



# Backpropagation in Perceptrons and Convolutions

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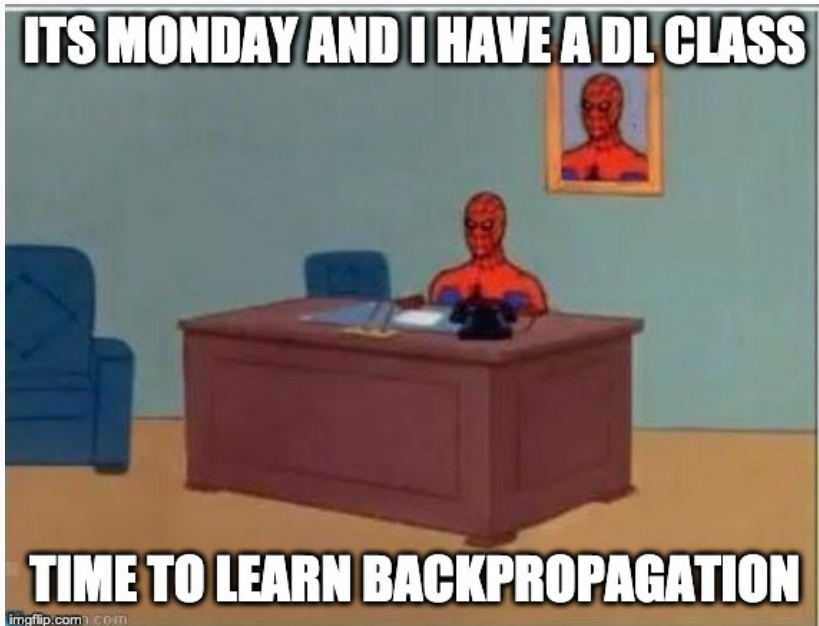
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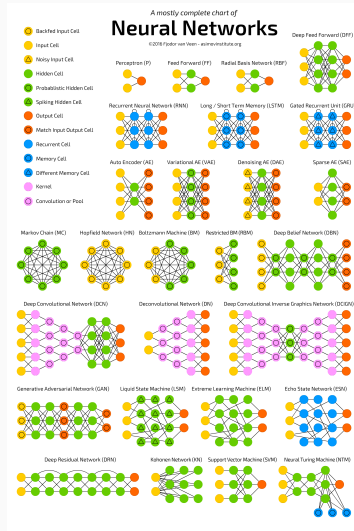
Hello World!

**ITS MONDAY AND I HAVE A DL CLASS**

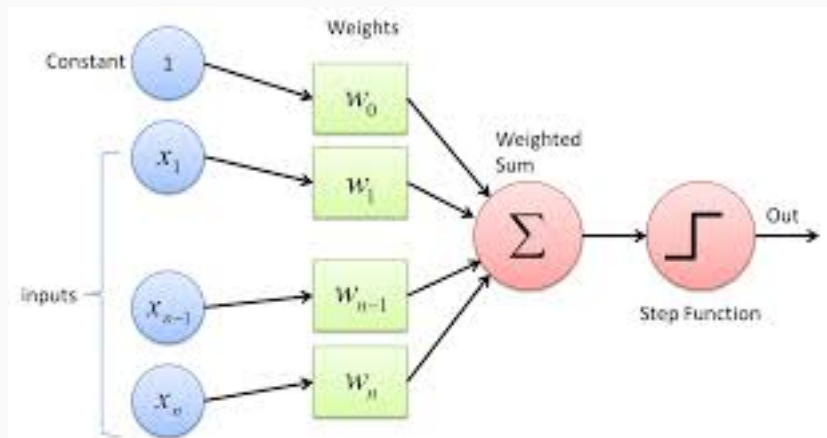


**TIME TO LEARN BACKPROPAGATION**

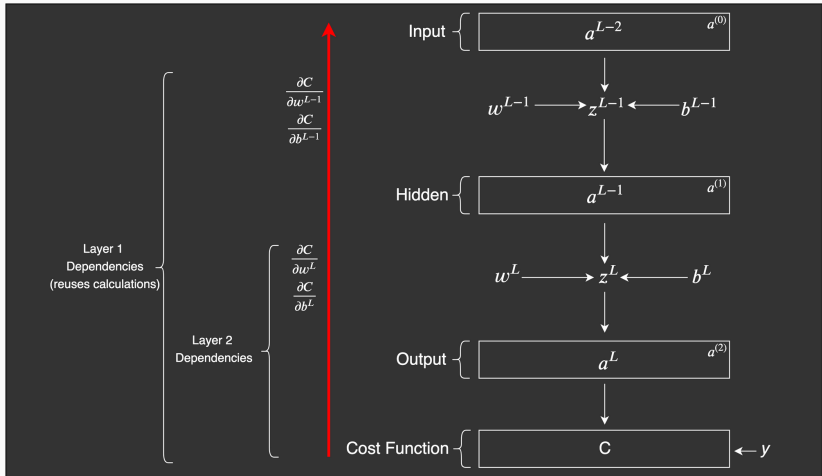
# Its not just CNN...



# Perceptron: The Artificial Neuron

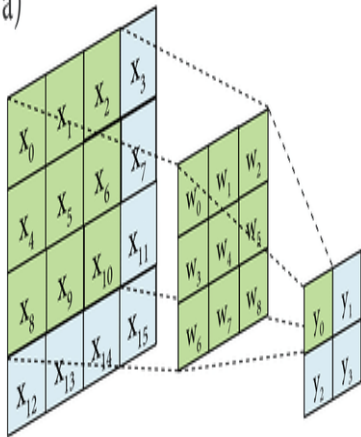


# Backpropagation in Perceptron

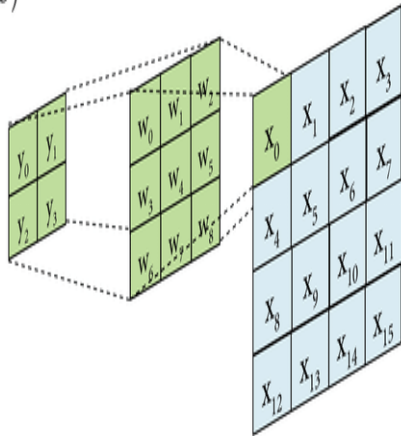


# A Convolution

a)



b)



# Backprop Maths: A simple example

**Input:**

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$b$

**Output:**

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

**Forward pass — convolution with one filter  $w$ , stride=1, padding=0**

$$y_1 = w_1x_1 = w_2x_2 + b$$

$$y_2 = w_1x_2 = w_2x_3 + b$$

$$y_3 = w_1x_3 = w_2x_4 + b$$



## And more ...

Gradient of loss function w.r.t  $y$ :

$$dy = \frac{\partial L}{\partial y}$$

Jacobian notation:

$$dy = \begin{bmatrix} \frac{\partial L}{\partial y_1} & \frac{\partial L}{\partial y_2} & \frac{\partial L}{\partial y_3} \end{bmatrix}$$

$$dy = \begin{bmatrix} dy_1 & dy_2 & dy_3 \end{bmatrix}$$

We want:

$$dx = \frac{\partial L}{\partial x} \quad dw = \frac{\partial L}{\partial w} \quad db = \frac{\partial L}{\partial b}$$

**db:**

$$db = \frac{\partial L}{\partial b} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial b} = dy \cdot \frac{\partial y}{\partial b}$$

$$= \begin{bmatrix} dy_1 & dy_2 & dy_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$db = dy_1 + dy_2 + dy_3$$

**dw:**

$$dw = \frac{\partial L}{\partial w} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial w} = dy \cdot \frac{\partial y}{\partial w}$$

$$\frac{\partial y}{\partial w} = \begin{bmatrix} \frac{\partial y_1}{\partial w_1} & \frac{\partial y_1}{\partial w_2} \\ \frac{\partial y_2}{\partial w_1} & \frac{\partial y_2}{\partial w_2} \\ \frac{\partial y_3}{\partial w_1} & \frac{\partial y_3}{\partial w_2} \end{bmatrix}$$

$$\frac{\partial y}{\partial w} = \begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \\ x_3 & x_4 \end{bmatrix}$$

$$dw = \begin{bmatrix} dy_1 & dy_2 & dy_3 \end{bmatrix} \cdot \begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \\ x_3 & x_4 \end{bmatrix}$$

$$dw_1 = x_1 dy_1 + x_2 dy_2 + x_3 dy_3$$

$$dw_2 = x_2 dy_2 + x_3 dy_3 + x_4 dy_4$$

$$dw = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} * \begin{bmatrix} dy_1 \\ dy_2 \\ dy_3 \end{bmatrix}$$

**It is a convolution operation**

**dx**

$$dx = \frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial x} = dy \cdot \frac{\partial y}{\partial x}$$

$$\frac{\partial y}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \frac{\partial y_1}{\partial x_3} & \frac{\partial y_1}{\partial x_4} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \frac{\partial y_2}{\partial x_3} & \frac{\partial y_2}{\partial x_4} \\ \frac{\partial y_3}{\partial x_1} & \frac{\partial y_3}{\partial x_2} & \frac{\partial y_3}{\partial x_3} & \frac{\partial y_3}{\partial x_4} \\ \frac{\partial y_4}{\partial x_1} & \frac{\partial y_4}{\partial x_2} & \frac{\partial y_4}{\partial x_3} & \frac{\partial y_4}{\partial x_4} \end{bmatrix}$$

$$= \frac{\partial y}{\partial x} = \begin{bmatrix} w_1 & w_2 & 0 & 0 \\ 0 & w_1 & w_2 & 0 \\ 0 & 0 & w_1 & w_2 \end{bmatrix}$$

$$dx_1 = w_1 dy_1$$

$$dx_2 = w_2 dy_1 + w_1 dy_w$$

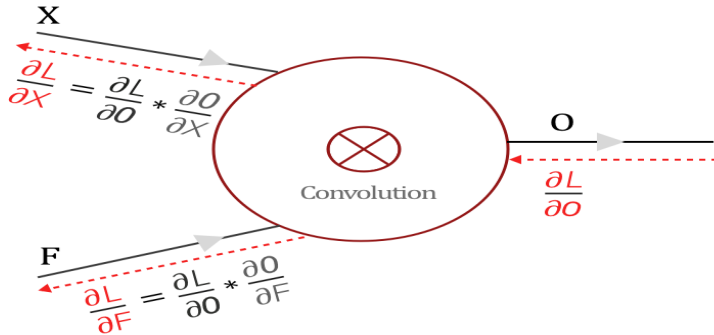
$$dx_3 = w_2 dy_2 + w_1 dy_3$$

$$dx_4 = w_2 dy_3$$

$$dw = \begin{bmatrix} 0 \\ dy_1 \\ dy_2 \\ dy_3 \\ 0 \end{bmatrix} * \begin{bmatrix} w_2 \\ w_1 \end{bmatrix}$$

**It is a convolution operation**

# Back pass



$\frac{\partial O}{\partial X}$  &  $\frac{\partial O}{\partial F}$  are local gradients

$\frac{\partial L}{\partial Z}$  is the loss from the previous layer which has to be backpropagated to other layers

Code time!!!





## Backpropagation in a Convolutional Layer of a CNN

Finding the gradients:

$$\frac{\partial L}{\partial F} = \text{Convolution} \left( \text{Input } X, \text{ Loss gradient } \frac{\partial L}{\partial O} \right)$$

$$\frac{\partial L}{\partial X} = \text{Full Convolution} \left( \begin{array}{l} 180^\circ \text{rotated} \\ \text{Filter } F \end{array}, \text{ Loss Gradient } \frac{\partial L}{\partial O} \right)$$

