MA 322 (2021) **Scientific Computing Lab** **Lab 01**

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Q1.
Starting element chosen is: **x1 = 10**
Number of iterations such that $|x_{n+1} - x_n| \leq 10^{-5}$ is: **5**
The order of convergence was initially assumed to be 1. Setting p = 1, the ratio $\frac{\alpha - x_{n+1}}{(\alpha - x_n)^1}$ was calculated. It could be seen that after several iterations, the sequence converges to a fixed constant c. Hence, the order of convergence is 1

Q2.
The roots obtained for various n are as follows:

For **n = 1**, the root obtained is = **3.0**
For **n = 5**, the root obtained is = **2.16**
For **n = 20**, the root obtained is = **2.02**
For **n = 100**, the root obtained is = **2.034**
For **n = 200**, the root obtained is = **2.027**
For **n = 400**, the root obtained is = **2.0305**
For **n = 1000**, the root obtained is = **2.0284**
For **n = 10000**, the root obtained is = **2.02882**
For **n = 50000**, the root obtained is = **2.0287640000000002**

With increasing n, the approximate root converged to the actual root value.

Q3.
Using ε **= 0.1**, an approximate root was found using the Bisection method.
Approximate root of **f(x) = x/2 – sin x** in the interval [π/2, π]: **1.8653206380689396**

Then, the newton's method was applied, using the approximate root as the initial value. This time, ε **= 0.5 x** 10^{-7}. The accurate root up to 7 decimal places is: **1.895494267033981**

Q4.
Similar to Q3,
Using ε **= 0.1**, an approximate root was found using the Bisection method.
Approximate root of f(x) = x/2 – sin x in the interval [π/2, π]: **1.8653206380689396**

To apply Fixed Point Method, suitable g(x) was to be found.

$$g(x) = \frac{2sinx}{x} + x - 1$$

g(x) is continuous for all x in [π/2, π]
π /2 <= g(x) <= π for all x in [π/2, π]

Hence, the fixed-point method can be applied.
Root obtained (with $\varepsilon = 10^{-15}$): 1.895494267033981

It can be seen that g is a contraction with **L = 0.363**.

The ratio $\dfrac{\log|\frac{x_{n+1}-\alpha}{x_n-\alpha}|}{\log|\frac{x_n-\alpha}{x_{n-1}-\alpha}|}$ was calculated at each stage.

Theoretically, the above ratio should converge to the order of convergence.
Last three calculated ratios are as follows:
0.99992 1.00017 0.99972
The ratio converges to 1 after several iterations, and **hence order of convergence = 1.**

Q5.
x_0 was set to -1, and x_1 was set to 0. Secant's method was used to find the root. The results are as follows:

Method Used: Secant Method
Approximate root found: **-0.5791589060508369**
Number of iterations taken: 11

Q6.
Similar to Q3, using the bisection method with **ε = 0.1**, approximate root was calculated.
Approximate root of f(x) in the interval **[-1,0]: -0.5625**

Using the approximate root as **x_0**, the iterative procedure as per the question was applied.

Root obtained (with $\varepsilon = 10^{-15}$): **1.895494267033981**

The ratio $\frac{\log|\frac{x_{n+1}-\alpha}{x_n-\alpha}|}{\log|\frac{x_n-\alpha}{x_{n-1}-\alpha}|}$ was calculated at each stage.

Theoretically, the above ratio should converge to the order of convergence.
Last three calculated ratios are as follows:
1.00000 1.00000 0.99945
The ratio converges to 1 after several iterations, and hence **order of convergence = 1.**

Q7.
Similar to Q6. **Order of convergence = 1**

Q8.
Similar to Q6. **Order of convergence = 2**

Order of Convergence:

If p is the order of convergence, then $\lim_{n\to\infty} \frac{\alpha - x_{n+1}}{(\alpha - x_n)^p} = C$
So, for large values of n,

$$\alpha - x_{n+1} \approx C(\alpha - x_n)^p$$
$$\alpha - x_n \approx C(\alpha - x_{n-1})^p$$

Dividing both the equations, we obtain:

$$\frac{\log|\frac{x_{n+1}-\alpha}{x_n-\alpha}|}{\log|\frac{x_n-\alpha}{x_{n-1}-\alpha}|} = p$$

Hence, as x_n tends to α, $\frac{\log|\frac{x_{n+1}-\alpha}{x_n-\alpha}|}{\log|\frac{x_n-\alpha}{x_{n-1}-\alpha}|}$ tends to the order of convergence. So, this ratio was calculated at each step.
The ratio, after several iterations, converged to the required order of convergence.