EAS508-HW4

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2022-10-20

Lab Code Homework

5.3 Cross Validation Labs

5.3.1 Validation Set Approach

```
# Setting the seed and loading the data
library(ISLR2)
## Warning: package 'ISLR2' was built under R version 4.0.5
set.seed(1)
train <- sample(392,196)</pre>
# Fitting a linear regression on the train data using subset option
lm.fit <- lm(mpg ~ horsepower, data = Auto, subset = train)</pre>
# Predicting the estimates for the 392 observations and calculate the MSE for 192 observations
mean((Auto$mpg - predict(lm.fit, Auto))[-train]^2)
## [1] 23.26601
# Fitting cubic regression and calculating the MSE
lm.fit2 <- lm(mpg ~poly(horsepower, 2), data = Auto, subset = train)</pre>
mean((Auto$mpg - predict(lm.fit2, Auto))[-train]^2)
## [1] 18.71646
# Fitting uadratic regression and calculating the MSE
lm.fit3 <- lm(mpg ~ poly(horsepower, 3), data = Auto, subset = train)</pre>
mean((Auto$mpg - predict(lm.fit3, Auto))[-train]^2)
```

```
## [1] 18.79401
# Using different seed and calculating the values for all the three regressions - will result into dif
set.seed(2)
train <- sample(392,196)</pre>
# Linear regression MSE
lm.fit <- lm(mpg ~ horsepower, data = Auto, subset = train)</pre>
mean((Auto$mpg - predict(lm.fit, Auto))[-train]^2)
## [1] 25.72651
# Cubic regression MSE
lm.fit2 <- lm(mpg ~poly(horsepower, 2), data = Auto, subset = train)</pre>
mean((Auto$mpg - predict(lm.fit2, Auto))[-train]^2)
## [1] 20.43036
# Quadratic regression MSE
lm.fit3 <- lm(mpg ~poly(horsepower, 3), data = Auto, subset = train)</pre>
mean((Auto$mpg - predict(lm.fit3, Auto))[-train]^2)
## [1] 20.38533
5.3.2 Leave One-Out Cross-Validation
# LOOCV using glm() package
glm.fit <- glm(mpg ~ horsepower, data = Auto)</pre>
coef(glm.fit)
## (Intercept) horsepower
## 39.9358610 -0.1578447
# LOOCV using normal lm() function
```

lm.fit <- lm(mpg ~ horsepower, data = Auto)</pre>

coef(lm.fit)

```
## (Intercept) horsepower
## 39.9358610 -0.1578447
# Cross-validation error using glm() package
library(boot)
## Warning: package 'boot' was built under R version 4.0.5
glm.fit <- glm(mpg ~ horsepower, data = Auto)</pre>
cv.err <- cv.glm(Auto, glm.fit)</pre>
cv.err$delta
## [1] 24.23151 24.23114
# Calculating CV error for for polynomial of order 1 to 10 using a for loop.
cv.error \leftarrow rep(0,10)
for (i in 1:10) {
  glm.fit <- glm(mpg ~ poly(horsepower, i), data = Auto)</pre>
 cv.error[i] <- cv.glm(Auto, glm.fit)$delta[1]</pre>
}
cv.error
## [1] 24.23151 19.24821 19.33498 19.42443 19.03321 18.97864 18.83305 18.96115
```

[9] 19.06863 19.49093

5.3.3 k-Fold Cross Validation

```
# Calculating k-fold CV error for for polynomial of order 1 to 10 with k = 10
set.seed(17)
cv.error.10 <- rep(0,10)

for (i in 1:10) {
    glm.fit <- glm(mpg ~ poly(horsepower, i), data = Auto)
    cv.error.10[i] <- cv.glm(Auto, glm.fit, K = 10)$delta[1]
}
cv.error.10</pre>
```

```
## [1] 24.27207 19.26909 19.34805 19.29496 19.03198 18.89781 19.12061 19.14666
## [9] 18.87013 20.95520
```

6.5.3 PCR and PLS Regression

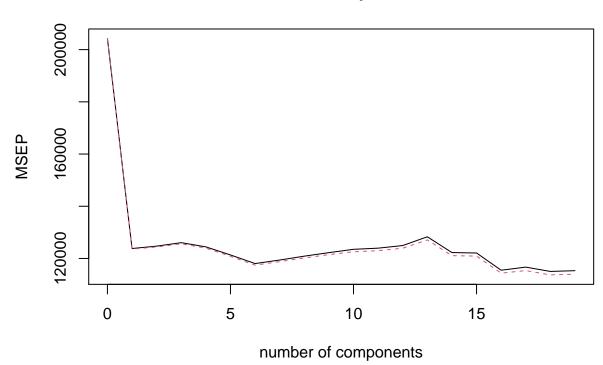
```
\# Creating model matrix for x and storing all salary values in y
# Omitting NA values
Hitters <- na.omit(Hitters)</pre>
x <- model.matrix(Salary ~ ., Hitters)[, -1]</pre>
y <- Hitters$Salary
# Creating train and test by setting the R seed
set.seed(1)
train <- sample(1:nrow(x), nrow(x) / 2)</pre>
test <- (-train)
y.test <- y[test]</pre>
# Applying PCR to Hitters data to predcit Salary
library(pls)
Principal Components Regression
##
## Attaching package: 'pls'
## The following object is masked from 'package:stats':
##
##
       loadings
set.seed(2)
pcr.fit <- pcr(Salary ~., data = Hitters, scale = TRUE, validation = "CV")</pre>
# Checking summary of our fit
summary(pcr.fit)
            X dimension: 263 19
## Data:
## Y dimension: 263 1
## Fit method: svdpc
## Number of components considered: 19
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##
          (Intercept) 1 comps 2 comps 3 comps 4 comps 5 comps 6 comps
## CV
                  452
                         351.9 353.2
                                           355.0
                                                     352.8
                                                              348.4
                                                                       343.6
                         351.6
                                           354.4
                                                                       342.7
## adjCV
                  452
                                  352.7
                                                    352.1
                                                              347.6
```

```
##
          7 comps 8 comps 9 comps 10 comps 11 comps 12 comps
                                                                    13 comps
## CV
            345.5
                     347.7
                               349.6
                                         351.4
                                                   352.1
                                                              353.5
                                                                        358.2
## adjCV
            344.7
                     346.7
                                         350.1
                                                   350.7
                                                              352.0
                                                                        356.5
                               348.5
##
                               16 comps
                                                   18 comps
                                                              19 comps
          14 comps
                    15 comps
                                         17 comps
## CV
             349.7
                       349.4
                                  339.9
                                            341.6
                                                       339.2
                                                                 339.6
## adjCV
             348.0
                       347.7
                                  338.2
                                            339.7
                                                       337.2
                                                                 337.6
## TRAINING: % variance explained
##
           1 comps 2 comps 3 comps 4 comps 5 comps 6 comps 7 comps
                                                                            8 comps
                                                                              94.96
## X
             38.31
                      60.16
                               70.84
                                         79.03
                                                  84.29
                                                            88.63
                                                                     92.26
## Salary
             40.63
                      41.58
                                42.17
                                         43.22
                                                  44.90
                                                            46.48
                                                                     46.69
                                                                              46.75
##
           9 comps
                                                  13 comps
                   10 comps
                              11 comps
                                        12 comps
                                                             14 comps
                                                                        15 comps
## X
             96.28
                       97.26
                                  97.98
                                            98.65
                                                      99.15
                                                                 99.47
                                                                           99.75
                                                      48.10
             46.86
                       47.76
                                  47.82
                                            47.85
                                                                 50.40
                                                                           50.55
## Salary
##
           16 comps 17 comps 18 comps 19 comps
                                            100.00
## X
              99.89
                        99.97
                                   99.99
## Salary
              53.01
                        53.85
                                   54.61
                                             54.61
```

${\it \# Plotting \ cross-validation \ MSE}$

validationplot(pcr.fit, val.type = "MSEP")

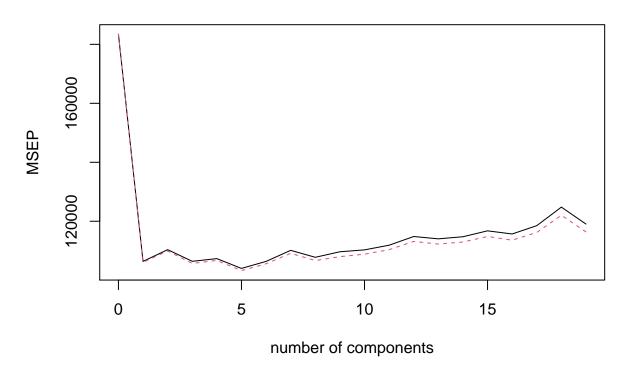
Salary



Performing PCR on the training data using subset function and plotting the CV MSE set.seed(1)

```
pcr.fit <- pcr(Salary ~., data = Hitters, subset = train, scale = TRUE, validation = "CV")
validationplot(pcr.fit, val.type = "MSEP")</pre>
```

Salary



Find the lowest CV error when M = 5

pcr.pred <- predict(pcr.fit, x[test,], ncomp = 5)

mean((pcr.pred - y.test)^2)</pre>

[1] 142811.8

```
# Fit PCR on complete dataset using M = 5 identified by CV

pcr.fit <- pcr(y~x, scale = TRUE, ncomp = 5)

summary(pcr.fit)</pre>
```

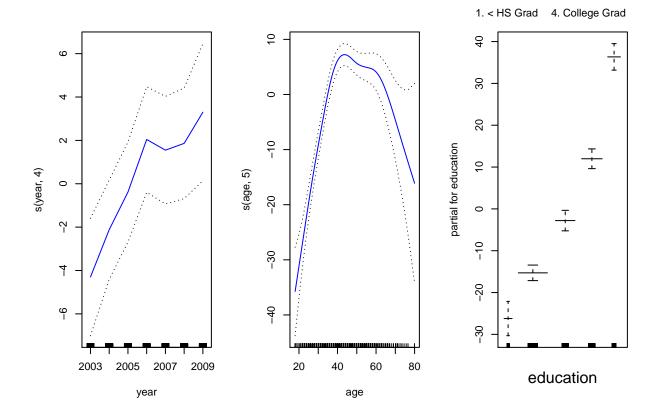
```
## Data: X dimension: 263 19
## Y dimension: 263 1
## Fit method: svdpc
## Number of components considered: 5
```

```
## TRAINING: % variance explained
      1 comps 2 comps 3 comps 4 comps
                                          5 comps
## X
        38.31
                 60.16
                          70.84
                                   79.03
                                             84.29
## y
        40.63
                 41.58
                          42.17
                                   43.22
                                             44.90
# Implement PLS using plsr() function
set.seed(1)
pls.fit <- plsr(Salary ~ ., data = Hitters, subset = train, scale = TRUE, validation = "CV")
summary(pls.fit)
Partial Least Squares
## Data:
            X dimension: 131 19
## Y dimension: 131 1
## Fit method: kernelpls
## Number of components considered: 19
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
          (Intercept) 1 comps 2 comps 3 comps 4 comps 5 comps
##
                                                                     6 comps
## CV
                428.3
                         325.5
                                  329.9
                                           328.8
                                                     339.0
                                                              338.9
                                                                       340.1
                         325.0
## adjCV
                428.3
                                  328.2
                                           327.2
                                                     336.6
                                                                       336.6
                                                              336.1
##
          7 comps 8 comps 9 comps 10 comps 11 comps 12 comps
                                                                   13 comps
                     347.1
                              346.4
## CV
            339.0
                                         343.4
                                                   341.5
                                                             345.4
                                                                       356.4
## adiCV
            336.2
                     343.4
                              342.8
                                         340.2
                                                   338.3
                                                             341.8
                                                                       351.1
                              16 comps 17 comps
                                                   18 comps
##
          14 comps
                    15 comps
                                                             19 comps
                                 350.0
## CV
             348.4
                       349.1
                                           344.2
                                                      344.5
                                                                345.0
             344.2
                       345.0
                                 345.9
                                           340.4
                                                      340.6
                                                                341.1
## adjCV
##
## TRAINING: % variance explained
##
           1 comps 2 comps 3 comps 4 comps 5 comps 6 comps 7 comps
                                                                           8 comps
## X
             39.13
                      48.80
                               60.09
                                        75.07
                                                  78.58
                                                           81.12
                                                                    88.21
                                                                             90.71
                               52.23
## Salary
             46.36
                      50.72
                                        53.03
                                                  54.07
                                                           54.77
                                                                    55.05
                                                                             55.66
           9 comps 10 comps 11 comps 12 comps 13 comps 14 comps 15 comps
                                 97.08
                                                      97.97
## X
             93.17
                       96.05
                                           97.61
                                                                98.70
                                                                          99.12
             55.95
                       56.12
                                 56.47
                                            56.68
                                                      57.37
                                                                57.76
                                                                          58.08
## Salary
##
           16 comps 17 comps 18 comps 19 comps
## X
              99.61
                        99.70
                                  99.95
                                           100.00
              58.17
                                             58.62
## Salary
                        58.49
                                  58.56
# Evaluating the coresponding test set MSE
pls.pred <- predict(pls.fit, x[test, ], ncomp = 1)</pre>
```

[1] 151995.3

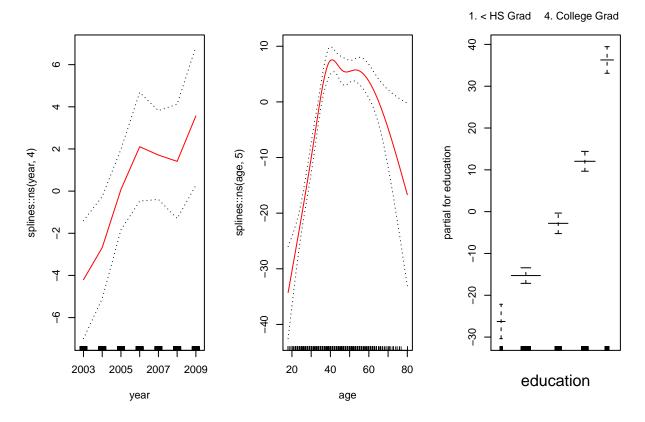
mean((pls.pred - y.test)^2)

```
\# Fitting PLS on complete dataset when M=1
pls.fit <- plsr(Salary ~ ., data = Hitters, scale = TRUE, ncomp = 1)</pre>
summary(pls.fit)
            X dimension: 263 19
## Data:
## Y dimension: 263 1
## Fit method: kernelpls
## Number of components considered: 1
## TRAINING: % variance explained
          1 comps
             38.08
## X
## Salary
             43.05
7.8.3 GAMs
# Fit a GAM or predict wage using natural spline functions of years and age.
gam1 <- lm(wage ~ splines::ns(year, 4) + splines::ns(age, 5) + education, data = Wage)
# Fit the model using smoothing splines
library(gam)
## Warning: package 'gam' was built under R version 4.0.5
## Loading required package: splines
## Loading required package: foreach
## Warning: package 'foreach' was built under R version 4.0.5
## Loaded gam 1.20.1
gam.m3 <- gam(wage ~ s(year, 4) + s(age, 5) + education, data = Wage)</pre>
# Plot the model
par(mfrow = c(1,3))
plot(gam.m3, se = TRUE, col = "blue")
```



```
# Plotting the GAM created using lm

par(mfrow = c(1,3))
plot.Gam(gam1, se = TRUE, col = "red")
```



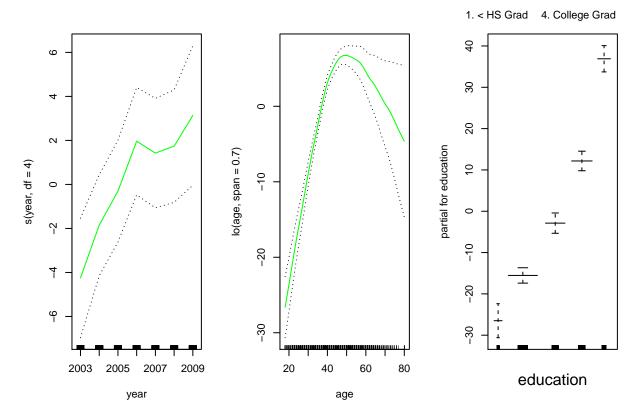
```
# Performing ANOVA test to determine the best model
gam.m1 <- gam(wage ~ s(age, 5) + education, data = Wage)</pre>
gam.m2 <- gam(wage ~ year + s(age, 5) + education, data = Wage)</pre>
anova(gam.m1, gam.m2, gam.m3, test = "F")
## Analysis of Deviance Table
## Model 1: wage ~ s(age, 5) + education
## Model 2: wage ~ year + s(age, 5) + education
## Model 3: wage ~ s(year, 4) + s(age, 5) + education
##
     Resid. Df Resid. Dev Df Deviance
                                                  Pr(>F)
## 1
          2990
                  3711731
## 2
          2989
                  3693842
                           1 17889.2 14.4771 0.0001447 ***
## 3
          2986
                  3689770
                          3
                               4071.1 1.0982 0.3485661
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
# summary of gam.m3
summary(gam.m3)
```

Call: gam(formula = wage ~ s(year, 4) + s(age, 5) + education, data = Wage)

10

##

```
## Deviance Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -119.43 -19.70 -3.33 14.17 213.48
##
## (Dispersion Parameter for gaussian family taken to be 1235.69)
##
      Null Deviance: 5222086 on 2999 degrees of freedom
## Residual Deviance: 3689770 on 2986 degrees of freedom
## AIC: 29887.75
##
## Number of Local Scoring Iterations: NA
##
## Anova for Parametric Effects
               Df Sum Sq Mean Sq F value
                                             Pr(>F)
                    27162
                            27162 21.981 2.877e-06 ***
## s(year, 4)
                1
## s(age, 5)
                1 195338 195338 158.081 < 2.2e-16 ***
## education
                4 1069726 267432 216.423 < 2.2e-16 ***
## Residuals 2986 3689770
                             1236
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##
              Npar Df Npar F Pr(F)
## (Intercept)
                    3 1.086 0.3537
## s(year, 4)
## s(age, 5)
                    4 32.380 <2e-16 ***
## education
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
# Using the predict method for class GAM
preds <- predict(gam.m2, newdata = Wage)</pre>
# Using local regression fits in GAM using lo()
gam.lo <- gam(wage ~ s(year, df = 4) + lo(age, span = 0.7) + education, data = Wage)
par(mfrow = c(1,3))
plot.Gam(gam.lo, se = TRUE, col = "green")
```



```
# Using lo() to create interactions before calling gam
gam.lo.i <- gam(wage ~ lo(year, age, span = 0.5) + education, data = Wage)

## Warning in lo.wam(x, z, wz, fit$smooth, which, fit$smooth.frame, bf.maxit, : liv
## too small. (Discovered by lowesd)

## Warning in lo.wam(x, z, wz, fit$smooth, which, fit$smooth.frame, bf.maxit, : lv
## too small. (Discovered by lowesd)

## Warning in lo.wam(x, z, wz, fit$smooth, which, fit$smooth.frame, bf.maxit, : liv
## too small. (Discovered by lowesd)

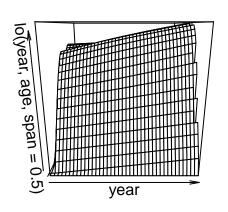
## Warning in lo.wam(x, z, wz, fit$smooth, which, fit$smooth.frame, bf.maxit, : lv
## too small. (Discovered by lowesd)

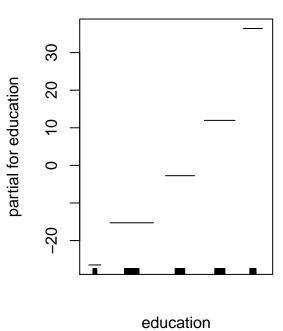
## Plotting the 2D surface using akima package
library(akima)</pre>
```

Warning: package 'akima' was built under R version 4.0.5

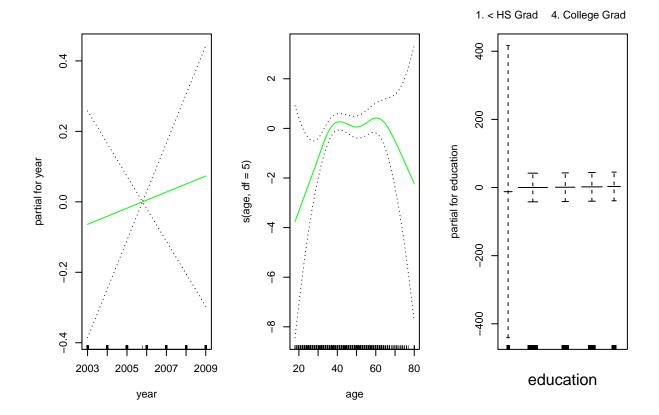
```
par(mfrow = c(1,2))
plot(gam.lo.i)
```

1. < HS Grad 4. College Grad





Fittinga logistic regression GAM using I() function
gam.lr <- gam(I(wage > 250) ~ year + s(age, df = 5) + education, family = binomial, data = Wage)
par(mfrow = c(1,3))
plot(gam.lr, se = TRUE, col = "green")



```
# Creeate a table with high earnes in the < HS category
attach(Wage)
table(education, I(wage > 250))
```

```
##
                         FALSE TRUE
## education
     1. < HS Grad
                            268
##
                                   0
##
     2. HS Grad
                            966
                                   5
##
     3. Some College
                            643
                                   7
##
     4. College Grad
                            663
                                  22
                                  45
     5. Advanced Degree
                            381
##
```

```
# Fitting a logsitic regression GAM by skipping the education category
gam.lr.s <- gam( I(wage > 250) ~ year + s(age, df = 5) + education, family = binomial, subset = (educat
par(mfrow = c(1,3))
plot(gam.lr.s, se = TRUE, col = "green")
```

