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Dept.: Mathematics and Computing

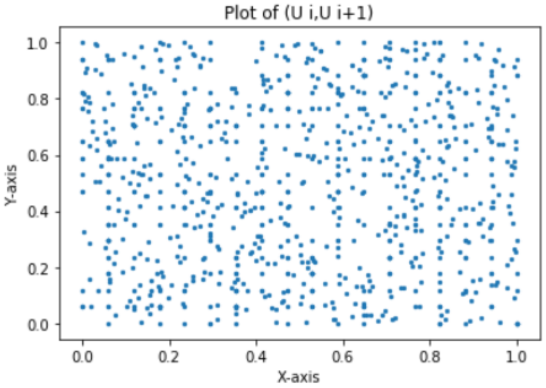
Q1.

The First seventeen Values of the required series was calculated using Linear Congruence Generator.

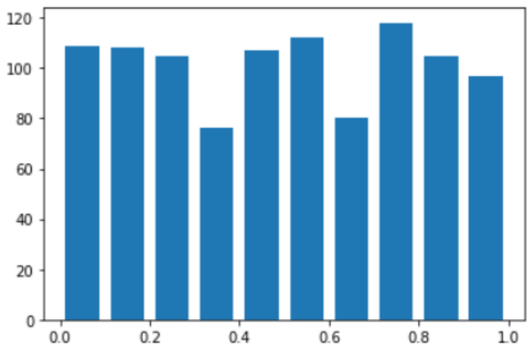
```
a = 6 , b = 0 , m = 17 , x0 = 1
The first 17 values of the sequence:
0.0588235 0.3529412 0.1176471 0.7058824 0.2352941 0.4117647 0.4705882 0.8235294 0.9411765 0.6470588
0.8823529 0.2941176 0.7647059 0.5882353 0.5294118 0.1764706 0.0588235
```

Then, 1000, 10000, and 100000 values of U_i were generated using the lagged Fibonacci generator.
For each case, Frequency Bar Diagram was created, and Plot of (U_i, U_{i+1}) was also created.

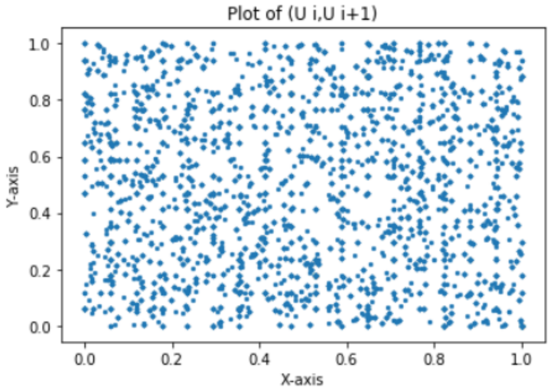
```
Case Number 1
Values Generated : 1000
Plot Scatter:
```



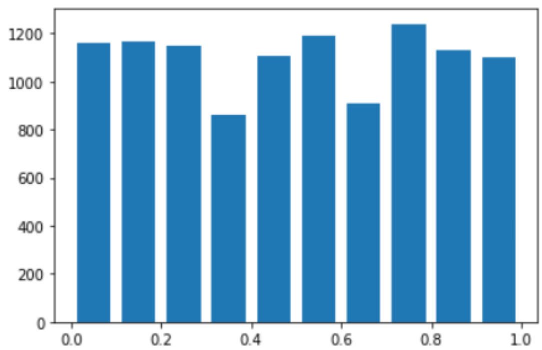

```
Frequency Histogram:
```



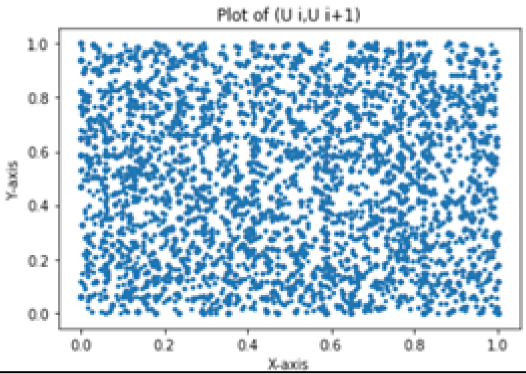
```
Case Number 2
Values Generated : 10000
Plot Scatter:
```



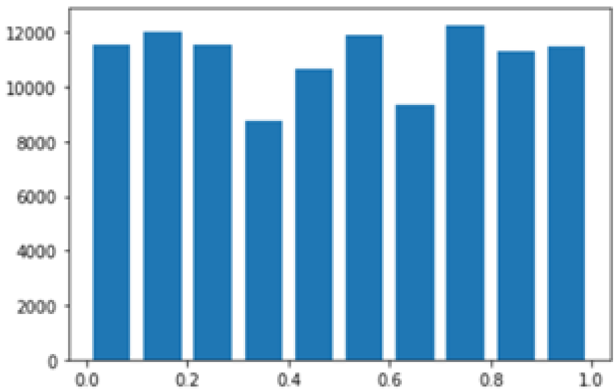

```
Frequency Histogram:
```



```
Case Number 3
Values Generated : 100000
Plot Scatter:
```




```
Frequency Histogram:
```



Observations: In contrast to Linear Congruence Generator, no clear pattern is observed in all the (U_i, U_{i+1}) plots, indicating that U_{i+1} is not directly dependent upon U_i (which is evident from Formula for lagged Fibonacci Generator). The points fill the entire 1*1 square almost uniformly, hence mimicking uniformity and randomness. Also, as the number of values generated increases, the density of the plot increases as well.
Also, frequencies of each interval (Total 10 intervals equally spaced between 0 and 1) are somewhat equal (with a slight deviation of about 10% from central value). This indicates that lagged Fibonacci Generators tend to have much greater period length (for appropriate input parameters) as compared to Linear Congruence Generators.

Q2.
The Values of U were generated using Linear Congruence Generator.

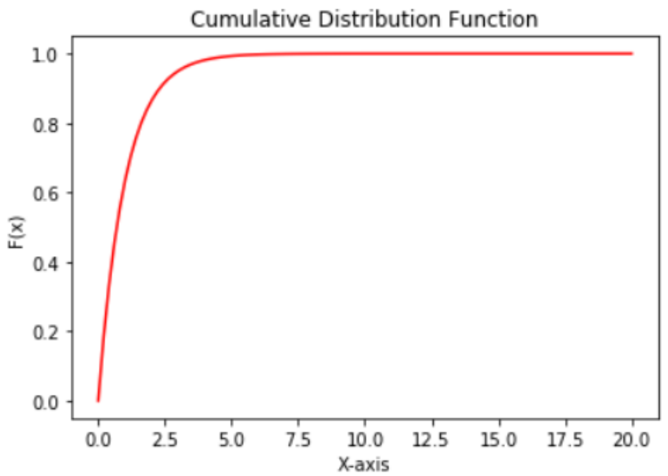
```
Values chosen for Linear Congruence Generator(for generating values of U)
a = 65793 b = 4282663 m = 8388608
Value of Theta Taken is: 1
```

In total, there are 8 cases. In each case, certain number of values of U were generated, which were further used to generate the values of X using the given formula. Then, the entire range of X was normalized to fit 0 to 1 Range, which was further divided into 20 equal intervals. Frequency of each interval was then calculated. Using the midpoints of interval as x coordinate, and frequency of the interval as y coordinate, a graph was plotted using the 20 points (joined with each other). This Graph denotes the Interval vs Frequency Graph. Using the midpoints of interval as x coordinate, and **cumulative** frequency of the interval as y coordinate, another graph was plotted using the 20 points (joined with each other). This Graph denotes the Interval vs Cumulative Frequency Graph. For each case, Sample mean and Sample Variance was also calculated using the 20 points. All the data is presented in table given on the next page.

$$F(x) = 1 - e^{-x/\theta}\ ,\ x \geq 0. \longrightarrow \text{Strictly Increasing Function}$$

Using the Cumulative Distribution Function given above, a graph of the actual distribution Function was created.

```
Actual Distribution Function F(x)
```

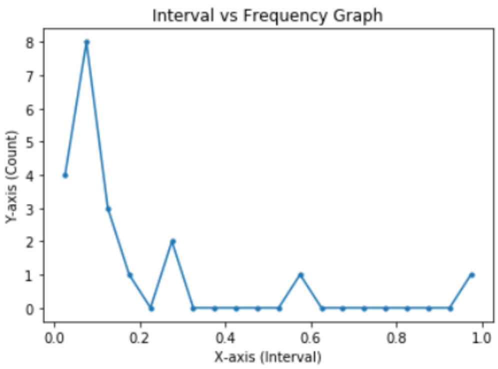
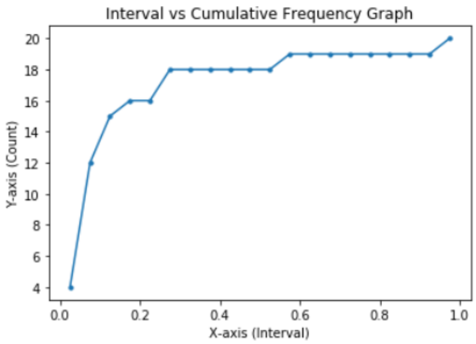
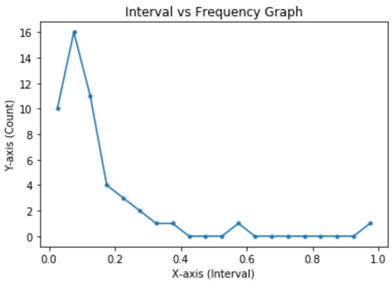
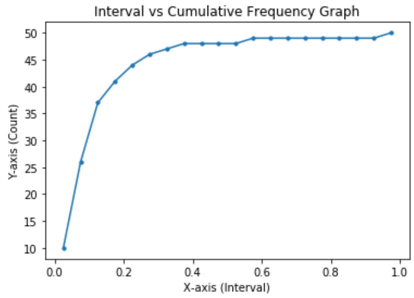
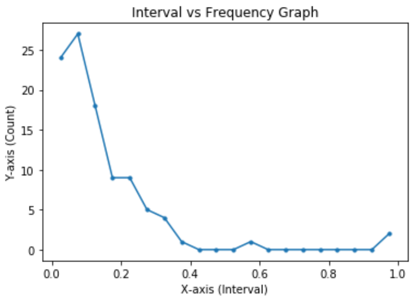
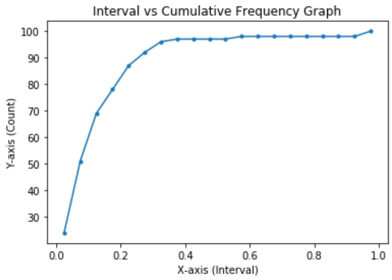
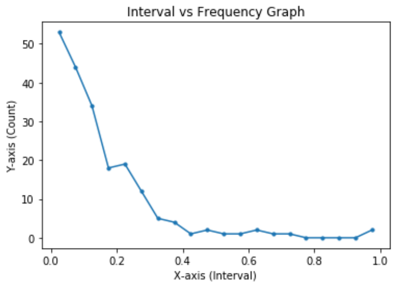
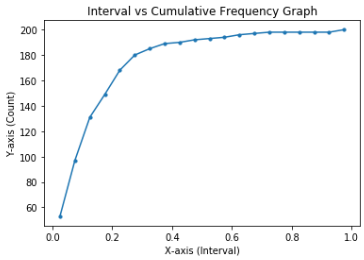
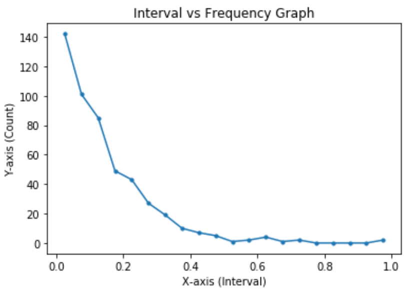
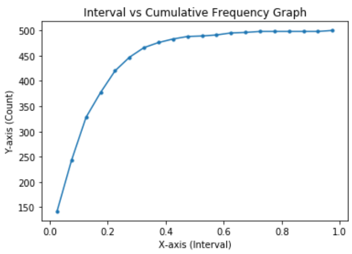
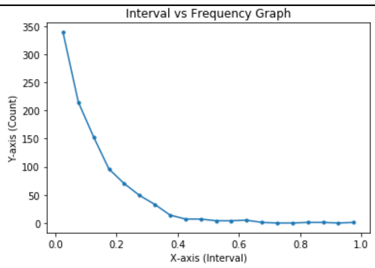
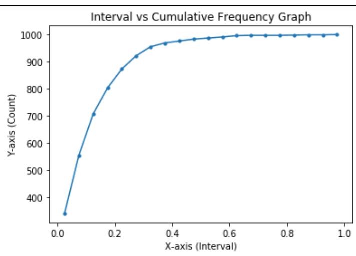


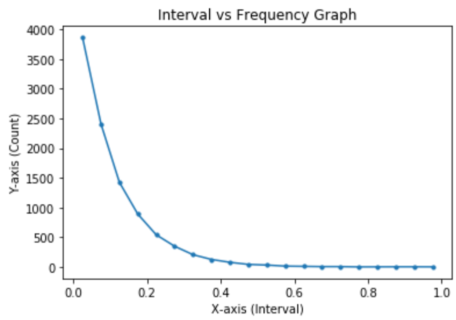
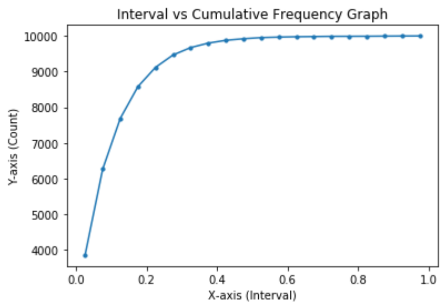
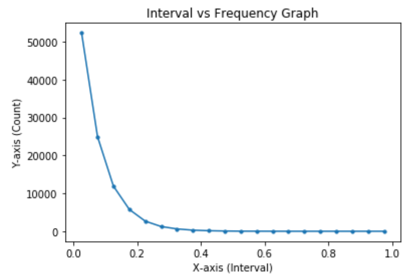
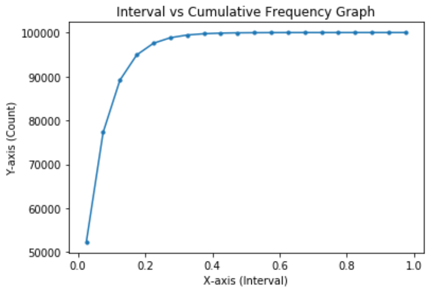
Actual Value of Mean = Theta = 1
Actual Value of Variance = $(\text{Theta})^2$ = 1

As depicted by the table given below, as the number of values generated increases, the cumulative Frequency vs Interval graph matches the graph of the Distribution function given above. The values of Sample Mean and Sample Variance converge to the Actual Mean and Actual Variance as the number of values generated increases.

The Interval vs Cumulative Frequency Graph converges to the above Cumulative Distribution Function because of the following result (stated in MA372 Stochastic Calculus):
Let X be a random variable uniformly distributed on [0,1]. If f is a strictly increasing distribution function, then the random variable $Z = f^{-1}(X)$ also follows the same distribution as that of f.

Table for Q2.

No. of Values	Frequency vs Interval	Cumulative Frequency vs Interval	Mean	Variance
20			1.213	2.786
50			1.002	1.389
100			1.007	1.328
200			1.083	1.326
500			1.035	1.032
1000			1.028	1.059

10000			1.005	1.023
100000			1.004	1.001

Q3.

The Values of U were generated using Linear Congruence Generator.

```
Values chosen for Linear Congruence Generator(for generating values of U)
a = 65793 b = 4282663 m = 8388608
```

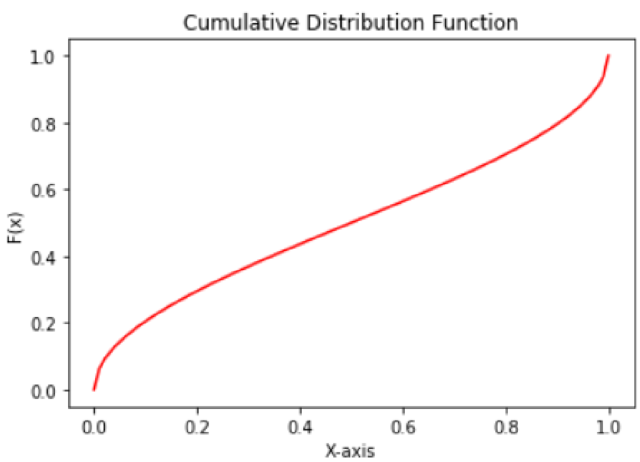
In total, there are 8 cases. In each case, certain number of values of U were generated, which were further used to generate the values of X using the given formula. Then, the entire range of X was normalized to fit 0 to 1 Range, which was further divided into 20 equal intervals. Frequency of each interval was then calculated. Using the midpoints of interval as x coordinate, and frequency of the interval as y coordinate, a graph was plotted using the 20 points (joined with each other). This Graph denotes the Interval vs Frequency Graph. Using the midpoints of interval as x coordinate, and **cumulative** frequency of the interval as y coordinate, another graph was plotted using the 20 points (joined with each other). This Graph denotes the Interval vs Cumulative Frequency Graph. For each case, Sample mean and Sample Variance was also calculated using the 20 points. All the data is presented in table given on the next page.

$$F(x) = \frac{2}{\pi} \arcsin \sqrt{x}\ ,\ 0 \le x \le 1.$$

Strictly Increasing Function

Using the Cumulative Distribution Function given above, a graph of the actual distribution Function was created.

Actual Distribution Function F(x)



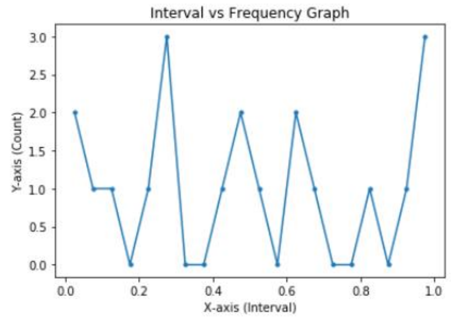
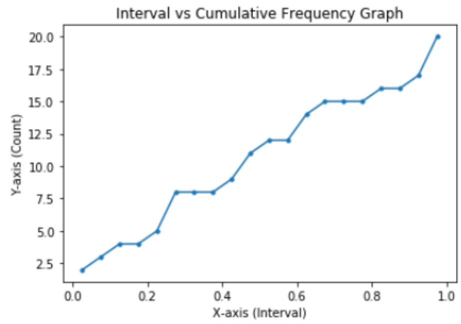
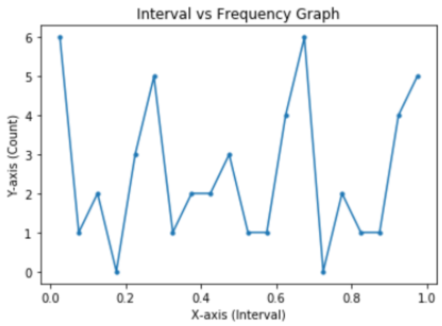
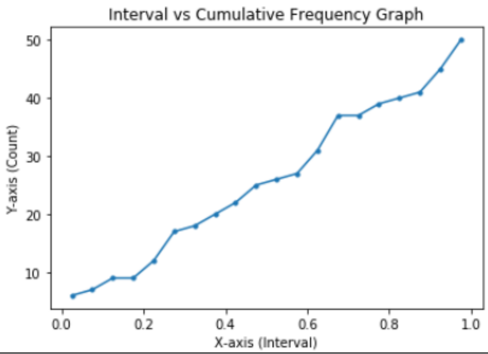
Actual Value of Mean = 0.5 (Calculated through Integration)

Actual Value of Variance = 0.125 (Calculated through Integration)

As depicted by the table given below, as the number of values generated increases, the cumulative Frequency vs Interval graph matches the graph of the Distribution function given above. The values of Sample Mean and Sample Variance converge to the Actual Mean and Actual Variance as the number of values generated increases.

The Interval vs Cumulative Frequency Graph converges to the above Cumulative Distribution Function because of the following result (stated in MA372 Stochastic Calculus):
Let X be a random variable uniformly distributed on [0,1]. If ***f*** is a strictly increasing distribution function, then the random variable Z = f^{-1}(X) also follows the same distribution as that of ***f***.

Table for Q3.

<table>
<tr><th>No. of Values</th><th>Frequency vs Interval</th><th>Cumulative Frequency vs Interval</th><th>Mean</th><th>Variance</th></tr>
<tr><td>20</td><td>
</td><td>
</td><td>0.491</td><td>0.108</td></tr>
<tr><td>50</td><td>
</td><td>
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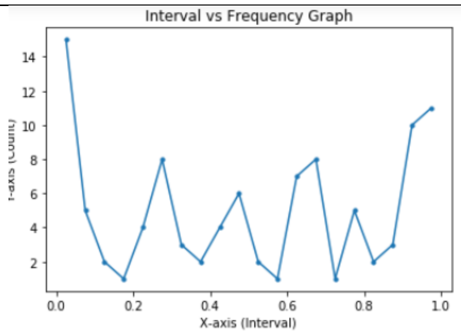
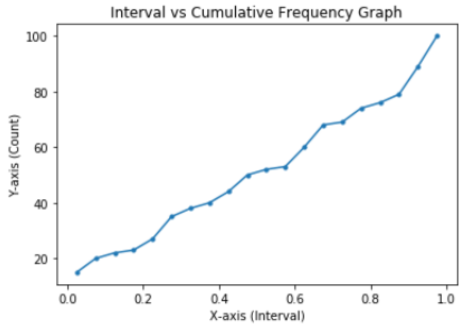
<tr><td>100</td><td>
</td><td>
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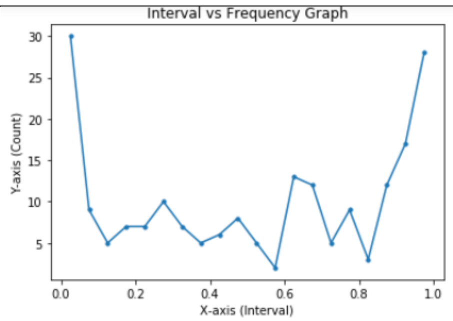
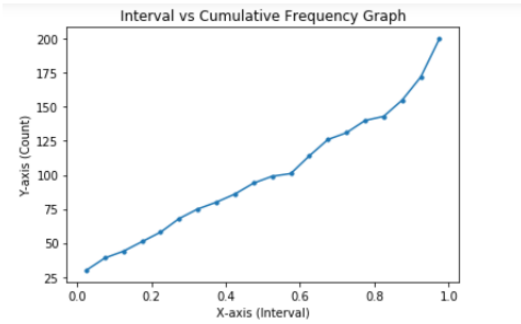
<tr><td>200</td><td>
</td><td>
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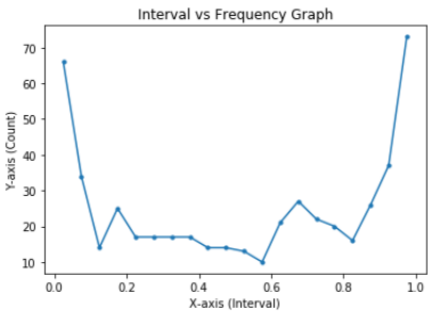
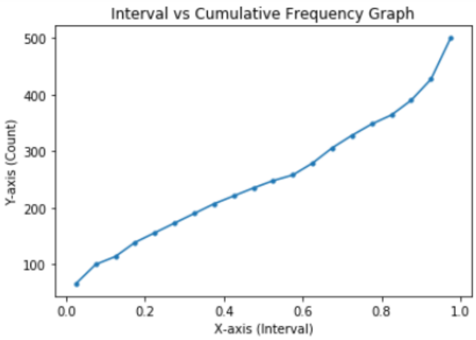
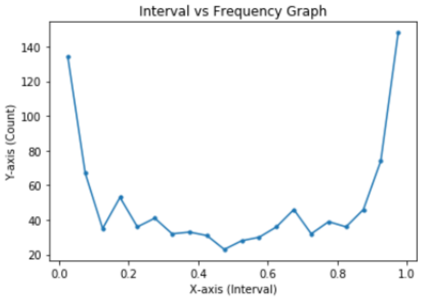
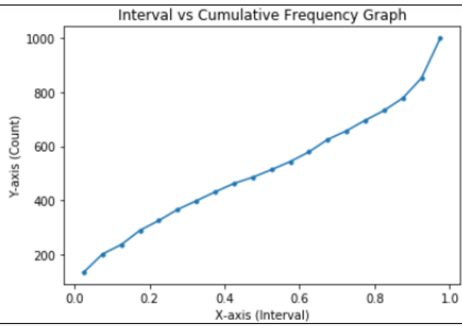
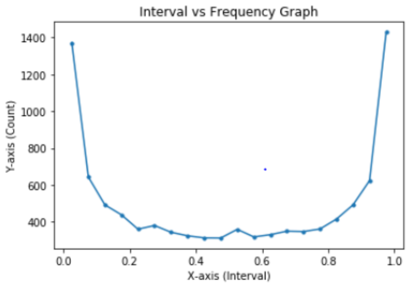
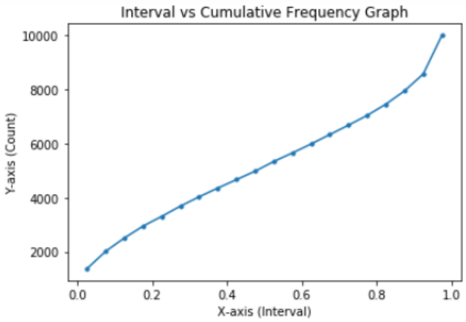
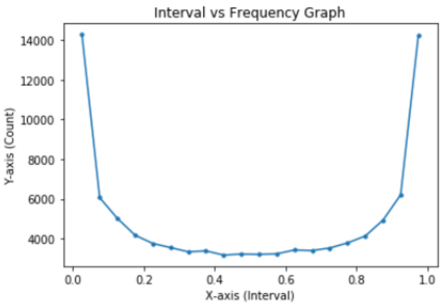
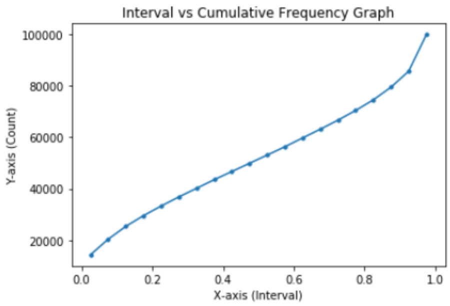
500			0.519	0.124
1000			0.510	0.126
10000			0.501	0.124
100000			0.500	0.125