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CSCI 154 - Project 3 Report (CHAOS Movie)

Introduction:

For our third and final project, we picked to discuss the movie CHAOS. This is a 9-part movie created by Jos Leys, Etienne Ghys, and Aurelien Alvarez. This report, we will discuss the 9 main chapters of the movie from Motion and Determinism to Research Today. The movie outlines various aspects of dynamical systems, the butterfly effects, and chaos theory.

Motivation:

The motivation of this movie was to help us understand how complex systems work and to grasp a better understanding of natural phenomena. We will also be able to learn about the boundaries of predictability as well as explore the origins of complex systems.

CHAOS 1 - Motion and Determinism:

The idea of Chaos was originally developed by the philosopher Heraclitus who came up with the idea around the 6th century B.C. The video outlined that in mathematics, everything is considered to be in motion. Hence, the idea of determinism was introduced. Determinism is the claim that the order of something happening happens due to causality. After doing some research it was found that some philosophers came up with the assumption that humans do not have free will and cannot be held responsible for their actions due to this idea. It is presumed that free will is a form of illusion and thus the behavior we exert is ruled by forces that we have no control over. This is different from fatalism as it describes itself to be that all events are destined to happen a certain way. An example of determinism would be that if my car is working, considering I start the ignition and everything functions normally, the car will start. This is an idea of determinism because there is only one outcome.

CHAOS 2 - Vector Field:

In the 17th century, Gottfried Wilhelm Leibniz and Issac Newton created infinitesimal calculus. This has then gone on to become an important tool in Mathematics from calculating the derivatives (differential) to the integral. With this introduction, we are able to predict the future with this new toolset. The second chapter of the movie puts us in a Lego world where it discusses various vector fields and movements. Vector fields are a function that is assigned to every point within a vector space. It can be pictured to be a collection of arrows that point to different points within a space. Moving on, the Cauchy-Lipschitz theorem claims that the starting points can determine the future trajectories, and for every point, there is a tangent present to their velocity vectors. James Clerk Maxwell also mentioned that there is a limit to determinism with the implied importance of the initial condition for any physical phenomena.

CHAOS 3 - Mechanics:

During the 17th century, Galileo Galilei was the one who studied falling objects but it was Issac Newton who proposed the universal law of gravitation. He came up with the idea that the gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. Besides, gravity is affecting everything around us. For example, apples or the Moon are attracted by the gravitational force of Earth and it modifies the speed of an object in a free fall condition. In Newton's formula, F= m.a is one of the most fundamental in the world of physics in which F represents force, m represents mass and a represents acceleration. The formula allows us to calculate the trajectory of objects based on their initial position, speed, mass, and the forces acting upon them. This has led to the ability to construct planetary choreographies, such as the periodic trajectory of three planets with only their mutual attraction acting upon them. Regarding the Moon's orbit around the Earth, it is falling, but the mutual attraction of gravitation bends its trajectory, allowing it to stay in orbit. The speed of the Moon relative to the Earth is what determines its orbit.

CHAOS 4 - Oscillations:

In this movie, the chaos concept discussed oscillations in a swinging pendulum. The main idea of this concept is that a pendulum has two main characteristics: its position, which is measured by

the angle it makes with a vertical line, and its speed, which indicates whether it's moving left or right. In the absence of friction, a pendulum would continue swinging indefinitely, but eventually, due to the presence of friction, it comes to a stop after a period of time. The motion of a pendulum is influenced by friction, which slows it down over time. However, with well-timed pushes, we can keep it swinging and create a phase portrait that forms a closed curve known as a limit cycle. The Lotka-Volterra model, which describes the interactions between two populations, is another example of a limit cycle. Poincaré's theorem confirms that such motions stabilize after a transition period, either stopping altogether or oscillating periodically, but this only holds for vector fields in a plane. For vector fields in space, the situation is more complex and can lead to chaos.

CHAOS 5 - Duhem's Bell:

The movie explores the concept of chaotic systems using the analogy of billiard balls on a surface with obstacles. The main idea is to understand the complexity of predicting the future behavior of such systems, whether it's the movement of celestial objects or billiard balls. An example of a billiard ball moving on a frictionless surface, where the path of the ball depends on the initial position and velocity is shown in the movie. The movie emphasizes the intricate nature of trajectories in chaotic systems, where small variations in initial conditions can lead to significantly different outcomes. The geodesic curves are defined as the path that would follow on a surface with no external forces affecting it. While mathematical deductions can determine trajectories and whether they diverge to infinity or not, the practical applicability of these predictions remains a challenge. The discussion raises questions about the possibility of predicting the future and emphasizes the complexity and uncertainty involved in understanding and foreseeing the outcomes of chaotic systems.

CHAOS 6 - Chaos & The Horseshoe:

The main concept discussed in the movie is the discovery of chaos and the exploration of its properties through the mathematical concept of a horseshoe. The movie begins by introducing Steve Smale, an American mathematician who made a significant discovery in the 1960s. He realized that chaos could be represented by abstract mathematical objects to simplify its complex nature such as imagining a horseshoe. The movie explains how the dynamics of a vector field

can be studied through a sequence of points which makes it easier to comprehend. It demonstrates various transformations such as scaling, contraction, and folding to showcase how trajectories behave differently depending on the properties of the transformation. The movie highlights the remarkable structural stability of the horseshoe, despite the individual trajectories being highly sensitive to initial conditions. It emphasizes that although chaos exists in the world, there is still the existence of stable points within the overall dynamics.

CHAOS 7 - Strange Attractors (The Butterfly Effect):

Chaos theory is a branch of mathematics that deals with the behavior of nonlinear dynamical systems that are highly sensitive to initial conditions. A small change in the initial conditions of such a system can result in vastly different outcomes, making their long-term behavior difficult to predict. Strange attractors are complex patterns that arise in certain nonlinear systems, which can exhibit chaotic behavior. They are called "strange" because they have a fractal structure, which means they have similar patterns at different scales. The attractor represents the long-term behavior of the system and determines where it will settle over time. The Butterfly Effect is a popular concept in Chaos theory, which describes the idea that a small change in the initial conditions of a system can have a large and unpredictable effect on its future behavior.

CHAOS 8 - Statistics (Lorenz' Mill):

The Lorenz mill is a simplified model of atmospheric convection, consisting of a set of three coupled ordinary differential equations. In statistics, chaos theory is relevant in the analysis of time series data, which is a set of measurements taken over time. The three equations describe the evolution of three variables: the temperature difference between the top and bottom of the system, the horizontal velocity of the fluid, and the vertical velocity of the fluid. The equations are as follows:

$$\mathbf{dx/dt = \sigma(y - x)}$$

$$\mathbf{dy/dt = x(\rho - z) - y}$$

$$\mathbf{dz/dt = xy - \beta z}$$

where x, y, and z represent the state variables of the system, and σ, ρ, and β are constants that determine the behavior of the system. When the parameters of the system are set to certain values, the solutions to the equations exhibit chaotic behavior, with trajectories that never repeat themselves and form a strange attractor. The Lorenz system is a well-known example of a chaotic system and has been studied extensively in chaos theory.

CHAOS 9 - Research Today:

Some of the current research topics include:

- Applications in biology: Chaos theory has been applied to study biological systems such as the dynamics of neurons in the brain.
- Complex networks: Many real-world systems can be modeled as complex networks, such as social networks, transportation networks, and power grids.
- Control of chaotic systems: Since chaotic systems are unpredictable, it can be challenging to control them.
- Machine learning: Chaos theory has also been applied to machine learning, particularly in the development of chaotic neural networks that can learn and adapt to new data.

Overall, chaos theory remains an active area of research with many potential applications in various fields.

Conclusion and Contributions:

To conclude this project, we can see that the movie has provided us with a lot of information regarding its application in different areas from mathematics to biology and various concepts from the Idea of Determinism to Lorenz' Mill. It begins by introducing the idea of determinism and how it can limit our understanding of the world. The movie then goes on to explain how infinitesimal calculus and vector fields have allowed us to predict the future trajectories of objects. It also explores the concept of oscillations and how small variations in initial conditions can lead to significantly different outcomes in chaotic systems. The movie emphasizes the complexity and uncertainty involved in understanding and predicting the outcomes of chaotic systems. Finally, the movie concludes by discussing how the mathematical concept of a horseshoe can be used to study the dynamics of a vector field and understand the properties of chaos. Overall, the movie provides an insightful exploration of the concept of chaos and its applications in different fields.

Feedback: The format and layout of the slides were good but the content we had could have been better as we were advised to do more research and look beyond what was provided in the videos to include in our report and to go into more detail about the concepts of Lorenz' Mill and the attractors.