Simulation Exercise - Report

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Part-1 - Exploring the Given Distribution

Normal Distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

μ and **σ** are parameters for this distribution and f(x) is the pdf function

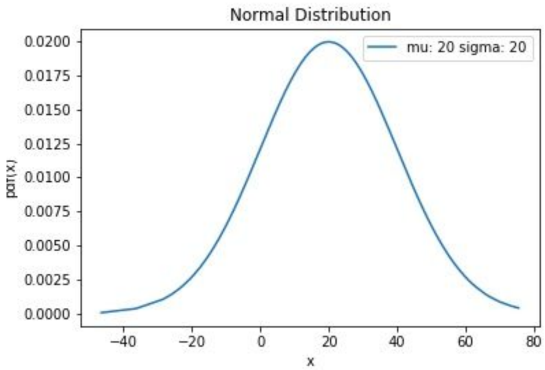
Mean of Distribution is μ
Variance of Distribution is σ^2

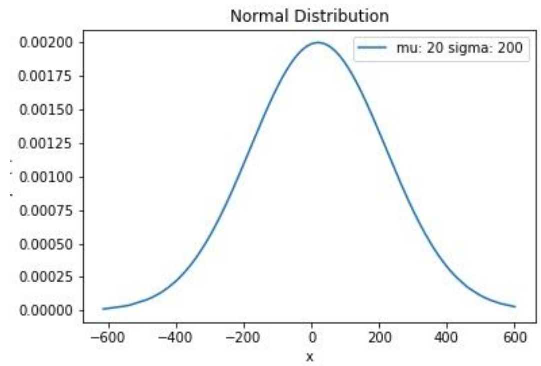
Values of Mean and Variance changing for different parameters

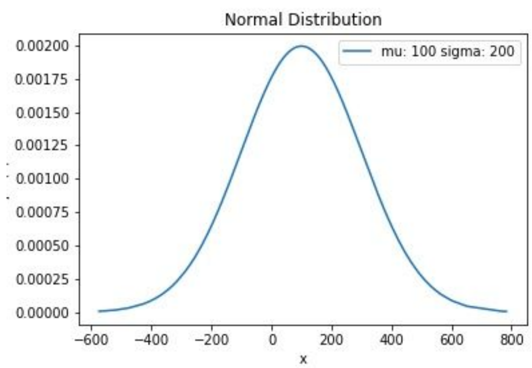
μ	σ	Mean	Variance
20	20	20	400
20	200	20	40000
100	200	100	40000
300	200	300	40000

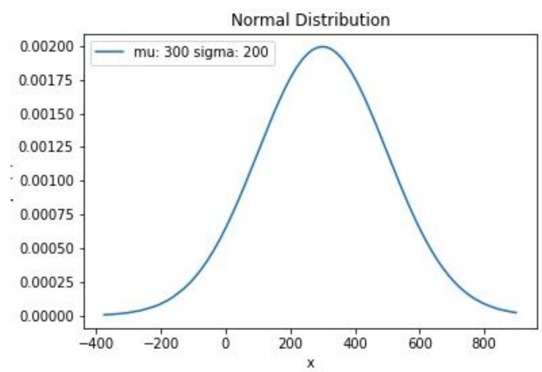
Distribution for above parameters:

Random variables were generated for above parameters and the pdf formula f(x) was applied to each of those points to get the corresponding distribution









<u>**Conclusion:**</u>

Mean and variance are directly proportional to **μ** and **σ**

Gamma Distribution:

$$f(x) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}$$

k and **θ** are parameters for this distribution and f(x) is the pdf function

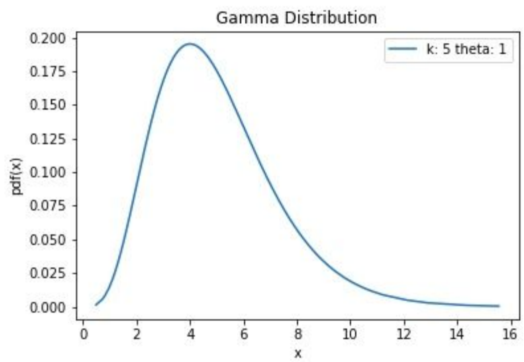
Mean of Distribution is k***θ**
Variance of Distribution is k***θ*****θ**

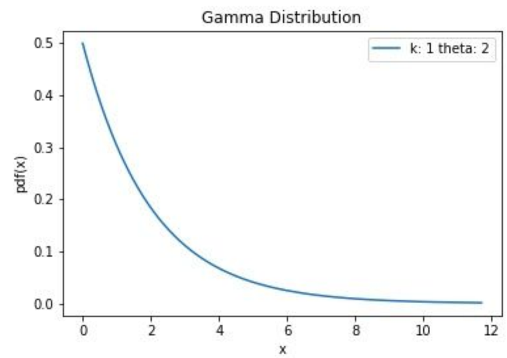
Values of Mean and Variance changing for different parameters

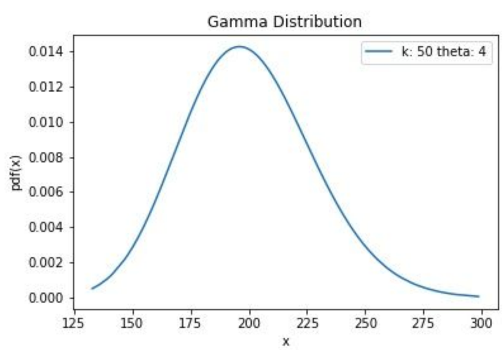
k	**θ**	Mean	Variance
5	1	5	5.00001
1	2	2	4.0
50	4	200	800.0001
25	1	25	25.0

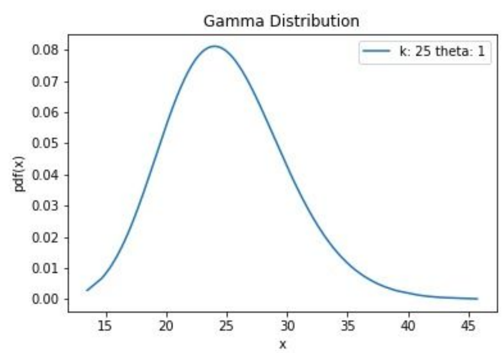
Distribution for above parameters:

Random variables were generated for above parameters and the pdf formula f(x) was applied to each of those points to get the corresponding distribution









Conclusion:

Mean and variance are directly proportional to k and **θ**

Geometric distribution:

$$P(X = n) = (1 - p)^{n-1}p$$

p is the parameters for this distribution and P(x) is the pmf function

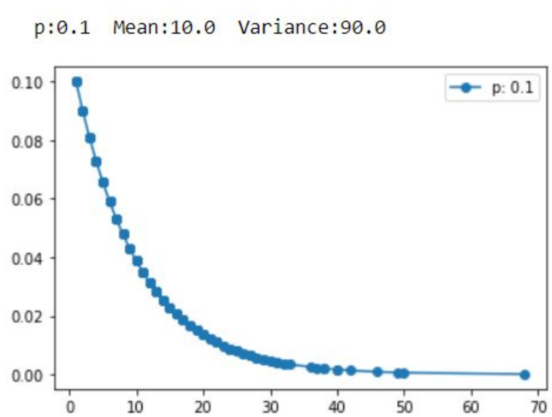
Mean of Distribution is 1/p
Variance of Distribution is $(1-p)/p^2$

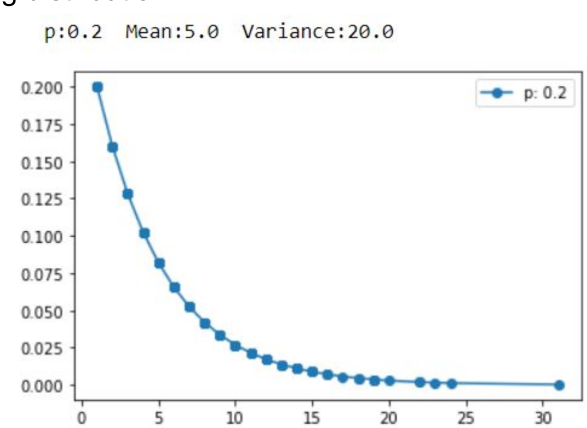
Values of Mean and Variance changing for different parameters

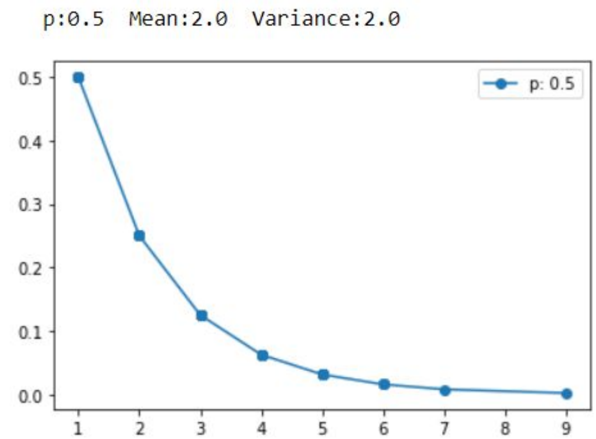
p	Mean	Variance
0.1	10	90
0.2	5	20
0.5	2	2
0.6	1.666	1.11

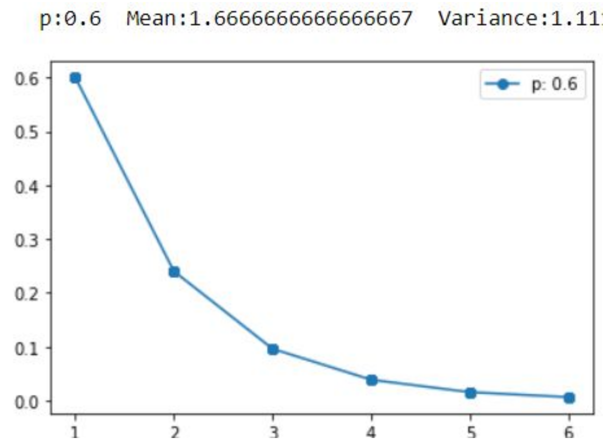
Distribution for above parameters:

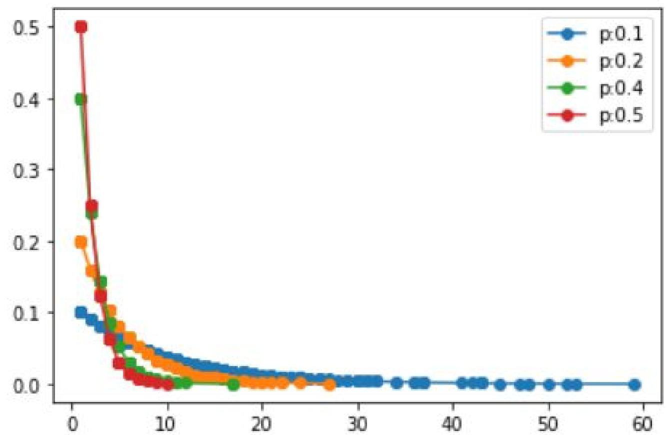
Random variables were generated for above parameters and the pmf formula f(x) was applied to each of those points to get the corresponding distribution











Conclusion: From the tabulation and the graphs, we can conclude that mean and variance decreases with increase in p for a geometric distribution.

Binomial Distribution:

$$P(X) = {}_{n}C_{x}\, p^{x}(1-p)^{n-x}$$

n and p are parameters for this distribution and P(x) is the pmf function

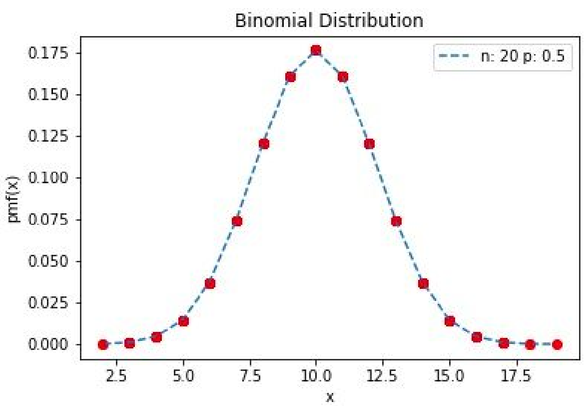
Mean of Distribution is n*p
Variance of Distribution is n*p*(1-p)

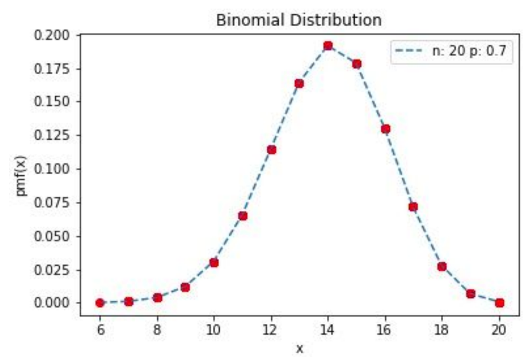
Values of Mean and Variance changing for different parameters

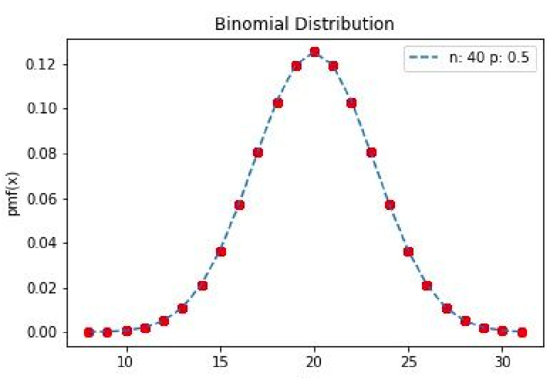
n	p	Mean	Variance
20	0.5	10	5.0
20	0.7	14	4.2
40	0.5	20	10
20	0.25	5	3.75

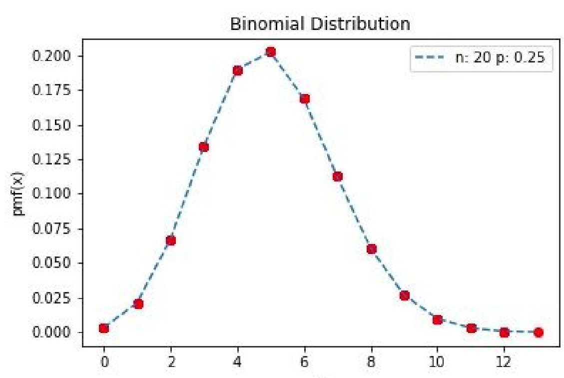
Distribution for above parameters:

Random variables were generated for above parameters and the pmf formula f(x) was applied to each of those points to get the corresponding distribution









Conclusion:
Mean is directly proportional to the n and p
Variance is directly proportional to n and inversely proportional to p

Negative Binomial Distribution:

$$f(k) = \binom{k+n-1}{n-1} p^n (1-p)^k$$

n and p are parameters for this distribution and P(x) is the pmf function

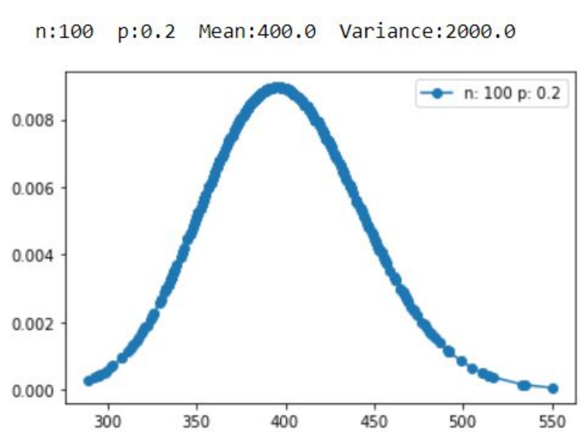
Mean of the distribution = p.r / 1-p
Variance of the distribution = p.r / $(1\text{-}p)^2$

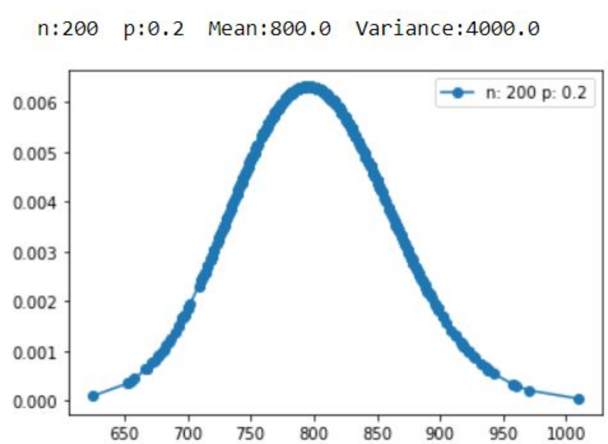
<u>Values of mean and variance for different parameters</u>

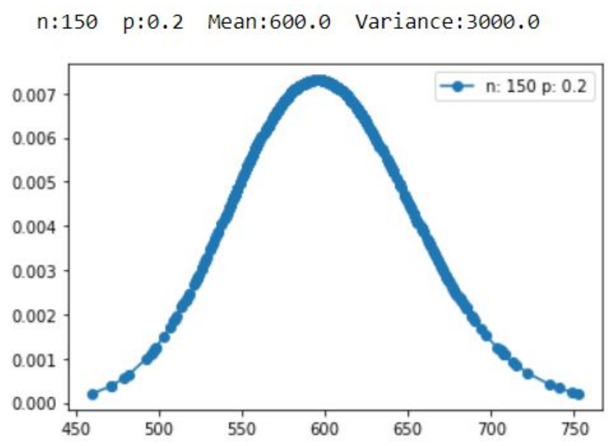
n	p	Mean	Variance
100	0.2	400	2000
200	0.2	800	4000
150	0.2	600	3000
150	0.5	150	300

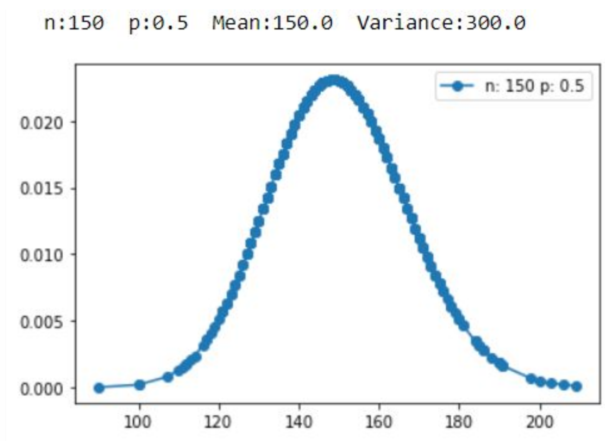
Distribution for above parameters:

Random variables were generated for above parameters and the pmf formula f(x) was applied to each of those points to get the corresponding distribution









Conclusion: From the tabulation and the graphs, we can conclude that

1. Mean and Variance increases with an increase in n
2. Mean and Variance decreases with an increase in p

Poisson Distribution:

$$\Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Here λ is a parameter in this distribution and Pr(X) is a pmf function .

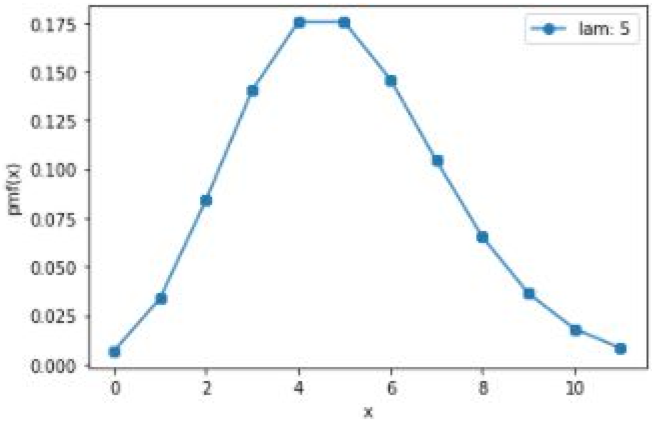
Mean of the distribution = λ
Variance of the distribution = λ

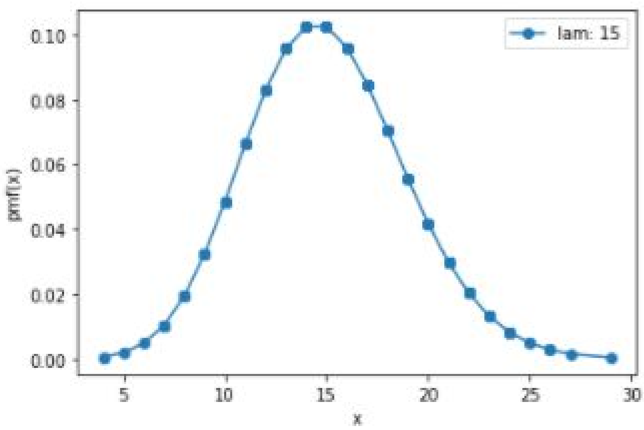
Values of mean and variance for different parameters

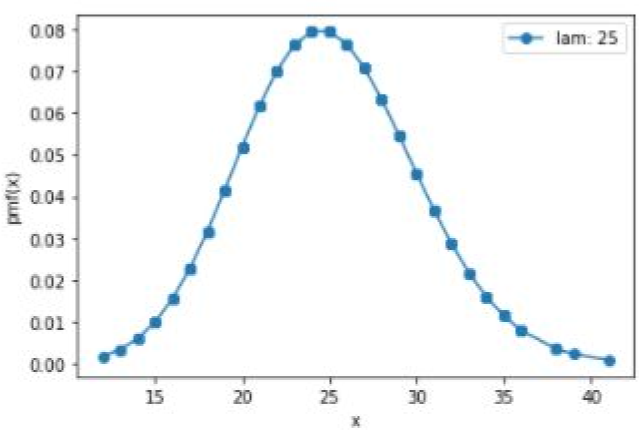
λ	Mean	Variance
5	5	5
15	15	15
25	25	25
35	35	35

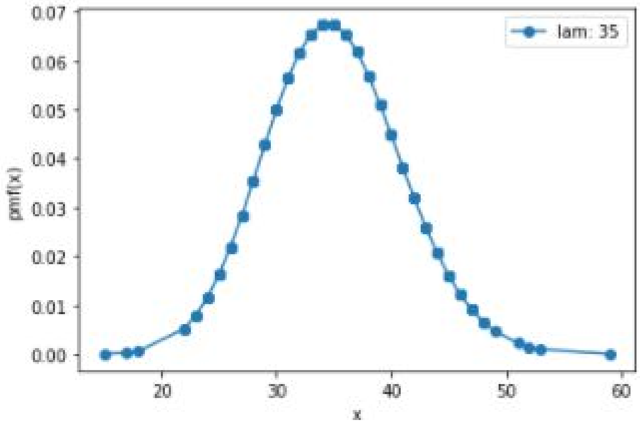
Distribution for above parameters

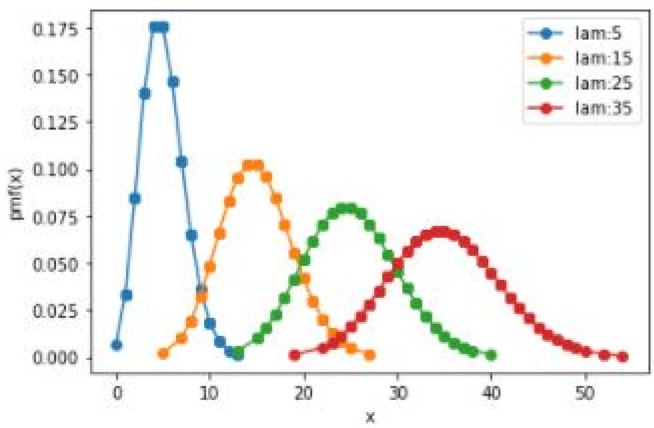
Random variables were generated for above parameters and the pmf formula f(x) was applied to each of those points to get the corresponding distribution .











Conclusion: From the tabulation and the graphs, we can conclude that mean and variance increases with increase in λ for a poisson distribution.

<u>Discrete Uniform Distribution:</u>

f(x) = 1/(b−a+1)

Here Parameters are a and b and f(x) is a pmf function .

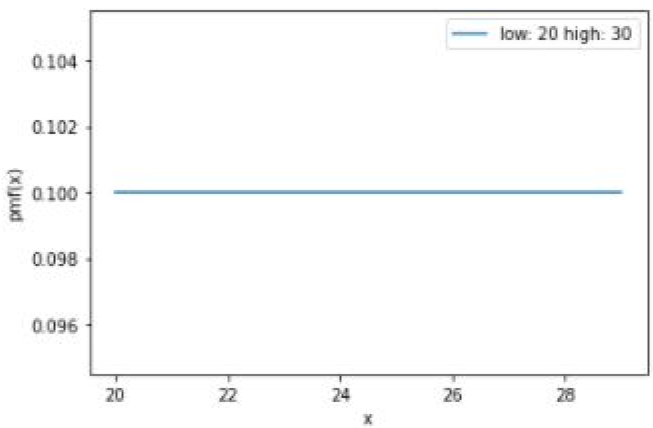
Mean of this distribution = (a+b)/2 .

Variance of this distribution = $((b - a + 1)^2 - 1)/12$.

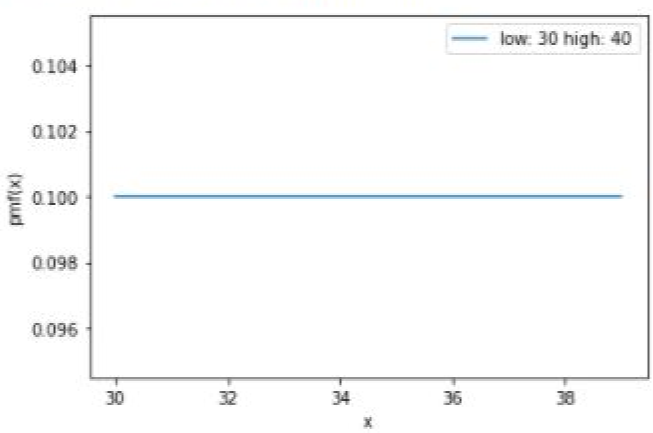
Values of mean and variance for different parameters

Low(a)	High(b+1)	Mean	Variance
20	30	24.5	8.25
30	40	34.5	8.25
40	55	47.0	18.666666666666668

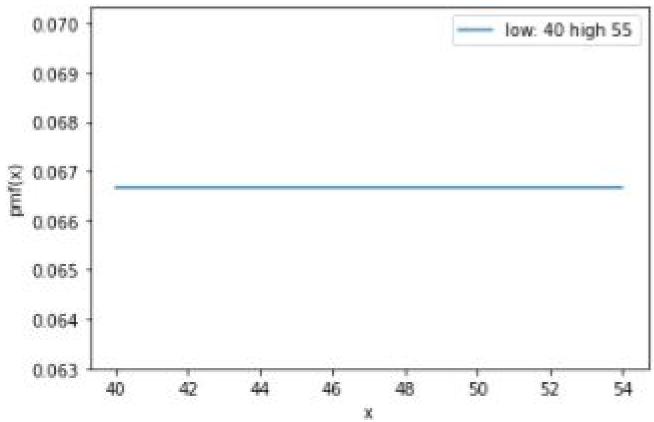
```
Mean of the distribution : 24.5
Variance of the distribution : 8.25
```



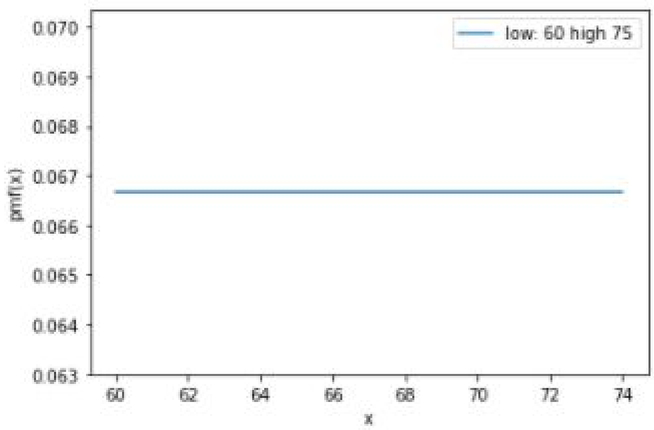

```
Mean of the distribution : 34.5
Variance of the distribution : 8.25
```



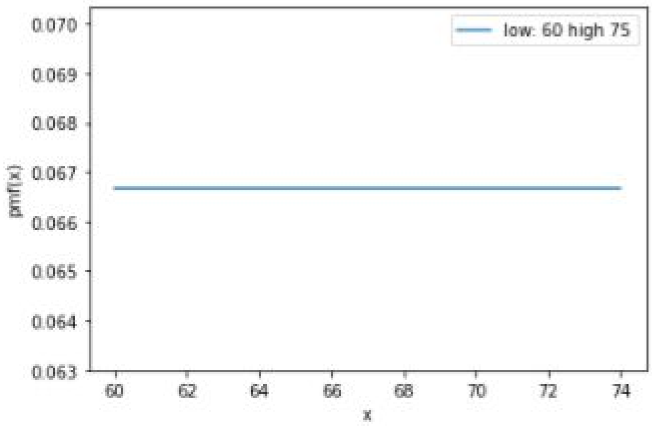
```
Mean of the distribution : 47.0
Variance of the distribution : 18.666666666666668
```

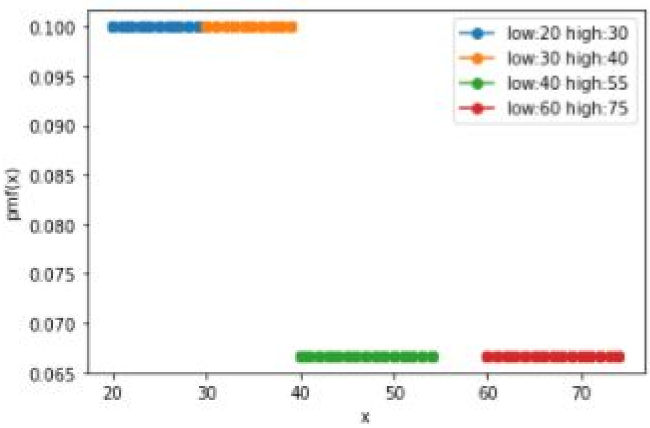



```
Mean of the distribution : 67.0
Variance of the distribution : 18.666666666666668
```



```
Mean of the distribution : 67.0
Variance of the distribution : 18.666666666666668
```





Conclusion: From the tabulation and the graphs, we can conclude that mean increases with increase in high and low values and variance increases by increasing the difference between high and low values for this discrete uniform distribution

Exponential Distribution:

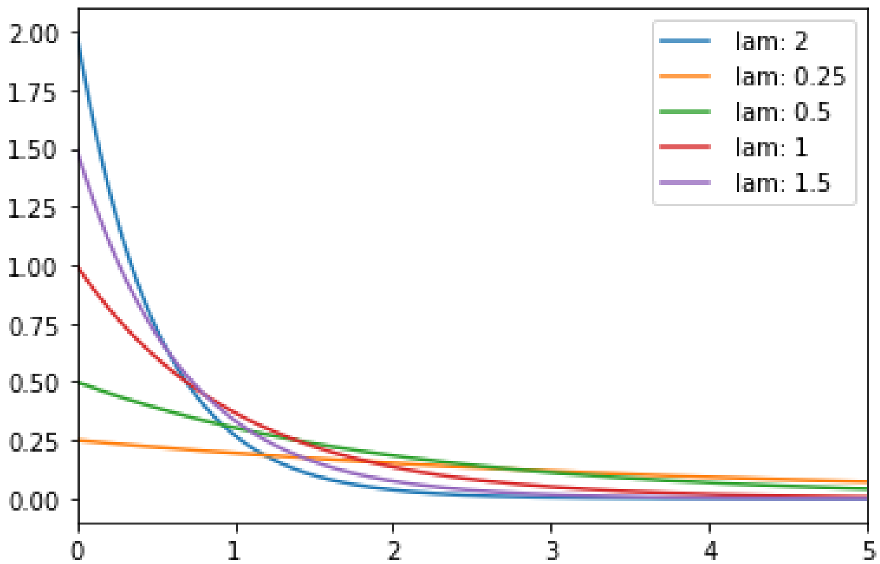
$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Lamda is the parameter and f(x) is pdf.
Mean of distribution= 1/ λ
Variance of distribution = $(1/ \lambda)^2$

Values of Mean and Variance changing for different parameters

λ	Mean	Variance
0.25	4.0	16.0
0.5	2	4.0
1	1	1
2	0.5	0.25

Distribution for above parameters:



Conclusion: From the tabulation and the graphs, we can conclude that mean decreases with increase in lambda (λ) and variance also decreases by increase in lambda (λ) in this exponential distribution.

Lognormal Distribution:

f(x) = $$\frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right)$$

μ and **σ** are parameters for this distribution and f(x) is the pdf function

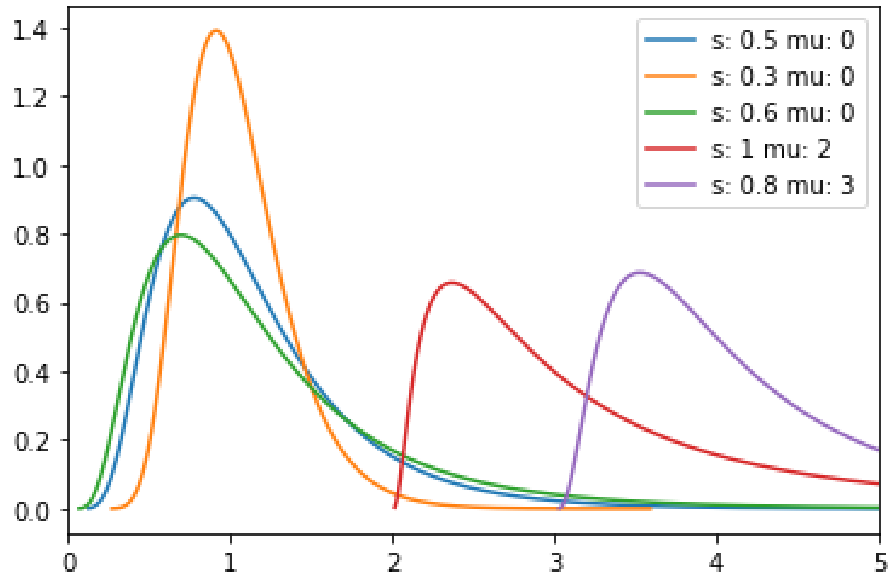
Mean of Distribution is = $\exp\left(\mu + \frac{\sigma^2}{2}\right)$

Variance of Distribution is = $[\exp(\sigma^2) - 1] \exp(2\mu + \sigma^2)$

Values of Mean and Variance changing for different parameters

μ	σ	Mean	Variance
0	0.5	1.133	0.364
0	0.3	1.046	0.103
0	0.6	1.197	0.621
2	1	12.182	255.01
3	0.8	27.660	685.89

<u>Distribution for above parameters:</u>



Conclusion: From the tabulation and the graphs, we can conclude that
both mean and variance increases by increase in σ (s). Also both mean and variance increases by increase in μ (mu) in this lognormal distribution.

Beta Distribution:

$$f(x,a,b) = \frac{\Gamma(a+b)x^{a-1}(1-x)^{b-1}}{\Gamma(a)\Gamma(b)}$$

f(x,a,b) is pdf and a,b are the parameters

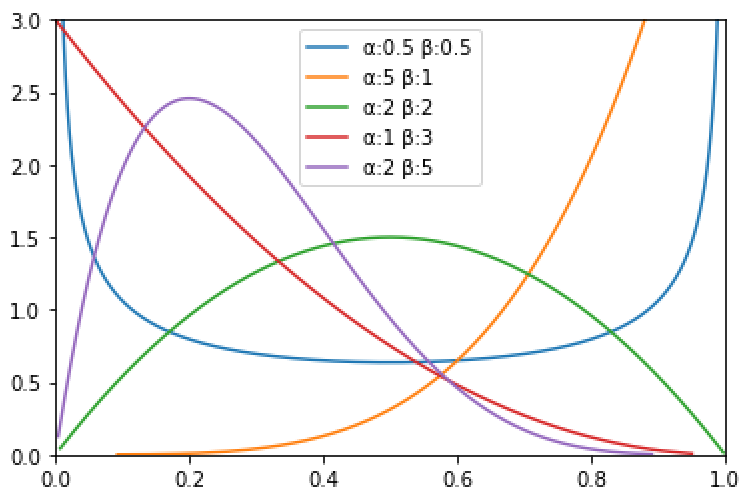
Mean of distribution is $\mathrm{E}[X] = \frac{\alpha}{\alpha+\beta}$

Variance of distribution is $\mathrm{var}[X] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Values of Mean and Variance changing for different parameters

α	β	Mean	Variance
0.5	0.5	0.5	0.125
1	3	0.25	0.0375
2	2	0.5	0.05
2	5	0.28	0.025
5	1	0.833	0.019

Plotted Distributions for above parameters:



Conclusion: From the tabulation and the graphs, we can conclude that
both mean and variance increases by increase in σ (s). Also both mean and variance increases by increase in μ (mu) in this lognormal distribution.

Part-2 - Verifying Central Limit Theorem

10000 Random Variables were generated for the given parameters to represent the population and thousand samples , each with 40 values , were considered for verifying Central Limit Theorem

Mean of each of those samples were calculated and stored and plotted to get probability distribution and then verification was performed

Normal Distribution:

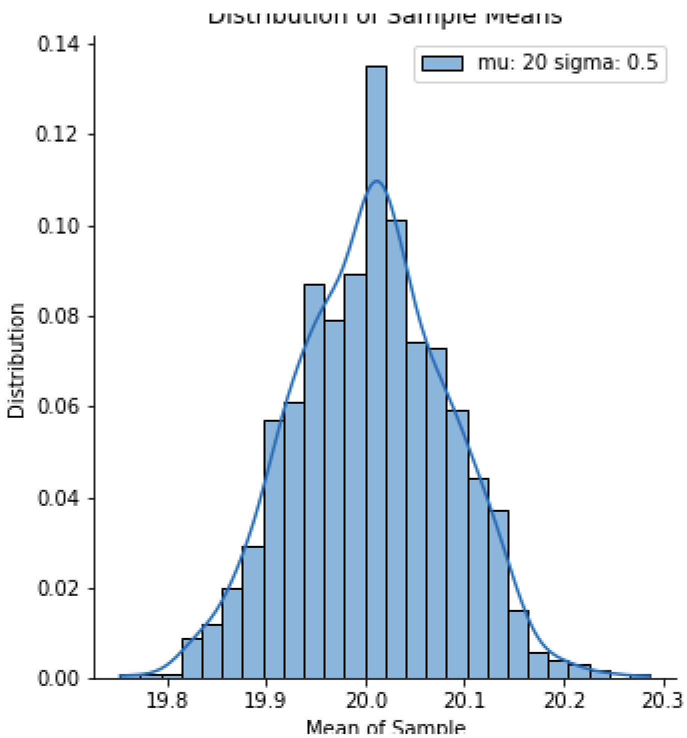
Parameters fixed for experiment:

$\mu = 20$
$\sigma = 0.5$
$n=40$

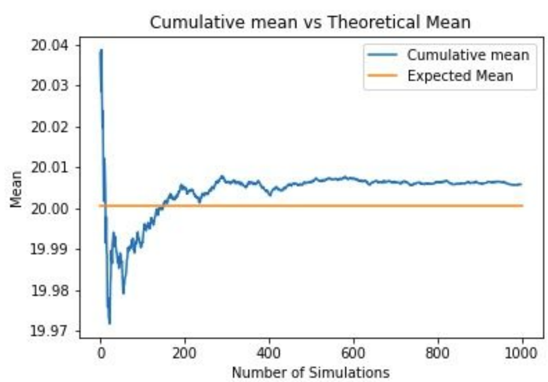
Observed Results

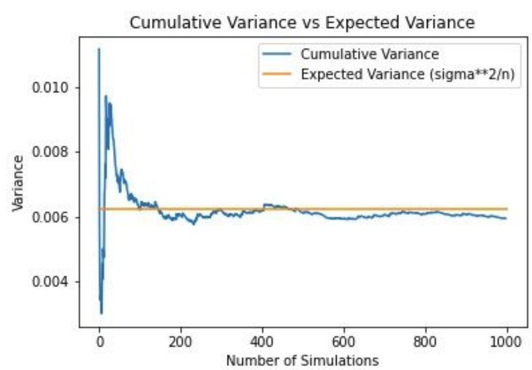
Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)
20.006	20.0006	0.0059	0.25

Distribution of Sample Mean



As it can be inferred from the plot , the means follow a Normal Distribution with mean = 20.006 and variance = 0.0059





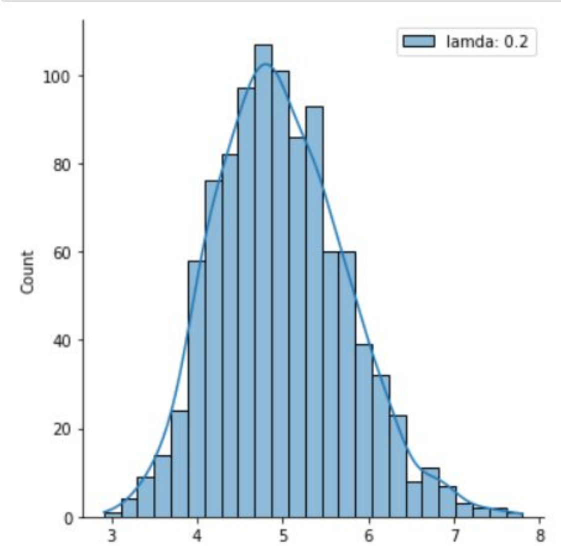
Exponential Distribution:

Parameters fixed for experiment:
lam=0.2

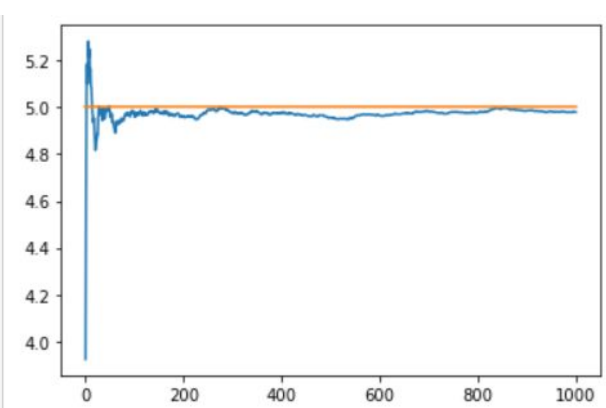
Observed Results

Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)	Theoretical Variance divided by sample size (σ^2/n)
4.979	5	0.6428	25	0.625

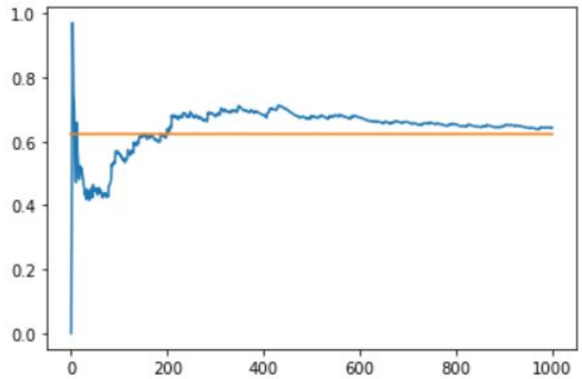
Distribution of Sample Mean



As it can be inferred from the plot , the distribution of sample means is approximately normal. Hence, we can conclude that CLT is verified.



From the above graph, we can infer that the expectation of sample means is approximately equal to the actual mean of the distribution.



From the above graph, we can infer that the Variance of Sample Means is approximately equal to Theoretical Variance / sample size.

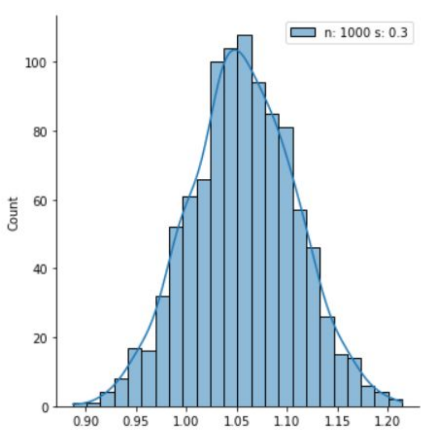
Lognormal Distribution:

Parameters fixed for experiment:
p=0.2

Observed Results

Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)	Theoretical Variance divided by sample size (σ^2/n)
1.027	1.046	0.00249	0.1030	0.0025

Distribution of Sample Mean



As it can be inferred from the plot , the distribution of sample means is approximately normal. Hence, we can conclude that CLT is verified.

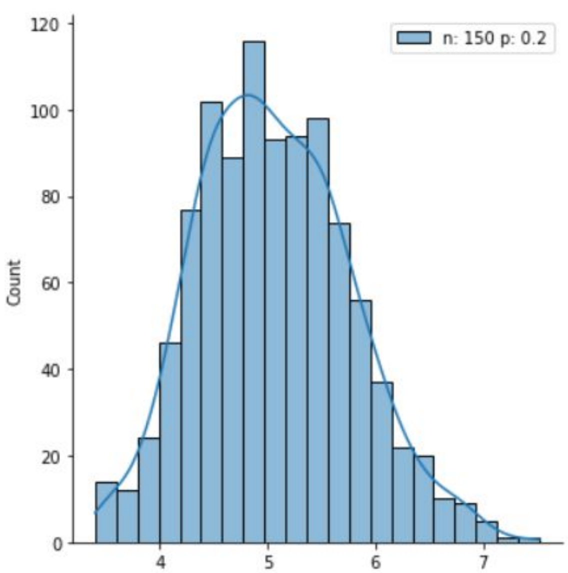
Geometric Distribution:

Parameters fixed for experiment:
p=0.2

Observed Results

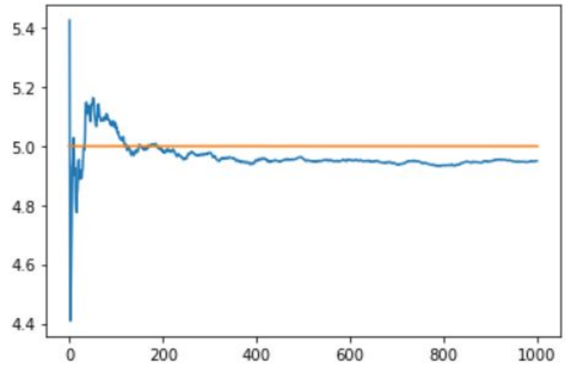
Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)	Theoretical Variance divided by sample size (σ^2/n)
4.9496	5	0.4921	20	0.5

Distribution of Sample Mean



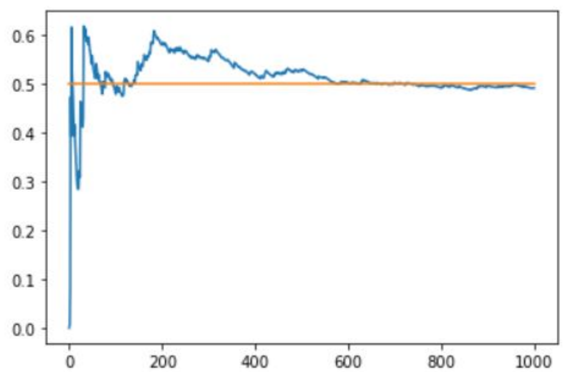
As it can be inferred from the plot , the distribution of sample means is approximately normal. Hence, we can conclude that CLT is verified.

```
Mean of the distribution : 5.0
Expectation of Sample Means 4.9496
```



From the above graph, we can infer that the expectation of sample means is approximately equal to the actual mean of the distribution.

```
Variance of the distribution : 20.0
Variance of Sample Means : 0.49216704749999995
σ/N : 0.5
```



From the above graph, we can infer that the Variance of Sample Means is approximately equal to Theoretical Variance / sample size.

Gamma Distribution:

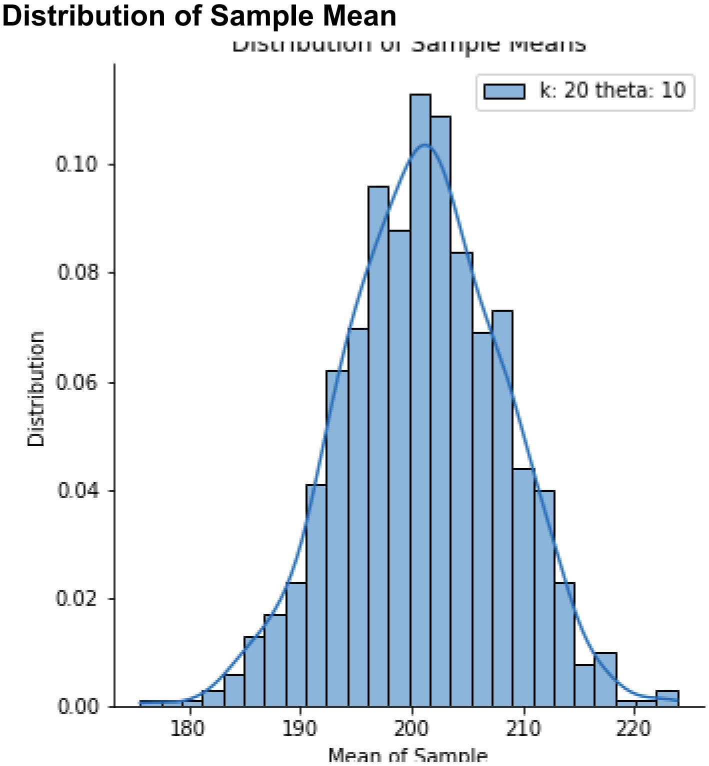
Parameters fixed for experiment:

k = 20
θ = 10
n=40

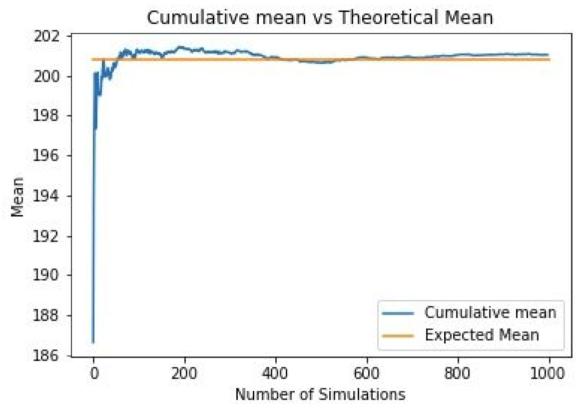
Observed Results

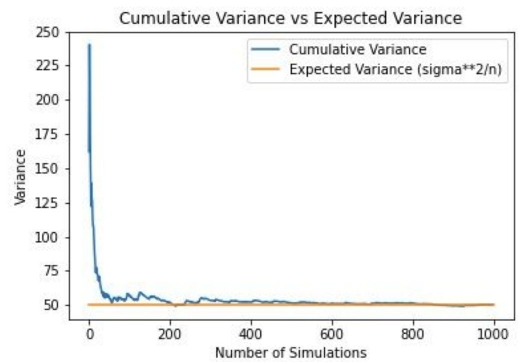
Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)
201.01	200.77	50.30	2000

Distribution of Sample Mean



As it can be inferred from the plot , the means follow a Normal Distribution with mean =201.01 and variance = 50.30





Beta Distribution:

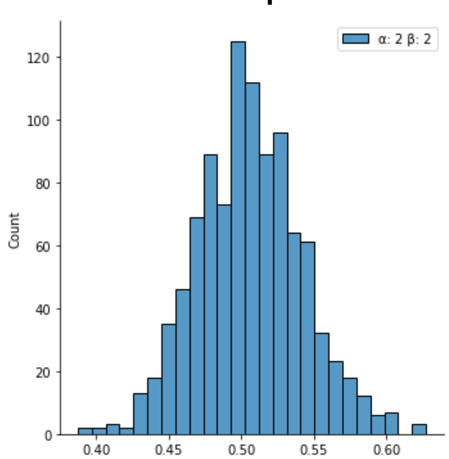
Parameters fixed for experiment:

α = 2
β= 2

Observed Results

Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)	Theoretical Variance(σ^2/n)
0.498	0.5	0.0011	0.05	0.00125

Distribution of Sample Mean:



It can be observed that distribution of sample mean follows Normal Distribution with mean = 0.498 and variance = 0.0011. Hence, CLT verified.

Binomial Distribution:

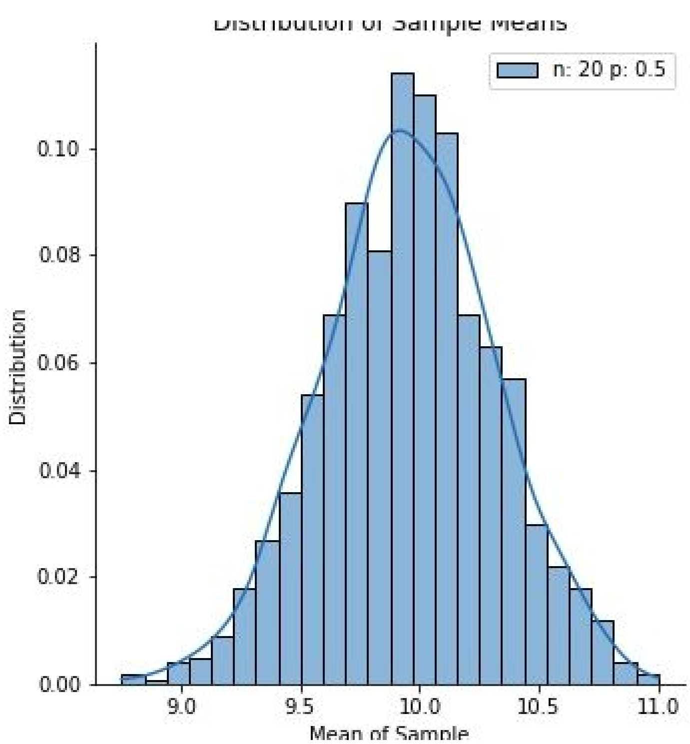
Parameters fixed for experiment:

n = 20
p = 0.5
n=40

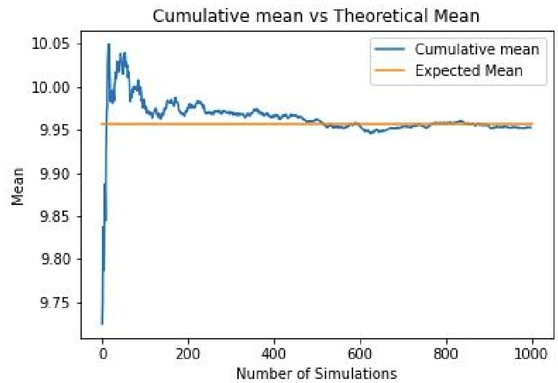
Observed Results

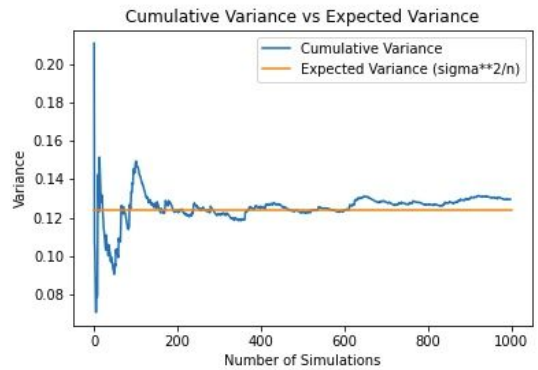
Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)
9.9527	9.9565	0.129	5.0

Distribution of Sample Mean



As it can be inferred from the plot , the means follow a Normal Distribution with mean = 9.952 and variance = 0.129





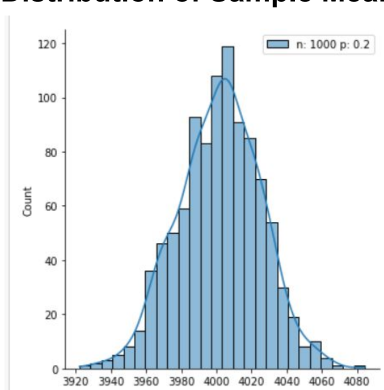
Negative Binomial Distribution:

Parameters fixed for experiment:

n = 1000
p = 0.2

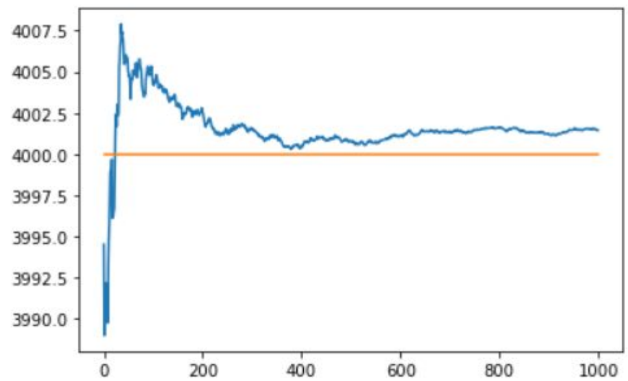
Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)	Theoretical Variance divided by sample size (σ^2/n)
4001.454	4000	497.018	20000	500

Distribution of Sample Mean



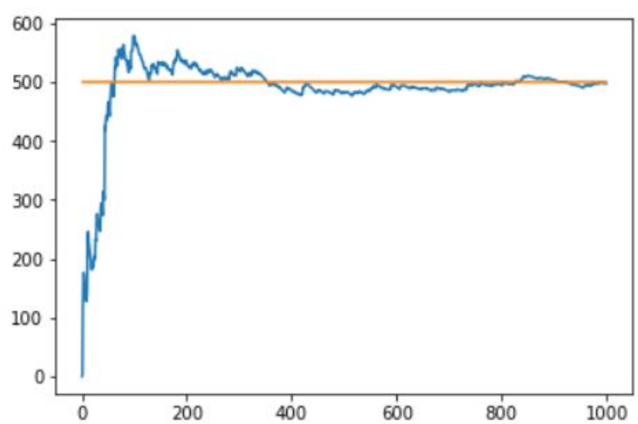
As it can be inferred from the plot , the distribution of sample means is approximately normal. Hence, we can conclude that CLT is verified.

```
Mean of the distribution : 4000.0
Expectation of Sample Means 4001.454
```



From the above graph, we can infer that the expectation of sample means is a good estimator of the actual mean of the distribution.

```
Variance of the distribution : 20000.0
Variance of Sample Means : 497.0184564993752
σ/N : 500.0
```



From the above graph, we can infer that the Variance of Sample Means is approximately equal to Theoretical Variance / sample size.

Poisson Distribution:

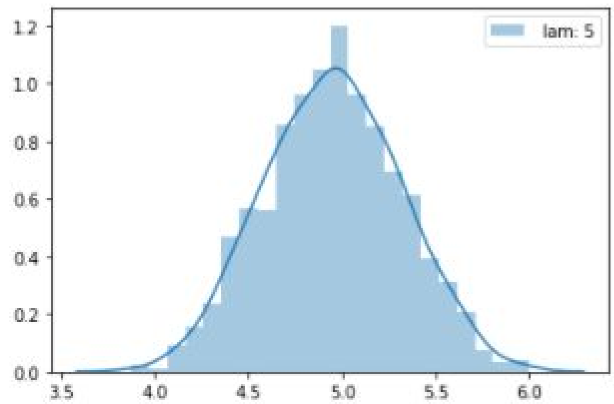
Parameters fixed for experiment:

1) $\lambda = 5$

Observed Results

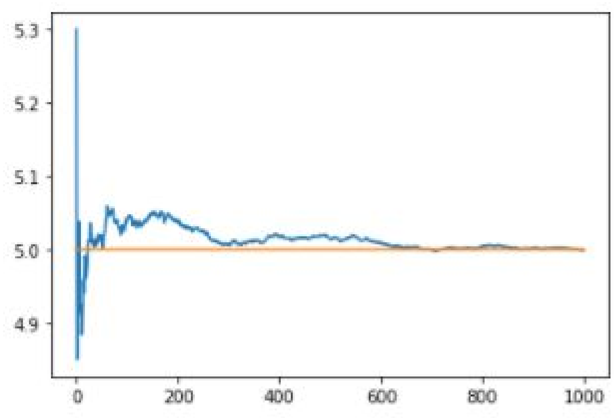
Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)	Theoretical Variance divided by sample size (σ^2/n)
4.998474999	5.0	0.1249124275	5.0	0.125

Distribution of Sample Mean:



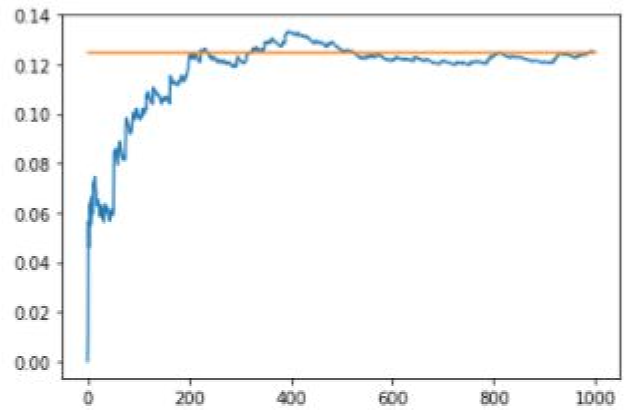
As it can be inferred from the plot , the distribution of sample means is approximately normal. Hence, we can conclude that CLT is verified.

```
Mean of the distribution : 5.0
Expectation of Sample Means 4.998474999999999
```



From the above graph, we can infer that the expectation of sample means is a good estimator of the actual mean of the distribution.

```
Variance of the distribution : 5.0
Variance of Sample Means : 0.1249124275
σ**2/N : 0.125
```



From the above graph, we can infer that the Variance of Sample Means is approximately equal to Theoretical Variance / sample size.

Discrete Uniform Distribution:

Parameters fixed for experiment:

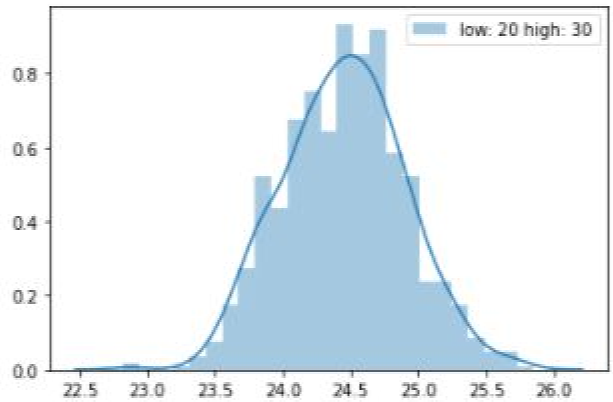
1)low = 20

2)high=30

Observed Results

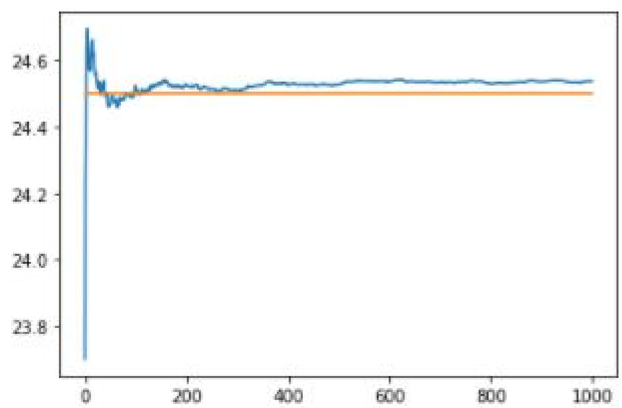
Expectation of Sample Means (x-bar)	Mean of Population (μ)	Variance of Sample Means (σ^2/n)	Theoretical Variance of Population (σ^2)	Theoretical Variance divided by sample size (σ^2/n)
24.53715	24.5	0.200273494375	8.25	0.20625

Distribution of Sample Mean:



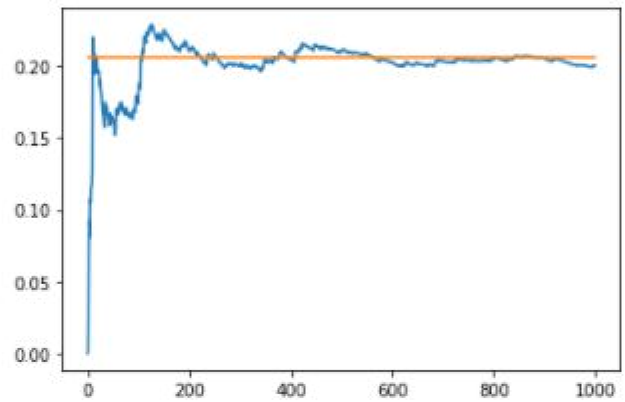
As it can be inferred from the plot , the distribution of sample means is approximately normal. Hence, we can conclude that CLT is verified.

```
Mean of the distribution : 24.5
Expectation of Sample Means 24.53715
```



From the above graph, we can infer that the expectation of sample means is a good estimator of the actual mean of the distribution.

```
Variance of the distribution : 8.25
Variance of Sample Means : 0.20027349437500003
σ**2/N : 0.20625
```



From the above graph, we can infer that the Variance of Sample Means is approximately equal to Theoretical Variance / sample size .