Neural Networks Assignment 1

This is a submission by 18MCMT28

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Import necessary packages

matplotlib for plotting and numpy for managing the huge arrays

In [405]:

```
import matplotlib.pyplot as plt
import numpy as np
from IPython.display import Image
```

In [362]:

```
%matplotlib notebook
```

Creating the datasets

In [402]:

```
dataset1_x = np.array([1, 7, 8, 9, 4, 8], dtype=np.float128)
dataset1_y = np.array([6, 2, 9, 9, 8, 5], dtype=np.float128)
dataset2_x = np.array([2, 3, 2, 7, 1, 5], dtype=np.float128)
dataset2_y = np.array([1, 3, 4, 1, 3, 2], dtype=np.float128)

# Create Set A for all points with output one
set_A = np.column_stack((dataset1_x, dataset1_y))
set_A_outputs = np.ones(len(set_A))

# Create Set B similarly with output zero
set_B = np.column_stack((dataset2_x, dataset2_y))
set_B_outputs = np.zeros(len(set_B))

# Mash everything together
data = np.concatenate((set_A, set_B))
desired = np.concatenate((set_A_outputs, set_B_outputs))

# Input for Widrow Hoff
desired_linear = np.copy(desired)
for index, a in enumerate(desired):
    if a == 0:
        desired_linear[index] = -1
data
```

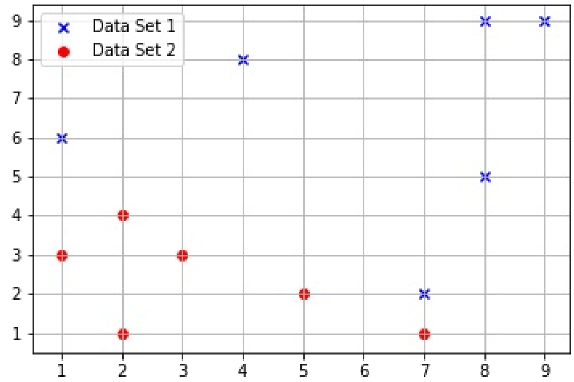
Out[402]:

```
array([[1., 6.],
       [7., 2.],
       [8., 9.],
       [9., 9.],
       [4., 8.],
       [8., 5.],
       [2., 1.],
       [3., 3.],
       [2., 4.],
       [7., 1.],
       [1., 3.],
       [5., 2.]], dtype=float128)
```

Plotting the dataset

In [364]:

```python
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
plt.grid(True)
ax.scatter(x=dataset1_x, y=dataset1_y, color='blue', marker='x', label='Data Set 1')
ax.scatter(x=dataset2_x, y=dataset2_y, color='red', marker='o', label='Data Set 2')
plt.legend(loc='upper left')
```



Out[364]:

```
<matplotlib.legend.Legend at 0x7f31fbd7a8d0>
```

Question A

Draw the feasible region for two points each, from the classes in the dataset

Yes, the data is linearly separable

Points choosen are:

Set A: (1, 6); (7, 2)

Set B: (2, 1); (3, 3)

In [403]:

```python
hom = data - data.mean()
hom
```

Out[403]:

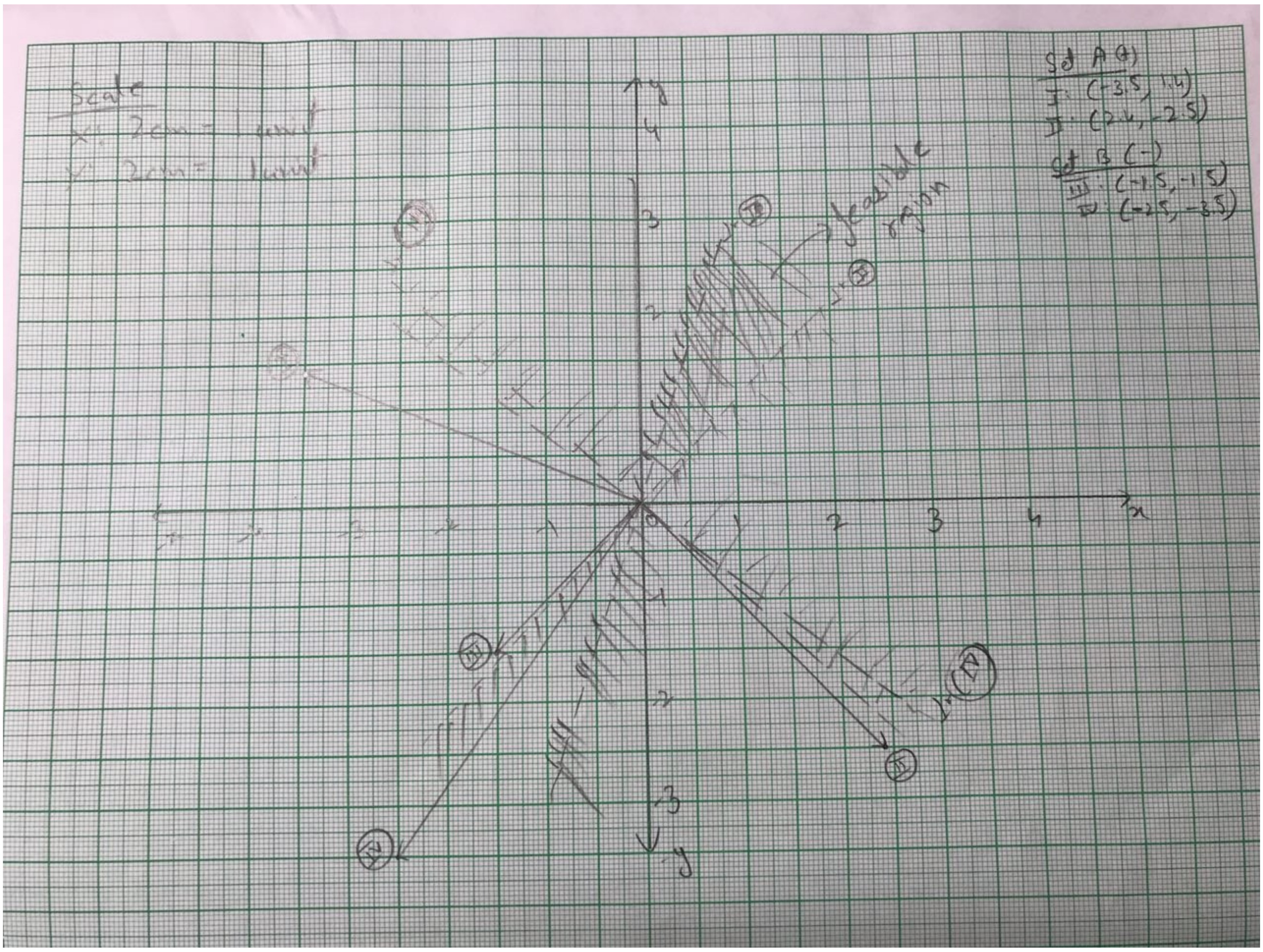
```
array([[-3.58333333,  1.41666667],
       [ 2.41666667, -2.58333333],
       [ 3.41666667,  4.41666667],
       [ 4.41666667,  4.41666667],
       [-0.58333333,  3.41666667],
       [ 3.41666667,  0.41666667],
       [-2.58333333, -3.58333333],
       [-1.58333333, -1.58333333],
       [-2.58333333, -0.58333333],
       [ 2.41666667, -3.58333333],
       [-3.58333333, -1.58333333],
       [ 0.41666667, -2.58333333]], dtype=float128)
```

The feasible region for the homogeneous points is as show below:

In [407]:

```
Image("solution.jpg")
```

Out[407]:



Implementation of the Perceptron

In [365]:

```
def sigmoid(x):
  return 1 / (1 + np.exp(-x))
```

In [366]:

```
class Perceptron():
    """Implements a single general perceptron"""
    def __init__(self, input_dimensions,  Weights=np.array([]), w_=False, learning_rate=1, epochs=500, fn='b
inary'):
        if not w_:
            self.Weights = np.zeros(input_dimensions + 1, dtype=np.float128) # An extra one for bias
        else:
            self.Weights = Weights
        self.epochs = epochs
        self.eta = learning_rate
        self.fn = fn

    # The activation function
    def activation_fn(self, y):
        if self.fn == 'binary':
            return 1 if y >= 0 else 0
        elif self.fn == 'linear':
            return y
        else: # fn == 'sigmoid'
            return sigmoid(y)

    def find_output(self, input_matrix):
        z = self.Weights.T.dot(input_matrix)
        return self.activation_fn(z)

    def learn(self, input_vector, desired_output):
        errors = []
        for _ in range(self.epochs):
            total_error = 0
            for i in range(desired_output.shape[0]):
                # Insert the weight 1 for every input for the bais
                x = np.insert(input_vector[i], 0, 1)
                actual_output = self.find_output(x)
                error = desired_output[i] - actual_output
                # Weight update rules
                if self.fn == 'binary':
                    self.Weights = self.Weights + self.eta * error * x
                    if error <= 0:
                        total_error += int(error != 0.0)
                elif self.fn == 'linear':
                    self.Weights = self.Weights + self.eta * error * x
                    total_error += (error ** 2) / 2
                else:
                    self.Weights = self.Weights + self.eta * error * x * actual_output * (1 - actual_output)
                    total_error += (error ** 2) / 2
            errors.append(total_error)
        return errors

    def predict(self, X):
        if self.fn == 'binary':
            return 1 if self.find_output(X) > 0.0 else 0
        elif self.fn == 'linear':
            return 1 if self.find_output(X) >= 0.0 else -1
        else:
            return 1 if self.find_output(X) >= 0.5 else 0
```

Question B

In each case, show the weight vector learned

In [367]:

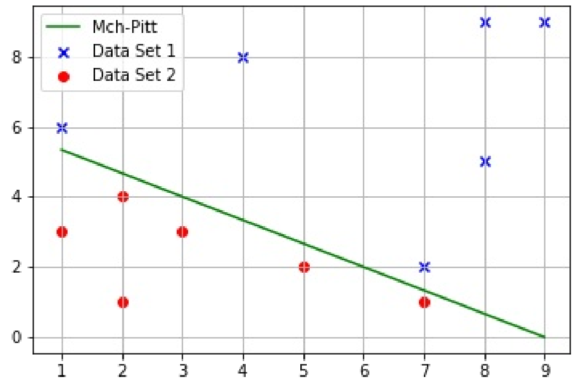
```
# Binary threshlod perceptron
perceptron_binary = Perceptron(input_dimensions=2)
perceptron_binary.learn(data, desired)
print(perceptron_binary.Weights)

# Linear perceptron
linear_perceptron = Perceptron(input_dimensions=2, fn='linear', epochs=4000, learning_rate=0.0001)
linear_perceptron.learn(data, desired_linear)
print(linear_perceptron.Weights)

# Sigmooid Perceptron
sigmoid_perceptron = Perceptron(input_dimensions=2, fn='sigmoid', epochs=6000, learning_rate=0.05)
sigmoid_perceptron.learn(data, desired)
print(sigmoid_perceptron.Weights)
```

```
[-18.   2.   3.]
[-0.90642431  0.04759966  0.17821521]
[-12.89433122   1.32953172   2.20958104]
```

Perceptron with Threshold activation function

In [368]:

```
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
plt.grid(True)
ax.scatter(x=dataset1_x, y=dataset1_y, color='blue', marker='x', label='Data Set 1')
ax.scatter(x=dataset2_x, y=dataset2_y, color='red', marker='o', label='Data Set 2')

# Sample points
i = np.linspace(np.amin(data, axis=0)[0], np.amax(data, axis=0)[0], 2000)

# To plot the boundary learned by the perceptron algorithm
weights = perceptron_binary.Weights
slope = -(weights[0]/weights[2])/(weights[0]/weights[1])
intercept = -weights[0]/weights[2]
y1 = (slope * i) + intercept
ax.plot(i, y1, color='green', label='Mch-Pitt')
plt.legend()
```



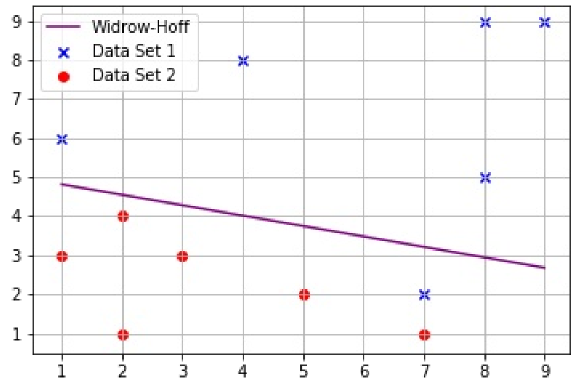
Out[368]:

```
<matplotlib.legend.Legend at 0x7f31fb8f9780>
```

Perceptron with Widrow Hoff learning or Least mean squared rule

In [369]:

```
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
plt.grid(True)
ax.scatter(x=dataset1_x, y=dataset1_y, color='blue', marker='x', label='Data Set 1')
ax.scatter(x=dataset2_x, y=dataset2_y, color='red', marker='o', label='Data Set 2')

# Sample points
i = np.linspace(np.amin(data, axis=0)[0], np.amax(data, axis=0)[0], 2000)

# To plot the boundary learned by the perceptron algorithm
weights = linear_perceptron.Weights
slope = -(weights[0]/weights[2])/(weights[0]/weights[1])
intercept = -weights[0]/weights[2]
y1 = (slope * i) + intercept
plt.plot(i, y1, color='purple', label='Widrow-Hoff')
plt.legend()
```



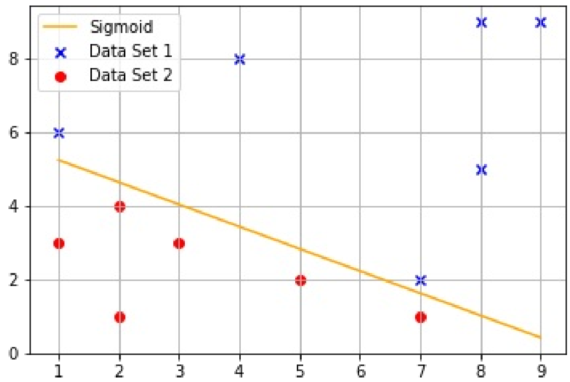
Out[369]:

```
<matplotlib.legend.Legend at 0x7f31fb869ba8>
```

Perceptron with Sigmoid activation function

In [370]:

```
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
plt.grid(True)
ax.scatter(x=dataset1_x, y=dataset1_y, color='blue', marker='x', label='Data Set 1')
ax.scatter(x=dataset2_x, y=dataset2_y, color='red', marker='o', label='Data Set 2')

# Sample points
i = np.linspace(np.amin(data, axis=0)[0], np.amax(data, axis=0)[0], 2000)

# To plot the boundary learned by the perceptron algorithm
weights = sigmoid_perceptron.Weights
slope = -(weights[0]/weights[2])/(weights[0]/weights[1])
intercept = -weights[0]/weights[2]
y1 = (slope * i) + intercept
plt.plot(i, y1, color='orange', label='Sigmoid')
plt.legend()
```



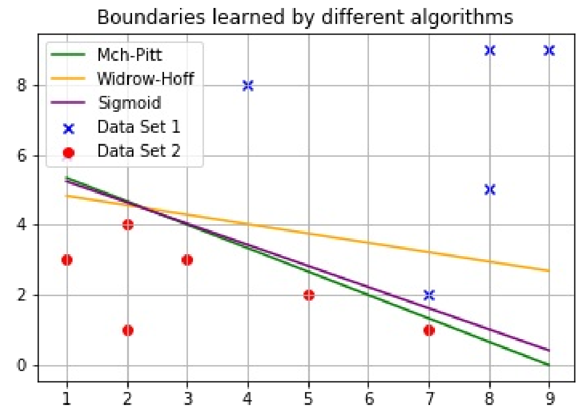
Out[370]:

```
<matplotlib.legend.Legend at 0x7f3203479ef0>
```

Boundaries learnt by all the three perceptrons

In [371]:

```
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
plt.grid(True)
ax.scatter(x=dataset1_x, y=dataset1_y, color='blue', marker='x', label='Data Set 1')
ax.scatter(x=dataset2_x, y=dataset2_y, color='red', marker='o', label='Data Set 2')

# Sample points
i = np.linspace(np.amin(data, axis=0)[0], np.amax(data, axis=0)[0], 2000)

# To plot the boundary learned by the perceptron algorithm
weights = perceptron_binary.Weights
slope = -(weights[0]/weights[2])/(weights[0]/weights[1])
intercept = -weights[0]/weights[2]
y1 = (slope * i) + intercept
ax.plot(i, y1, color='green', label='Mch-Pitt')

# To plot boundary learned by using the Widrow-Hoff learning rule
weights = linear_perceptron.Weights
slope = -(weights[0]/weights[2])/(weights[0]/weights[1])
intercept = -weights[0]/weights[2]
y2 = (slope * i) + intercept
ax.plot(i, y2, color='orange', label='Widrow-Hoff')

# To plot the boundary of the sigmoid perceptron
weights = sigmoid_perceptron.Weights
slope = -(weights[0]/weights[2])/(weights[0]/weights[1])
intercept = -weights[0]/weights[2]
y3 = (slope * i) + intercept
ax.plot(i, y3, color='purple', label='Sigmoid')

plt.legend()
plt.title('Boundaries learned by different algorithms')
```



Out[371]:

```
Text(0.5,1,'Boundaries learned by different algorithms')
```

Question C

Create a test set and show the accuracy of the perceptrons

The following additions are made to the data set

Data Set 1 = (6,5); (8,8); (8, 4)

Data Set 2 = (1,1); (4,2); (8, -1)

In [372]:

```
# Make the data set
test_a = np.array([[6, 5], [8, 8], [8, 4]])
test_b = np.array([[1, 1], [4, 2], [8, -1]])
test_set = np.concatenate((test_a, test_b))
test_labels = [1, 1, 1, 0, 0, 0]
test_linear = [1, 1, 1, -1, -1, -1]
```

For Binary threshold perceptron

In [373]:

```
correct = 0
for index, arr in enumerate(test_set):
    if perceptron_binary.predict(np.insert(arr, 0, 1)) == test_labels[index]:
        correct += 1
accuracy = correct / len(test_set)
error = 1 - accuracy
print("Accuracy = " + str(accuracy * 100) + "%\nError = " + str(error * 100) + '%')
```

```
Accuracy = 100.0%
Error = 0.0%
```

For Widrow-Hoff perceptron

In [374]:

```
correct = 0
for index, arr in enumerate(test_set):
    if linear_perceptron.predict(np.insert(arr, 0, 1)) == test_linear[index]:
        correct += 1
accuracy = correct / len(test_set)
error = 1 - accuracy
print("Accuracy = " + str(accuracy * 100) + "%\nError = " + str(error * 100) + '%')
```

```
Accuracy = 100.0%
Error = 0.0%
```

For Sigmoid Perceptron

In [375]:

```
correct = 0
for index, arr in enumerate(test_set):
    if sigmoid_perceptron.predict(np.insert(arr, 0, 1)) == test_labels[index]:
        correct += 1
accuracy = correct / len(test_set)
error = 1 - accuracy
print("Accuracy = " + str(accuracy * 100) + "%\nError = " + str(error * 100) + '%')
```

```
Accuracy = 100.0%
Error = 0.0%
```

Comparison table

Perceptron	Accuracy
Binary	100%
Widrow-Hoff	100%
Sigmoid	100%

Question D

Comment on the convergence time of the algorithms

We shall initialise the weight vectors and and run the algorithm until they acheive results similar to that we have attained above for each of the algorithms respectively

We consider the following initialization

1. $x_0 = 1, x_1 = 1, x_2 = 1$
2. $x_0 = -1, x_1 = -1, x_2 = -1$
3. $x_0 = -10, x_1 = 5, x_2 = 10$
4. $x_0 = -20, x_1 = -20, x_2 = -20$
5. $x_0 = 0.011, x_1 = 1.23, x_2 = 2.34$

Here, x_0 is the bais term

In [388]:

```
w1 = np.array([1, 1, 1])
w2 = np.array([-1, -1, -1])
w3 = np.array([-10, 5, 10])
w4 = np.array([-20, -20, 20])
w5 = np.array([0.011, 1.23, 2.34])

def get_accuracy(X, y):
    correct = 0
    for index, arr in enumerate(X):
        if sigmoid_perceptron.predict(np.insert(arr, 0, 1)) == y[index]:
            correct += 1
    accuracy = correct / len(X)
    error = 1 - accuracy
    print("Accuracy = " + str(accuracy * 100) + "%\nError = " + str(error * 100) + '%')
```

We define an instance of a perceprtron for each weight, for each type of learnig algorithm

In [389]:

```
# Binary threshlod perceptrons
p1 = Perceptron(input_dimensions=2, Weights=w1, w_=True)
p1_errors = p1.learn(data, desired)
p2 = Perceptron(input_dimensions=2, Weights=w2, w_=True)
p2_errors = p2.learn(data, desired)
p3 = Perceptron(input_dimensions=2, Weights=w3, w_=True)
p3_errors = p3.learn(data, desired)
p4 = Perceptron(input_dimensions=2, Weights=w4, w_=True)
p4_errors = p4.learn(data, desired)
p5 = Perceptron(input_dimensions=2, Weights=w5, w_=True)
p5_errors = p5.learn(data, desired)

# Linear perceptrons
l1 = Perceptron(input_dimensions=2, fn='linear', epochs=4000, learning_rate=0.0001, Weights=w1, w_=True)
l1_errors = l1.learn(data, desired_linear)
l2 = Perceptron(input_dimensions=2, fn='linear', epochs=4000, learning_rate=0.0001, Weights=w2, w_=True)
l2_errors = l2.learn(data, desired_linear)
l3 = Perceptron(input_dimensions=2, fn='linear', epochs=4000, learning_rate=0.0001, Weights=w3, w_=True)
l3_errors = l3.learn(data, desired_linear)
l4 = Perceptron(input_dimensions=2, fn='linear', epochs=4000, learning_rate=0.0001, Weights=w4, w_=True)
l4_errors = l4.learn(data, desired_linear)
l5 = Perceptron(input_dimensions=2, fn='linear', epochs=4000, learning_rate=0.0001, Weights=w5, w_=True)
l5_errors = l5.learn(data, desired_linear)

# Sigmoid Perceptrons
s1 = Perceptron(input_dimensions=2, fn='sigmoid', epochs=6000, learning_rate=0.05, Weights=w1, w_=True)
s1_errors = s1.learn(data, desired)
s2 = Perceptron(input_dimensions=2, fn='sigmoid', epochs=6000, learning_rate=0.05, Weights=w2, w_=True)
s2_errors = s2.learn(data, desired)
s3 = Perceptron(input_dimensions=2, fn='sigmoid', epochs=6000, learning_rate=0.05, Weights=w3, w_=True)
s3_errors = s3.learn(data, desired)
s4 = Perceptron(input_dimensions=2, fn='sigmoid', epochs=6000, learning_rate=0.05, Weights=w4, w_=True)
s4_errors = s4.learn(data, desired)
s5 = Perceptron(input_dimensions=2, fn='sigmoid', epochs=6000, learning_rate=0.05, Weights=w5, w_=True)
s5_errors = s5.learn(data, desired)
```

Out[389]:

```
array([-10.01420659,   4.97158682,   9.9857934 ], dtype=float128)
```

In [395]:

```
btu = ['BTU', p1_errors.index(min(p1_errors))+1, p2_errors.index(min(p2_errors))+1, p3_errors.index(min(p3_e
rrors))+1,
      p4_errors.index(min(p4_errors))+1, p5_errors.index(min(p5_errors))+1]
wid = ['Widrow-Hoff', l1_errors.index(min(l1_errors))+1, l2_errors.index(min(l2_errors))+1, l3_errors.index(
min(l3_errors))+1,
      l4_errors.index(min(l4_errors))+1, l5_errors.index(min(l5_errors))+1]
sig = ['Sigmoid', s1_errors.index(min(s1_errors))+1, s2_errors.index(min(s2_errors))+1, s3_errors.index(min(
s3_errors))+1,
      s4_errors.index(min(s4_errors))+1, s5_errors.index(min(s5_errors))+1]
titles = ['Perceptron', 'Weights1', 'Weights2', 'Weights3', 'Weights4', 'Weights5']
d = []
d.append(titles)
d.append(btu)
d.append(wid)
d.append(sig)
for i in range(4):
    line = '|'.join(str(x).ljust(12) for x in d[i])
    print(line)
    if i == 0:
        print('-' * len(line))
```

```
Perceptron  |Weights1    |Weights2    |Weights3    |Weights4    |Weights5    
-----------------------------------------------------------------------------
BTU         |38          |34          |11          |12          |31          
Widrow-Hoff |4000        |4000        |4000        |4000        |4000        
Sigmoid     |6000        |6000        |6000        |6000        |6000        
```

Analysis

Each element in the above table gives the number of epochs that the algorithm took to **find the best solution**, for each set of weights that we have defined eairlier.

Stopping condition

The alorithm is run for the number of epochs given for each instance of the weights. We choose the same learning rate and epochs to facilitate a fair comaparison of the convergence time

Some intresting observations

1. For the choosen weights, both Widrow-Hoff and Sigmoid perceptrons do not converge before the stoppping condition
2. The Perceptron learning algorithm converges after a set number of epochs as it is gaurenteed to...

The convergence speed is faster if, the *slope and intercept* for the intial weights choosen, is **closer** to the *slope and intercept* of w^*, the solution vector

Question F

Make the dataset linearly non-sepatable and find the solution vector for Widrow-Hoff and Sigmoid perceptrons

We choose some points from set A(+) and exchange it with the set B(-) as follows...

1. (1, 6) <--> (2, 1)
2. (9, 9) <--> (2, 4)
3. (7, 2) <--> (3, 3)

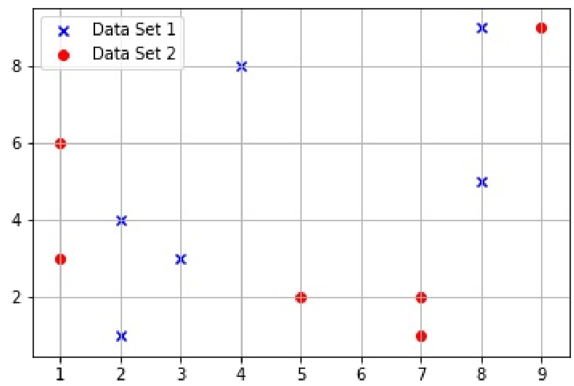
In [396]:

```
dataset1_x_ = np.array([2, 3, 8, 2, 4, 8], dtype=np.float128)
dataset1_y_ = np.array([1, 3, 9, 4, 8, 5], dtype=np.float128)
dataset2_x_ = np.array([1, 7, 9, 7, 1, 5], dtype=np.float128)
dataset2_y_ = np.array([6, 2, 9, 1, 3, 2], dtype=np.float128)

# Create Set A for all points with output one
set_A_ = np.column_stack((dataset1_x_, dataset1_y_))
set_A_outputs_ = np.ones(len(set_A_))

# Create Set B similarly with output zero
set_B_ = np.column_stack((dataset2_x_, dataset2_y_))
set_B_outputs_ = np.zeros(len(set_B_))

# Mash everything together
data_ = np.concatenate((set_A_, set_B_))
desired_ = np.concatenate((set_A_outputs_, set_B_outputs_))

# Input for Widrow Hoff
desired_linear_ = np.copy(desired_)
for index, a in enumerate(desired):
    if a == 0:
        desired_linear_[index] = -1
```

Plotting the new dataset

In [398]:

```
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
plt.grid(True)
ax.scatter(x=dataset1_x_, y=dataset1_y_, color='blue', marker='x', label='Data Set 1')
ax.scatter(x=dataset2_x_, y=dataset2_y_, color='red', marker='o', label='Data Set 2')
plt.legend(loc='upper left')
```



Out[398]:

```
<matplotlib.legend.Legend at 0x7f31fb607e80>
```

An attempt to learn the decision boundary by the perceptrons

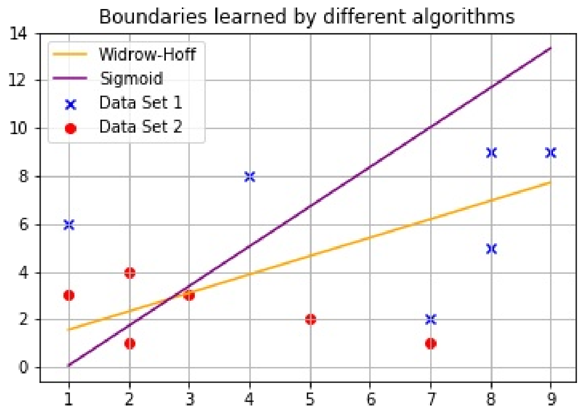
In [399]:

```
# Linear perceptron
linear_perceptron_ = Perceptron(input_dimensions=2, fn='linear', epochs=4000, learning_rate=0.0001)
linear_perceptron_.learn(data_, desired_linear_)
print(linear_perceptron.Weights)

# Sigmoid Perceptron
sigmoid_perceptron_ = Perceptron(input_dimensions=2, fn='sigmoid', epochs=6000, learning_rate=0.05)
sigmoid_perceptron_.learn(data_, desired_)
print(sigmoid_perceptron.Weights)
```

```
[-0.90642431  0.04759966  0.17821521]
[-12.89433122   1.32953172   2.20958104]
```

In [400]:

```
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
plt.grid(True)
ax.scatter(x=dataset1_x, y=dataset1_y, color='blue', marker='x', label='Data Set 1')
ax.scatter(x=dataset2_x, y=dataset2_y, color='red', marker='o', label='Data Set 2')

# Sample points
i = np.linspace(np.amin(data, axis=0)[0], np.amax(data, axis=0)[0], 2000)

# To plot boundary learned by using the Widrow-Hoff learning rule
weights = linear_perceptron_.Weights
slope = -(weights[0]/weights[2])/(weights[0]/weights[1])
intercept = -weights[0]/weights[2]
y2 = (slope * i) + intercept
ax.plot(i, y2, color='orange', label='Widrow-Hoff')

# To plot the boundary of the sigmoid perceptron
weights = sigmoid_perceptron_.Weights
slope = -(weights[0]/weights[2])/(weights[0]/weights[1])
intercept = -weights[0]/weights[2]
y3 = (slope * i) + intercept
ax.plot(i, y3, color='purple', label='Sigmoid')

plt.legend()
plt.title('Boundaries learned by different algorithms')
```



Out[400]:

```
Text(0.5,1,'Boundaries learned by different algorithms')
```

The solutions found in both the cases is not correct as a single layer perceptron cannot learn a linearly non-seperable dataset, **even if the function used is non-linear as in the case of Sigmoid**.

Both the algorithms fail to find a viable soultion.

In [ ]: