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Q1.

Using Mid-point, Trapezoidal and Simpson's methods, the integral values were estimated. The results are as follows:

```
For (a) part:
Using Midpoint Method, estimated value of the integral = 0.74048
Difference between actual and calculated value: 0.117255
Using Trapezoidal Method, estimated value of the integral = 0.392699
Difference between actual and calculated value: 0.230526
Using Simpsons Method, estimated value of the integral = 0.624553
Difference between actual and calculated value: 0.001328

For (b) part:
Using Midpoint Method, estimated value of the integral = 0.628319
Difference between actual and calculated value: 0.418879
Using Trapezoidal Method, estimated value of the integral = 1.745329
Difference between actual and calculated value: 0.698131
Using Simpsons Method, estimated value of the integral = 1.000655
Difference between actual and calculated value: 0.046543

For (c) part:
Using Midpoint Method, estimated value of the integral = 0.778801
Difference between actual and calculated value: 0.031977
Using Trapezoidal Method, estimated value of the integral = 0.68394
Difference between actual and calculated value: 0.062884
Using Simpsons Method, estimated value of the integral = 0.74718
Difference between actual and calculated value: 0.000356
```

Q2. Normal methods were followed (Not Composite).

```
For (a) part:
Using Trapezoidal Method, estimated value of the integral = 7.5

For (b) part:
Using Simpsons Method, estimated value of the integral = 7.166667
```

Q3. Since the upper limit is not bounded, the upper limit must be converted to some finite value so that the composite trapezoidal and the composite Simpson's method can be applied to estimate the integral. The value of x is substituted by $\frac{t}{1-t}$.

The integral now becomes:

$$\int_0^1 \frac{dt}{t^2+9(1-t)^2}$$

The value of **n** has been set to **4**.

The results are as follows:

```
The estimate using composite trapezoidal method:  0.5098915989159891
The estimate using composite Simpson's method:  0.5205962059620596
The Actual value of the integral: 0.5235987755982988
```

Q4.

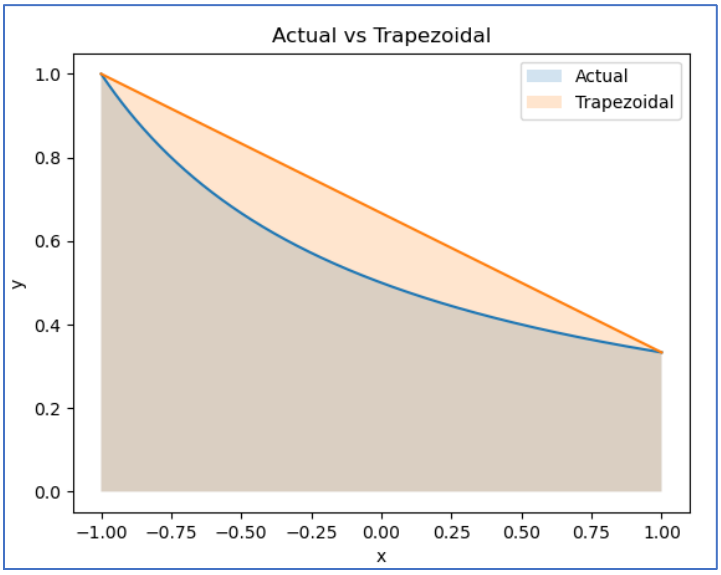
Using the trapezoidal and Simpson's method, the value of the integral was estimated.

```
The actual value of the Integral = 1.09861228866811
The estimate using trapezoidal method:  1.3333333333333333
The estimate using Simpson's method:  1.1111111111111112
```

Then, the graph of 1/(x+2) was plotted in the range [-1,1]. The area under the graph is to be estimated. (depicted by blue shaded region)

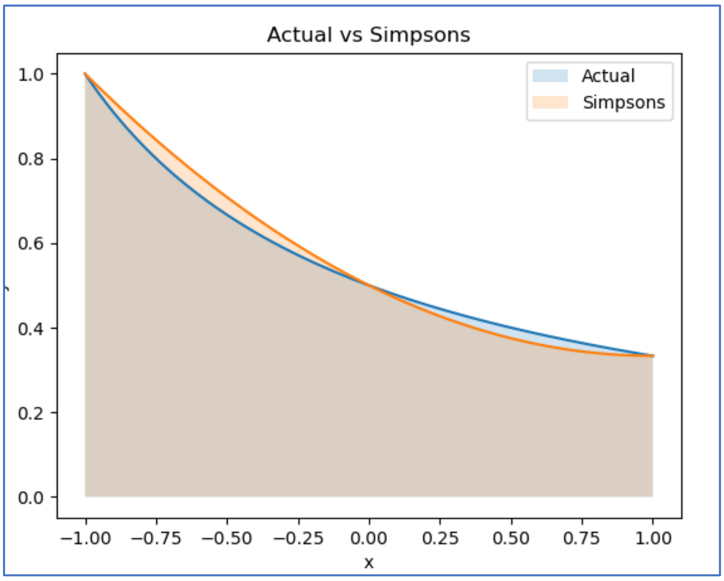
Then, the areas calculated for trapezoidal rule and Simpson's rule was individually plotted out. (orange shaded region). The Trapezoidal plot was made through direct linear estimation, whereas the Simpson's plot was made through quadratic estimation (using 3 points).

The plots are as follows:



As shown in the figure, **Trapezoidal Estimate is greater than the actual estimate.**

Equation of line: y = -0.33x + 0.66



As shown in the figure, **Simpson's Estimate is almost same as the actual estimate.**

Equation of line: y = 0.1667x*x - 0.33x + 0.5

Q5.

For this question, we will use the error estimation for composite Trapezoidal, Simpson, and Midpoint rules.

- Composite Trapezoidal rule: $|E| \leq \frac{b-a}{12} h^2 \left\| f^{(2)} \right\|_\infty$
- Composite Simpson rule: $|E| \leq \frac{b-a}{180} h^4 \left\| f^{(4)} \right\|_\infty$
- Composite Midpoint rule: $|E| \leq \frac{b-a}{6} h^2 \left\| f^{(2)} \right\|_\infty$

4^{th} derivative = $24/4^5$, 2^{nd} derivative = 1/32

Using the above-mentioned error formulas, the results are as follow:

```
Part (a)
Constraints: h <= 0.0438178046004133 and n >= 46
Required value of n: 46
Corresponding value of h: 0.043478260869565216
The estimate using composite trapezoidal method: 0.4054705778040843
The difference between actual and calculated value: 5.469695920301554e-06

Part (b)
Constraints: h <= 0.44267276788012866 and n >= 6
Required value of n: 6
Corresponding value of h: 0.3333333333333333
The estimate using composite Simpsons method: 0.4054663745840217
The difference between actual and calculated value: 1.2664758576863555e-06

Part (c)
Constraints: h <= 0.030983866769659634 and n >= 65
Required value of n: 65
Corresponding value of h: 0.03076923076923077
The estimate using composite midpoint method: 0.40546373841574274
The difference between actual and calculated value: 1.3696924212602823e-06
```

Q6.

The functional calls were minimized (by avoiding repetitive calls).

```
Part (a)
Table:
```

h	T(h/2)	Error value
1.5000000000	0.9173076923	0.5094339623
0.7500000000	1.0970043616	0.1638067045
0.3750000000	1.1384585664	0.0364125722
0.1875000000	1.1481180340	0.0084133053
0.0937500000	1.1505008862	0.0020711433
0.0468750000	1.1510947524	0.0005159143
0.0234375000	1.1512431055	0.0001288634

0.0117187500	**1.1512801867**	**0.0000322087**
0.0058593750	**1.1512894566**	**0.0000080517**
0.0029296875	**1.1512917740**	**0.0000020129**
0.0014648438	**1.1512923534**	**0.0000005032**

Estimated value: 1.1512923533779342
Number of iterations taken: 11
Number of function calls: 2049
Difference between actual and estimated value: 1.9311908872055028e-07

Part (b)
Table:

h	T(h/2)	Error value
0.4750000000	5.8922619048	0.6928982726
0.2375000000	4.0836933187	0.4428757120
0.1187500000	3.3570758478	0.2164435669
0.0593750000	3.1017719812	0.0823090376
0.0296875000	3.0241335026	0.0256729667
0.0148437500	3.0029962433	0.0070387232
0.0074218750	2.9975598200	0.0018136163
0.0037109375	2.9961899090	0.0004572177
0.0018554687	2.9958467297	0.0001145517
0.0009277344	2.9957608905	0.0000286535
0.0004638672	2.9957394280	0.0000071644
0.0002319336	2.9957340622	0.0000017912
0.0001159668	2.9957327207	0.0000004478

Estimated value: 2.9957327207096545
Number of iterations taken: 13
Number of function calls: 8193
Difference between actual and estimated value: 4.471556636076457e-07
Part (c)

For m = 0.5
Table:

h	T(h/2)	Error value
0.7853981634	1.8549591311	0.0221890424
0.3926990817	1.8540752278	0.0004767354
0.1963495408	1.8540746773	0.0000002969

Estimated value: 1.8540746773016665
Number of iterations taken: 3
Number of function calls: 9

For m = 0.8
Table:

h	T(h/2)	Error value

```
0.7853981634 2.2847455921 0.1124222561
0.3926990817 2.2576215270 0.0120144430
0.1963495408 2.2572054615 0.0001843277
0.0981747704 2.2572053268 0.0000000596

Estimated value: 2.257205326820873
Number of iterations taken: 4
Number of function calls: 17

For m = 0.95
Table:

      h          T(h/2)      Error value
0.7853981634 3.2328552103 0.3294147895
0.3926990817 2.9426673463 0.0986138866
0.1963495408 2.9089732769 0.0115828047
0.0981747704 2.9083375614 0.0002185838
0.0490873852 2.9083372484 0.0000001076

Estimated value: 2.9083372484446572
Number of iterations taken: 5
Number of function calls: 33
```

Q7.

(a) This part is entirely theoretical. The process for finding the error using the inexact function is as follows:

$$T(f) = h\left[\frac{f(x_0)}{2} + f(x_1) + \ldots + f(x_{n-1}) + \frac{f(x_n)}{2}\right] + h\left[\frac{\delta(x_0)}{2} + \delta(x_1) + \ldots + \delta(x_{n-1}) + \frac{\delta(x_n)}{2}\right]$$

So,

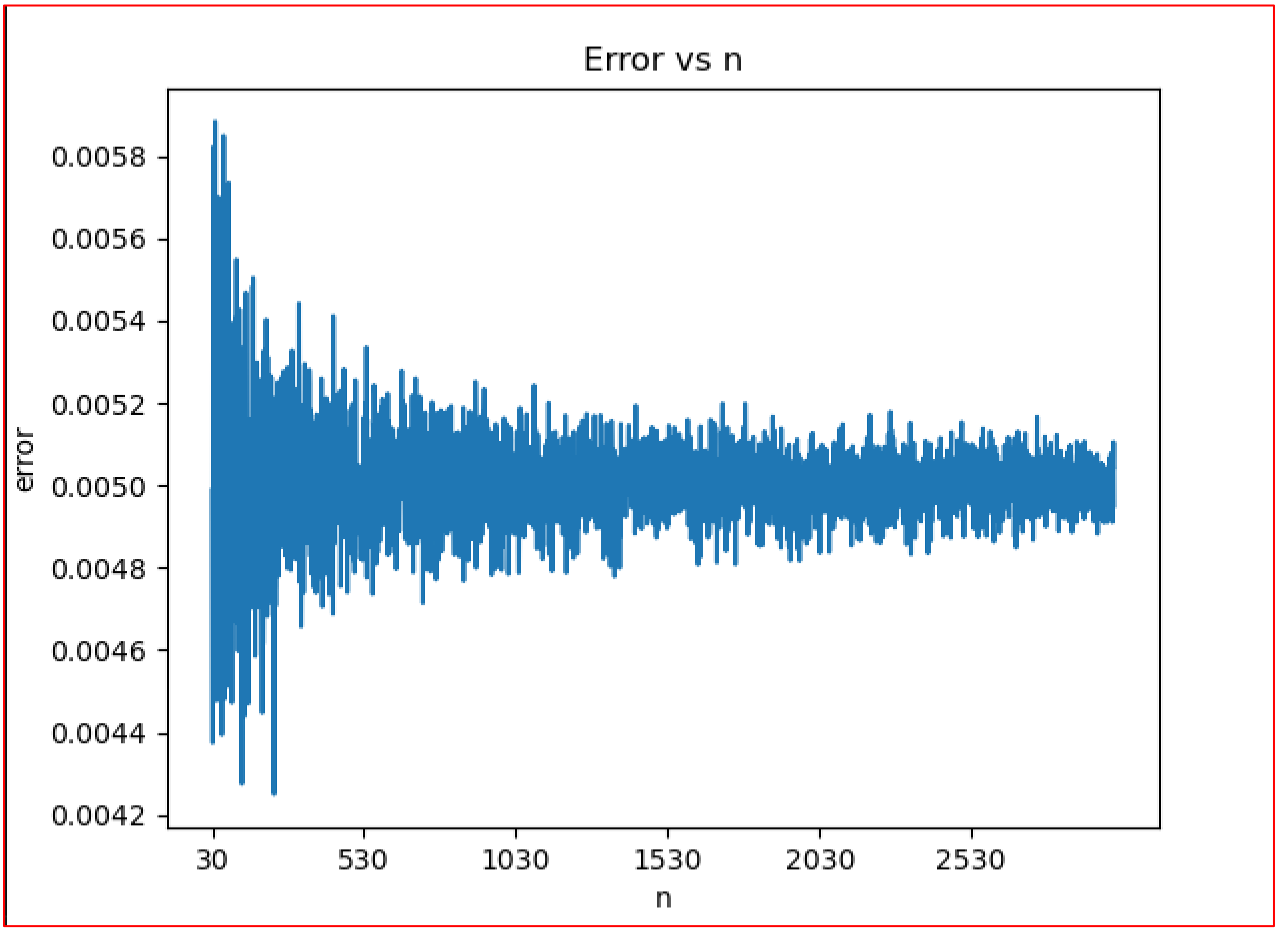
$$\begin{aligned}
\int_a^b \widehat{f}(x)dx - T(\widehat{f}) &= \left(\int_a^b f(x)dx - T(f)\right) + h\left[\frac{\delta(x_0)}{2} + \delta(x_1) + \ldots + \delta(x_{n-1}) + \frac{\delta(x_n)}{2}\right] \\
&\leq \left(\int_a^b f(x)dx - T(f)\right) + \delta(b-a)
\end{aligned}$$

$$\left|\int_a^b \widehat{f}(x)dx - T(\widehat{f})\right| \leq \frac{h^2}{12}(b-a)\left\|f\right\|_\infty + \delta(b-a)$$

The second term remains fixed irrespective of the value of n and h. Hence, there will always be an error of approximately **(δ(b-a) = 0.005)** in the estimation.

(b)

For values of n ranging between **[30,3000]**, the plot of **Error vs n** was plotted out.



As done in **7a**, the maximum error possible is **δ(b-a) + error due to the composite trapezoidal rule**. The **error δ(b-a)** is independent of **h**. Additionally, the expected error was observed to be **0.005**. This is due to the fact that the random [0,1) function of python has the mean 0.5.
From the graph, it can be seen that **n $\geq$ 1500, h $\leq$ (1/1500)** is a reasonable choice for the step size. (This is because after **n>=1500**, the error stabilizes around **0.005** (as shown in the plot)).