Indian Institute of Technology Kharagpur
Department of Mechanical Engineering

Data-driven methods in thermal and fluid sciences-ME41201

End-sem examination-Part A

Time: 8:20 to 9:50 AM | November 14th, 2023 | Max mark: 100

Instructions

1. A working (automated) python code with proper syntax should be constructed.
2. Figures (or plots) should have appropriate labels. Axes scales should be justified with reasonable limits.
3. Work it out individually and maintain work ethics

---

1. The Land-Ocean surface temperature index (source: NASA/GISS), which indicates global warming process, is given in a file *surfaceTemp.txt*. In the file, the first column is for the year (x), the second column is for the index (y), and the third column is an arbitrary fit. Contruct a best 10th degree polynomial fit in the form $y = \Sigma_0^{10} \alpha_k x^k$, where the loadings α_k are to be determined by four regression techniques: least-squares, LASSO, ridge, and elastic net. Compare the models for each against each other. Randomly pick any time point and corrupt the temperature measurement at that location. For instance, the temperature reading at that location could be zero. Investigate the resulting model and E_2 error for the four regression techniques considered. Identify the models that are robust to such an outlier and those that are not. Explicitly calculate the variance of the loading coefficients α_k for each method for a number of random trials with one or more corrupt data points. **(50 M)**

2. The process of phase separation in multi-component alloy systems, including order-disorder transitions is described by the following partial differential equation,
$$u_t - 0.0001u_{xx} + 5u^3 - 5u = 0, \quad x \in [-1, 1], \quad t \in [0, 1],$$
$$u(0, x) = x^2 \cos(\pi x),$$
$$u(t, -1) = u(t, 1),$$
$$u_x(t, -1) = u_x(t, 1),$$
where t is the time, x is the space co-ordinate, and suffixes are the partial derivatives. Construct a data-driven solution using Physics Informed Neural Networks approach with your chosen parameters such as the number of hidden layers, neurons, optimizers, learning rates, and the other. Make sure that the solution converges even if a small perturbation is carried to the network parameters. Error minimzation should include the differential equation, boundary and the initial conditions. Training process should be with in time $t \leq 0.5$, testing can be upto $t < 0.7$ and prediction should be for $t > 2$. Finally, (a) develop a space-time time contour plot of $u(x, t)$ upto $t = 2$. (b) Compare solutions $u(x, t = 1)$, $u(x, t = 1.1)$, $u(x, t = 2)$ in a single plot. **(50 M)**

—