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$\frac{1}{10}$

Probability Assignment - CSU44062

Q1

	gw	¬gw
ps	140	10
¬ps	700	150

(a) According to conditional probability from lectures:

$$\boxed{P(A|B) = \frac{P(A \wedge B)}{P(B)}}$$

Therefore, we can calculate $P(gw|ps)$ with:

$$\boxed{P(gw|ps) = \frac{P(gw \wedge ps)}{P(ps)}}$$

From the table above, we know that

$P(gw \wedge ps) = \frac{140}{1000}$ (Since 140 on the table is when gw AND ps, and all the numbers in the table add to 1000)

$P(ps) = \frac{150}{1000}$ (Since the row of ps equates to $140+10=150$. Also, all numbers in the table add to 1000)

$$\therefore P(gw \mid ps) = \frac{\left(\frac{140}{1000}\right)}{\left(\frac{150}{1000}\right)} = \frac{14}{15}$$

$$\approx 0.9\dot{3}$$

Which counts are irrelevant to this calculation:

- $\neg ps \land gw = 700$
- $\neg ps \land \neg gw = 150$

Both these counts (i.e. the entire $\neg ps$ row) are irrelevant to this calculation.

(b) According to conditional probability

$$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$$

Therefore, we can calculate $P(ps \mid gw)$ with:

$$\boxed{P(ps \mid gw) = \frac{P(ps \land gw)}{P(gw)}}$$

From the table, we know that:

$$P(ps \land gw) = \frac{140}{1000}$$

(since 140 on the table is when $ps \land gw$, and all the numbers on the table add to 1000)

$P(gw) = \frac{840}{1000}$ (since the ~~row~~ column of gw equates to $140 + 700 = 840$. Again, all numbers in the table add to 1000)

$\therefore P(ps \mid gw) = \frac{\left(\frac{140}{1000}\right)}{\left(\frac{840}{1000}\right)} = \frac{1}{6}$

$\approx 0.1\dot{6}$

Which counts are irrelevant to this calculation:

- $\neg gw \wedge ps = 10$
- $\neg gw \wedge \neg ps = 150$

Both of these counts (i.e. the entire $\neg gw$ column) are irrelevant to this calculation)

Q2

(i) $P(A \wedge B) = P(A) * P(B)$

(ii) $P(A \mid B) = P(A)$

To prove: (i) and (ii) are equivalent

Step 1: Prove (i) $\rightarrow$ (ii) [(i) implies (ii)]

(1) $P(A \wedge B) = P(A) \times P(B)$ (i)

(2) Recall from conditional probability that:

$$P(A \mid B) = \frac{P(A \wedge B)}{P(B)}$$

Now substitute our equation for $P(A \wedge B)$ from (1) into the equation for $P(A|B)$.

$$\therefore P(A|B) = \frac{P(A) \times P(B)}{P(B)}$$

(3) $$P(A|B) = P(A) \times \frac{P(B)}{P(B)} = P(A) \times 1$$

(4) $$P(A|B) = P(A) \quad \text{(ii)}$$

Hence proved (i) → (ii)

Step ② : Prove (ii) → (i) [(ii) implies (i)]

(1) $$P(A|B) = P(A) \quad \text{(ii)}$$

(2) We know from conditional probability that:

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

Now we have two equations for $P(A|B)$ from (1) and (2). Let's ~~combine~~ make them equal to each other.

$$\therefore \frac{P(A \wedge B)}{P(B)} = P(A)$$

(3) $$P(A \wedge B) = P(A) \times P(B) \quad \text{(ii)}$$

Hence proved (ii) → (i)

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Q3

yng : Age = "smaller than 30"
¬yng : Age = "not smaller than 30"
pb : PB = true

(a) Supposing
$P(yng) = 0.25$
$P(pb \mid yng) = 0.95$
$P(pb \mid \neg yng) = 0.01$

Recall from lectures that the Bayesian rule states:

$$\text{Choose } \arg\max_{\omega} P(x \mid \omega) P(\omega)$$

(i.e. picking the ω which makes the combination you are looking at as likely as possible)

In this case:

yng: $P(pb \mid yng) P(yng) = 0.95 \times 0.25 = 0.2375$

¬yng: $P(pb \mid \neg yng) P(\neg yng) = 0.01 \times (1 - 0.25) = 0.01 \times 0.75$
$= 0.0075$

∴ The best guess is yng since $0.2375 > 0.0075$

(b) Supposing

$p(yng) = 0.01$

$p(pb | yng) = 0.95$

$p(pb | \neg yng) = 0.01$

The bayesian rule from (a) applied:

yng: $p(pb | yng) \times P(yng) = 0.95 \times 0.01 = 0.0095$

¬yng: $p(pb | \neg yng) \times P(\neg yng) = 0.01 \times (1 - 0.01) = 0.01 \times 0.99$

$= 0.0099$

∴ The best guess is ¬yng since $0.0099 > 0.0095$

(c) Supposing

$p(yng) = 0.01$

$p(pb | yng) = 0.95$

$p(pb | \neg yng) = 0.0001$

The bayesian rule from (a) applied:

yng: $p(pb | yng) \times P(yng) = 0.95 \times 0.01 = 0.0095$

¬yng: $p(pb | \neg yng) \times P(\neg yng) = 0.0001 \times (1 - 0.01)$

$= 0.0001 \times 0.99$

$= 0.000099$

∴ The best guess is yng since $0.0095 > 0.000099$

Q4

	noisy: +	noisy: −
cool: +	62	108
cool: −	38	292

$$p(\text{cool}:+) = \frac{62+108}{500} = 0.34$$

$$p(\text{cool}:+ \mid \text{noisy}:+) = \frac{62}{62+38} = 0.62$$

From lectures, we know that ~~the~~ one of the formulations of independence can be written as:

$$P(X \mid Y) = P(X)$$

We can apply this to cool and noisy, in fact to the "+" settings of these variables. This is done above with $p(\text{cool}:+)$ and $p(\text{cool}:+ \mid \text{noisy}:+)$. However we realize that:

$$P(\text{cool}:+ \mid \text{noisy}:+) \neq p(\text{cool}:+)$$

Since

$$0.62 \neq 0.34$$

therefore

$$P(X \mid Y) \neq P(X)$$

Thus, we can say that Cool: + is NOT independent of noisy: +.

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Q5

open: +	noisy: +	noisy: −
cool: +	54	36
cool: −	6	4

open: −	noisy: +	noisy: −
cool: +	8	72
cool: −	32	288

$p(\text{cool}:+ \mid \text{open}:+) = \frac{54+36}{100} = 9/10$

$p(\text{cool}:+ \mid \text{open}:+, \text{noisy}:+) = \frac{54}{54+6} = 9/10$

From lectures, we discussed that an alternative definition for conditional independence is:

$$P(X \mid Y, Z) = P(X \mid Z) \quad (1)$$

We can rewrite this equation using some basic commutativity:

$P(X \mid Y, Z) = P(X \mid Z, Y)$ [probability of X given Y AND Z = probability of X given Z AND Y]

$P(X \mid Z, Y) = P(X \mid Y)$ [using (1)]

$\therefore P(X \mid Y, Z) = P(X \mid Y)$ [proves conditional independence]

Therefore, if we can prove the equation ~~P(X|Z~~, $P(X|Y,Z) = P(X|Y)$, then we can prove that conditional independence is true.

$$P(X|Y,Z) = P(X|Y)$$

$$P(\text{cool}:+|\text{open}:+, \text{noisy}:+) = P(\text{cool}:+|\text{open}:+)$$

$$9/10 = 9/10$$

$\therefore$ Cool:+ is conditionally independent of noisy:+ given open:+.

Q6

4 tosses of a coin : H H H T
where H= heads
T= tails

Let's say θh = probability of a toss of the coin giving H.

For $\theta h = 0.1$

$$P(H,H,H,T) = (0.1)\times(0.1)\times(0.1)\times(1-0.1)$$
$$= 0.0009$$

For $\theta h = 0.5$

$$P(H,H,H,T) = (0.5)\times(0.5)\times(0.5)\times(1-0.5)$$
$$= 0.0625$$

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For $\theta h = 0.75$

$$P(H,H,H,T) = (0.75) \times (0.75) \times (0.75) \times (1-0.75)$$
$$= 0.10546875$$

For $\theta h = 0.9$

$$P(H,H,H,T) = (0.9) \times (0.9) \times (0.9) \times (1-0.9)$$
$$= 0.0729$$

N.B. For this sequence, the general formula for the probability of the sequence using θh would be:

$$P(H,H,H,T) = (\theta h) \times (\theta h) \times (\theta h) \times (1-\theta h)$$

where θh = probability of H

Since coin flips are independent, the probability of the sequence HHHT can be found by multiplying the probability of each event in the sequence.