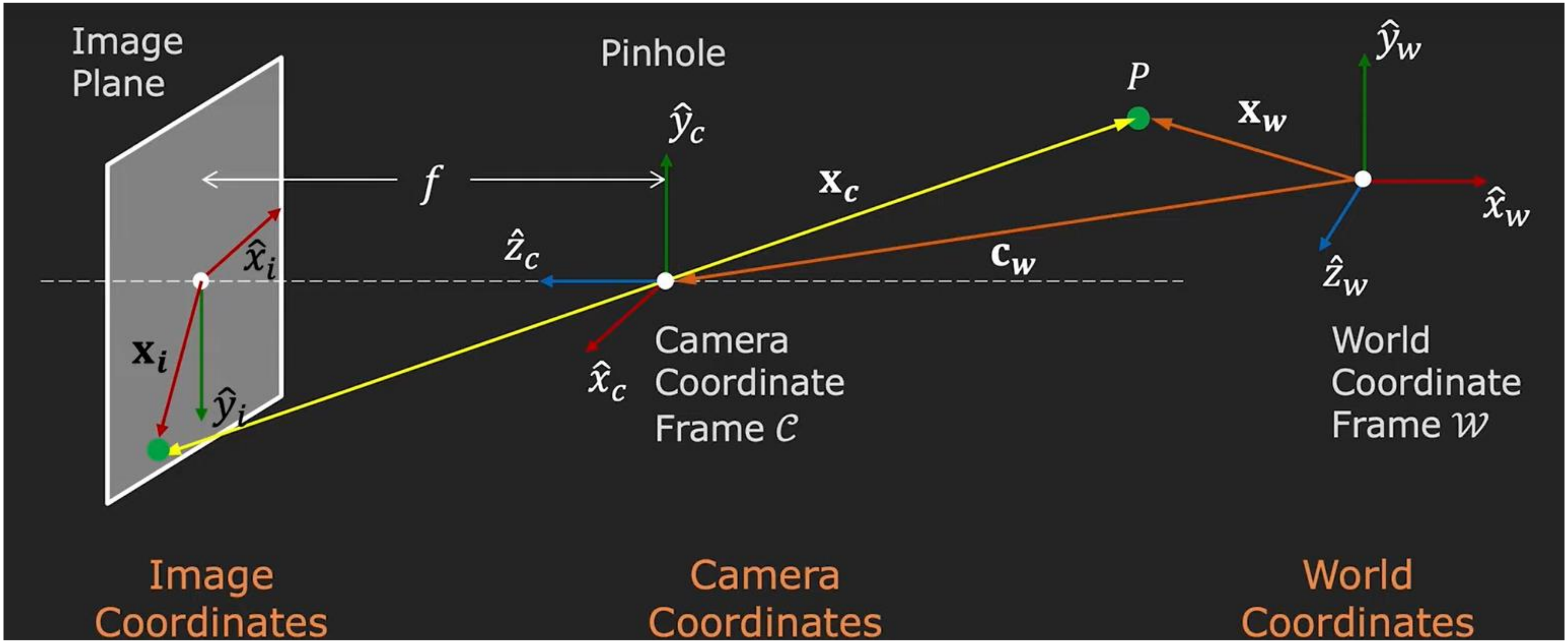
Intrinsic and Extrinsic Parameters

- Perspective equations so far in terms of *camera's* frame.
- Camera's *intrinsic* (Camera) and *extrinsic* (World) parameters needed to calibrate geometry.



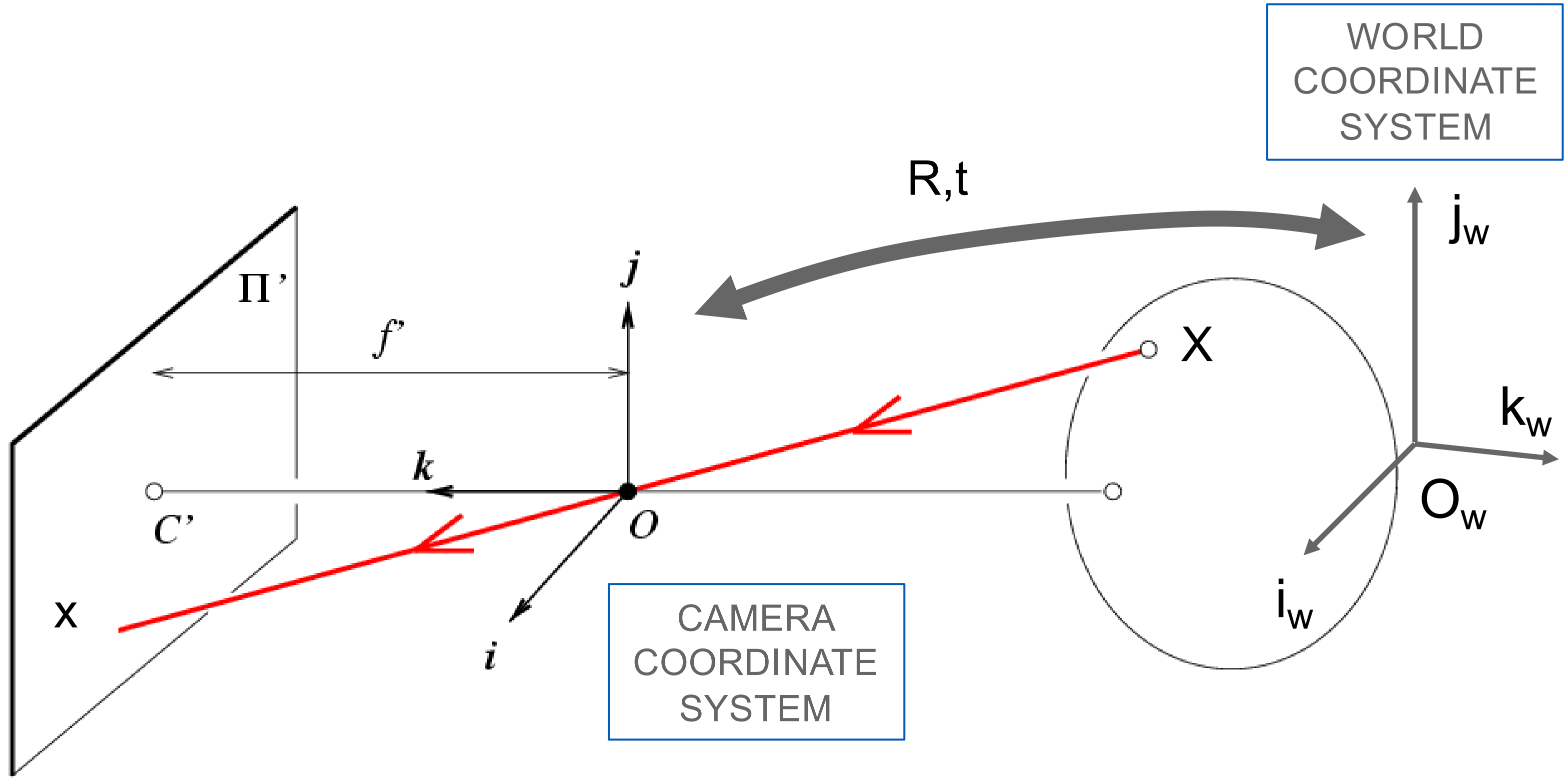
K. Grauman

Three different coordinate systems



$\mathbf{X}_i$ is the projection of point P in the world

Projection Matrix



$$\mathbf{x} = \mathbf{K}\underbrace{[\mathbf{R} \quad \mathbf{t}]}_{\text{Extrinsic Matrix}}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Intrinsic and Extrinsic Parameters

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

2D Image Coordinates

Intrinsic properties (Optical Centre, scaling)

Extrinsic properties (Camera Rotation and translation)

3D World Coordinates

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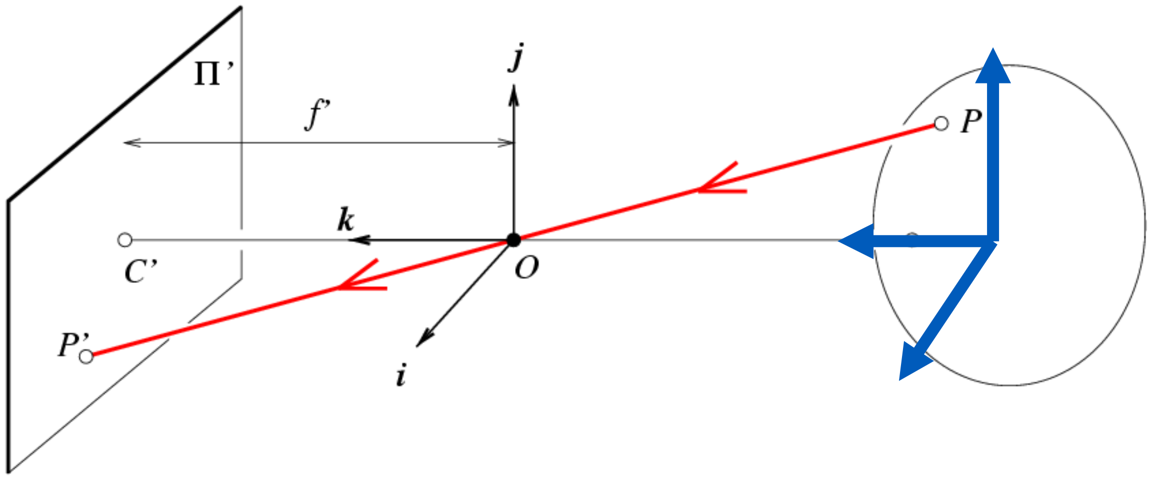
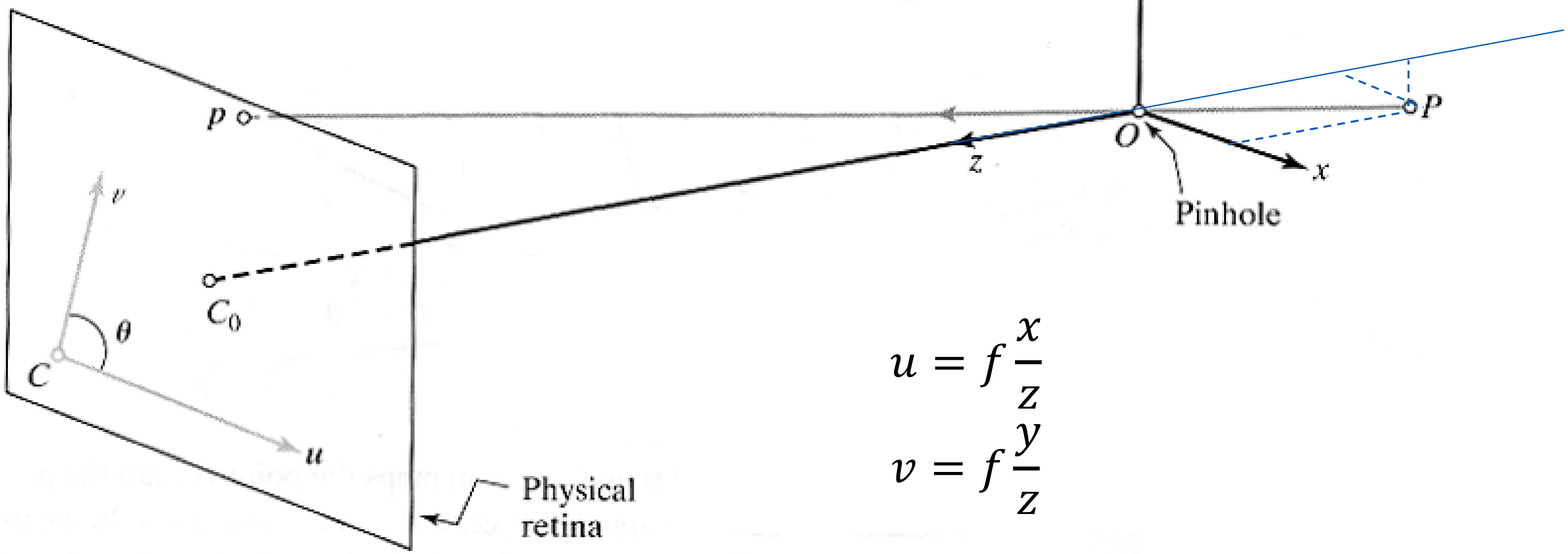
Idealized (Pinhole) Assumptions

- Everything is a unit length
  - Focal Length is 1 unit, pixels are 1 unit
- The camera and the world coordinate systems are aligned with the image plane.
  - No Rotation or Translation



Intrinsic parameters: from World to Pixel

$p = (u,v)$ focal length $= f$ $P = (x,y,z)$

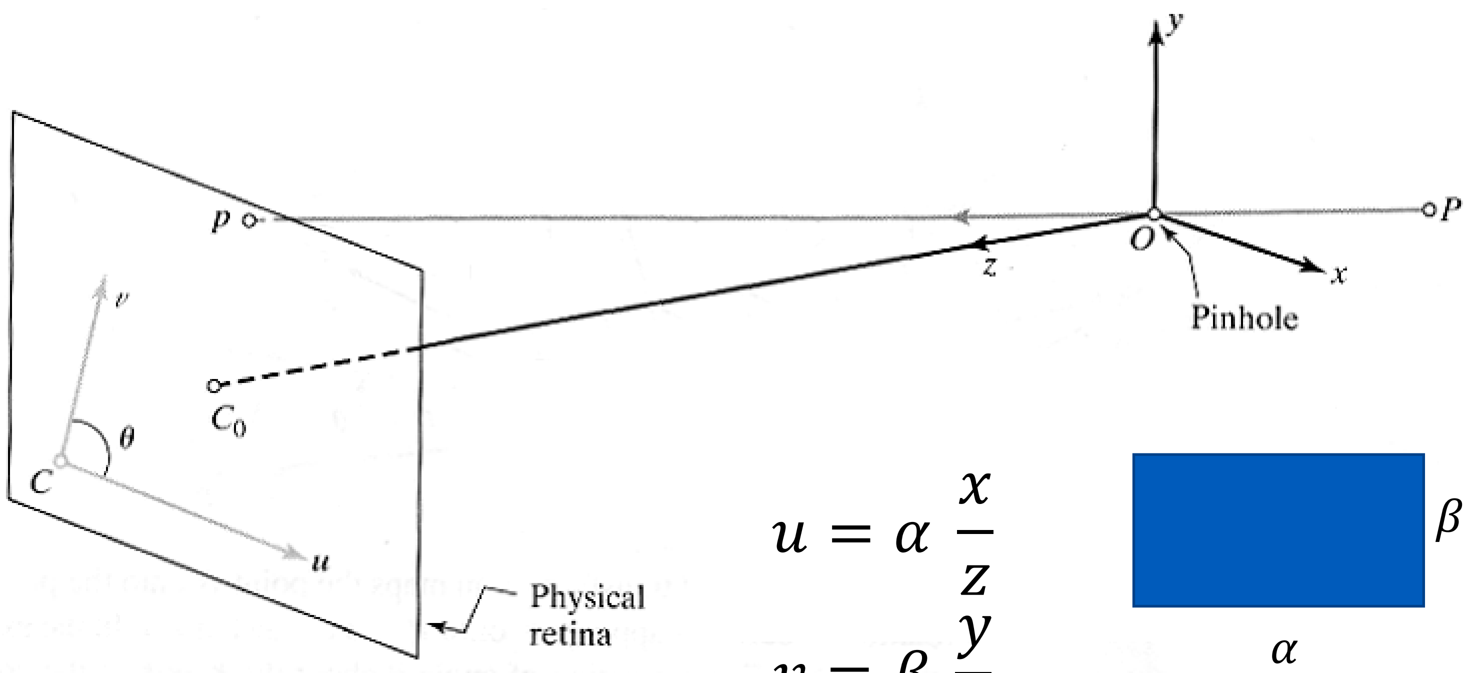


$$u = f\frac{x}{z}$$
$$v = f\frac{y}{z}$$

Perspective projection

Forsyth&Ponce

Intrinsic parameters: Relax pixels aspect



$$u = \alpha \frac{x}{z}$$

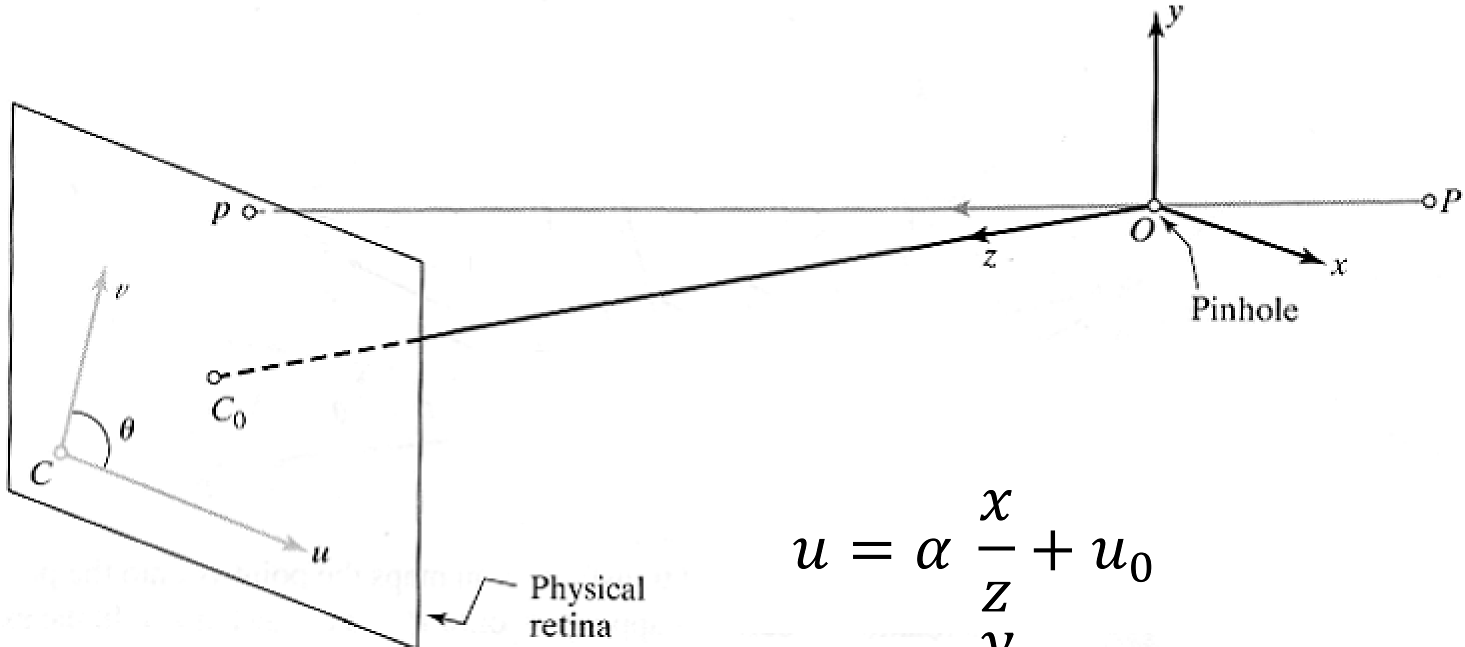
$$v = \beta \frac{y}{z}$$

β

α

Pixels may not be square

Intrinsic parameters: Relax image center



$$u = \alpha \frac{x}{z} + u_0$$

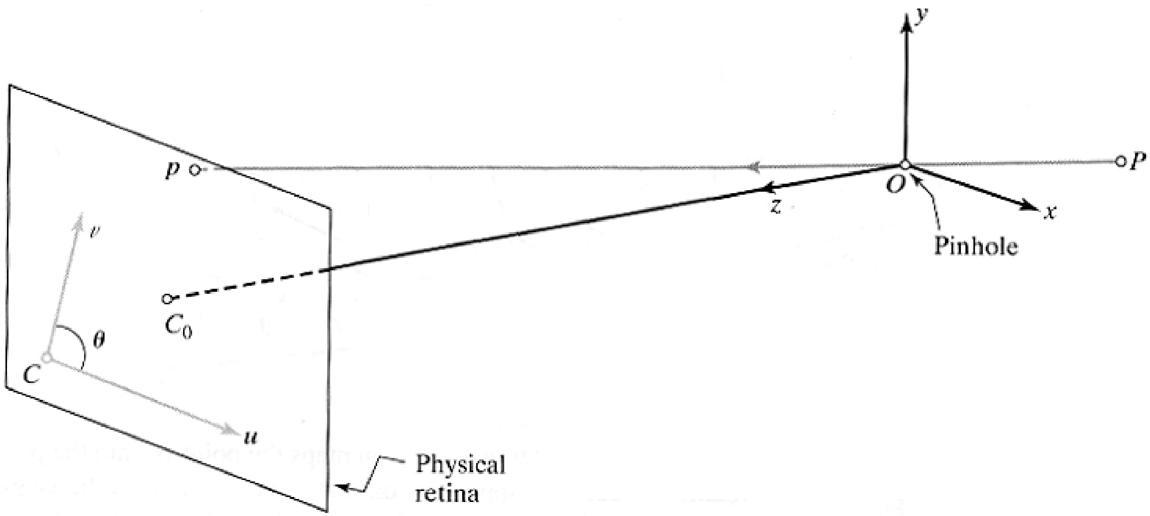
$$v = \beta \frac{y}{z} + v_0$$

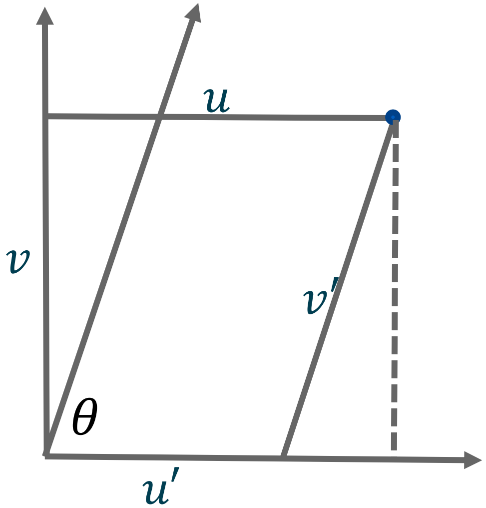
Origin of our camera pixel coordinates is offset

Intrinsic parameters

Relax the image plane is orthogonal to the camera axis:

May be skew between camera pixel axes





$$v' \sin(\theta) = v$$
$$u' = u - \cos(\theta) v' = u - \cot(\theta) v$$

$$u = \alpha \frac{x}{z} - \alpha \cot(\theta) \frac{y}{z} + u_0$$
$$v = \frac{\beta}{\sin(\theta)} \frac{y}{z} + v_0$$

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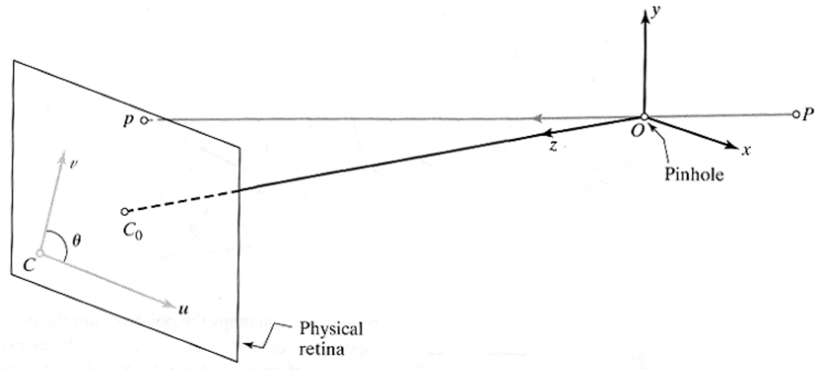
W. Freeman

Intrinsic parameters: Matrix form

- Using homogenous coordinates



$$u = \alpha \frac{x}{z} - \alpha \cot(\theta) \frac{y}{z} + u_0$$

$$v = \frac{\beta}{\sin(\theta)} \frac{y}{z} + v_0$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & -\alpha\cot(\theta) & u_0 & 0 \\ 0 & \dfrac{\beta}{\sin(\theta)} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

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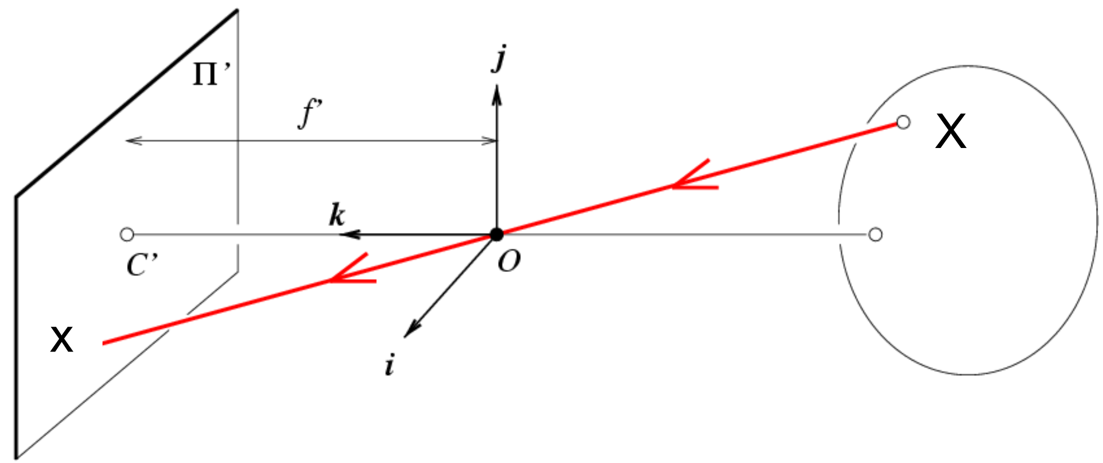
Projection matrix

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)



Intrinsic Assumptions

- Optical center at (0,0)
- Unit aspect ratio
- No skew

K

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \left[\begin{array}{ccc|c} f & 0 & 0 & 0\\ 0 & f & 0 & 0\\ 0 & 0 & 1 & 0\end{array}\right]\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}$$

Slide Credit: Savarese

Remove assumption: known optical center

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

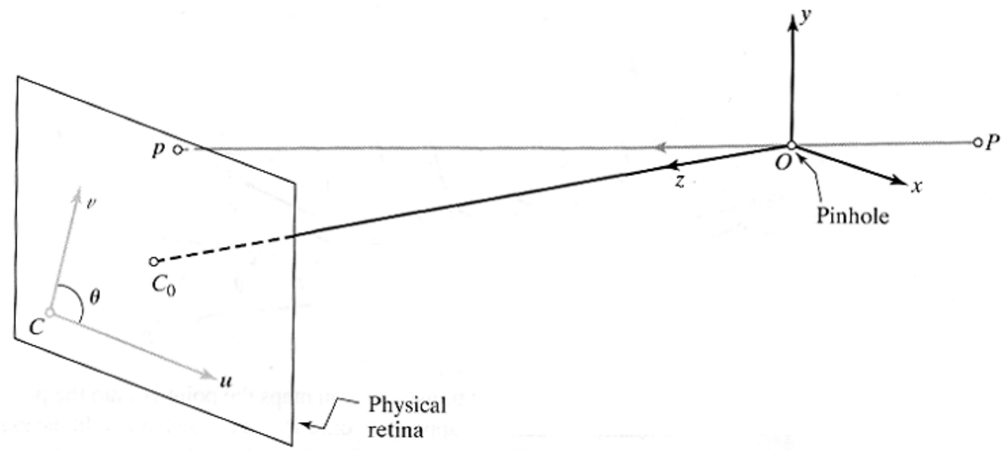
x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

Intrinsic Assumptions

- ~~Optical center at (0,0)~~
- Unit aspect ratio
- No skew



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \quad \Rightarrow \quad w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix} f & 0 & u_0 & 0 \\ 0 & f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Remove assumption: square pixels

$$\mathbf{x} = \mathbf{K}\left[\mathbf{R} \quad \mathbf{t}\right]\mathbf{X}$$

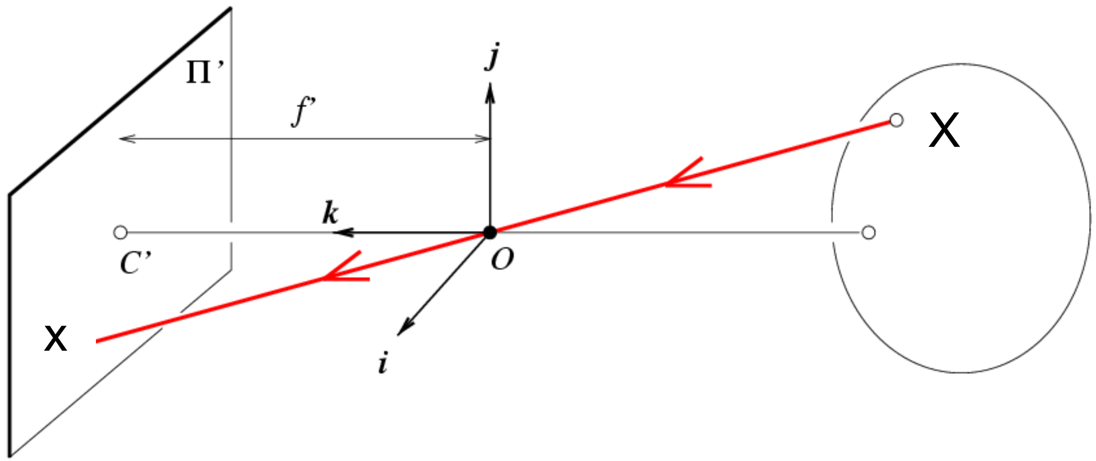
x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

Intrinsic Assumptions

- ~~Optical center at (0,0)~~
- ~~Unit aspect ratio~~
- No skew

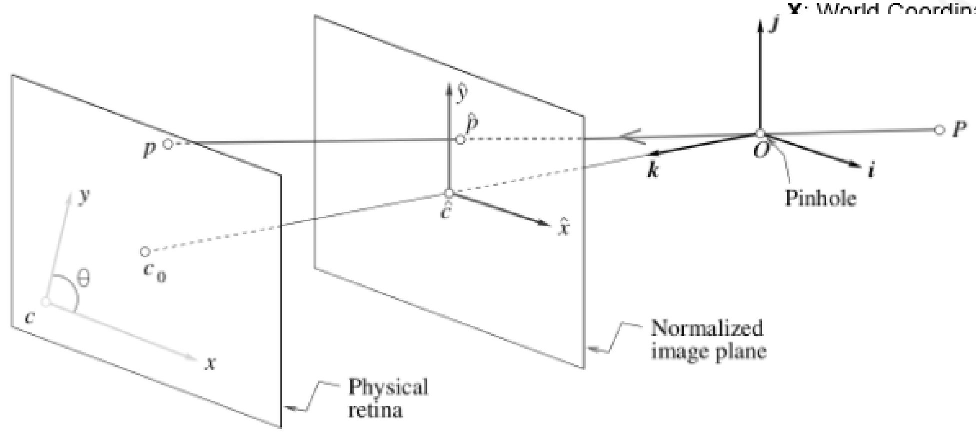


$$\mathbf{x} = \mathbf{K}\left[\mathbf{I} \quad \mathbf{0}\right]\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Remove assumption: non-skewed pixels

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

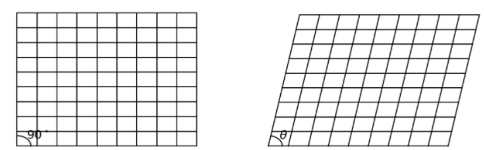


Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

Intrinsic Assumptions

- ~~Optical center at (0,0)~~
- ~~Unit aspect ratio~~
- ~~No skew~~



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0 & 0\\0 & \beta & v_0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Note: different books use different notation for parameters

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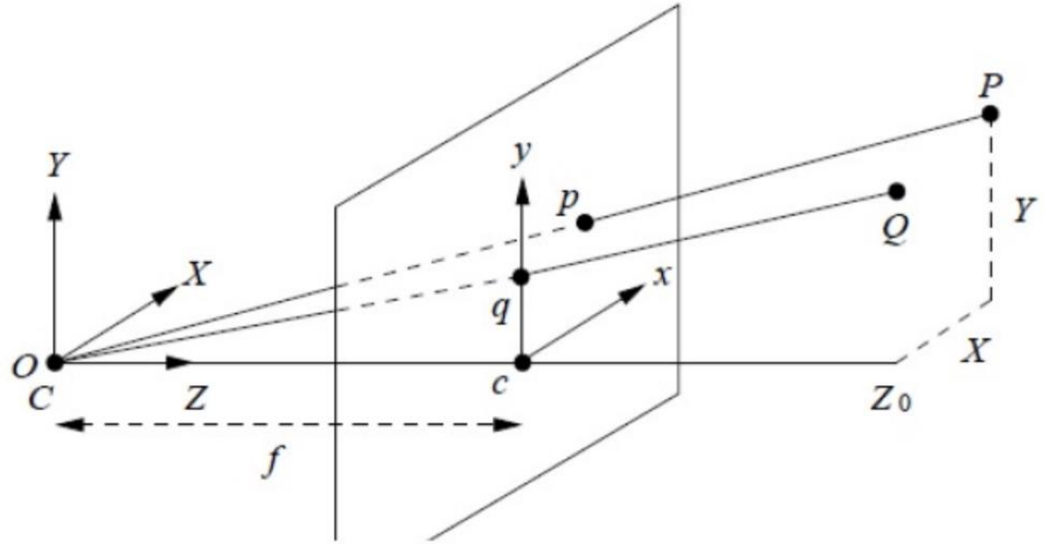
IMAGE FORMATION I

Other types of projection

Weak Perspective Projection

- When you zoom your camera very far, the depth changes in the scene are negligible when compared to the distance from camera



$$u = f\frac{x}{z}$$
$$v = f\frac{y}{z}$$

- Scene depth << distance to camera.
- Z is the same for all scene points, say Z_0

$$x = sX, \quad y = sY, \quad s = \frac{f}{Z_0} \text{ for all scene points.}$$

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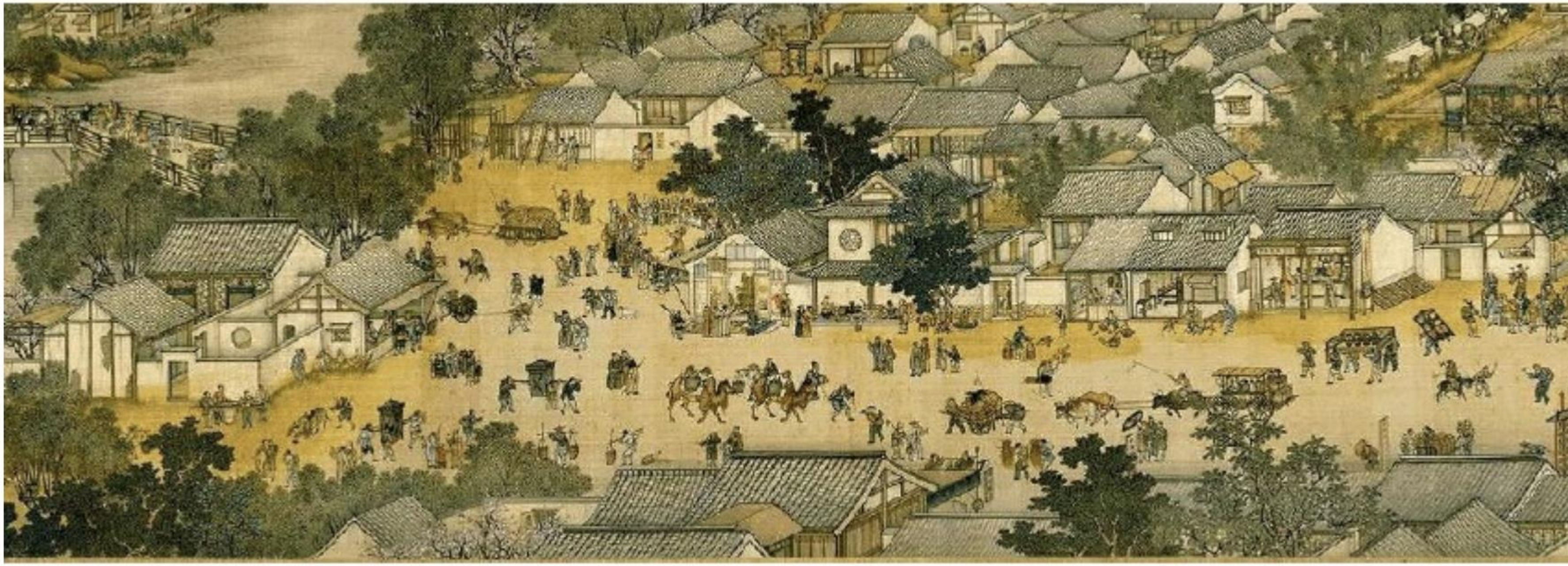
Weak Perspective Projection - Example

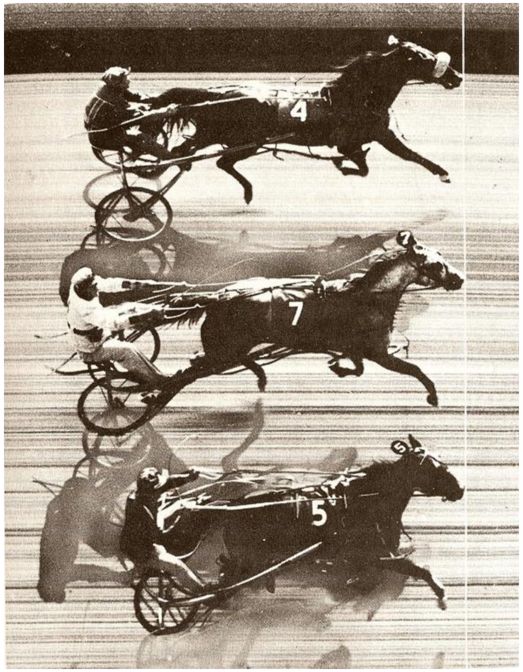


Qingming Festival by the Riverside   Zhang Zeduan ~900 AD

As a comparison

Photo Finish Photography (strip photography)

- A photo-finish image is made by a camera with an extremely thin, vertical slit.
- A photo-finish camera records an object moving past its exposure slit over time. 1D array



1960 Olympic tryouts at Palo Alto, CA

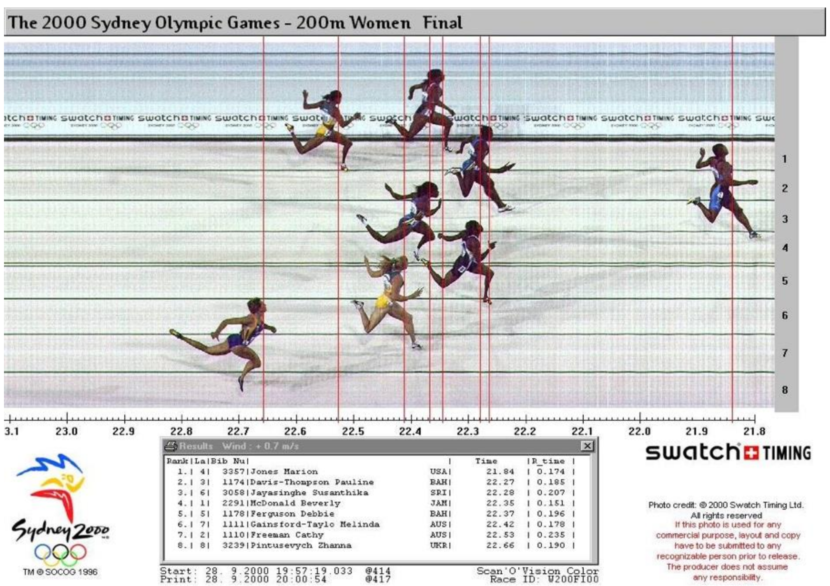
Photo Finish Example

SprintTimer - Photo Finish 4+

Professional Sports Timer

Sten Kaiser

2,490.00₸ · Offers In-App Purchases



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2.	3	1174	Davis-Thompson Pauline	BAH	22.27	0.185
3.	6	3058	Jayasinghe Susanthika	SRI	22.28	0.207
4.	1	2291	McDonald Beverly	JAM	22.35	0.151
5.	5	1178	Ferguson Debbie	BAH	22.37	0.196
6.	7	1111	Gainsford-Taylo Melinda	AUS	22.42	0.178
7.	2	1110	Freeman Cathy	AUS	22.53	0.235
8.	8	3239	Pintusevych Zhanna	UKR	22.66	0.190

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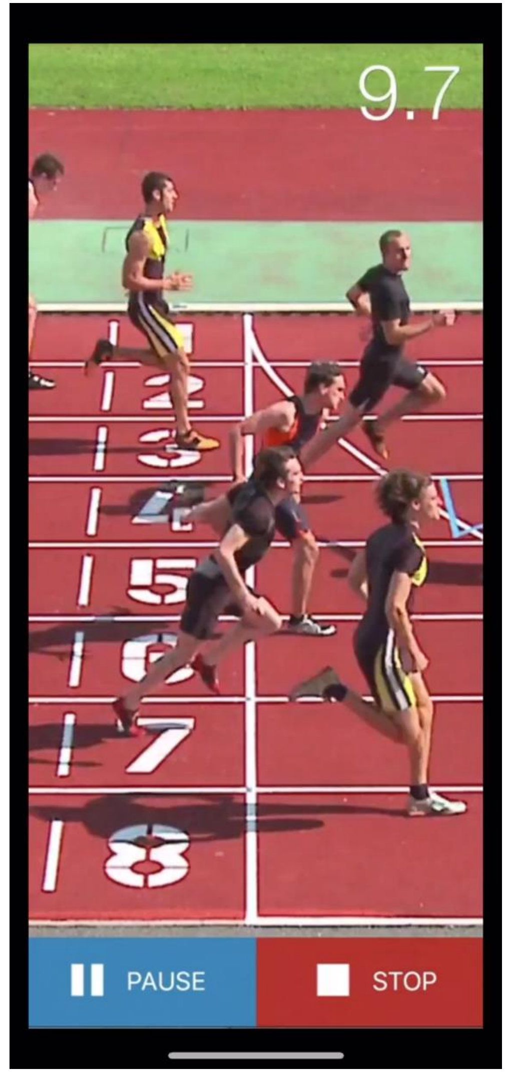
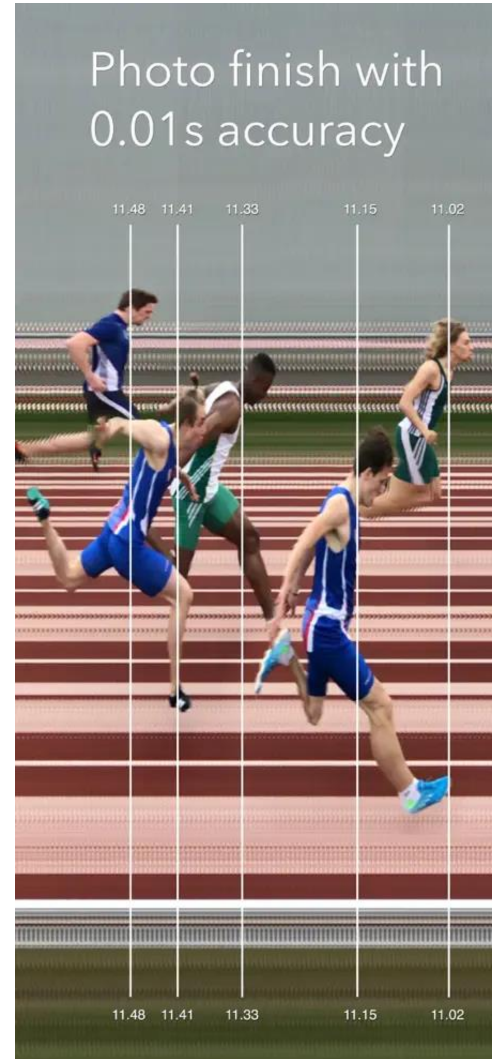
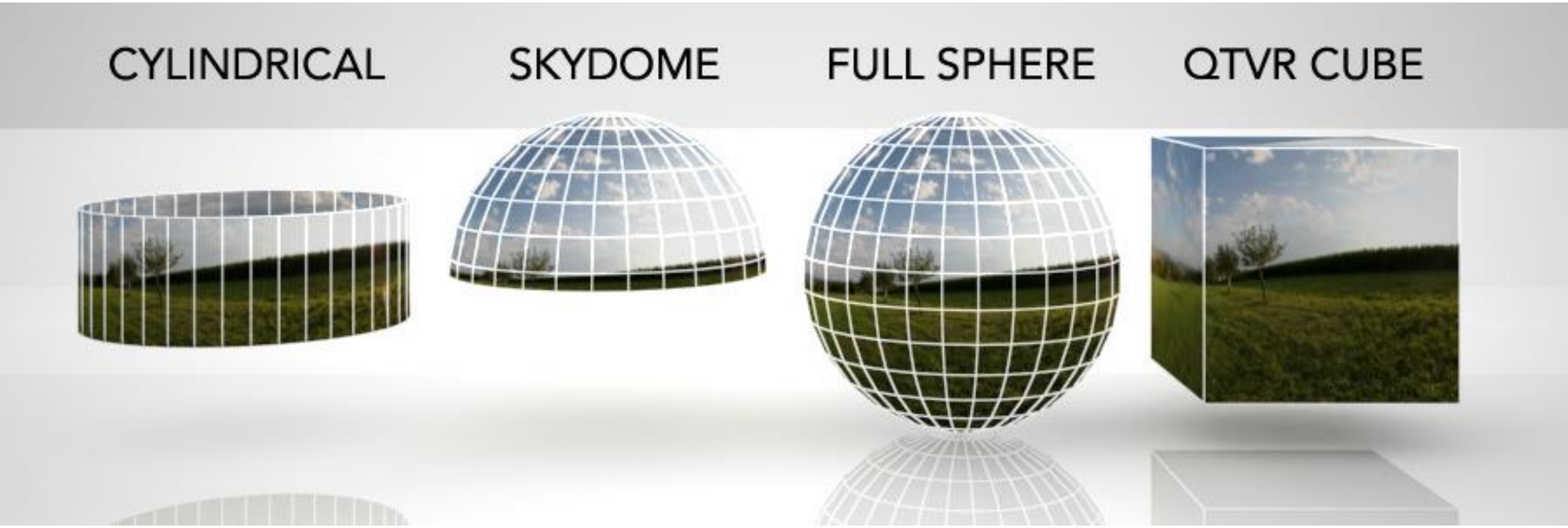




360 Imaging

360 Camera and Projection Mode

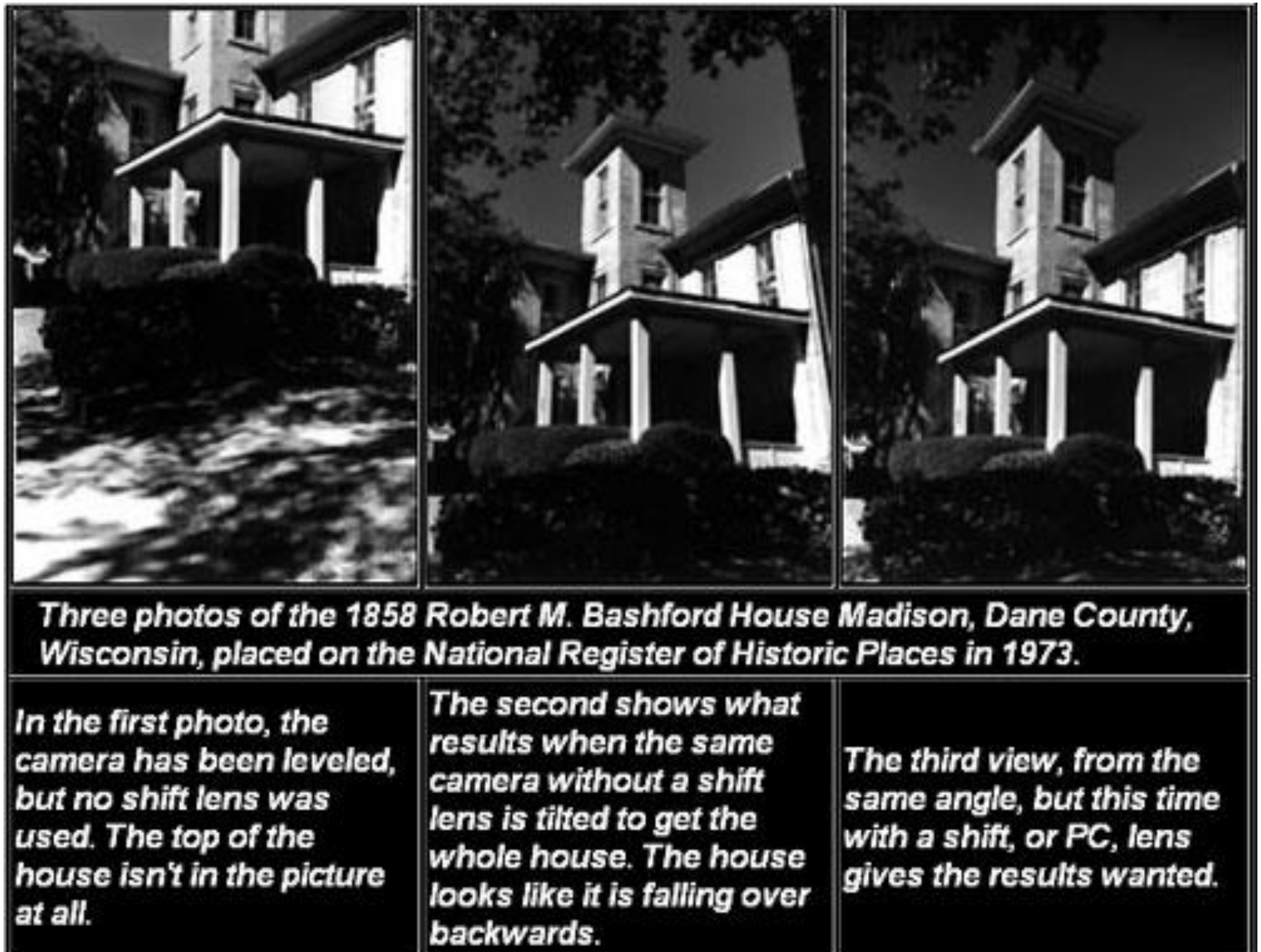


Tilt-shift Imaging

Note how the focus plane is along the train, and how the blurring of the background proceeds from left to right.

Often used to simulate a miniature scene

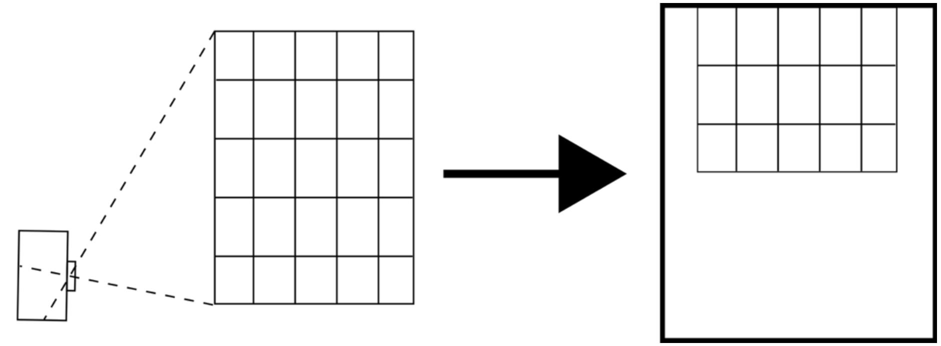
Tilt-shift perspective correction



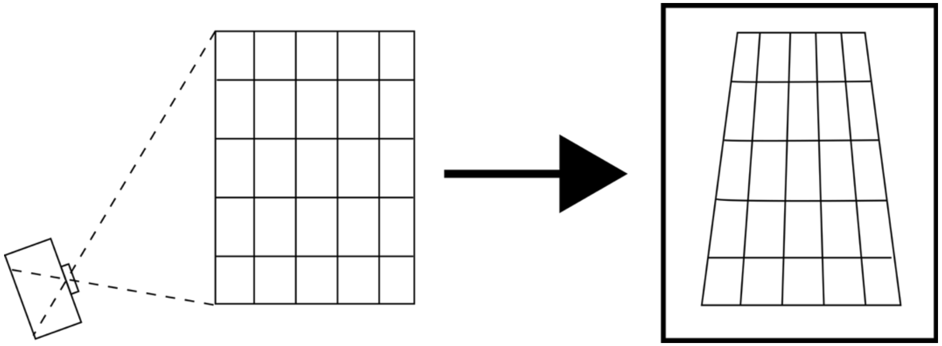
Tilt-shift Camera

http://en.wikipedia.org/wiki/Tilt-shift_photography

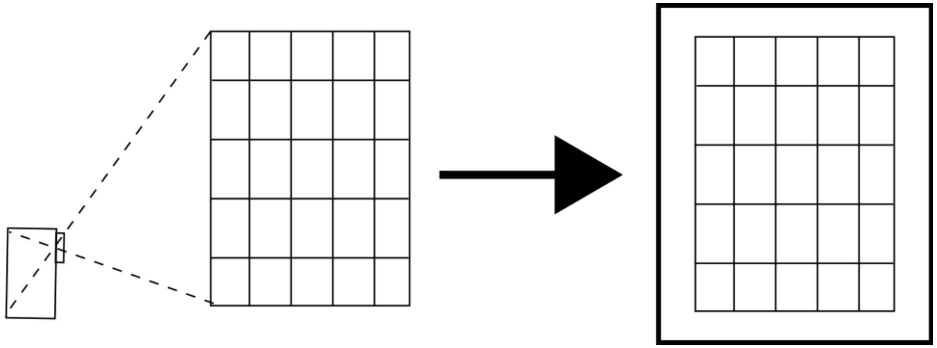
Tilt-shift Correction



Keeping the camera level, with an ordinary lens, captures only the bottom portion of the building.



Tilting the camera upwards results in perspective distortion.



Shifting the lens upwards results in a picture of the entire subject without perspective distortion.

Tilt-shift Software Correction

Normal Camera

Tilt-shift Camera

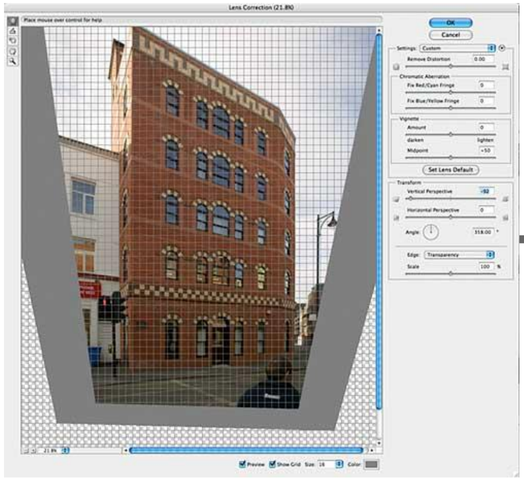


Image transformation is in the next lecture.

IMAGE FORMATION

Camera Calibration

Camera Calibration (Simplest form)

- Intrinsic + Extrinsic combined

Camera to Pixel

$$\begin{bmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{w}\end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0\\ 0 & f_y & o_y & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix} x_c\\ y_c\\ z_c\\ 1\end{bmatrix}$$

$$\tilde{\mathbf{u}} = M_{int}\,\tilde{\mathbf{x}}_c$$

World to Camera

$$\begin{bmatrix} x_c\\ y_c\\ z_c\\ 1\end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x\\ r_{21} & r_{22} & r_{23} & t_y\\ r_{31} & r_{32} & r_{33} & t_z\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix} x_w\\ y_w\\ z_w\\ 1\end{bmatrix}$$

$$\tilde{\mathbf{x}}_c = M_{ext}\tilde{\mathbf{x}}_w$$

Combining the above two equations, we get the full projection matrix P:

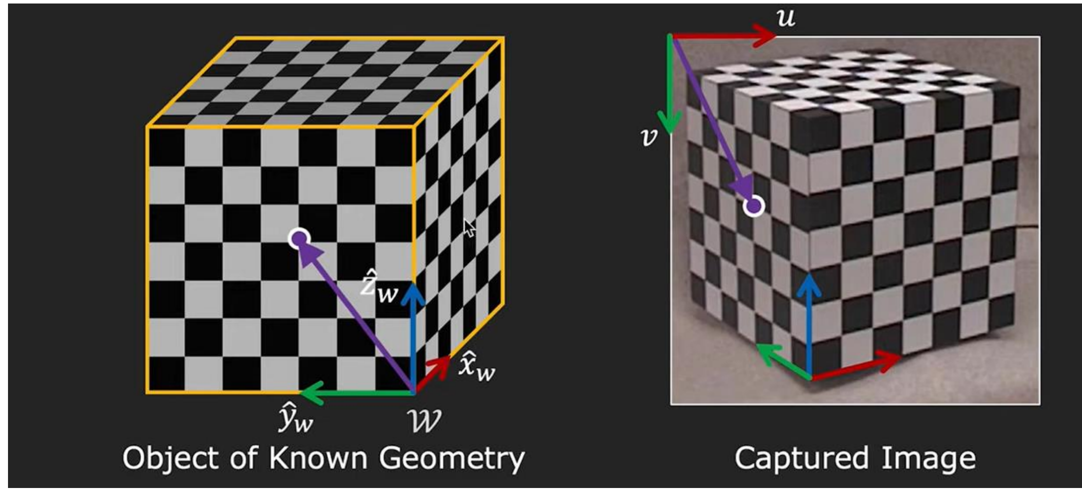
$$\boxed{\tilde{\mathbf{u}} = M_{int}M_{ext}\,\tilde{\mathbf{x}}_w = P\,\tilde{\mathbf{x}}_w}$$

$$\begin{bmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{w}\end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14}\\ p_{21} & p_{22} & p_{23} & p_{24}\\ p_{31} & p_{32} & p_{33} & p_{34}\end{bmatrix}\begin{bmatrix} x_w\\ y_w\\ z_w\\ 1\end{bmatrix}$$

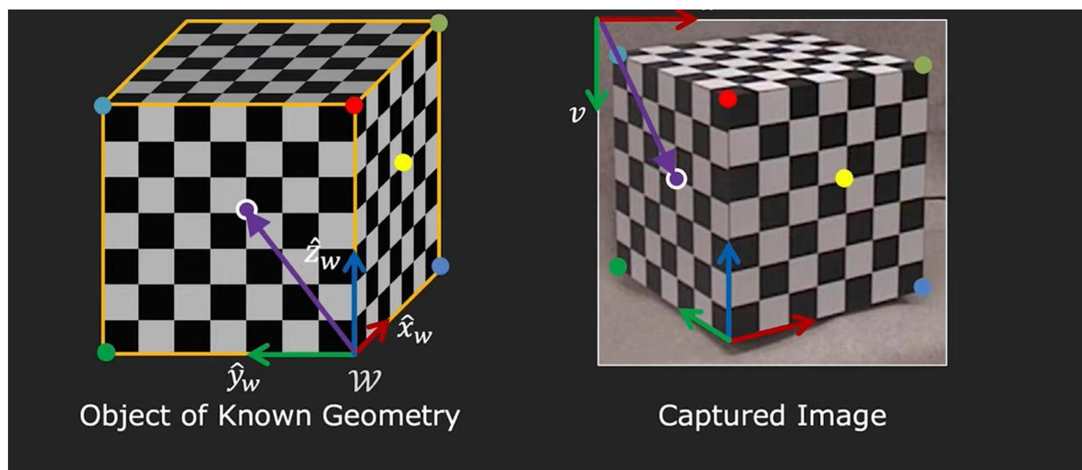
Point Correspondence

- Get **correspondence** of **world points** and **image points**

$$\mathbf{u} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 56 \\ 115 \end{bmatrix}$$
(pixels)



$$\mathbf{x}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$
(inches)



Expanding Projection Matrix

$$w \begin{bmatrix} u^{(i)} \\ v^{(i)} \\ 1 \end{bmatrix} \equiv \underbrace{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}}_{\text{Unknown}} \underbrace{\begin{bmatrix} x_w^{(i)} \\ y_w^{(i)} \\ z_w^{(i)} \\ 1 \end{bmatrix}}_{\text{Known}}$$

Known (applies to $w\begin{bmatrix} u^{(i)} \\ v^{(i)} \\ 1 \end{bmatrix}$) | Unknown | Known

Expanding the matrix as linear equations:

$$u^{(i)} = \frac{p_{11}x_w^{(i)} + p_{12}y_w^{(i)} + p_{13}z_w^{(i)} + p_{14}}{p_{31}x_w^{(i)} + p_{32}y_w^{(i)} + p_{33}z_w^{(i)} + p_{34}}$$

$$v^{(i)} = \frac{p_{21}x_w^{(i)} + p_{22}y_w^{(i)} + p_{23}z_w^{(i)} + p_{24}}{p_{31}x_w^{(i)} + p_{32}y_w^{(i)} + p_{33}z_w^{(i)} + p_{34}}$$

Expanded Projection matrix

$$
\begin{bmatrix}
x_w^{(1)} & y_w^{(1)} & z_w^{(1)} & 1 & 0 & 0 & 0 & 0 & -u_1 x_w^{(1)} & -u_1 y_w^{(1)} & -u_1 z_w^{(1)} & -u_1 \\
0 & 0 & 0 & 0 & x_w^{(1)} & y_w^{(1)} & z_w^{(1)} & 1 & -v_1 x_w^{(1)} & -v_1 y_w^{(1)} & -v_1 z_w^{(1)} & -v_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_w^{(i)} & y_w^{(i)} & z_w^{(i)} & 1 & 0 & 0 & 0 & 0 & -u_i x_w^{(i)} & -u_i y_w^{(i)} & -u_i z_w^{(i)} & -u_i \\
0 & 0 & 0 & 0 & x_w^{(i)} & y_w^{(i)} & z_w^{(i)} & 1 & -v_i x_w^{(i)} & -v_i y_w^{(i)} & -v_i z_w^{(i)} & -v_i \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_w^{(n)} & y_w^{(n)} & z_w^{(n)} & 1 & 0 & 0 & 0 & 0 & -u_n x_w^{(n)} & -u_n y_w^{(n)} & -u_n z_w^{(n)} & -u_n \\
0 & 0 & 0 & 0 & x_w^{(n)} & y_w^{(n)} & z_w^{(n)} & 1 & -v_n x_w^{(n)} & -v_n y_w^{(n)} & -v_n z_w^{(n)} & -v_n
\end{bmatrix}
\begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Solution space is scale-invariant

$$
\underbrace{\begin{bmatrix}
x_w^{(1)} & y_w^{(1)} & z_w^{(1)} & 1 & 0 & 0 & 0 & 0 & -u_1 x_w^{(1)} & -u_1 y_w^{(1)} & -u_1 z_w^{(1)} & -u_1 \\
0 & 0 & 0 & 0 & x_w^{(1)} & y_w^{(1)} & z_w^{(1)} & 1 & -v_1 x_w^{(1)} & -v_1 y_w^{(1)} & -v_1 z_w^{(1)} & -v_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_w^{(i)} & y_w^{(i)} & z_w^{(i)} & 1 & 0 & 0 & 0 & 0 & -u_i x_w^{(i)} & -u_i y_w^{(i)} & -u_i z_w^{(i)} & -u_i \\
0 & 0 & 0 & 0 & x_w^{(i)} & y_w^{(i)} & z_w^{(i)} & 1 & -v_i x_w^{(i)} & -v_i y_w^{(i)} & -v_i z_w^{(i)} & -v_i \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_w^{(n)} & y_w^{(n)} & z_w^{(n)} & 1 & 0 & 0 & 0 & 0 & -u_n x_w^{(n)} & -u_n y_w^{(n)} & -u_n z_w^{(n)} & -u_n \\
0 & 0 & 0 & 0 & x_w^{(n)} & y_w^{(n)} & z_w^{(n)} & 1 & -v_n x_w^{(n)} & -v_n y_w^{(n)} & -v_n z_w^{(n)} & -v_n
\end{bmatrix}}_{\substack{A \\ \text{Known}}}
\underbrace{\begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix}}_{\substack{\mathbf{p} \\ \text{Unknown}}}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Solve for $\mathbf{p}$

$$\boxed{A\,\mathbf{p} = \mathbf{0}}$$

Objective/Loss Function

We want $A\mathbf{p}$ as close to 0 as possible and $||\mathbf{p}||^2 = 1$:

$$\min_{\mathbf{p}} ||A\mathbf{p}||^2 \quad \text{such that } ||\mathbf{p}||^2 = 1$$

$$\min_{\mathbf{p}} (\mathbf{p}^T A^T A\mathbf{p}) \quad \text{such that } \mathbf{p}^T\mathbf{p} = 1$$

Loss function $L(\mathbf{p}, \lambda)$:

$$L(\mathbf{p}, \lambda) = \mathbf{p}^T A^T A\mathbf{p} - \lambda(\mathbf{p}^T\mathbf{p} - 1)$$

Solve Objective Function

Taking derivatives of $L(\mathbf{p},\lambda)$ w.r.t $\mathbf{p}$: $2A^TA\mathbf{p} - 2\lambda\mathbf{p} = \mathbf{0}$

$\boxed{A^TA\mathbf{p} = \lambda\mathbf{p}}$ Eigenvalue Problem

Eigenvector $\mathbf{p}$ with smallest eigenvalue λ of matrix A^TA minimizes the loss function $L(\mathbf{p})$.

- An [explanation]

Estimation K, R

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M_{int}} \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{M_{ext}}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \boxed{\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}} = KR$$

Given that K is an Upper Right Triangular matrix and R is an Orthonormal matrix, it is possible to uniquely "decouple" K and R from their product using "QR factorization".

Wiki: QR Factorization

Estimating Translation

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M_{int}} \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{M_{ext}}$$

That is:

$$\begin{bmatrix} p_{14} \\ p_{24} \\ p_{34} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = K\mathbf{t}$$

Therefore:

$$\boxed{\mathbf{t} = K^{-1} \begin{bmatrix} p_{14} \\ p_{24} \\ p_{34} \end{bmatrix}}$$

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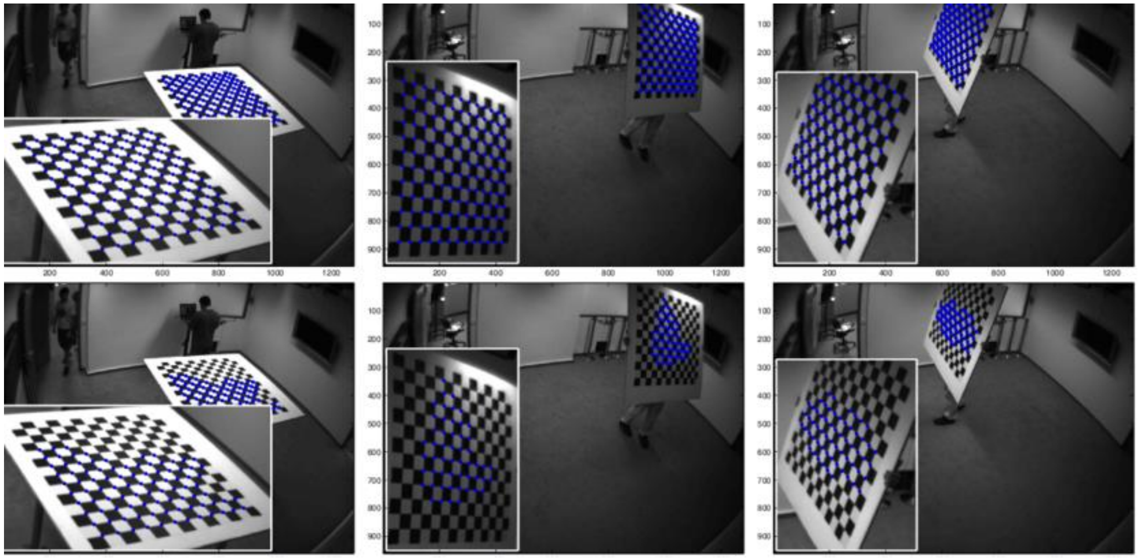
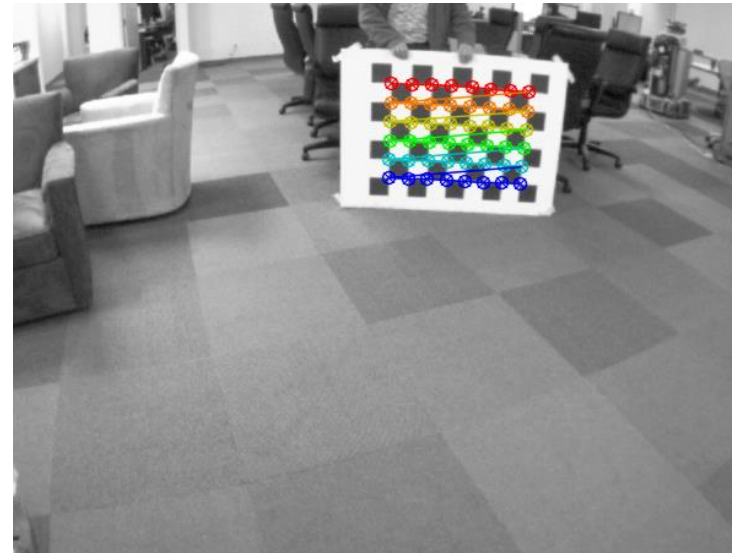
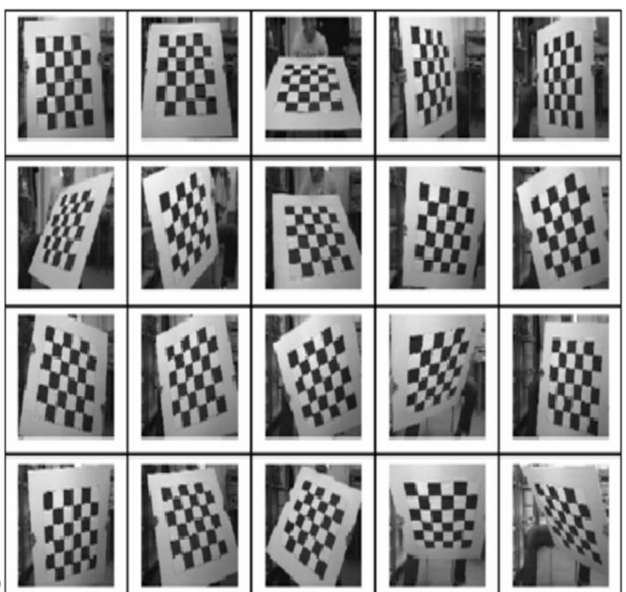
Steps of Camera Calibration

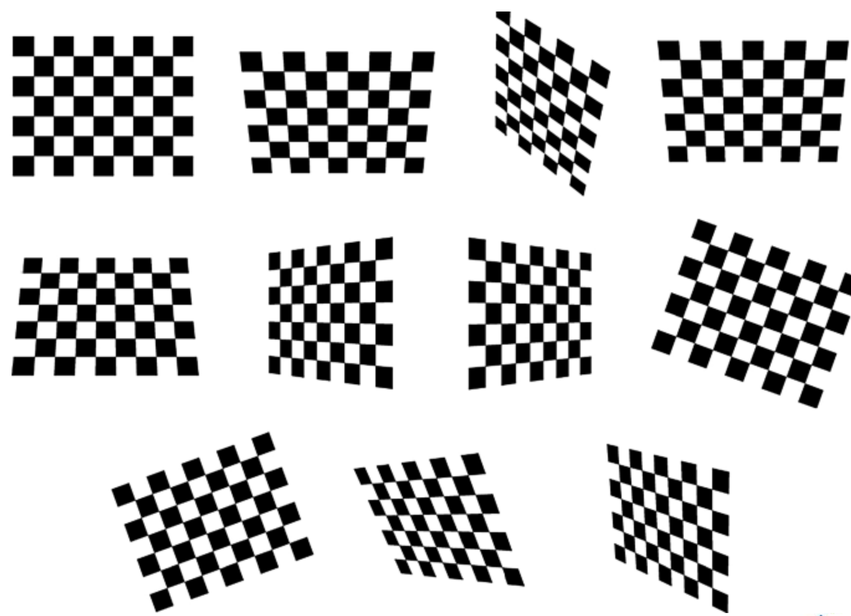
- From a set of corresponding points build A
- Compute the eigenvector **p** of A^TA for the smallest eigenvalue.
- Rearrange **p** as **P** as a 3x4 matrix
- Take the 3 x 3 matrix and compute the QR decomposition
  - Q is K (Calibration matrix)
  - R is rotation matrix
- Compute $T = K^{-1}P_{i4}$

Cover more poses









Why rotation matrix is orthogonal

- Assume a point x, a rotation matrix R;
- A rotated point Rx has the same length with x;
  - So $||Rx||^2 = ||x||^2$
  - $(Rx)^T(Rx) = x^T R^T R x^T = x^T(R^T \mathrm{R})x = x^T \mathrm{I} x$
  - So $R^T R = \mathrm{I}$
- A rotated point Rx can be rotated back to the original point x with the "reversed rotation" R^{-1}.
  - So, $R^{-1}(Rx) = x$, then $R^{-1} = R^T$.

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How many degrees of freedom?

- Extrinsic Parameters
  - 6 = 3 translation + 3 rotation
- Intrinsic Parameters
  - 5 = 2 focal length, 2 image center, 1 skew
  - = 6 (3D transform) – 1 depth losing
- Camera Calibration
  - 11 = 6 Extrinsic + 5 Intrinsic
  - = 12 (3x4 Projection matrix) – 1 scale factor

Project 1: Camera Calibration

- The only **algorithm libraries** that are allowed to use.
  - PyTorch: Tensor operations.
    - Ready for modern deep learning frameworks
  - Solution can be a little different from slides.
- Find the project access link in Piazza.