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Q1.

```
The Number of Values Generated is : 10
LCG         X
0.0000041   1
0.2177886   2177
0.0195922   195
0.5000653   5001
0.8155864   8155
0.7027810   7027
0.5525957   5525
0.7065942   7065
0.6421427   6421
0.7131508   7131
```

Case1:
10 values were generated from the Discrete Distribution.
Firstly, 10 values were generated (within the range [0,1)) using Linear Congruence Generator. The parameters for the LCG are as follows: a = 1597, b = 51749, m = 244944, x_0 = 1
From the given discrete distribution, the array q (Cumulative Distribute for the Discrete RV) and the array c (Probability Mass Function) were created. For each random float number (let it be denoted by **U**) generated within the range [0,1), the value of **k** was obtained such that q[k-1] < U <= q[k]. Then, c[k] was chosen to be the required X value.

Case2: 100 values were generated from the Discrete Distribution.

LCG	X
0.0000041	1
0.2177886	2177
0.0195922	195
0.5000653	5001
0.8155864	8155
0.7027810	7027
0.5525957	5525
0.7065942	7065
0.6421427	6421
0.7131508	7131
0.1130340	1131
0.7265661	7265
0.5372861	5373
0.2571282	2571
0.8449401	8449

0.5805572	5805
0.3610989	3611
0.8861536	8861
0.3985197	3985
0.6471683	6471
0.7390955	7391
0.5467290	5467
0.3375588	3375
0.2926546	2927
0.5807409	5807
0.6544925	6545
0.4357200	4357
0.0561108	561
0.8201916	8201
0.0571804	571
0.5283942	5283
0.0568783	569

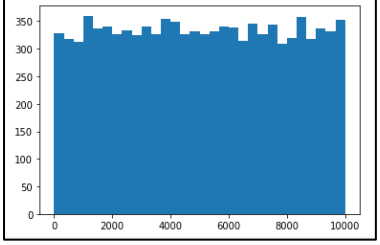
0.8201916	8201
0.0571804	571
0.5283942	5283
0.0568783	569
0.0459248	459
0.5531468	5531
0.5867749	5867
0.2908420	2909
0.6859241	6859
0.6321037	6321
0.6808209	6809
0.4822817	4823
0.4150867	4151
0.1047505	1047
0.4977750	4977
0.1579463	1579
0.4515195	4515

0.2879597	2879
0.0829047	829
0.6100170	6101
0.4083913	4083
0.4121840	4121
0.4691317	4691
0.4146254	4147
0.3680066	3681
0.9177526	9177
0.8622175	8623
0.1725946	1725
0.8447686	8447
0.3067232	3067
0.0481702	481
0.1390685	1391
0.3036980	3037
0.2169557	2169
0.6895413	6895
0.4086975	4087
0.9011733	9011

0.3850758	3851
0.1772773	1773
0.3230453	3231
0.1145609	1145
0.1649928	1649
0.7047937	7047
0.7668855	7669
0.9273997	9273
0.2686410	2687
0.2309385	2309
0.0200536	201
0.2368092	2369
0.3956088	3957
0.9985099	9985
0.8315207	8315
0.1497853	1497
0.4183242	4183
0.2749976	2749
0.3823568	3823
0.8350521	8351
0.7894621	7895
0.9822123	9823

0.8042491	8043
0.5971365	5971
0.8382406	8383
0.8814709	8815
0.9202430	9203
0.8393306	8393
0.6222728	6223
0.9810038	9811
0.8743713	8743
0.5822106	5823
0.0016412	17
0.8322515	8323
0.3168398	3169

```
The Number of Values Generated is : 10000
```



The table above shows the generated values of X (right) and the corresponding value generated by LCG (left). Same Procedure was followed as Case-1.

Case3: 10000 values of X were generated. A histogram to the left depicts that the generated distribution is mimicking uniformity (i.e. the probability of each value from the discrete distribution being chosen is approximately same).

Q2. (a) $f(x) = 20x(1-x)^3$. Now, g(x) denotes the probability density function of $U[0,1]$. So, g(x) = 1, for all x within the range [0,1]. Now, we are required to find the minimum value of c such that the inequality $f(x) \leq cg(x)$ is satisfied for all x within the range [0,1]. So, we are to find the maximum value that f(x)/g(x) attains within the range [0,1]. Upon differentiating, we see that f(x)/g(x) attains its max value at x = ¼. [0 <= x <= 1]. And, the maximum value of f(x) turns out be 2.109375. So, minimum possible value of c is **2.109375**.

```
The value of c chosen is: 2.109375
The number of values taken is 10
X generated   Iterations Reqd.
0.0950132           3
0.4819215           3
0.3019922           1
0.4803007           2
0.5676850           2
0.4763818           4
0.4282096           1
0.3256589           1
0.3330060           1
0.4380397           1
The Averge Value obtained for Nummber of iterations is 1.9
```

(b) Keeping c = 2.109375, random variables were generated from f(x) using the acceptance-rejection method. The following table to the left shows 10 generated values of f(x) (and the number of iterations required in each case). Using python inbuilt distribution builder, a set of coordinates (x_i, y_i) were obtained using the generated X values (similar procedure followed as the Frequency histograms in the previous assignments).

```
The value of c chosen is: 2.109375
The number of values taken is 10
The Averge Value obtained for Nummber of iterations is 1.9
The error value is 3.8649059449206113

The number of values taken is 100
The Averge Value obtained for Nummber of iterations is 1.73
The error value is 0.4233404294398935

The number of values taken is 1000
The Averge Value obtained for Nummber of iterations is 2.03
The error value is 0.05767565635325676

The number of values taken is 10000
The Averge Value obtained for Nummber of iterations is 2.093
The error value is 0.02208315717230523

The number of values taken is 100000
The Averge Value obtained for Nummber of iterations is 2.10838
The error value is 0.021290921155687044
```

Using the coordinates and the given function f(x), the error was calculated (in a similar fashion to regression) using the formula: **error** = $\frac{1}{m}\sum_{k=1}^{m}(f(x_i)-y_i)^2$. As shown in the screenshots, the error value diminishes to 0 as the number of values generated increases. Hence, the distribution formed by the sample X values converges to the distribution of f(x).

(c) The experimented was repeated for different number of total values generated. The outcomes can be seen in the screenshots. It can be seen that as the number of generated values increases, the avg. number of iterations converges to c. The outcome of this experiment signifies that the expected number of iterations required for a generated value to be accepted is c. This is true, because, it can be proved that the Acceptance probability (Probability that a certain value is accepted) is 1/c. (So, the expected value of the iterations required is c).

(d) The above experiment was repeated with 2 higher values of c (5 and 10). In both these cases, the average number of iterations converge to their respective values of c (supporting the fact that the acceptance probability is 1/c as mentioned above). Since, it takes longer times to generate the values of X when values of are higher, it is always more practical to ensure that value of c is closer to the minimum possible value. Larger values of c decrease the acceptance probability, and hence lead to larger run times. Also, as number of generated values increases, less the is the error between the sample distribution and f(x).

```
The value of c chosen is: 5
The number of values taken is 10
The Averge Value obtained for Nummber of iterations is 7.2
The error value is 9.80851581608693

The number of values taken is 100
The Averge Value obtained for Nummber of iterations is 4.85
The error value is 0.37986993273738373

The number of values taken is 1000
The Averge Value obtained for Nummber of iterations is 4.854
The error value is 0.020266620296368963

The number of values taken is 10000
The Averge Value obtained for Nummber of iterations is 4.9024
The error value is 0.021364165075157482

The number of values taken is 100000
The Averge Value obtained for Nummber of iterations is 4.99773
The error value is 0.02295345335572417
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```
The value of c chosen is: 10
The number of values taken is 10
The Averge Value obtained for Nummber of iterations is 6.6
The error value is 6.480300106865036

The number of values taken is 100
The Averge Value obtained for Nummber of iterations is 10.89
The error value is 0.12195885344770299

The number of values taken is 1000
The Averge Value obtained for Nummber of iterations is 9.637
The error value is 0.04169592911840443

The number of values taken is 10000
The Averge Value obtained for Nummber of iterations is 10.0782
The error value is 0.028506292726774435

The number of values taken is 100000
The Averge Value obtained for Nummber of iterations is 9.99132
The error value is 0.021686855945622058
```

Q3.

The values of c chosen for are: c = 2, and c = 3. These values are possible (all values of c >= 1.2 are possible). For each value of c, the experiment is conducted for 10,100 and 1000 values (number of values generated using acceptance rejection method).

Note: During performing acceptance and rejection method, Random values for U [0,1] have been obtained though random module of python. For generating the values of distribution **g**, the technique for generating values for Discrete distributions was employed.

After generating the values, a frequency table was also generated.

The first observation is that higher values of c implies higher Avg. Acceptance Iterations.

The second observation is that as the number of generated values increases, the avg. value of acceptance iterations converges to the value of c.

Also, from the frequency tables, as the number of generated values reaches larger values, the generated values seem to follow the distribution followed by f(x). (i.e. (frequency of certain value)/ (number of values) is approximately equal to f (certain value) where f is the probability mass function of X).

The outcome of the experiments is as follows:

The value of c is 2

```
The value of c chosen is: 2
The number of values taken is 10
8 7 3 7 3 1 5 2 1 3
The Frequency Table is as follows:
Value Count
  1     2
  2     1
  3     3
  4     0
  5     1
  6     0
  7     2
  8     1
  9     0
  10     0
Áverage iterations taken 1.2

The number of values taken is 100
10 1 9 9 4 10 7 5 4 5 10 3 4 4 10 8 5 7 5 4 6 7 6 4 9 6 5 7 9 8 3 3 5 7 7 7 1 8 10 6 2 8 4 6 6 1 7 7 4 1
0 7 9 2 2 9 1 9 8 3 8 9 2 7 1 6 5 2 6 10 9 6 5 2 4 6 5 3 9 9 3 3 2 7 8 2 10 2 9 6 3 7 3 3 3 2 4 5 2 7 9
The Frequency Table is as follows:
Value Count
  1     5
  2     11
  3     11
  4     10
  5     10
  6     11
  7     14
  8     7
  9     13
  10     8
Áverage iterations taken 1.97

The number of values taken is 1000
4 10 3 5 9 2 7 10 9 8 3 3 10 1 8 8 2 3 1 9 2 5 10 4 2 8 6 9 6 10 3 7 4 4 9 2 3 1 10 4 10 1 8 6 2 6 10 10
 3 2 10 7 8 10 1 1 5 3 7 3 8 2 7 6 2 2 8 10 7 8 5 7 8 7 1 10 1 2 8 7 1 7 2 7 7 4 10 9 4 4 2 8 1 10 7 1 7
 4 5 4 10 2 8 10 8 4 1 5 1 7 6 3 1 5 6 3 1 7 5 4 9 2 6 9 1 1 5 2 1 6 9 2 8 5 1 10 8 5 7 2 1 8 6 5 9 7 10
 4 1 6 5 9 5 8 1 8 1 9 1 2 6 5 1 6 1 5 9 4 5 6 7 8 7 8 2 3 6 10 9 8 6 10 3 10 6 10 6 9 3 8 1 1 2 4 6 8 2
 7 4 1 1 7 2 4 8 4 6 9 1 1 9 9 8 9 2 7 3 2 7 7 6 2 6 6 5 10 5 2 3 8 4 9 6 5 5 6 5 4 10 3 9 5 7 5 5 2 1 2
 2 7 6 9 5 6 3 4 10 8 5 1 4 9 10 1 10 4 7 10 7 1 4 2 5 3 9 2 8 2 8 8 7 1 6 9 2 5 5 10 6 6 8 3 1 2 8 7 1
6 3 5 8 1 8 8 1 8 10 8 6 8 5 10 9 10 2 9 7 10 6 4 2 5 3 10 6 10 3 10 2 8 7 2 3 6 9 9 5 6 3 5 8 1 8 4 5 3
 4 8 7 1 10 2 5 5 1 8 1 9 9 5 5 9 7 10 8 5 8 9 6 10 7 7 10 2 1 9 5 6 7 1 2 9 9 3 1 4 9 9 1 6 6 8 2 5 4 4
 1 9 8 5 7 9 7 3 10 4 9 9 4 5 3 2 10 8 9 9 5 9 4 3 6 3 3 5 5 10 7 6 2 8 1 8 9 4 1 8 7 9 1 4 10 5 3 6 6 9
 1 10 4 6 10 5 10 5 10 8 4 7 1 10 6 3 2 6 10 8 3 3 8 9 9 3 9 10 5 10 8 4 3 4 10 4 8 7 3 6 6 8 4 7 1 4 9
7 8 1 7 8 6 3 8 5 2 9 9 2 4 7 5 6 1 5 2 3 1 3 1 2 10 5 2 7 8 10 4 2 3 4 10 3 9 4 8 9 2 5 6 7 9 8 5 9 7 1
0 4 9 10 1 5 6 9 3 9 6 5 5 3 1 4 3 2 9 2 1 8 10 6 2 1 2 1 8 1 10 4 6 7 1 7 10 2 8 9 9 4 2 10 6 9 4 10 5
5 2 4 10 2 1 8 5 8 1 8 8 3 4 10 5 8 6 8 6 5 3 10 9 10 3 4 4 7 1 5 7 10 9 9 10 6 3 1 4 8 7 6 5 1 5 10 8 3
 2 3 1 7 9 1 10 6 2 2 10 1 2 1 8 9 10 4 7 3 6 9 7 1 6 9 3 8 10 3 1 2 9 9 9 7 8 6 2 6 1 7 10 4 6 2 5 3 2
8 7 8 8 3 7 4 5 5 10 9 5 8 8 6 7 3 10 9 1 1 1 1 2 3 1 3 6 2 5 1 9 10 5 9 5 1 3 10 5 6 1 1 6 5 1 2 4 1 2
1 4 1 6 9 1 1 4 6 5 9 8 4 6 9 8 4 9 7 9 6 9 10 5 1 2 7 1 9 9 5 5 5 10 10 10 1 4 4 8 6 3 1 1 4 4 8 5 2 2
1 2 6 4 6 3 10 2 6 4 4 1 9 9 1 3 1 6 1 5 10 8 10 10 1 5 9 2 3 1 7 5 2 2 2 2 4 7 2 10 8 6 8 10 5 3 7 9 2
4 8 9 2 5 6 6 2 9 10 9 4 4 6 4 5 1 7 1 2 1 1 1 8 8 5 10 2 9 8 4 8 5 7 9 4 3 5 6 5 8 6 1 2 9 5 5 9 5 10 4
 1 5 4 5 6 1 8 6 9 1 2 9 2 5 9 4 6 5 7 7 9 5 1 10 8 1 3 1 2 6 5 4 1 7 5 5 3 1 4 6 6 5 10 2 7 10 8 6 4 1
6 9 7 7 9 8 2 10 7 5 9 1 7 8 10 4 3 1 5 3 6 10 5 2 6 1 5 5 1 5 4 5 4 9 1 2 2 6 10 5 2 7 8 1 7 9 6 5 6 1
2 9 4 2 7 9 2 4 9 2
The Frequency Table is as follows:
Value Count
  1     128
  2     103
  3     72
  4     89
  5     115
  6     99
  7     82
  8     102
  9     110
  10     100
Áverage iterations taken 2.046
```

The value of c is 3

```
The value of c chosen is: 3
The number of values taken is 10
3 2 5 4 10 4 3 1 1 10
The Frequency Table is as follows:
Value Count
  1     2
  2     1
  3     2
  4     2
  5     1
  6     0
  7     0
  8     0
  9     0
  10     2
Áverage iterations taken 3.3

The number of values taken is 100
8 4 8 8 1 1 5 4 6 8 7 6 1 3 10 8 10 8 6 6 3 2 2 6 1 1 9 10 10 9 4 9 5 2 9 6 9 8 7 2 6 7 7 1 9 5 7 1 1 1
9 10 3 3 10 5 8 8 4 6 7 8 4 1 8 10 3 7 6 10 6 10 7 8 6 6 6 2 8 8 8 6 5 5 1 10 5 5 2 5 2 2 4 4 1 7 6 5 7
3
The Frequency Table is as follows:
Value Count
  1     12
  2     8
  3     6
  4     7
  5     10
  6     15
  7     10
  8     15
  9     7
  10     10
Áverage iterations taken 3.05

The number of values taken is 1000
10 5 6 3 1 6 1 5 6 6 2 10 6 2 10 7 8 9 2 8 10 8 3 7 6 10 7 2 6 2 9 2 5 4 10 8 1 5 6 3 1 5 6 10 6 2 7 9 7
 5 6 2 2 3 3 2 8 5 5 4 3 9 3 5 2 8 5 1 5 9 7 3 4 3 6 4 1 9 10 9 8 6 3 8 7 1 2 3 8 9 1 3 9 10 7 2 4 10 8
4 8 3 9 5 7 10 7 10 1 8 7 5 9 1 5 6 7 5 7 1 10 7 9 6 4 10 1 10 3 2 7 9 1 10 6 3 4 4 9 1 4 9 10 6 9 2 4 8
 10 8 8 9 5 9 8 9 3 1 9 9 2 7 5 9 10 2 2 6 2 5 4 1 4 1 8 3 1 7 9 8 9 9 8 2 8 1 1 1 5 9 10 10 8 5 10 2 4
3 9 10 3 8 9 8 8 1 5 10 6 7 3 6 5 3 6 5 5 2 6 4 10 2 5 2 8 4 9 1 5 10 4 10 10 7 1 5 1 8 1 2 2 3 10 2 4 1
0 7 4 5 6 4 3 10 1 5 7 8 6 8 3 1 6 10 6 10 10 1 4 10 7 3 7 2 3 1 10 10 10 10 5 4 2 10 2 7 9 1 4 8 7 2 2
7 8 10 3 2 4 2 3 2 10 8 10 2 7 10 9 1 9 6 8 5 6 5 2 3 2 2 6 1 5 9 1 3 6 8 8 7 8 4 7 10 1 8 6 5 7 3 2 2 8
 6 1 1 3 2 7 7 1 5 5 2 8 1 8 3 3 6 9 6 4 9 6 2 9 8 2 2 1 8 8 2 6 7 2 1 5 9 1 9 6 10 6 6 10 6 1 3 7 5 1 2
 5 1 6 7 1 2 1 8 2 6 5 10 8 4 9 7 5 2 10 7 7 3 6 7 4 3 2 9 9 10 3 8 2 9 5 8 8 8 10 2 7 8 9 9 1 5 3 7 1 2
 8 5 1 2 4 8 2 4 9 2 10 9 1 1 1 5 6 5 3 2 9 9 1 7 8 10 2 1 8 7 9 2 3 1 1 2 9 10 4 6 1 4 8 10 7 10 1 9 3
5 5 2 2 1 1 5 5 4 5 6 7 9 7 2 7 9 8 1 7 6 2 1 1 1 1 5 9 6 5 7 10 2 9 1 9 5 7 3 8 7 1 5 8 5 2 9 10 2 6 8
10 2 2 7 7 7 5 9 2 5 10 3 2 1 9 3 2 1 5 10 6 2 1 9 7 9 7 1 4 10 3 9 9 5 6 6 3 5 9 3 1 2 7 9 1 10 9 3 3 3
 5 2 5 3 5 5 5 10 1 9 5 9 8 5 9 3 8 6 9 2 5 10 5 9 3 5 6 5 1 10 6 7 4 10 6 6 7 9 3 8 3 7 7 5 9 1 2 7 2 2
 3 3 8 6 6 1 1 6 4 6 10 9 2 2 10 4 5 6 6 10 5 2 1 9 5 7 7 4 10 10 10 5 1 6 4 5 8 5 9 2 9 10 10 5 10 2 10
 5 2 10 6 3 2 1 8 6 7 7 6 4 9 8 3 9 2 7 1 10 1 7 10 4 1 2 7 8 8 4 7 10 4 6 10 2 9 3 1 6 6 8 5 10 4 1 5 6
 2 8 1 10 6 7 8 6 7 4 2 8 8 9 1 7 4 9 2 1 9 3 5 9 3 7 2 10 5 5 7 6 7 2 5 5 3 5 3 10 3 7 6 9 7 10 5 9 6 1
 1 7 5 4 2 8 4 8 2 8 7 9 10 2 1 5 6 6 2 4 4 10 1 5 8 5 7 9 5 8 10 9 3 6 10 10 2 1 2 5 2 1 2 9 3 5 8 2 7
7 9 1 8 1 3 2 4 5 5 3 9 4 9 4 2 4 9 10 7 9 2 9 3 5 5 4 9 2 9 8 10 3 2 1 7 6 8 3 7 5 8 6 2 7 6 7 7 3 8 2
9 10 9 4 9 3 4 7 2 3 5 9 10 3 9 6 4 3 7 9 1 3 1 10 10 2 8 3 7 2 1 8 2 10 4 3 6 9 8 2 5 2 1 4 4 4 5 9 7 1
0 5 4 4 8 9 2 9 2 4 9 1 5 6 2 1 8 3 9 7 8 9 2 5 6 4 2 4 1 8 5 3 1 8 1 4 8 6 8 6 2 1 8 5 3 7 6 3 2 1 2 5
4 8 6 8 7 10 9 7 1 3
The Frequency Table is as follows:
Value Count
  1     111
  2     127
  3     86
  4     69
  5     111
  6     90
  7     97
  8     96
  9     112
  10     101
Áverage iterations taken 2.905
```