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Dept.: Mathematics and Computing

Q1. The Table given below shows the sequence generated for:

i)  $a = 6, b = 0, m = 11$

a	b	m	$x_0$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	Period Length
6	0	11	0	0	0	0	0	0	0	0	0	0	0	1
6	0	11	1	6	3	7	9	10	5	8	4	2	1	10
6	0	11	2	1	6	3	7	9	10	5	8	4	2	10
6	0	11	3	7	9	10	5	8	4	2	1	6	3	10
6	0	11	4	2	1	6	3	7	9	10	5	8	4	10
6	0	11	5	8	4	2	1	6	3	7	9	10	5	10
6	0	11	6	3	7	9	10	5	8	4	2	1	6	10
6	0	11	7	9	10	5	8	4	2	1	6	3	7	10
6	0	11	8	4	2	1	6	3	7	9	10	5	8	10
6	0	11	9	10	5	8	4	2	1	6	3	7	9	10
6	0	11	10	5	8	4	2	1	6	3	7	9	10	10

We can observe that for  $x_0 = 0$ , the period length is 1.

We can observe that for all other values of  $x_0$ , the period length is 10.

Since  $10 = (m-1)$ , this is the maximum period length that can be achieved.

ii)  $a = 3, b = 0, m = 11$

a	b	m	$x_0$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	Period Length
3	0	11	0	0	0	0	0	0	0	0	0	0	0	1
3	0	11	1	3	9	5	4	1	3	9	5	4	1	5
3	0	11	2	6	7	10	8	2	6	7	10	8	2	5
3	0	11	3	9	5	4	1	3	9	5	4	1	3	5
3	0	11	4	1	3	9	5	4	1	3	9	5	4	5
3	0	11	5	4	1	3	9	5	4	1	3	9	5	5
3	0	11	6	7	10	8	2	6	7	10	8	2	6	5
3	0	11	7	10	8	2	6	7	10	8	2	6	7	5
3	0	11	8	2	6	7	10	8	2	6	7	10	8	5
3	0	11	9	5	4	1	3	9	5	4	1	3	9	5
3	0	11	10	8	2	6	7	10	8	2	6	7	10	5

We can observe that for  $x_0 = 0$ , the period length is 1.

We can observe that for all other values of  $x_0$ , the period length is 5.

So, the best choice is:  $a = 6, b = 0, m = 11, x_0 = 1$  to 10

If we take any one of the above choices, 10 distinct values (full period) appear before repetition. This is because the given values of  $a, b$  and  $m$  follow the condition for full period.

- $b=0$
- $m=11$  (prime number)
- 11 divides  $6^{10}-1$
- 11 does not divide  $6^j-1$  for  $j \in \{1, 2, \dots, 9\}$

Q2.

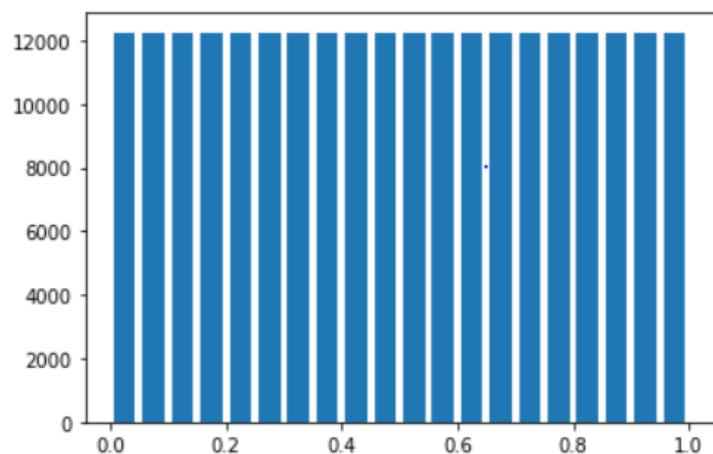
5 random values of  $x_0$  (for each case) were chosen using random module in python. From the histograms and the frequency tables, it can be observed that frequency of each category is approximately equal, indicating that the values generated by the linear congruence generator is uniformly distributed between 0 and 1. The fraction of values falling in any subinterval is approximately equal to its length. Hence, the generator mimics uniformity.

**Note:** For the first graph of each case,  $a=1597, b=51749$ , and second graph of each case,  $a=51749, b=1597$ .

Case : 1

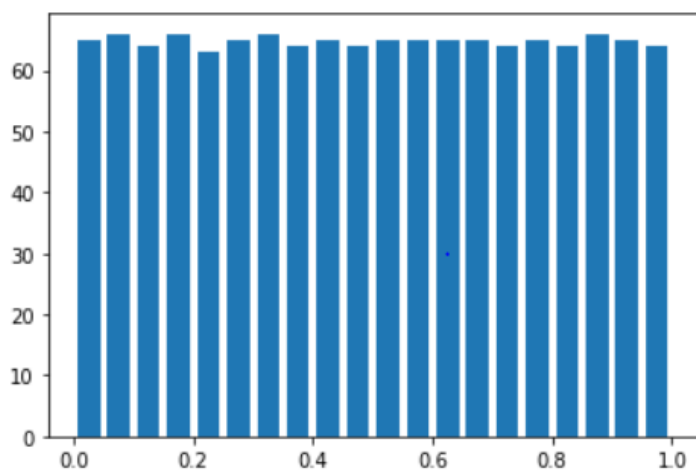
The value of  $x_0$  chosen is : 21555

The value of  $a$  is : 1597



Interval	Count
0.0 - 0.05	12248
0.05 - 0.1	12247
0.1 - 0.15	12247
0.15 - 0.2	12247
0.2 - 0.25	12247
0.25 - 0.3	12248
0.3 - 0.35	12247
0.35 - 0.4	12247
0.4 - 0.45	12247
0.45 - 0.5	12247
0.5 - 0.55	12248
0.55 - 0.6	12247
0.6 - 0.65	12247
0.65 - 0.7	12247
0.7 - 0.75	12247
0.75 - 0.8	12248
0.8 - 0.85	12247
0.85 - 0.9	12247
0.9 - 0.95	12247
0.95 - 1.0	12247

The value of  $a$  is : 51749

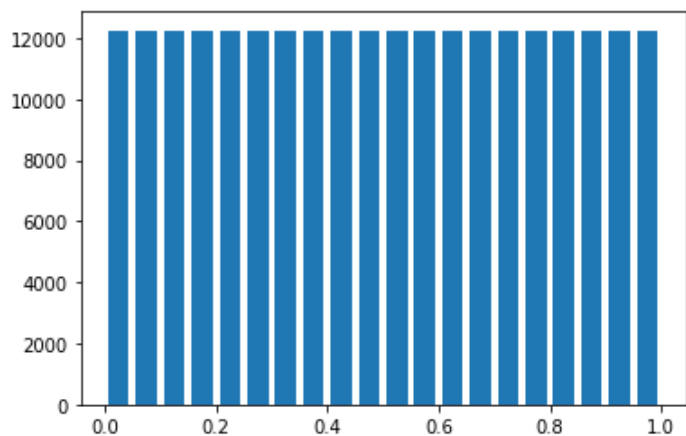


Interval	Count
0.0 - 0.05	65
0.05 - 0.1	64
0.1 - 0.15	66
0.15 - 0.2	65
0.2 - 0.25	64
0.25 - 0.3	65
0.3 - 0.35	64
0.35 - 0.4	66
0.4 - 0.45	65
0.45 - 0.5	64
0.5 - 0.55	65
0.55 - 0.6	64
0.6 - 0.65	66
0.65 - 0.7	65
0.7 - 0.75	64
0.75 - 0.8	65
0.8 - 0.85	64
0.85 - 0.9	66
0.9 - 0.95	65
0.95 - 1.0	64

Case : 2

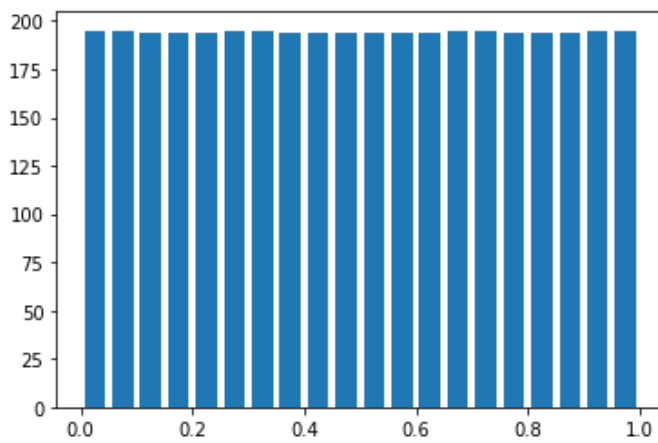
The value of  $x_0$  chosen is : 76392

The value of  $a$  is : 1597



Interval	Count
0.0 - 0.05	12248
0.05 - 0.1	12247
0.1 - 0.15	12247
0.15 - 0.2	12247
0.2 - 0.25	12247
0.25 - 0.3	12248
0.3 - 0.35	12247
0.35 - 0.4	12247
0.4 - 0.45	12247
0.45 - 0.5	12247
0.5 - 0.55	12248
0.55 - 0.6	12247
0.6 - 0.65	12247
0.65 - 0.7	12247
0.7 - 0.75	12247
0.75 - 0.8	12248
0.8 - 0.85	12247
0.85 - 0.9	12247
0.9 - 0.95	12247
0.95 - 1.0	12247

The value of a is : 51749

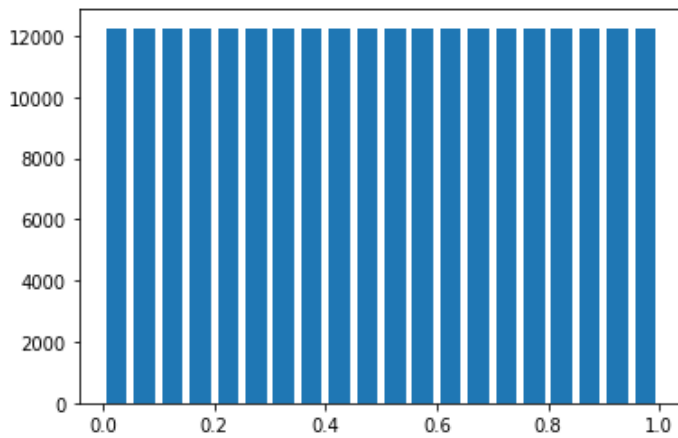


Interval	Count
0.0 - 0.05	195
0.05 - 0.1	195
0.1 - 0.15	194
0.15 - 0.2	194
0.2 - 0.25	194
0.25 - 0.3	195
0.3 - 0.35	195
0.35 - 0.4	194
0.4 - 0.45	194
0.45 - 0.5	194
0.5 - 0.55	195
0.55 - 0.6	195
0.6 - 0.65	194
0.65 - 0.7	194
0.7 - 0.75	194
0.75 - 0.8	195
0.8 - 0.85	195
0.85 - 0.9	194
0.9 - 0.95	194
0.95 - 1.0	194

Case : 3

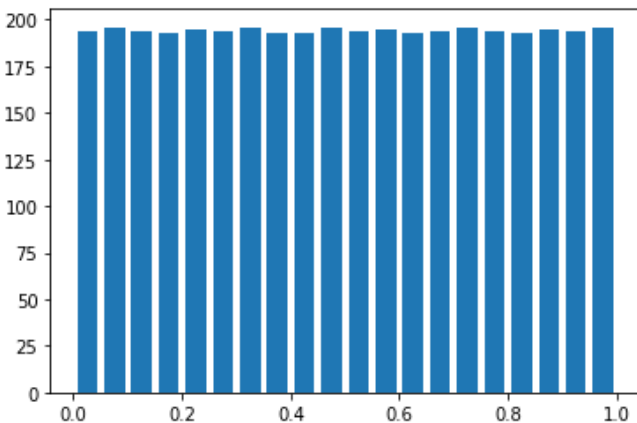
The value of x0 chosen is : 115212

The value of a is : 1597



Interval	Count
0.0 - 0.05	12248
0.05 - 0.1	12247
0.1 - 0.15	12247
0.15 - 0.2	12247
0.2 - 0.25	12247
0.25 - 0.3	12248
0.3 - 0.35	12247
0.35 - 0.4	12247
0.4 - 0.45	12247
0.45 - 0.5	12247
0.5 - 0.55	12248
0.55 - 0.6	12247
0.6 - 0.65	12247
0.65 - 0.7	12247
0.7 - 0.75	12247
0.75 - 0.8	12248
0.8 - 0.85	12247
0.85 - 0.9	12247
0.9 - 0.95	12247
0.95 - 1.0	12247

The value of a is : 51749

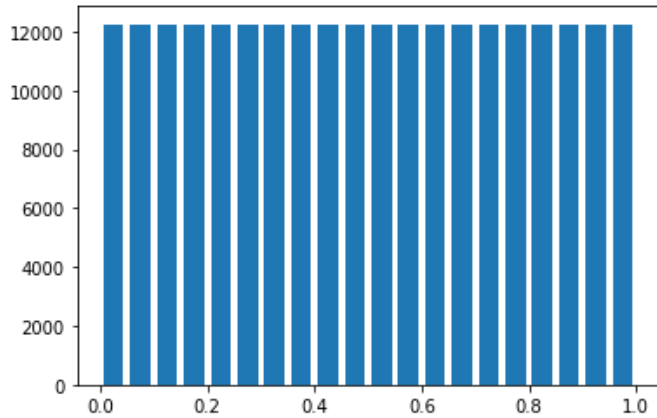


Interval	Count
0.0 - 0.05	193
0.05 - 0.1	194
0.1 - 0.15	195
0.15 - 0.2	194
0.2 - 0.25	196
0.25 - 0.3	193
0.3 - 0.35	194
0.35 - 0.4	195
0.4 - 0.45	194
0.45 - 0.5	196
0.5 - 0.55	193
0.55 - 0.6	194
0.6 - 0.65	195
0.65 - 0.7	194
0.7 - 0.75	196
0.75 - 0.8	193
0.8 - 0.85	194
0.85 - 0.9	195
0.9 - 0.95	194
0.95 - 1.0	196

Case : 4

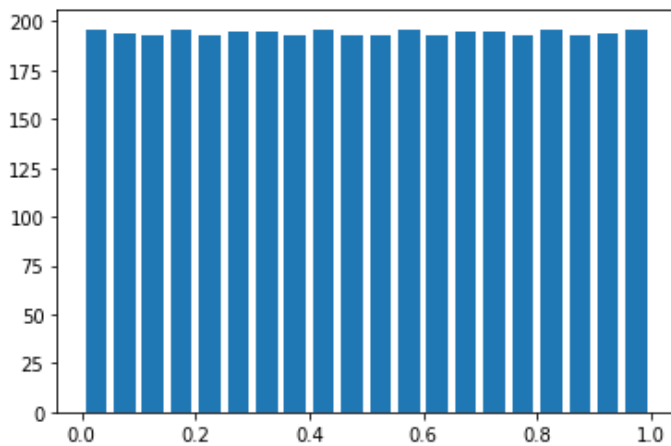
The value of  $x_0$  chosen is : 78652

The value of  $a$  is : 1597



Interval	Count
0.0 - 0.05	12248
0.05 - 0.1	12247
0.1 - 0.15	12247
0.15 - 0.2	12247
0.2 - 0.25	12247
0.25 - 0.3	12248
0.3 - 0.35	12247
0.35 - 0.4	12247
0.4 - 0.45	12247
0.45 - 0.5	12247
0.5 - 0.55	12248
0.55 - 0.6	12247
0.6 - 0.65	12247
0.65 - 0.7	12247
0.7 - 0.75	12247
0.75 - 0.8	12248
0.8 - 0.85	12247
0.85 - 0.9	12247
0.9 - 0.95	12247
0.95 - 1.0	12247

The value of  $a$  is : 51749

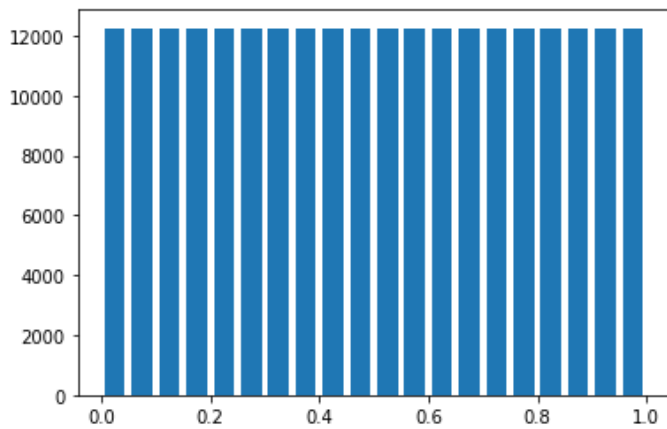


Interval	Count
0.0 - 0.05	195
0.05 - 0.1	195
0.1 - 0.15	193
0.15 - 0.2	196
0.2 - 0.25	193
0.25 - 0.3	195
0.3 - 0.35	195
0.35 - 0.4	193
0.4 - 0.45	196
0.45 - 0.5	193
0.5 - 0.55	195
0.55 - 0.6	195
0.6 - 0.65	193
0.65 - 0.7	196
0.7 - 0.75	193
0.75 - 0.8	195
0.8 - 0.85	195
0.85 - 0.9	193
0.9 - 0.95	196
0.95 - 1.0	193

Case : 5

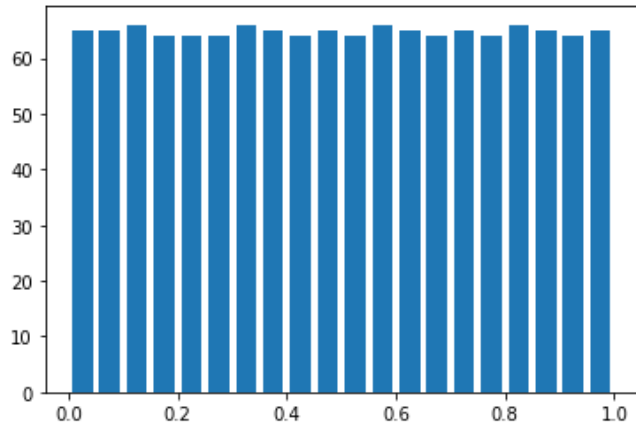
The value of  $x_0$  chosen is : 153026

The value of  $a$  is : 1597



Interval	Count
0.0 - 0.05	12248
0.05 - 0.1	12247
0.1 - 0.15	12247
0.15 - 0.2	12247
0.2 - 0.25	12247
0.25 - 0.3	12248
0.3 - 0.35	12247
0.35 - 0.4	12247
0.4 - 0.45	12247
0.45 - 0.5	12247
0.5 - 0.55	12248
0.55 - 0.6	12247
0.6 - 0.65	12247
0.65 - 0.7	12247
0.7 - 0.75	12247
0.75 - 0.8	12248
0.8 - 0.85	12247
0.85 - 0.9	12247
0.9 - 0.95	12247
0.95 - 1.0	12247

The value of a is : 51749



Interval	Count
0.0 - 0.05	64
0.05 - 0.1	66
0.1 - 0.15	65
0.15 - 0.2	64
0.2 - 0.25	65
0.25 - 0.3	64
0.3 - 0.35	66
0.35 - 0.4	65
0.4 - 0.45	64
0.45 - 0.5	65
0.5 - 0.55	64
0.55 - 0.6	66
0.6 - 0.65	65
0.65 - 0.7	64
0.7 - 0.75	65
0.75 - 0.8	64
0.8 - 0.85	66
0.85 - 0.9	65
0.9 - 0.95	64
0.95 - 1.0	65

### Q3.

As specified by the question, 2-D plot was created using the points  $(u_{i-1}, u_i)$  until repetition. We are getting a pattern (a series of straight lines with same slope) as shown below.

