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Q1.

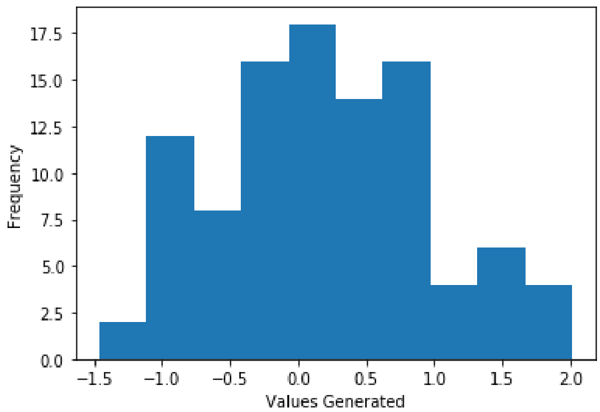
(a) A sample of 100 and 10000 values were generated from $\mathcal{N}$ (0,1) distribution, once using the Box-Muller Method, and once using the Marsaglia and Bray Method. These methods are actually used to generate samples from bivariate distribution from $\mathcal{N}$ (0, I_2). So, each component of the bivariate values generated is considered to be a sample generated from the $\mathcal{N}$ (0,1) distribution. (So, the program generates 50 and 5000 values from the bivariate distribution in both methods). Sample Mean and Sample Variance have also been calculated. They approximately are equal to 0 and 1 respectively (as seen in the below screenshot)

(b) A two-dimensional plot was created in each case using matlplotlib library of python. The x-axis represents the generated values and the y-axis represents the frequency values corresponding to those values. (A frequency histogram was used to display the data). We can see that the frequency histograms resemble a normal distribution with mean 0 and variance 1.

Values Generated = 100	Values Generated = 10000

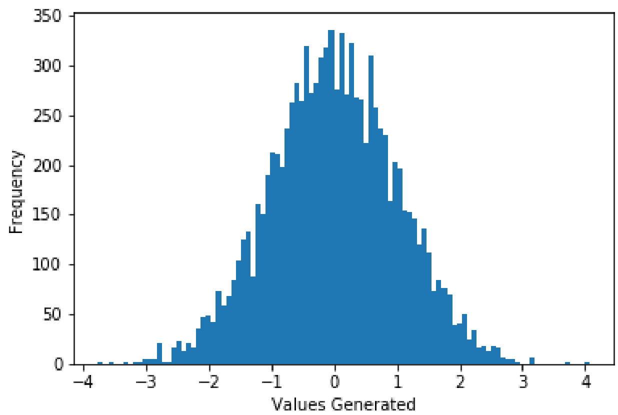
Values Generated = 100 (Box Muller):

```
Method used: Box Muller Method
Sample Mean is 0.20354931898783746
Sample Variance is 0.60702123036487
The plot obtained is as
Number of bins (bars) is 10
```



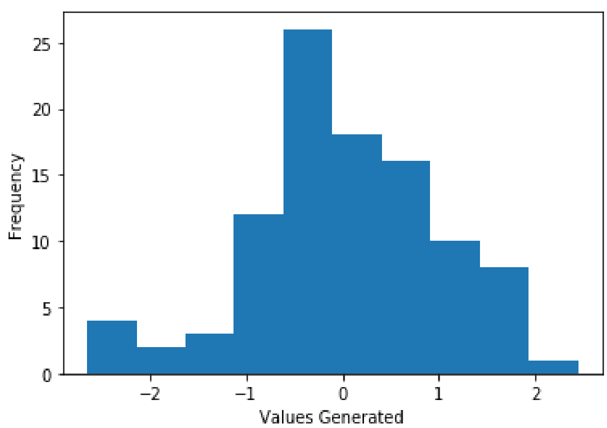
Values Generated = 10000 (Box Muller):

```
Method used: Box Muller Method
Sample Mean is -0.002191000758322
Sample Variance is 1.0118473348800183
The plot obtained is as
Number of bins (bars) is 100
```



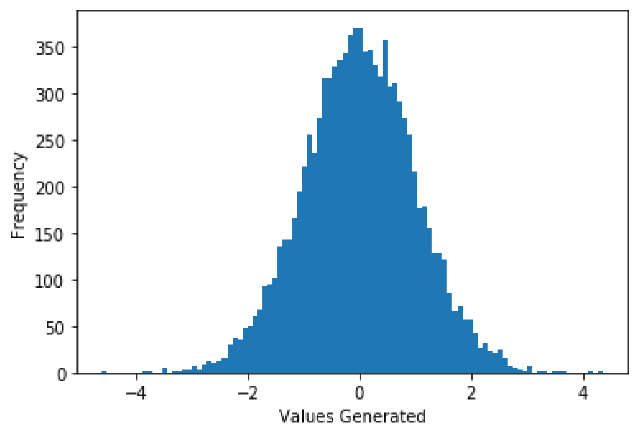
Values Generated = 100 (Marsaglia and Bray):

```
Method used: Marsaglia and Bray Method
Sample Mean is 0.04171919707702867
Sample Variance is 0.9790504898565984
The plot obtained is as
Number of bins (bars) is 10
```



Values Generated = 10000 (Marsaglia and Bray):

```
Method used: Marsaglia and Bray Method
Sample Mean is 0.0010587635612166388
Sample Variance is 0.9985361520078424
The plot obtained is as
Number of bins (bars) is 100
```

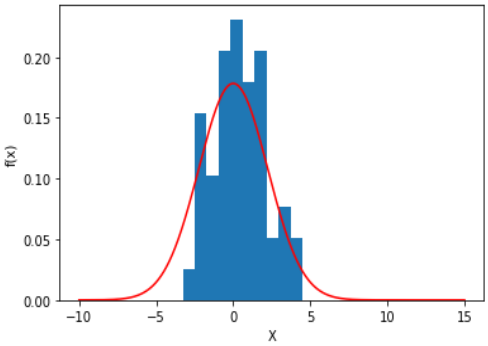


(c)

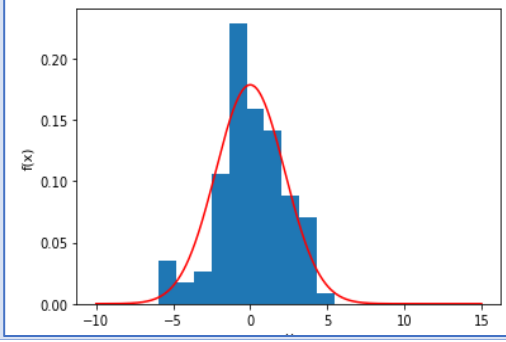
N(0,5)	N(5,5)

N(0,5)

```
Normal distrivution with mean 0 and variance 5 is to be generated
Method used: Box-Muller Method
Number of values generated: 100
```

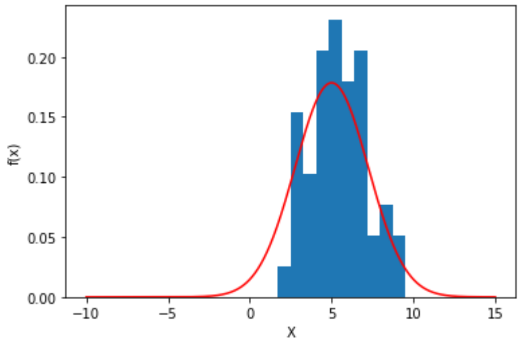



```
Method used: Marsaglia and Bray Method
Number of values generated: 100
```

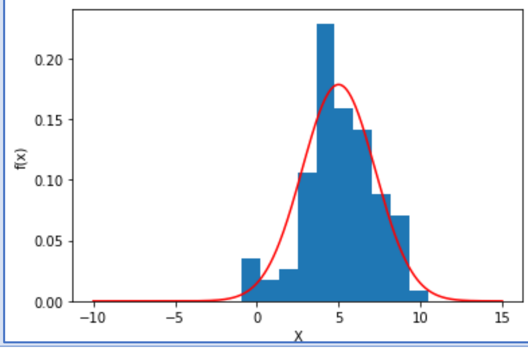


N(5,5)

```
Normal distrivution with mean 5 and variance 5 is to be generated
Method used: Box-Muller Method
Number of values generated: 100
```

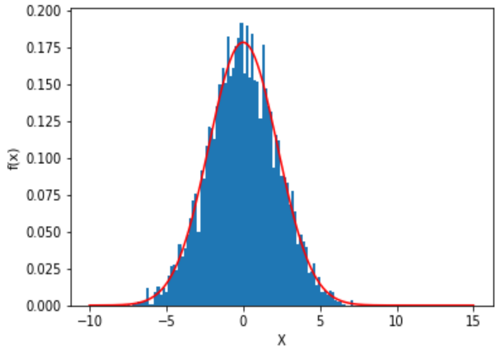



```
Method used: Marsaglia and Bray Method
Number of values generated: 100
```

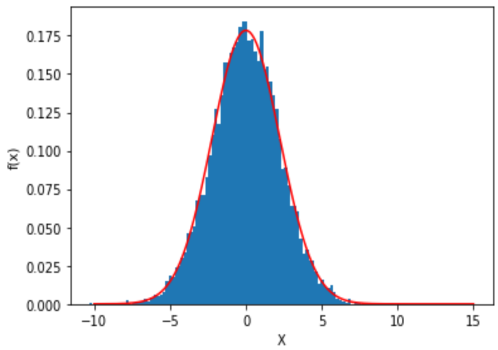


N(0,5)

```
Normal distrivution with mean 0 and variance 5 is to be generated
Method used: Box-Muller Method
Number of values generated: 10000
```

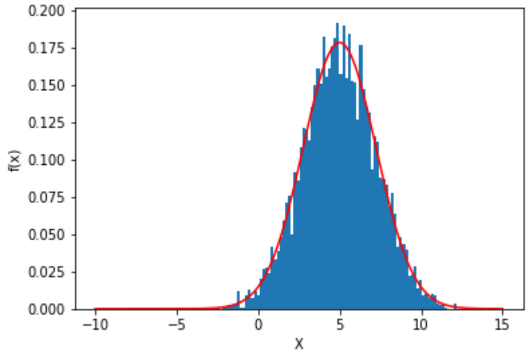



```
Method used: Marsaglia and Bray Method
Number of values generated: 10000
```

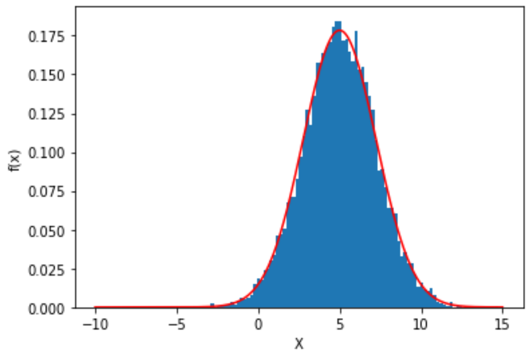


N(5,5)

```
Normal distrivution with mean 5 and variance 5 is to be generated
Method used: Box-Muller Method
Number of values generated: 10000
```




```
Method used: Marsaglia and Bray Method
Number of values generated: 10000
```



Here, we have created distribution plots using the generated values (in blue color) for each case. Here the actual distribution function f(x) has also been plotted (in red color) using the formula for pdf of the normal distribution with given mean and variance. We see that the PDFs created from both the generated samples and the actual formula approximately match. The resemblance is much more strong when the sample size is 10000. It shows the greater the sample size, the better the resemblance. The distributions also are symmetrically distributed about the mean. Both methods mimic randomness equally well.

<u>*Note:</u> $N(\mu,\sigma^2)$ has been obtained from the normal distribution $N(0,1)$ using the transformation formula $\mu + \sigma X$. (where X represents the samples distributed from the $N(0,1)$ distribution)

Q2.

```
Time taken for Box Muller method for generating 10000 values 0.025457
Time taken for Marsaglia and Bray for generating 10000 values 0.016954
```

Time taken(in seconds) for each method was obtained using the time module in python. (This experiment was performed only for 10000 sample size only, as no significant time change was observed when the sample size was 100). It was observed that the Marsaglia and Bray method is faster than compared to the Box Muller method. This is because the evaluation of math.sin and math.cos functions in python is generally done with the help of Taylor expansions (the number of terms calculated is large in order to improve accuracy). It usually takes larger computing times. Marsaglia and Bray method does not use sin and cos functions, and hence usually have smaller computing times as compared to the Box-Muller method.

Q3.

```
The proprtion of values rejected while using Marsaglia and Bray Method:
When 10 values were generated: 0.0
When 100 values were generated: 0.20634920634920634
When 1000 values were generated: 0.2
When 10000 values were generated: 0.22287845819086105
When 100000 values were generated: 0.21611664184369367
When 1000000 values were generated: 0.21400702363323681
```

The rejection proportions were calculated for the Marsaglia and Bray method for different values of the sample size. It could be seen that as the number of values increases, the rejection proportion converges to $1-\frac{\pi}{4} \approx 0.2146$. This is because this method only accepts those values who lie within the circle of radius 1 centered at the origin. The total possible range is represented by the square of length 2 units with the center as (0,0) with sides parallel to the X-axis and the Y-axis. Hence, the acceptance probability is Area of circle/Are of Square = $\frac{\pi}{4}$ and subsequently, the rejection proportion is $1-\frac{\pi}{4}$.