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Q1. Using the values of the parameters (given in the question), the up factor (**u**) and down factor **(d)** was calculated for each value of M by using the below formulas:

$$\Delta t = \frac{T}{M}$$

$$u = e^{\sigma\sqrt{\Delta t} + \left(r - \frac{1}{2}\sigma^2\right)\Delta t}$$

$$d = e^{-\sigma\sqrt{\Delta t} + \left(r - \frac{1}{2}\sigma^2\right)\Delta t}$$

The up probability (**p**) was calculated as follows:

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

The Option Price (at t = 0) was calculated as follows:

$$H(0) = \frac{1}{e^{rT}} \sum_{k=0}^{M} \binom{M}{k} p^k (1-p)^{M-k} f(S(0)u^k d^{M-k})$$

$where\ f\ represents\ Payoff\ of\ the\ option$
$for\ European\ call\ option: f(x) = \max(x - K, 0)$
$for\ European\ put\ option: f(x) = \max(K - x, 0)$

The Option Prices for various values of M is as follows:

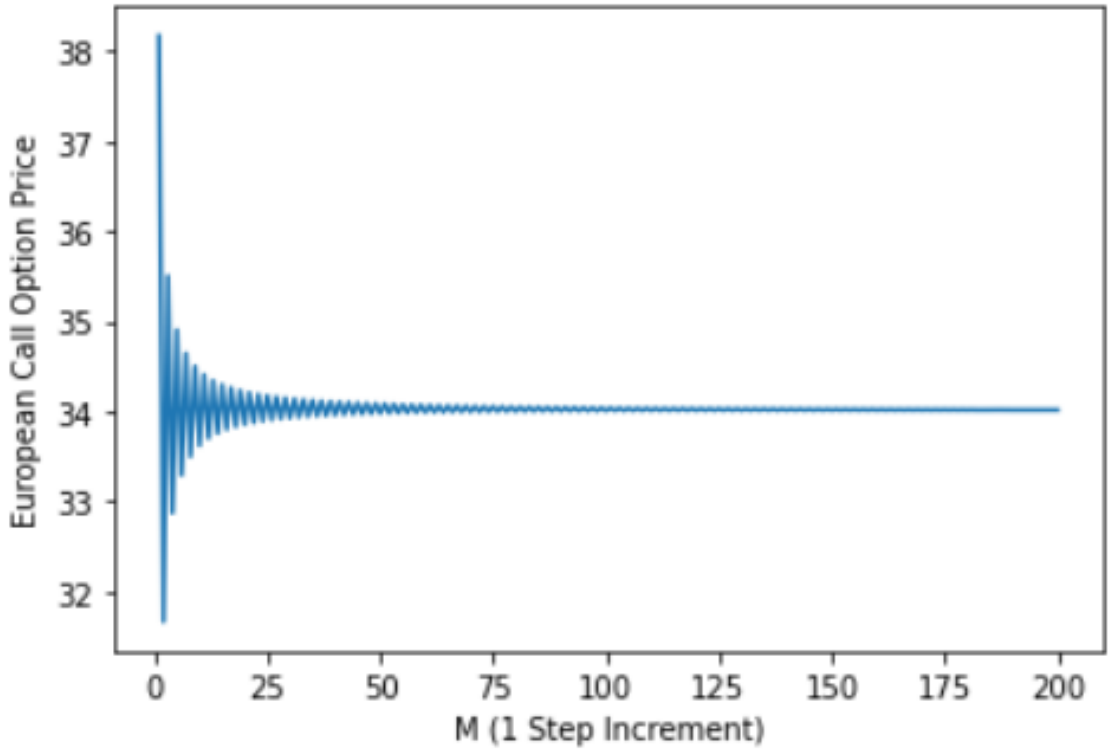
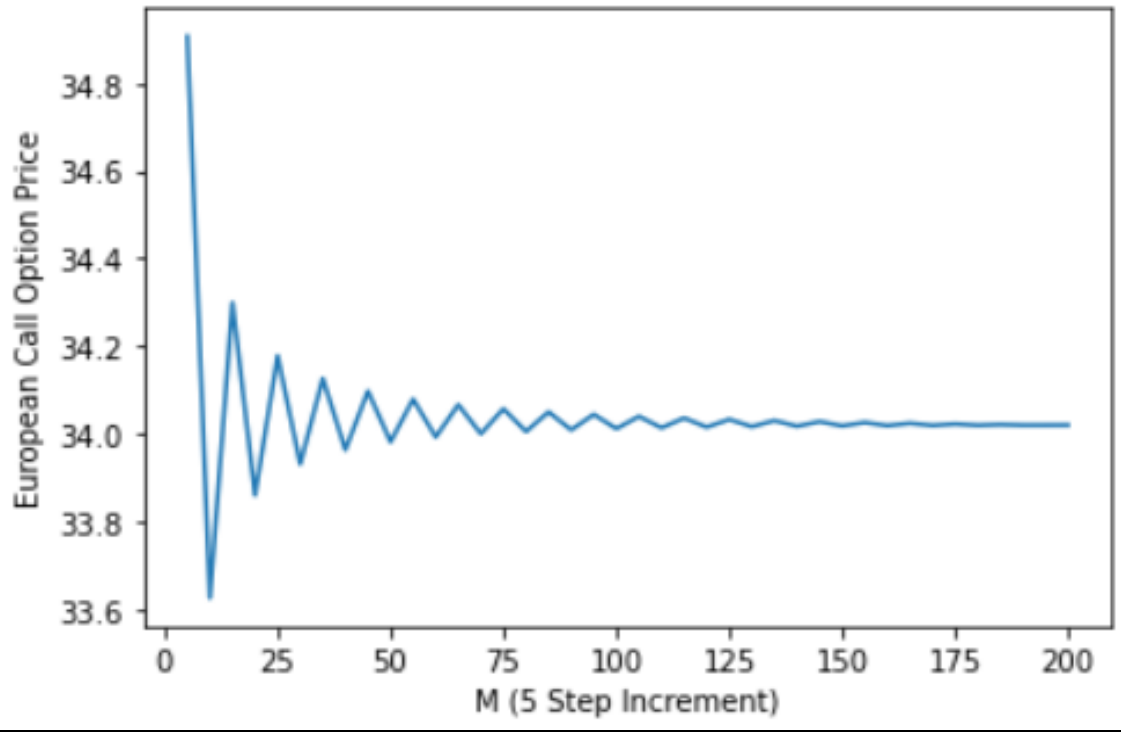
M	European Call Option Price	European Put Option Price
1	38.16763503	19.94171725
5	34.90653251	16.68061473
10	33.62502175	15.39910398
20	33.85944949	15.63353171
50	33.98118437	15.75526659
100	34.01116098	15.78524321
200	34.01957870	15.79366093
400	34.01913177	15.79321399

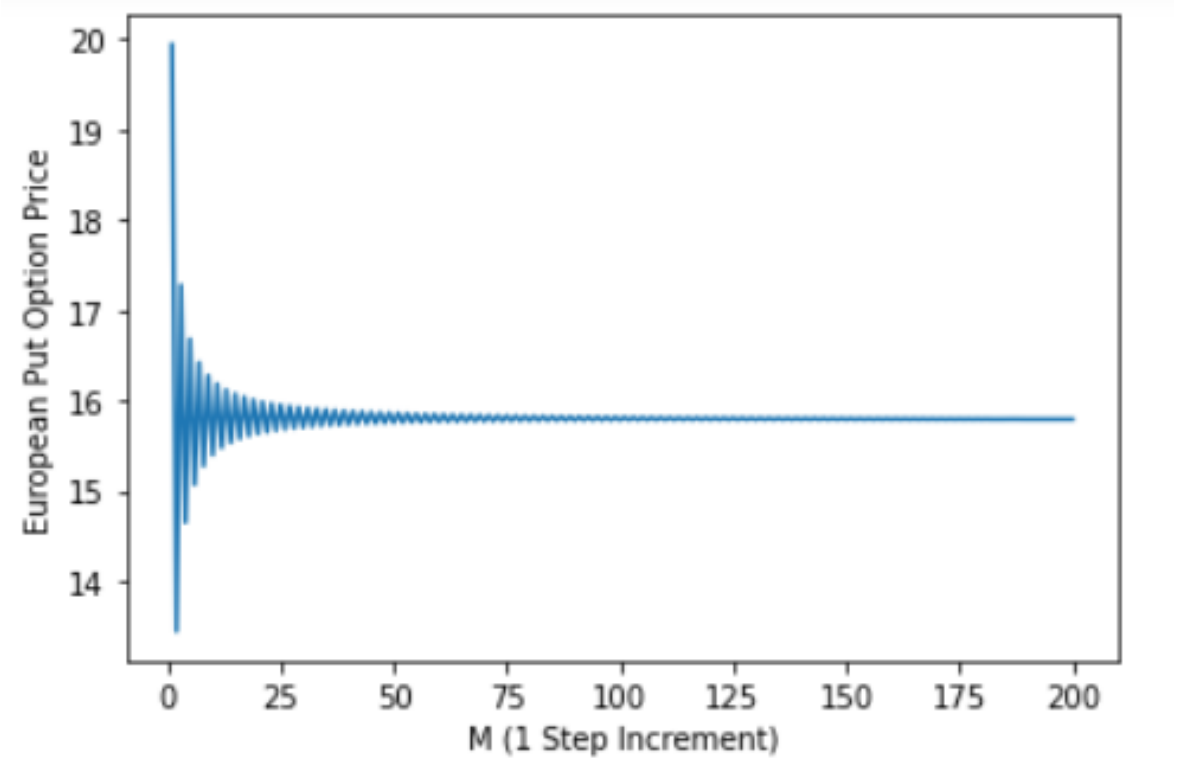
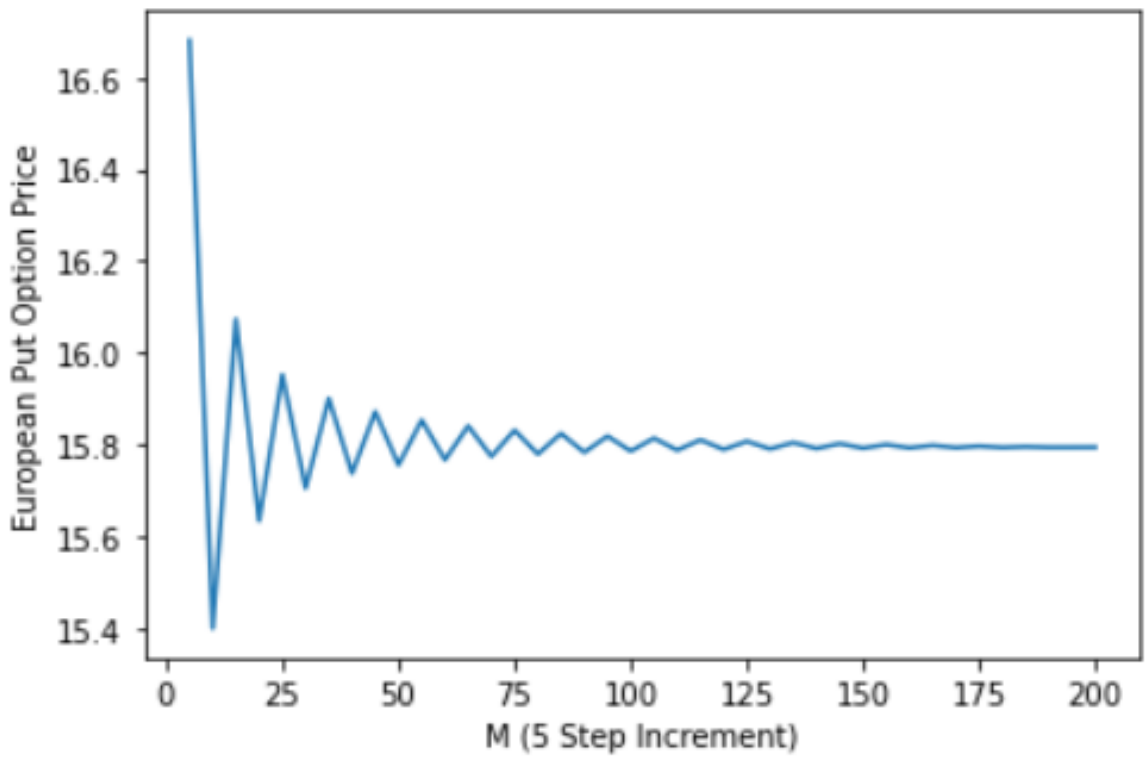
Q2.

Two sets of input arrays (containing various values of M) were used.

The **first array** contained the values from **1 to 200** varying **M** in **steps of 1**.

The **second array** contained the values from **1 to 200** varying **M** in **steps of 5**.

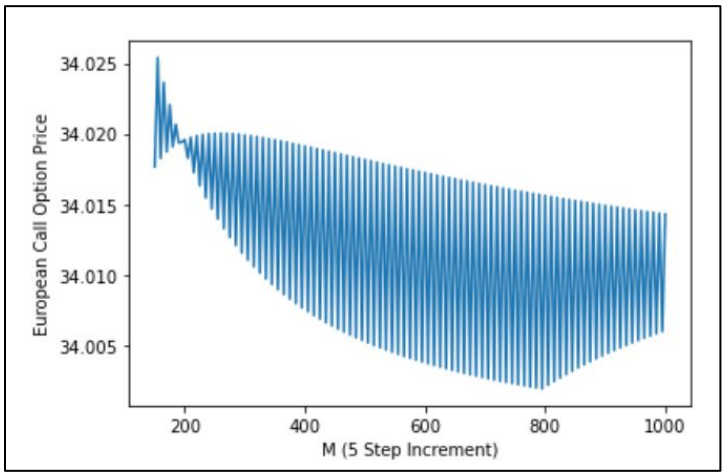
M	European Call Option Price Vs M (Binomial Model Framework)
M with 1 Step Increments	
M with 5 Step Increments	

M	European Put Option Price Vs M (Binomial Model Framework)
M with 1 Step Increments	
M with 5 Step Increments	

Observations:

Both the Call option prices and the Put option prices (at t=0) seem to oscillate about a central value. If we only consider even values of M separately, the corresponding option prices form a gradually increasing sequence **(0<M<200)** converging to the central value from the bottom. Similarly, if we consider only odd values of M separately, the corresponding option prices form a gradually decreasing sequence **(0<M<200)** converging to the central value from the above.

The absolute difference between consecutive values of option prices decreases as M increases. The difference reaches zero as M reaches higher values **(0<M<200)**. The European Call Option price seems to converge to the point **34.01** and the European Call Option price converges to **15.78**.



Note: The graph matches theoretical conclusions when M remains below 200. When M surpasses 200, the option prices graph shows some irregularity, which may be explained through accumulating errors, or computational discrepancies. However, for higher values of M (~1800), these irregularities decrease and the option price converges to the expected values.

Q3.

The value of M has been set to **20**.

For each value of t, the number of remaining time intervals and the number of time intervals that have already occurred were calculated.

All possible case scenarios till time **t** were taken into consideration.

For each possible case, the number of ups and the number of downs that have already occurred have been noted. Then, the stock price at the current time (**t**) was calculated. Considering this as the initial stock price, the option price was calculated (for the remaining time period).

The results obtained are given in the next page.

Given below are all the European Call Option Prices at different times (for all scenarios).
UPS represents the number of ups that have already occurred.
DOWNS represents the number of downs that have already occurred.

M=20

t = 0

UPS	DOWNS	CALL OPTION PRICE
0	0	33.85944948849384

t = 0.5

UPS	DOWNS	CALL OPTION PRICE
0	2	15.095872513879758
1	1	31.893253222246376
2	0	59.95876890092259

t = 1

UPS	DOWNS	CALL OPTION PRICE
0	4	5.154831129992469
1	3	13.469716242796965
2	2	29.80395512132694
3	1	57.69999468717516
4	0	100.66266571336136

t = 1.5

UPS	DOWNS	CALL OPTION PRICE
0	6	1.1250032145209163
1	5	4.121404621027414
2	4	11.767496962598837
3	3	27.5732042363838
4	2	55.29535567856727
5	1	98.43886924880034
6	0	160.6113877530171

t = 3

UPS	DOWNS	CALL OPTION PRICE
0	12	0.0
1	11	0.0
2	10	0.0
3	9	0.1183301448516881
4	8	1.235971133857899
5	7	6.14852046342653
6	6	19.725206220102642
7	5	46.97618778485102
8	4	91.19343329629588
9	3	154.84169905359852
10	2	242.03018282001364
11	1	359.93418379078935
12	0	519.099688850719

t = 4.5

UPS	DOWNS	CALL OPTION PRICE
0	18	0.0
1	17	0.0
2	16	0.0
3	15	0.0
4	14	0.0
5	13	0.0
6	12	0.0
7	11	0.0
8	10	0.0
9	9	8.14917387261674
10	8	36.25149449124525
11	7	83.95057683153279
12	6	149.1496056352558
13	5	237.15908891136283
14	4	355.95946506182884
15	3	516.3231991518774
16	2	732.791598029106
17	1	1024.993372815408
18	0	1419.424512100038

Given below are all the European Put Option Prices at different times (for all scenarios).
UPS represents the number of ups that have already occurred.
DOWNS represents the number of downs that have already occurred.

M=20

t = 0

UPS	DOWNS	PUT OPTION PRICE
0	0	15.633531710991276

t = 0.5

UPS	DOWNS	PUT OPTION PRICE
0	2	24.672817161536063
1	1	15.48714343140138
2	0	8.479204228539846

t = 1

UPS	DOWNS	PUT OPTION PRICE
0	4	35.965303616397534
1	3	24.983286569394075
2	2	15.26943210857484
3	1	8.00422345974074
4	0	3.5041738979719717

t = 1.5

UPS	DOWNS	PUT OPTION PRICE
0	6	48.30495083519326
1	5	36.970072066516465
2	4	25.270959639777367
3	3	14.96337187269708
4	2	7.436262009137822
5	1	2.9982497452660692
6	0	0.9424265244113356

t = 3

UPS	DOWNS	PUT OPTION PRICE
0	12	78.22822279375715
1	11	72.35769482612885
2	10	64.43331094390453
3	9	53.85484171072243
4	8	40.53331384641619
5	7	25.955023925263923
6	6	13.221828652306376
7	5	4.958185582926971
8	4	1.2357022342387138
9	3	0.17210275688518661
10	2	0.008705281628291726
11	1	0.0
12	0	0.0

t = 4.5

UPS	DOWNS	PUT OPTION PRICE
0	18	95.53406311515673
1	17	93.12931642139075
2	16	89.88324791682146
3	15	85.50151375593353
4	14	79.58679130640233
5	13	71.60275111353513
6	12	60.82542413915252
7	11	46.27755440065571
8	10	26.639984302677398
9	9	8.281211219146925
10	8	0.6015461682626682
11	7	0.0
12	6	0.0
13	5	0.0
14	4	0.0
15	3	0.0
16	2	0.0
17	1	0.0
18	0	0.0

Note: For every question and every subcase, the program initially checks for **No Arbitrage Condition** before proceeding further. The following check is carries out:

No Arbitrage Condition: $\boldsymbol{d} < \boldsymbol{e^{\frac{rT}{m}}} < \boldsymbol{u}$