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Q1:

Using the Runge-Kutta method of order 4, the number of units of KOH that will be formed after 0.2 seconds has been determined.

Units of KOH after 0.2s is: 53731.39630040521

Q2:

Using the Runge-Kutta method of order 2 and Modified Euler Method, the approximations to the IVP was calculated for different values of h.

$$\frac{dy}{dt} = -y + t + 1, \quad 0 \le t \le 1, \quad y(0) = 1$$

h	x	2nd order Range-Kutta	Modified Euler Method
0.1	0.1	1.005	1.005
0.1	0.2	1.0190249999999998	1.0190249999999998
0.1	0.3	1.0412176249999998	1.0412176249999998
0.1	0.4	1.070801950625	1.070801950625
0.1	0.5	1.107075765315625	1.107075765315625
0.1	0.6	1.1494035676106404	1.1494035676106404
0.1	0.7	1.1972102286876296	1.1972102286876296
0.1	0.8	1.2499752569623048	1.2499752569623048
0.1	0.9	1.3072276075508857	1.3072276075508857
0.1	1	1.3685409848335515	1.3685409848335515
0.2	0.2	1.02	1.02
0.2	0.4	1.0724	1.0724
0.2	0.6	1.151368	1.151368
0.2	0.8	1.25212176	1.25212176
0.2	1	1.3707398432	1.3707398432
0.25	0.25	1.03125	1.03125
0.25	0.50	1.1103515625	1.1103515625
0.25	0.75	1.226837158203125	1.226837158203125
0.25	1	1.3725290298461914	1.3725290298461914
0.5	0.5	1.125	1.125
0.5	1	1.390625	1.390625

In the code, we approximate the value of y for various values of h. We find that the Runge-Kutta Method (Order 2) and the Modified Euler Method almost produce the equal approximations. This is because both the methods boil down to the same formulas.

***Runge-Kutta Method* Order 2:**

$$y_{n+1} = y_n + h \times f(t + \frac{h}{2}, y_n + \frac{h}{2} f(t, y_n))$$

Modified Euler Method

$$y_{n+1} = y_n + \frac{h}{2}(f(t, y_n) + f(t + h, y_n + hf(t, y_n)))$$

We have $f(t, y) = -y + t + 1$. Thus, $f(t, y)$ is linear in both t and y.

$$\therefore\ y_n + \frac{h}{2}\big(f(t, y_n) + f(t + h, y_n + hf(t, y_n))\big) = y_n + \frac{h}{2}(f(2t + h, 2y_n + hf(t, y_n)))$$

$$\therefore\ y_n + \frac{h}{2}\big(f(2t + h, 2y_n + hf(t, y_n))\big) = y_n + h \times f(t + \frac{h}{2}, y_n + \frac{h}{2} f(t, y_n))$$

$\therefore$ Modified Euler Method is equivalent to Runge $-$ Kutta Method of Order 2

Q3:

(a) Using the Modified Euler method, the solutions was approximated at certain points. It was then compared with the actual value at those points.

h	x	Approximated value	Actual Value
0.5	1.5	2.3541666666666665	2.3541019662496847
0.5	2.0	2.7417450827887775	2.7416573867739413

(b) Using the Modified Euler method, the solutions was approximated at certain points. It was then compared with the actual value at those points.

h	x	Approximated value	Actual Value
0.25	1.25	1.4160750785402427	1.4031989692799332
0.25	1.5	1.0310110697781514	1.0164101466785118
0.25	1.75	0.7522666785837252	0.7380097715499843
0.25	2.0	0.5432450024334279	0.5296870980395587

Q4:

Euler's method with h= 0.025, the Runge-Kutta second-order method with h=0.05, and the Runge-Kutta fourth-order method with h = 0.1 was employed and the approximate values were compared at the common mesh points of these methods 0.1,0.2,0.3,0.4, and 0.5.

x	Euler	Runge-Kutta O-2	Runge-Kutta O-4
0.1	0.655498	0.657373	0.657414
0.2	0.825338	0.829213	0.829298
0.3	1.008933	1.014939	1.015070
0.4	1.205635	1.213908	1.214087
0.5	1.414726	1.425409	1.425638

Q5:

Using the exact values of y(t) as the starting values and h as 0.2, the approximations (using explicit Adams-Bashforth 4 step method in part (a) and using implicit Adams-Bashforth 3 step method in part (b)). Difference in the values has also been calculated.

adam-bash	adam-mltn	differ	initial ys
0.500000000	0.500000000	0.000000000	0.500000000
0.829298621	0.829298621	0.000000000	0.829298621
1.214087651	1.214087651	0.000000000	1.214087651
1.648940600	1.648934632	0.000005968	1.648940600
2.127312354	2.127215333	0.000097021	2.127229536
2.641081018	2.640833980	0.000247038	2.640859086
3.180348021	3.179902125	0.000445896	3.179941539
3.733060128	3.732342008	0.000718120	3.732400017
4.284493130	4.283401805	0.001091325	4.283483788
4.816657482	4.815063578	0.001593904	4.815176268
5.307583810	5.305320149	0.002263661	5.305471951

Q6:

Initial/Starting values were obtained from the 4th order Runge-Kutta method. These starting values were further used to approximate the solutions of given IVPs using Adam Bashforth method.

(a)

$\frac{dy}{dt} = \frac{2-2ty}{t^2+1}, \quad 0 \leq t \leq 1, \quad y(0) = 1$, with $h = 0.1$; actual solution $y(t) = \frac{2t+1}{t^2+1}$.

x	Act.	Comp	Diff
0.00000	1.00000	1.00000	0.00000
0.10000	1.18812	1.16346	0.02466
0.20000	1.34615	1.29358	0.05258
0.30000	1.46789	1.38793	0.07996
0.40000	1.55172	1.47765	0.07407
0.50000	1.60000	1.53013	0.06987
0.60000	1.61765	1.55458	0.06307
0.70000	1.61074	1.55337	0.05736
0.80000	1.58537	1.53370	0.05167
0.90000	1.54696	1.50025	0.04671

(b)

$\frac{dy}{dt} = \frac{y^2}{t+1}, \quad 1 \leq t \leq 2, \quad y(1) - \ln(2)^{-1}$, with $h = 0.1$; actual solution $y(t) = \frac{-1}{\ln(t+1)}$.

x	Act.	Comp	Diff
1.00000	-1.44270	-1.44270	0.00000
1.10000	-1.34782	-1.35196	0.00414
1.20000	-1.26830	-1.27532	0.00702
1.30000	-1.20061	-1.20966	0.00905
1.40000	-1.14225	-1.15045	0.00820
1.50000	-1.09136	-1.09913	0.00778
1.60000	-1.04656	-1.05360	0.00704
1.70000	-1.00679	-1.01340	0.00660
1.80000	-0.97123	-0.97737	0.00614
1.90000	-0.93922	-0.94499	0.00577

(c)
$\frac{dy}{dt} = \frac{y^2+y}{t}$, $1 \le t \le 3$, $y(1) = -2$, with $h = 0.2$; actual solution $y(t) = \frac{2t}{1-t}$.

x	Act.	Comp	Diff
1.20000	-12.00000	-1.74999	10.25001
1.40000	-7.00000	-1.59999	5.40001
1.60000	-5.33333	-1.49999	3.83334
1.80000	-4.50000	-1.43505	3.06495
2.00000	-4.00000	-1.37715	2.62285
2.20000	-3.66667	-1.33556	2.33111
2.40000	-3.42857	-1.29809	2.13049
2.60000	-3.25000	-1.27029	1.97971
2.80000	-3.11111	-1.24570	1.86541

Q7:
Initial/Starting values were obtained from the 4th order Runge-Kutta method. These starting values were further used to approximate the solutions of given IVPs using Adam Bashforth 4th order predictor method.

x = 0.000	calculated	f(x) = 0.5
x = 0.200	calculated	f(x) = 0.5
x = 0.400	calculated	f(x) = 0.8118773333333333
x = 0.600	calculated	f(x) = 1.1599169675644445
x = 0.800	calculated	f(x) = 1.5282751347565926
x = 1.000	calculated	f(x) = 1.9069335861153958
x = 1.200	calculated	f(x) = 2.281113199350842
x = 1.400	calculated	f(x) = 2.6319130565890205
x = 1.600	calculated	f(x) = 2.936315198303456
x = 1.800	calculated	f(x) = 3.166342876495915
x = 2.000	calculated	f(x) = 3.2877981849970297