1. **Start with the diode/BJT relation**

$$I_C = I_S(T)\,\exp\!\Big(\frac{qV_{be}}{k_BT}\Big) \quad\Longrightarrow\quad V_{be} = V_T \ln\!\Big(\frac{I_C}{I_S}\Big).$$

2. **Differentiate V_{be} w.r.t. T (keep I_C constant):**

$$\frac{\partial V_{be}}{\partial T} = \frac{\partial V_T}{\partial T}\ln\!\Big(\frac{I_C}{I_S}\Big) + V_T\frac{\partial}{\partial T}\Big[\ln\!\Big(\frac{I_C}{I_S}\Big)\Big].$$

Since I_C is constant, $\partial \ln(I_C/I_S)/\partial T = -\dfrac{1}{I_S}\dfrac{\partial I_S}{\partial T}$. Thus

$$\boxed{\frac{\partial V_{be}}{\partial T} = \frac{\partial V_T}{\partial T}\ln\!\Big(\frac{I_C}{I_S}\Big) - V_T\frac{1}{I_S}\frac{\partial I_S}{\partial T}} \tag{1.2}$$

3. **Compute $\dfrac{1}{I_S}\dfrac{\partial I_S}{\partial T}$ for**

$$I_S(T) = I_{C,0}\,T^{\eta}\exp\!\Big(-\frac{qV_{gap,0}}{k_BT}\Big).$$

Take the log and differentiate:

$$\ln I_S = \ln I_{C,0} + \eta\ln T - \frac{qV_{gap,0}}{k_BT},$$

so

$$\frac{1}{I_S}\frac{\partial I_S}{\partial T} = \frac{\partial \ln I_S}{\partial T} = \frac{\eta}{T} + \frac{qV_{gap,0}}{k_BT^2}.$$

Define $A = \dfrac{qV_{gap,0}}{k_B}$; equivalently note $\dfrac{qV_{gap,0}}{k_BT^2} = \dfrac{V_{gap,0}}{V_T\,T}$. For now keep the simple form:

$$\boxed{\frac{1}{I_S}\frac{\partial I_S}{\partial T} = \frac{\eta}{T} + \frac{qV_{gap,0}}{k_BT^2}}. \tag{1.3}$$

4. **Plug (1.3) into (1.2):**

$$\frac{\partial V_{be}}{\partial T} = \frac{\partial V_T}{\partial T}\ln\!\Big(\frac{I_C}{I_S}\Big) - V_T\Big(\frac{\eta}{T} + \frac{qV_{gap,0}}{k_BT^2}\Big).$$

5. **Use $\ln\!\Big(\dfrac{I_C}{I_S}\Big) = \dfrac{V_{be}}{V_T}$ and $\dfrac{\partial V_T}{\partial T} = \dfrac{k_B}{q} = \dfrac{V_T}{T}$:**

$$\frac{\partial V_T}{\partial T}\ln\!\Big(\frac{I_C}{I_S}\Big) = \frac{V_T}{T}\cdot\frac{V_{be}}{V_T} = \frac{V_{be}}{T}.$$

Also note $V_T\cdot\frac{qV_{gap,0}}{k_BT^2} = \frac{V_{gap,0}}{T}$ because $V_T = \dfrac{k_BT}{q}$.

6. **Collect terms → final compact form (thesis eq. 1.5):**

$$\frac{\partial V_{be}}{\partial T} = \frac{V_{be}}{T} - \frac{\eta V_T}{T} - \frac{V_{gap,0}}{T} = \frac{V_{be} - \eta V_T - V_{gap,0}}{T}.$$
