
Name: Udandarao Sai Sandeep
Roll Number: 180123063
Dept.: Mathematics and Computing

Note: The following modules would be required to run the programs:
pip install **numpy**, pip install **matplotlib**, pip install **scipy**, pip install **prettytable**

Q1. In order to estimate the value of the Call/Put Option Price, **classical BSM framework** was used. The formula used to calculate **Call option price** is as follows:

$$c(t,x) = xN(d_+(T-t,x)) - Ke^{-r(T-t)}N(d_-(T-t,x)) \;\; 0 \leq t < T, \;\; x > 0$$

$$where\ d_\pm(T-t,x) = \frac{1}{\sigma\sqrt{T-t}}[\log(x/K) + (r \pm \frac{\sigma^2}{2})(T-t)]$$
$$and\ N\ is\ the\ CDF\ of\ N(0,1)$$

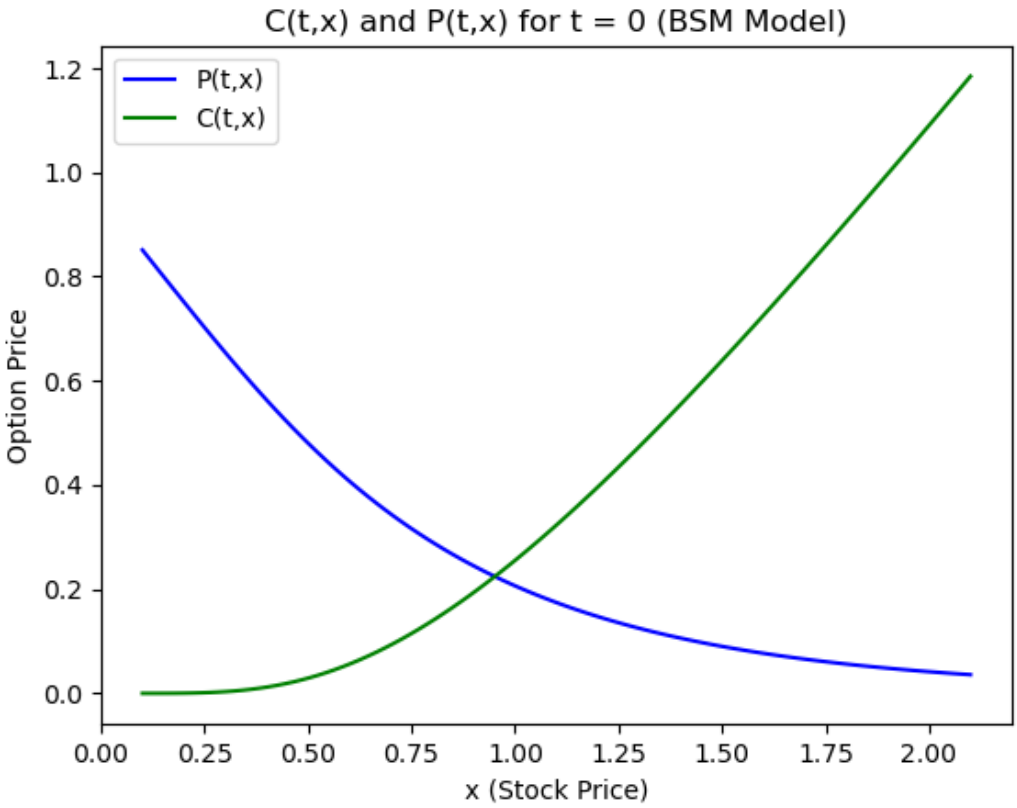
Then, the **put-call parity** was then employed to calculate the corresponding put option price.

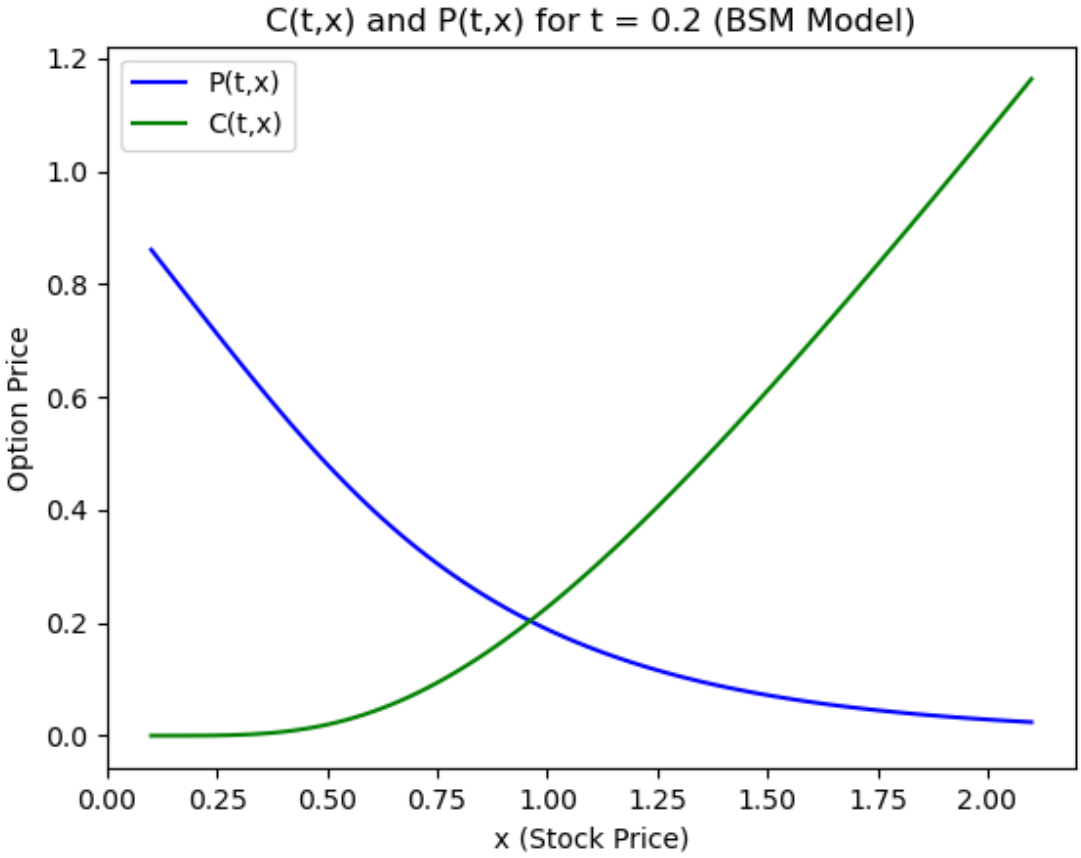
$$c(t,x) - p(t,x) = x - Ke^{-r(T-t)}$$

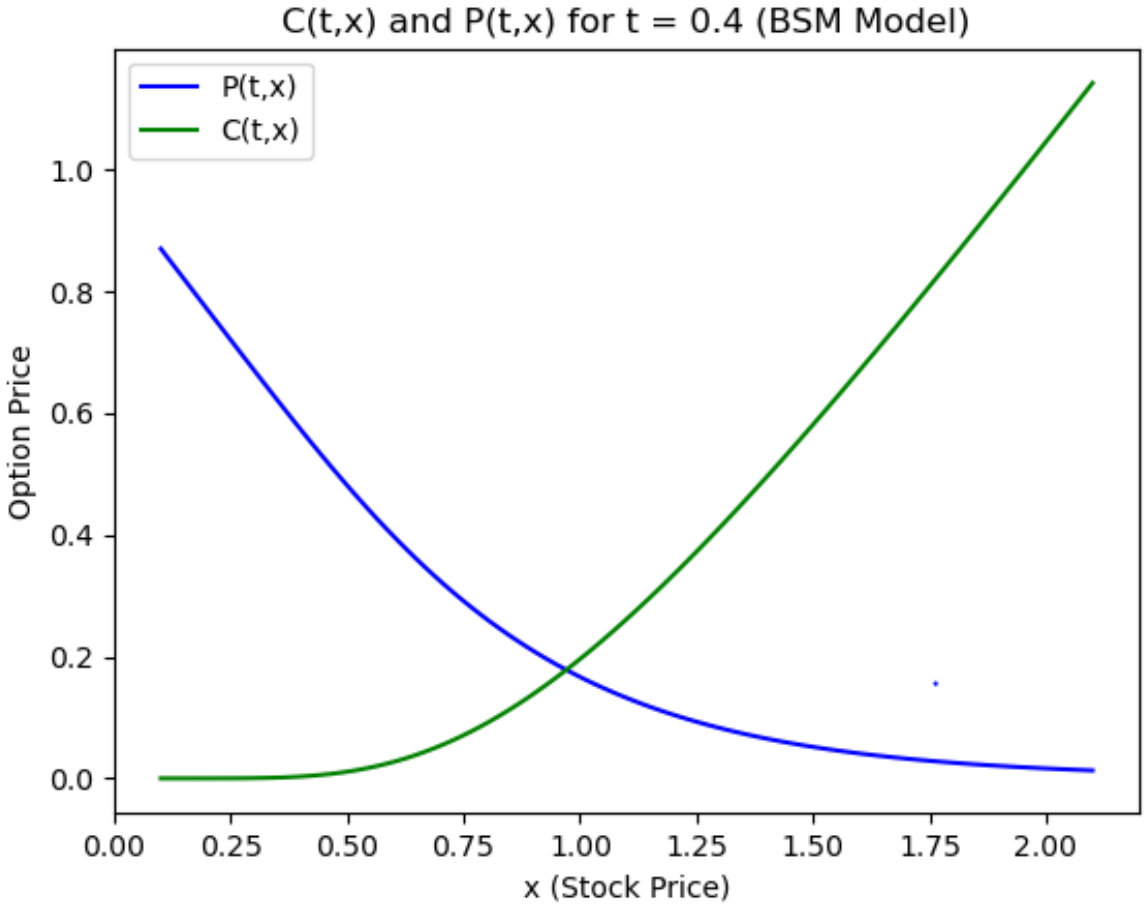
The implementation of the functions (in python) is as follows:

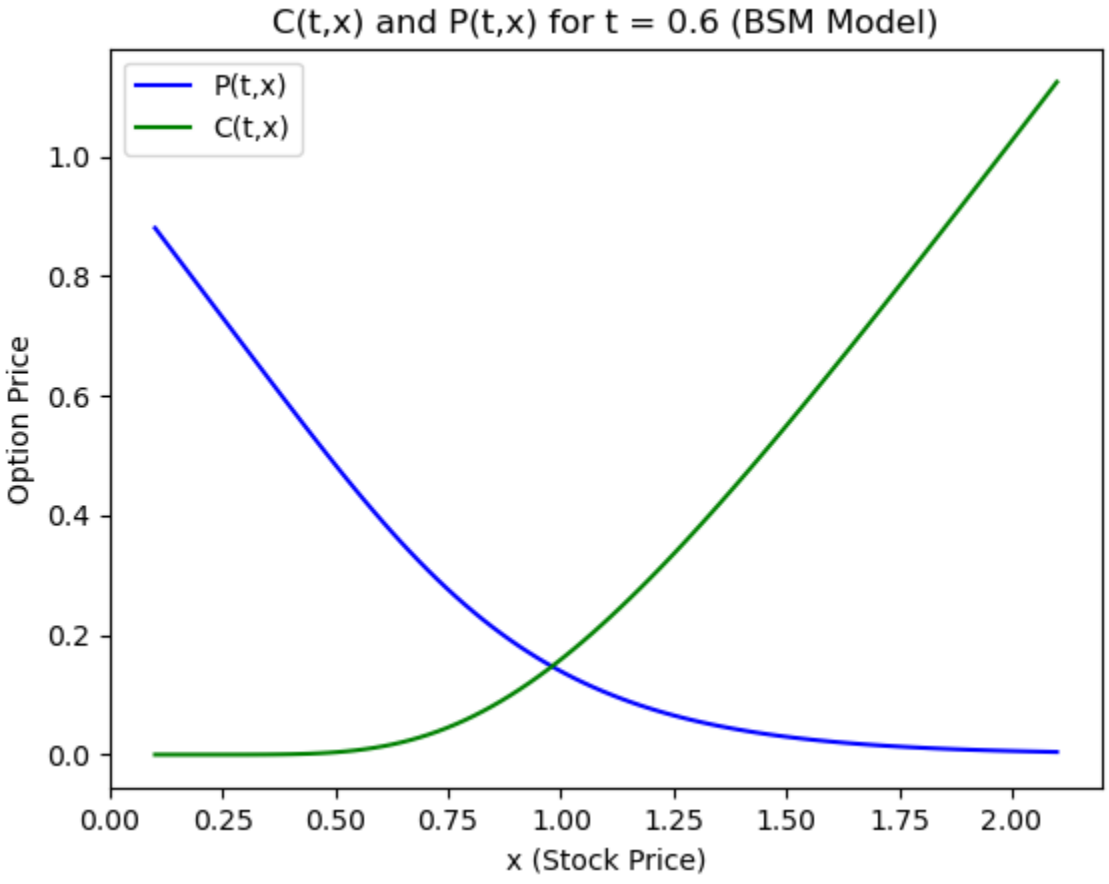
```
def d_positive(S,K,del_t,r,sigma):
    val = math.log(S/K) + (r+(sigma*sigma/2))*(del_t)
    return val/(sigma*math.sqrt(del_t))


def d_negative(S,K,del_t,r,sigma):
    val = math.log(S/K) + (r-(sigma*sigma/2))*(del_t)
    return val/(sigma*math.sqrt(del_t))

def bsm_call(S,K,T,t,r,sigma):
    if(t==T):
        return np.maximum(S-K,0)
    term1 = S*norm.cdf(d_positive(S,K,T-t,r,sigma))
    term2 = K*math.exp(-r*(T-t))*norm.cdf(d_negative(S,K,T-t,r,sigma))
    return term1-term2

def bsm_put(S,K,T,t,r,sigma):
    if(t==T):
        return np.maximum(K-S,0)
    return K*math.exp(-r*(T-t))-S+bsm_call(S,K,T,t,r,sigma)
```

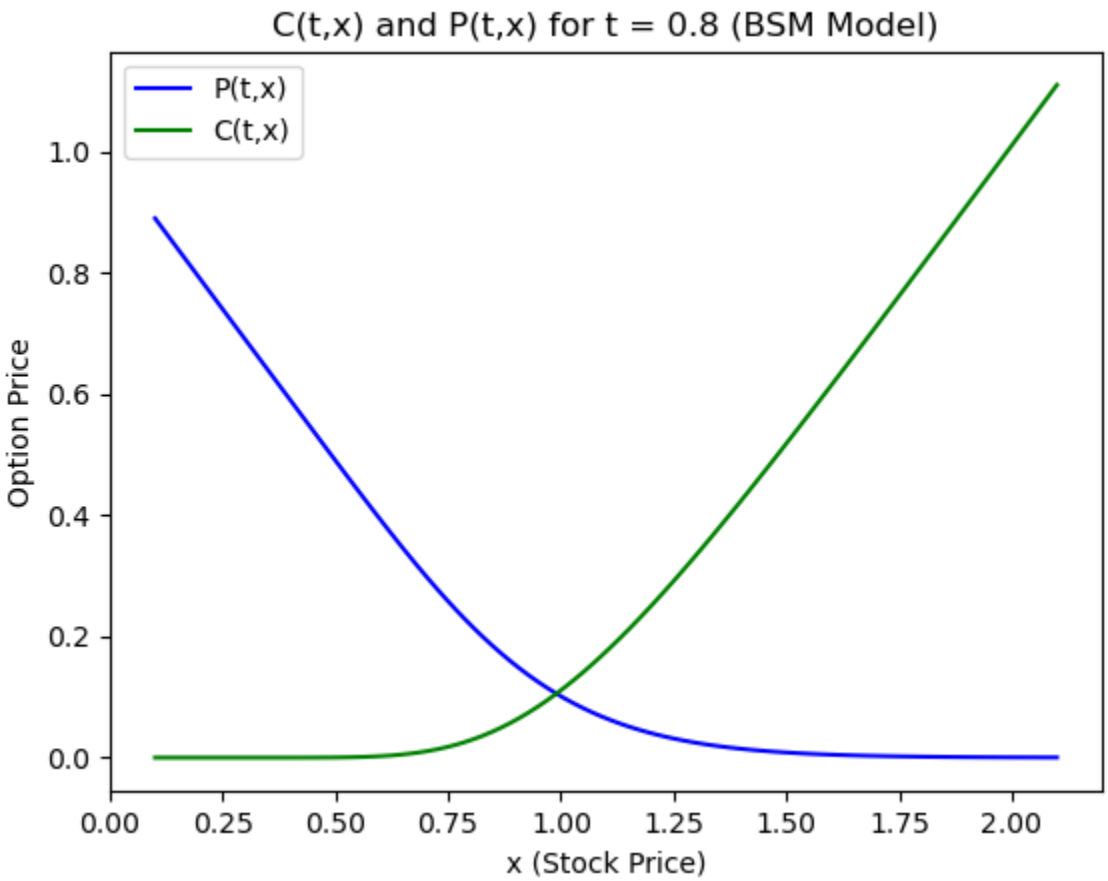
Q2. Call Option Prices (green) and **Put Option** Prices (blue) (as a function of x alone) have been calculated for the given **six** different values of t (as given in the assignment). The Stock Price has been varied from **0.1 to 2.1**. The plots are as follows:

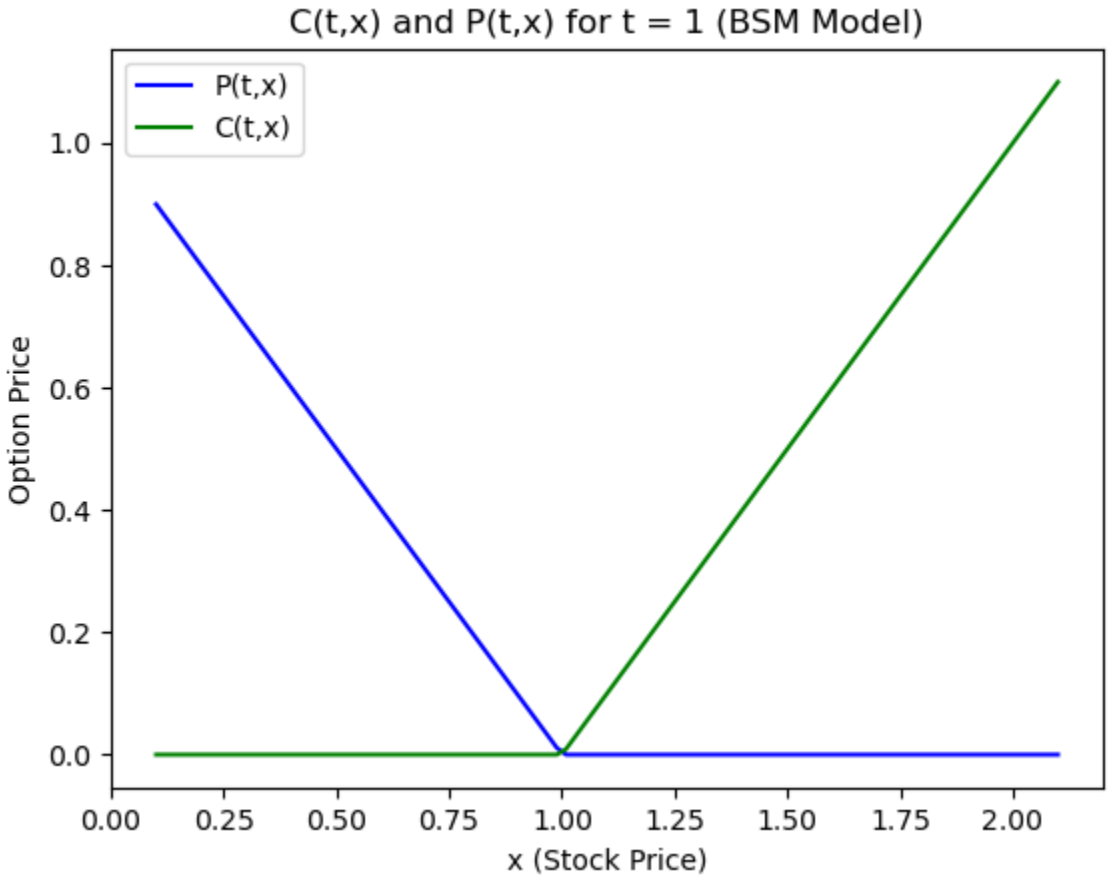




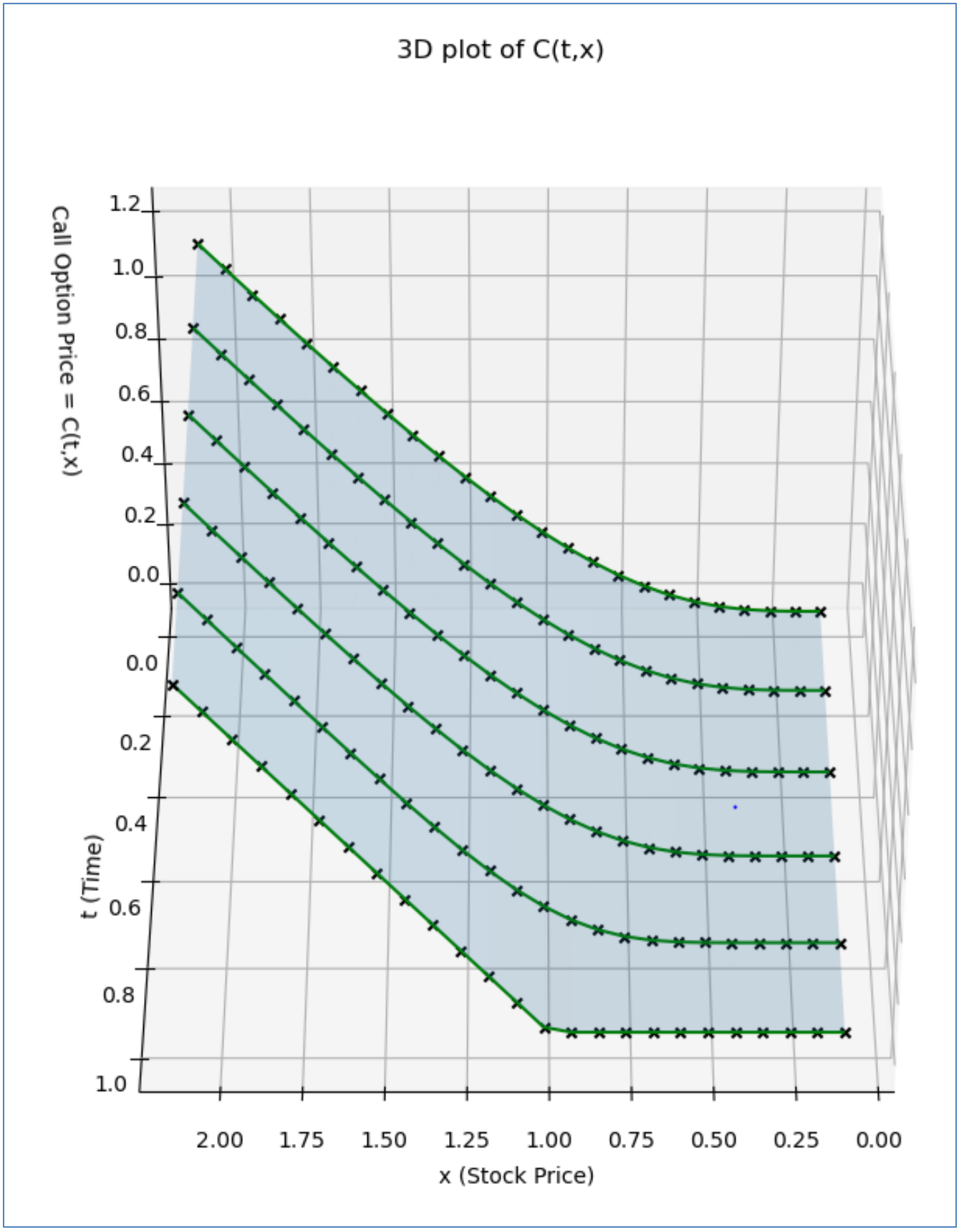

C(t,x) and P(t,x) for t = 0.4 (BSM Model)
P(t,x)
C(t,x)
Option Price
x (Stock Price)
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00


C(t,x) and P(t,x) for t = 0.6 (BSM Model)
P(t,x)
C(t,x)
Option Price
x (Stock Price)
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

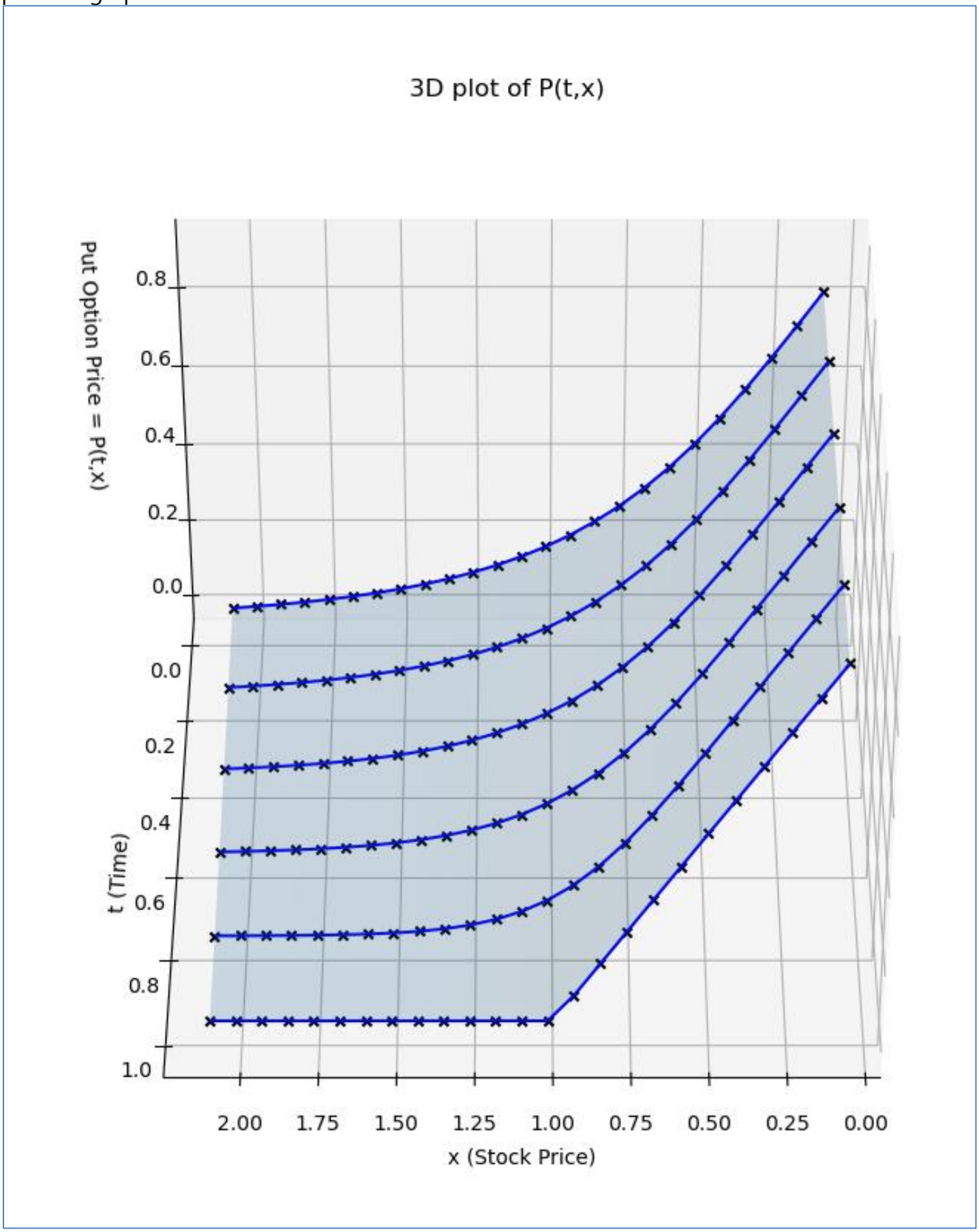

C(t,x) and P(t,x) for t = 0.8 (BSM Model)
P(t,x)
C(t,x)
Option Price
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x (Stock Price)


C(t,x) and P(t,x) for t = 1 (BSM Model)
P(t,x)
C(t,x)
Option Price
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x (Stock Price)

As expected, **the call option price increases with increase in stock price**, and **the put option price decreases with increase in stock price**. Now, the same information has been displayed in **3-D** format as follows (the edge lines have been emphasized with darker edge colors):

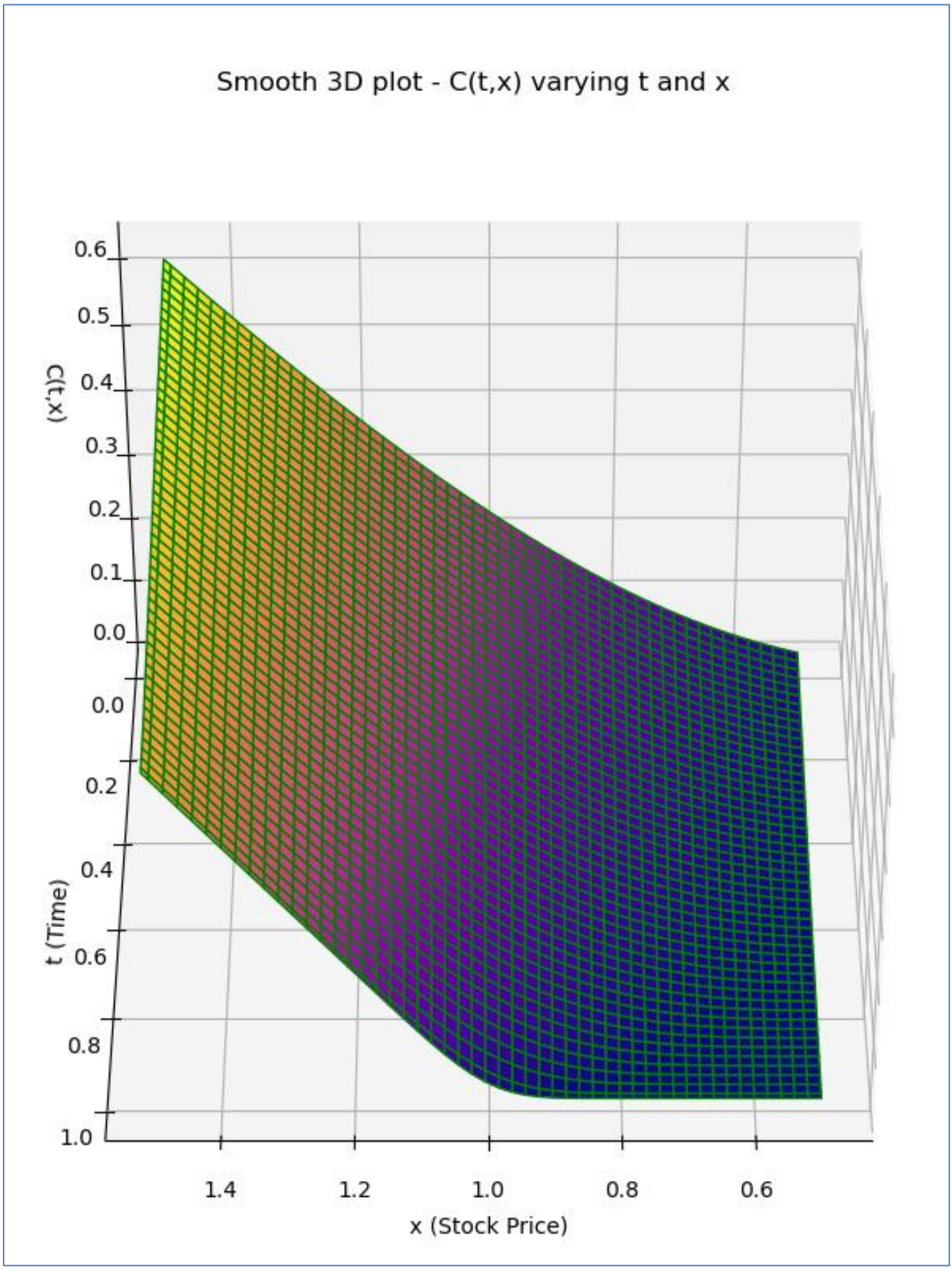


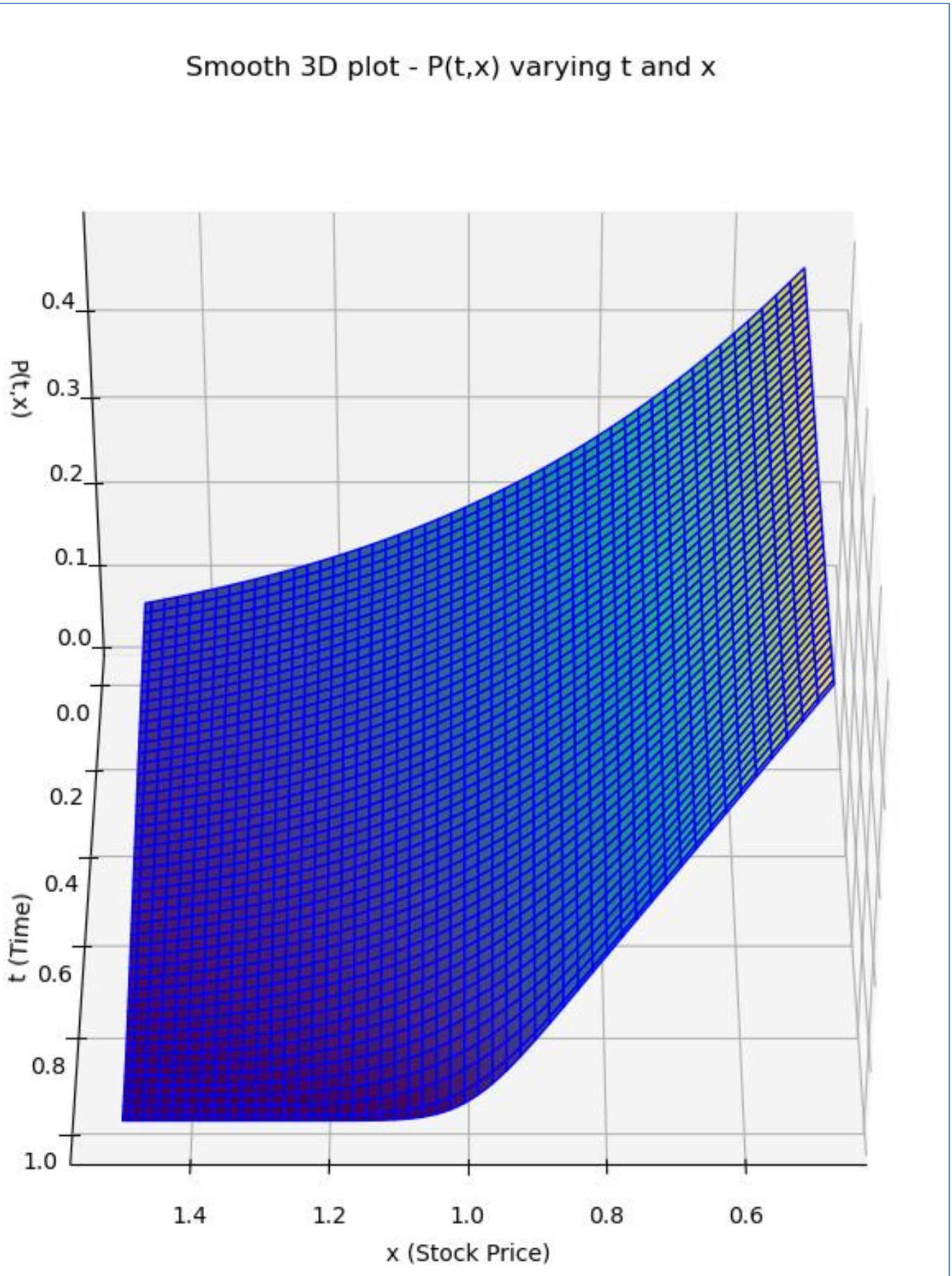
In both the plots taken, the **black X marker** represents the points **(x, t)** that have been taken to plot the graphs.



Q3.

Now, by using more input values of t and x, smooth 3D plots have been made.

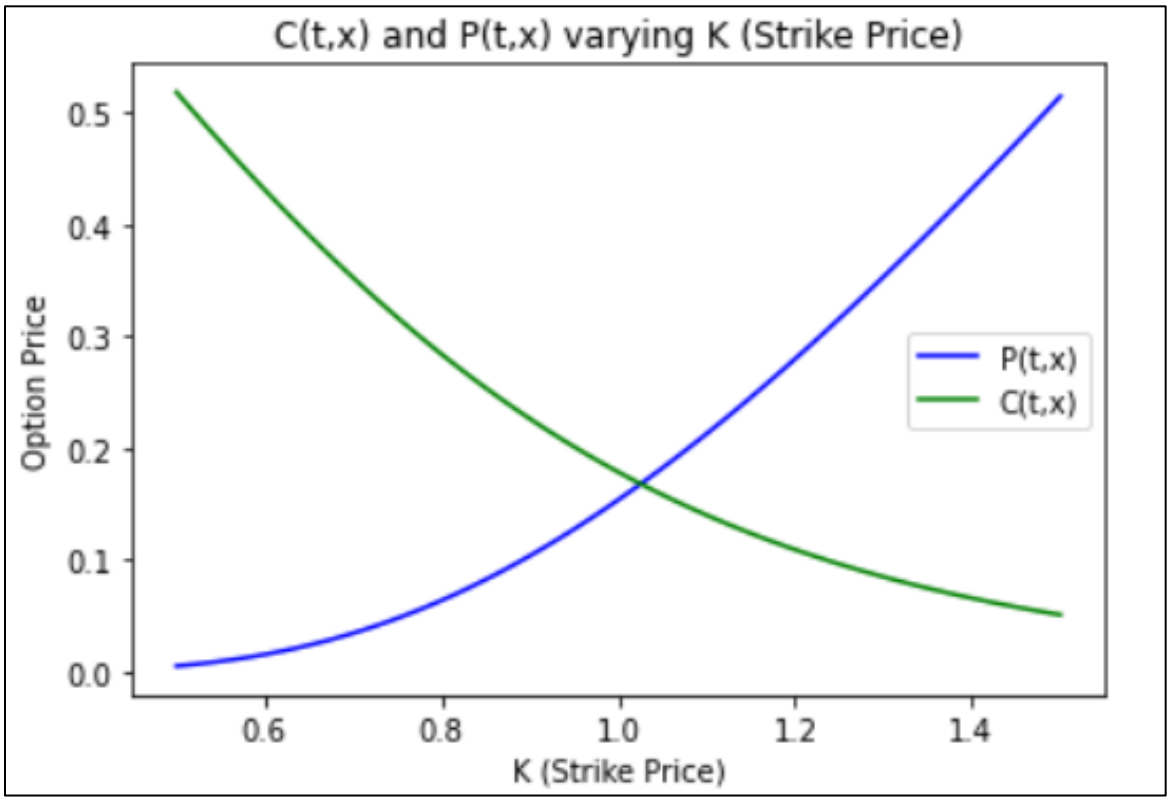


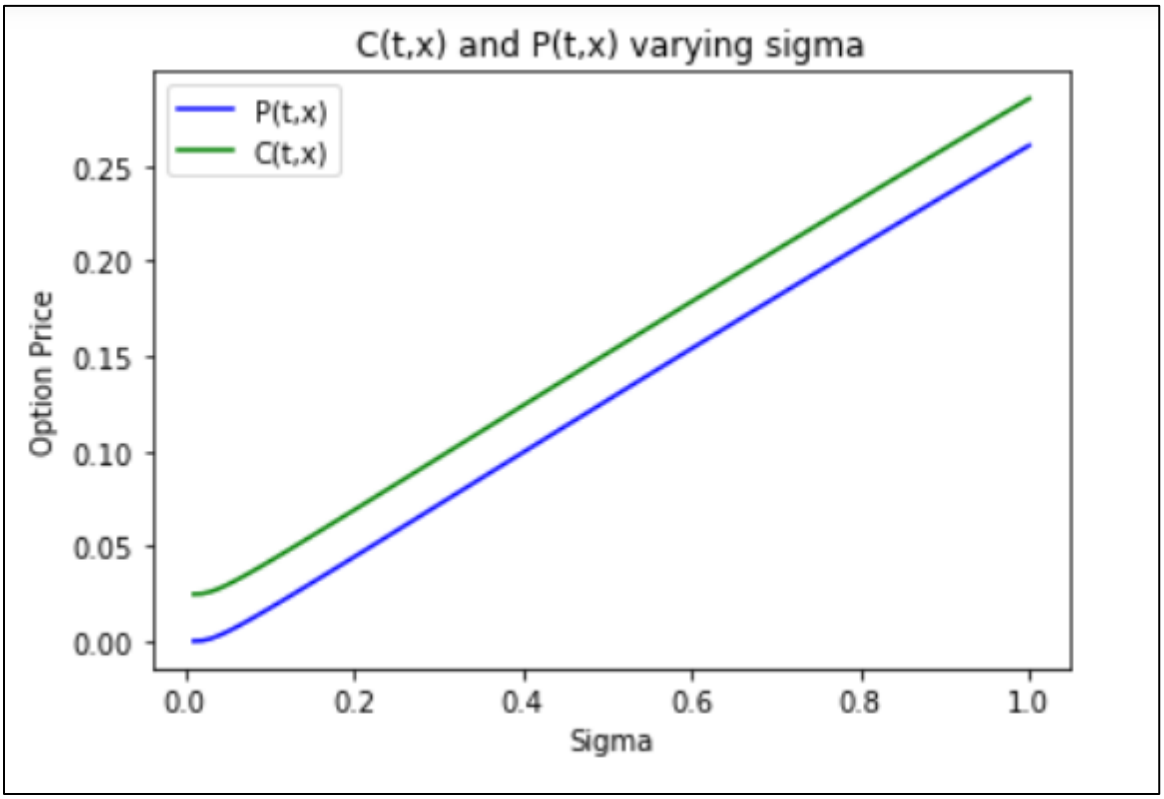

Smooth 3D plot - P(t,x) varying t and x
P(t,x)
0.4
0.3
0.2
0.1
0.0
0.0
0.2
0.4
0.6
0.8
1.0
t (Time)
1.4
1.2
1.0
0.8
0.6
x (Stock Price)

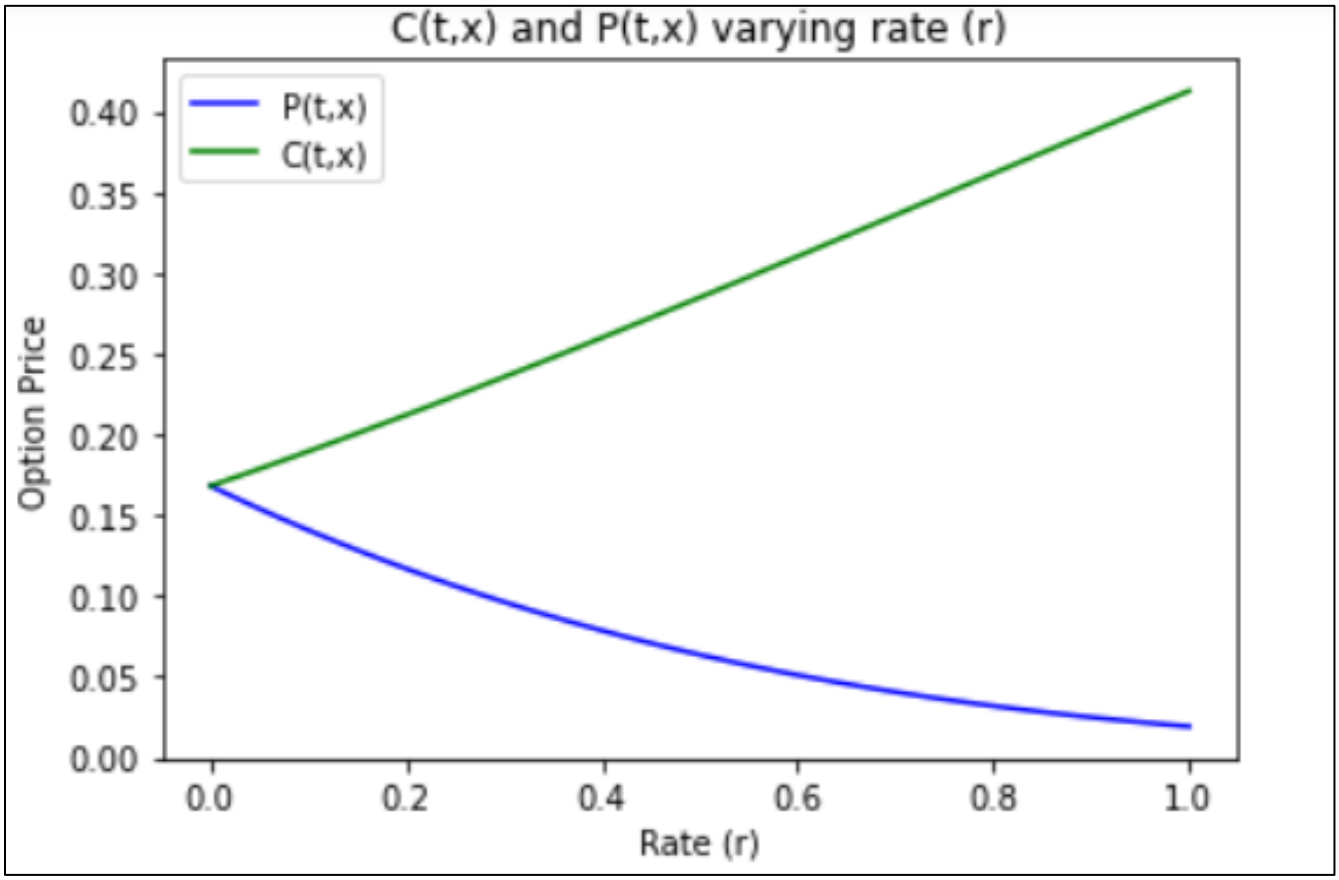
Q4. Sensitivity analysis of both the call and put option price on the basis of Model parameters (**Strike Price (K), sigma(σ), rate (r) and Final Time (T)**) has been done.

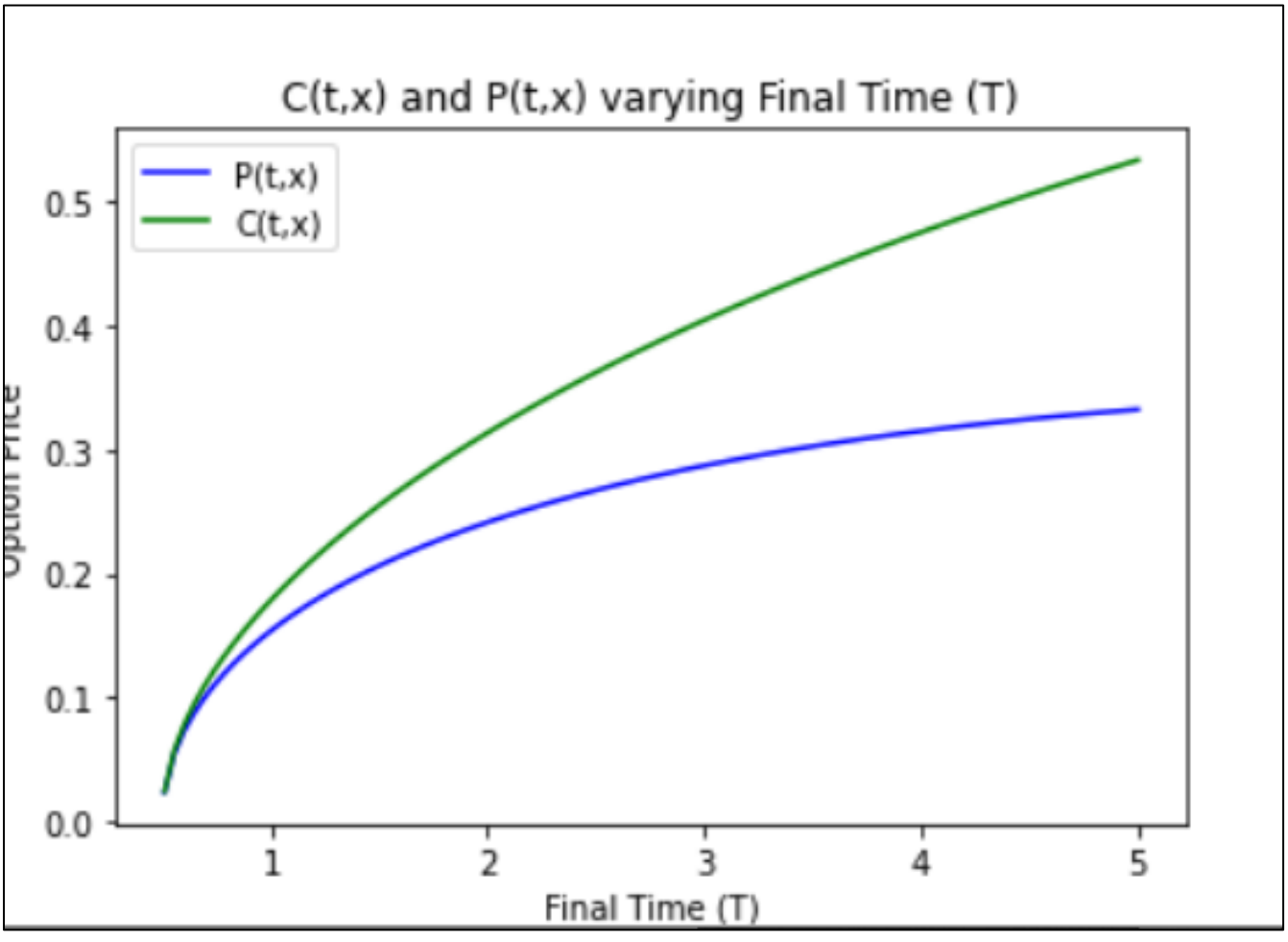
Note: **x** and **t** have been fixed to **1** and **0.5** respectively.

2-D graphs (varying one variable)

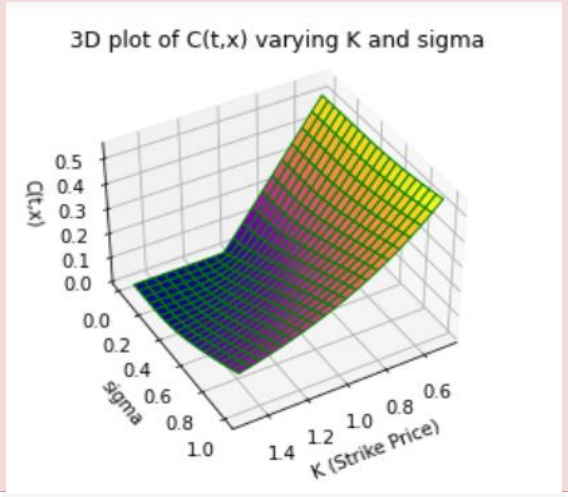
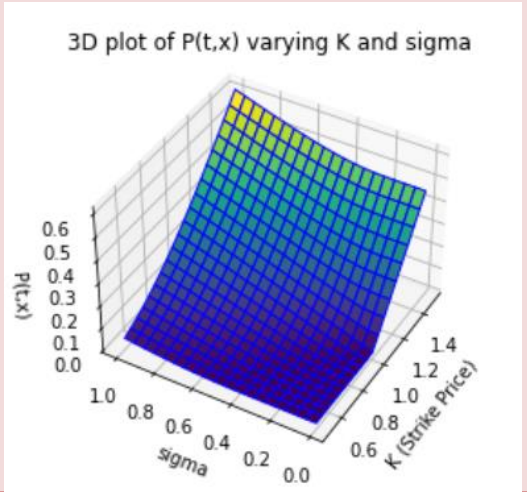
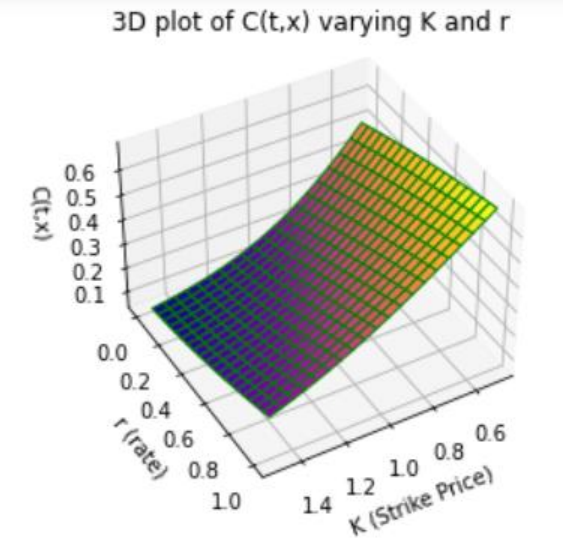
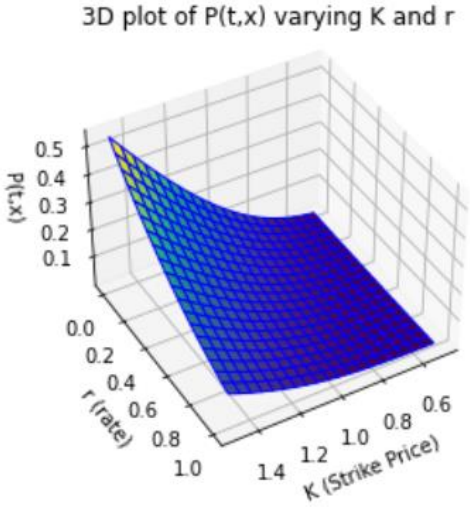
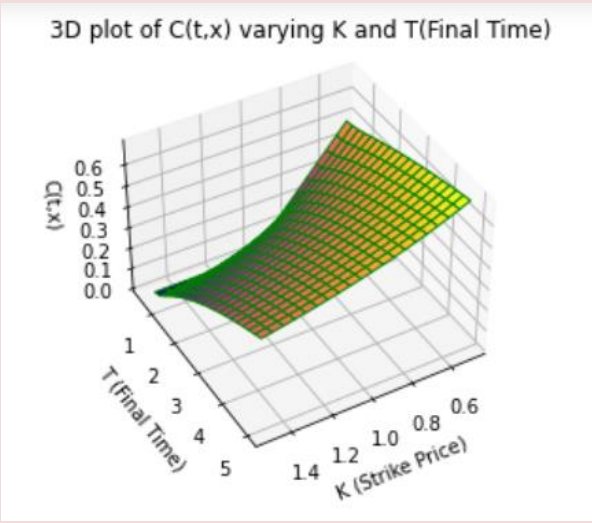
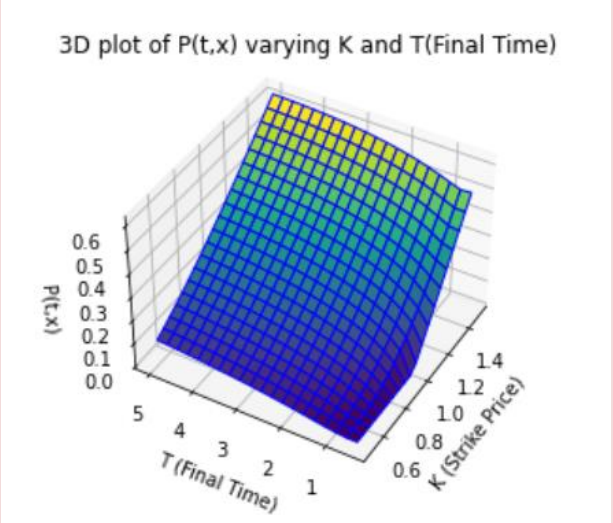


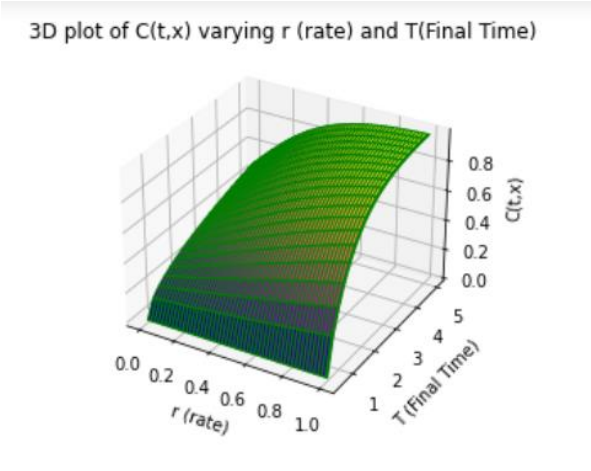
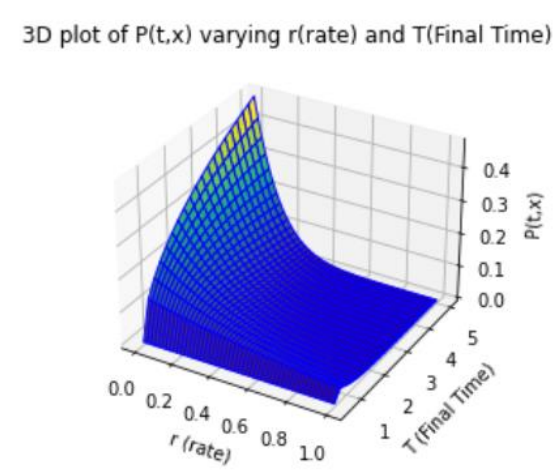
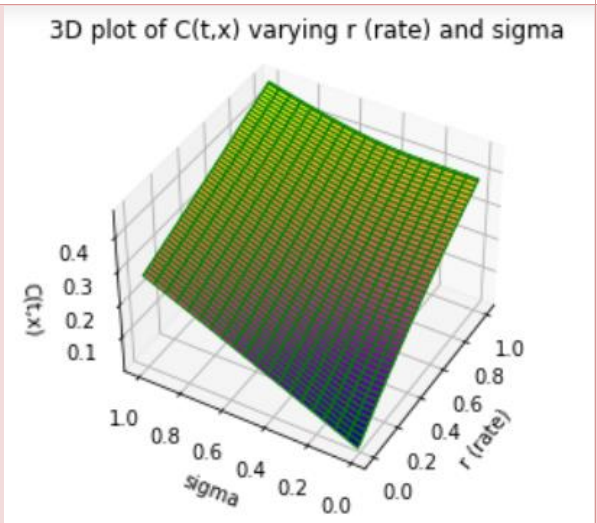
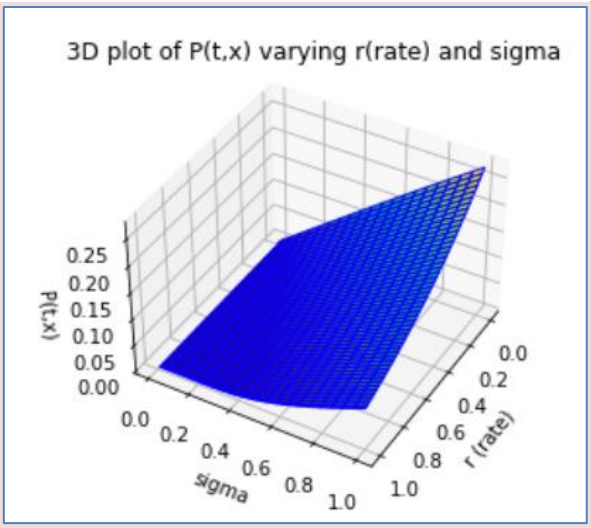
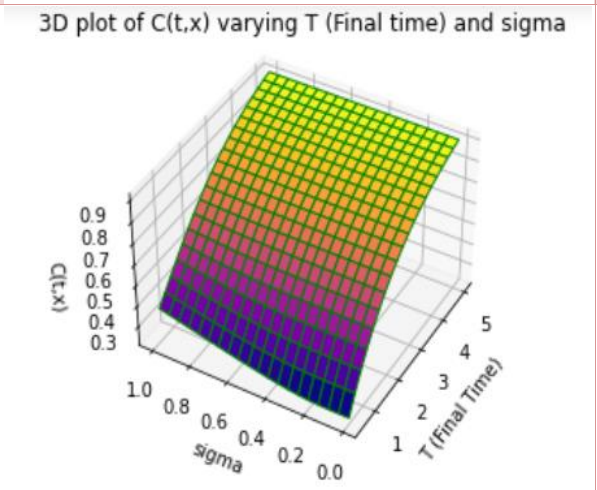
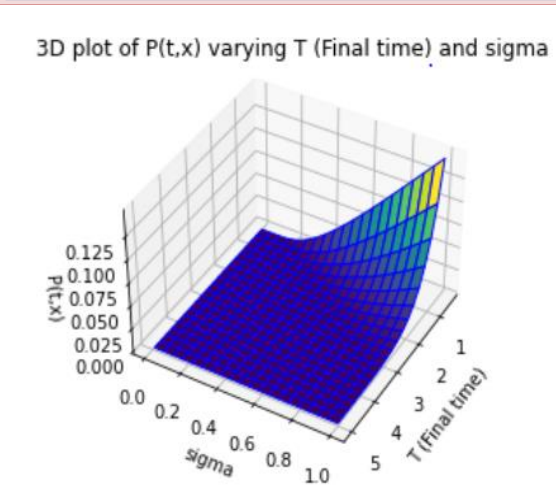



C(t,x) and P(t,x) varying rate (r)
P(t,x)
C(t,x)
Option Price
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.0
0.2
0.4
0.6
0.8
1.0
Rate (r)


C(t,x) and P(t,x) varying Final Time (T)
P(t,x)
C(t,x)
0.0
0.1
0.2
0.3
0.4
0.5
1
2
3
4
5
Final Time (T)

3-D graphs (varying 2 parameters)

Parameters	Call Option Price	Put Option Price
K and sigma		
K and r		
K and T		

r and T		
r and sigma		
T and sigma		

Tables (for different combinations (**2D Graphs**)):

Strike Price(K)	Call Option Price	Put Option Price
0.5	0.517768	0.005423
0.60101	0.429708	0.01588
0.70202	0.350549	0.035236
0.80303	0.281857	0.06506
0.90404	0.223993	0.105713
1.005051	0.176399	0.156635
1.106061	0.137972	0.216724
1.207071	0.107382	0.28465
1.308081	0.08329	0.359074
1.409091	0.064463	0.438763

Sigma	Call Option Price	Put Option Price
0.01	0.02469	0.0
0.11	0.044553	0.019863
0.21	0.071625	0.046935
0.31	0.0991	0.07441
0.41	0.126597	0.101907
0.51	0.154007	0.129317
0.61	0.181269	0.156579
0.71	0.208339	0.183649
0.81	0.23518	0.21049
0.91	0.261757	0.237067

Rate (r)	Call Option Price	Put Option Price
0.0	0.167996	0.167996
0.10101	0.189626	0.140376
0.20202	0.212403	0.116327
0.30303	0.236169	0.095574
0.40404	0.260751	0.07783
0.505051	0.285967	0.062804
0.606061	0.311628	0.050204
0.707071	0.337547	0.039748
0.808081	0.363543	0.03116
0.909091	0.389445	0.024182

Final Time (T)	Call Option Price	Put Option Price
0.51	0.024178	0.023678
0.963535	0.171697	0.148787
1.417071	0.244137	0.199319
1.870606	0.299811	0.233576
2.324141	0.346328	0.259157
2.777677	0.386733	0.279094
3.231212	0.422626	0.29498
3.684747	0.454987	0.30778
4.138283	0.484465	0.318138
4.591818	0.511527	0.326507

Tables for **3D graphs**:

Strike Price (K)	Rate (r)	Call Option Price	Put Option Price
0.5	0.0	0.50633	0.00633
0.605263	0.105263	0.439952	0.014183
0.710526	0.210526	0.385594	0.02513
0.815789	0.315789	0.341642	0.038277
0.921053	0.421053	0.306385	0.052583
1.026316	0.526316	0.278251	0.067098
1.131579	0.631579	0.255904	0.081066
1.236842	0.736842	0.238254	0.093933
1.342105	0.842105	0.224432	0.10533
1.447368	0.947368	0.213755	0.115037

Strike Price (K)	Sigma	Call Option Price	Put Option Price
0.5	0.01	0.512345	0.0
0.605263	0.114211	0.409681	0.0
0.710526	0.218421	0.307393	0.000377
0.815789	0.322632	0.221171	0.016819
0.921053	0.426842	0.171749	0.070061
1.026316	0.531053	0.148517	0.149493
1.131579	0.635263	0.139505	0.243145
1.236842	0.739474	0.138476	0.344781
1.342105	0.843684	0.142383	0.451352
1.447368	0.947895	0.149653	0.561285

Strike Price (K)	Final Time (T)	Call Option Price	Put Option Price
0.5	0.5	0.5	0.0
0.605263	0.973684	0.423979	0.015075
0.710526	1.447368	0.39334	0.070995
0.815789	1.921053	0.382929	0.142766
0.921053	2.394737	0.381812	0.219613
1.026316	2.868421	0.385726	0.297425
1.131579	3.342105	0.392647	0.374325
1.236842	3.815789	0.401475	0.449358
1.342105	4.289474	0.411559	0.52201
1.447368	4.763158	0.422484	0.591999

Rate (r)	Final Time (T)	Call Option Price	Put Option Price
0.0	0.5	0	0
0.10101	0.973684	0.184174	0.137454
0.20202	1.447368	0.307606	0.133419
0.30303	1.921053	0.443571	0.093675
0.40404	2.394737	0.585992	0.05107
0.505051	2.868421	0.719341	0.021691
0.606061	3.342105	0.828433	0.007054
0.707071	3.815789	0.905814	0.00171
0.808081	4.289474	0.953514	0.000299
0.909091	4.763158	0.979294	3.6e-05

Rate (r)	sigma	Call Option Price	Put Option Price
0.0	0.01	0.002821	0.002821
0.10101	0.114211	0.061988	0.012737
0.20202	0.218421	0.118702	0.022626
0.30303	0.322632	0.172574	0.031979
0.40404	0.426842	0.223708	0.040786
0.505051	0.531053	0.272211	0.049047
0.606061	0.635263	0.318188	0.056765
0.707071	0.739474	0.361744	0.063945
0.808081	0.843684	0.402976	0.070593
0.909091	0.947895	0.441982	0.076719

Final Time (T)	sigma	Call Option Price	Put Option Price
0.5	0.01	0.221199	0.0
0.973684	0.114211	0.385436	0.0
1.447368	0.218421	0.515038	0.0
1.921053	0.322632	0.617309	0.0
2.394737	0.426842	0.698013	1e-06
2.868421	0.531053	0.761699	3e-06
3.342105	0.635263	0.811956	5e-06
3.815789	0.739474	0.851614	6e-06
4.289474	0.843684	0.882909	8e-06
4.763158	0.947895	0.907605	9e-06