Assignment - I

Mathematics - I (for 2024-25 AB)

for sections - A1, A3 + A6.

(1) If $x^x y^y z^z = c$ then show that $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ at $x = y = z$

(2) show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ if $z = \log\left(\frac{x^2+y^2}{xy}\right)$

(3) If $u = \log(x^3+y^3+z^3-3xyz)$ Show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

(4) State Euler's theorem for three variables and Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ if $u = \frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y}$

(5) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

(6) If z is a homogeneous function of degree n in x and y

Show that $x^2\frac{\partial^2 z}{\partial x^2} + y^2\frac{\partial^2 z}{\partial y^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} = n(n-1)z$

(7) If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$ then find $J\left(\frac{u,v,w}{x,y,z}\right)$

(8) If $u = x^2-y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$

(9) Show that the functions $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$ are functionally related and find the relation

(10) Expand $f(x,y) = e^x \log(1+y)$ in powers of x and y upto Second degree terms.

(11) Expand $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-1)$ upto 2nd degree terms

(12) Expand $f(x,y) = x^2y + 3y - 2$ in about the point $(1,-2)$

(13) Find extreme values of $f(x,y) = x^3 + y^3 - 3axy$

(14) If $xyz = 8$ find the values of x and y for which $u = \frac{5xyz}{x+2y+4z}$ is maximum

(15) A rectangular box open at the top is to have volume 32 cubic ft. Find the dimensions of the box requiring least material for its construction

(16) In a plane triangle, find the maximum value of $\cos A \cos B \cos C$

(17) Find minimum value of $x^2+y^2+z^2$, given that $xyz = a^3$ by using Lagrange's method of undetermined multipliers

(18) Find equations of the tangent plane and normal to the surface at the given point of the following

(1) $2x^2+y^2-3+2z=0$ $(2,1,-3)$

(2) $a^2=xyz$, (x_1,y_1,z_1)

(3) $2xz^2-3xy-4x-7=0$, $(1,-1,2)$

(19) compute approximate values of the following

(1) $(1.04)^{3.01}$ (2) $[(3.82)^2+2(2.1)^3]^{1/5}$

(20) The work must be done to propel a ship of displacement D for a distance s in time t is propotional to $\left(\frac{s^2 D^{2/3}}{t^2}\right)$. Find approximately the increase of work necessary when the displacement is increased by 1%, the time is diminished by 1% and the distance diminished by 2%.

(21) The time of oscilations of a simple pendulum is given by the equation $T=2\pi\sqrt{\frac{l}{g}}$. If an experiment carried out to find the value of g, error of 1.5% and 0.5% are possible in the values of l and T respectively. Show that the error

in the calculated value of g is 0.5%.

22. If $pv^2 = K$ and relative error in p and v are respectively 0.05 and 0.025, show that error in K is 10%.

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