Backpropagation in Perceptrons and Convolutions

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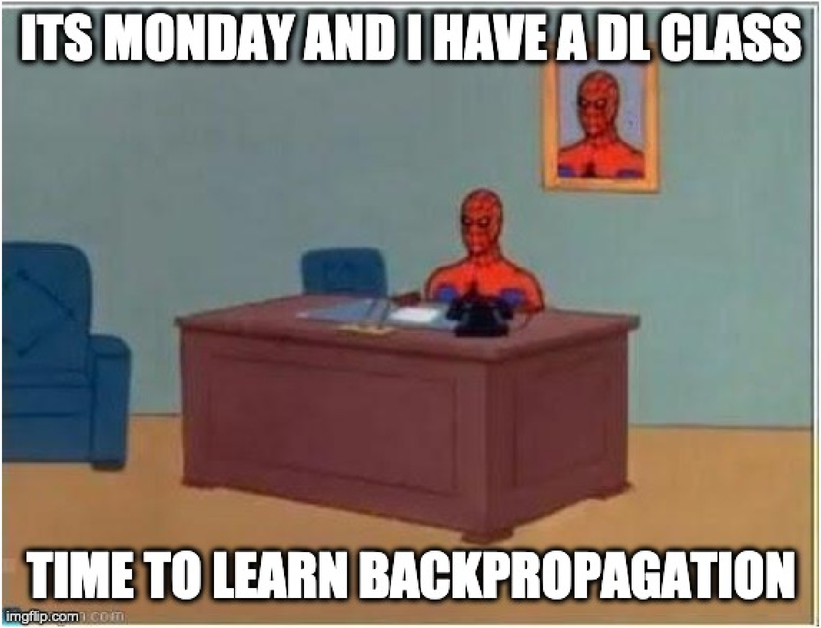
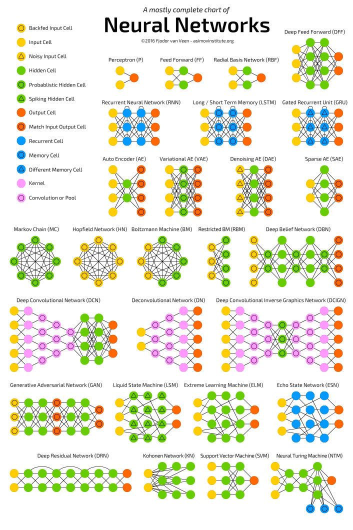
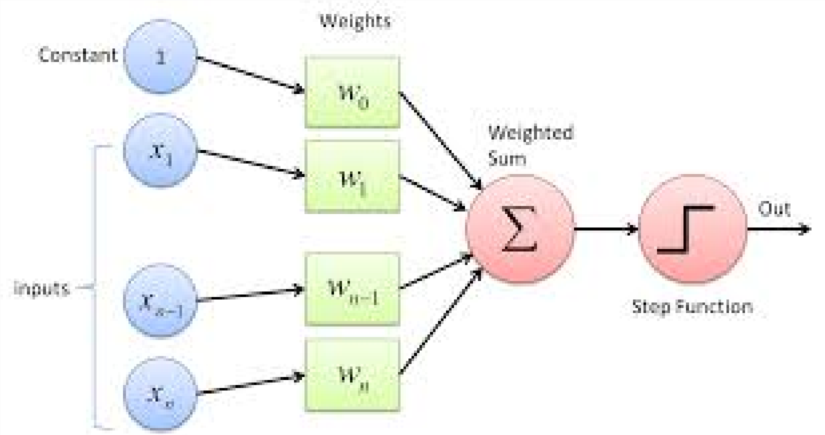
Hello World!


ITS MONDAY AND I HAVE A DL CLASS
TIME TO LEARN BACKPROPAGATION
imgflip.com

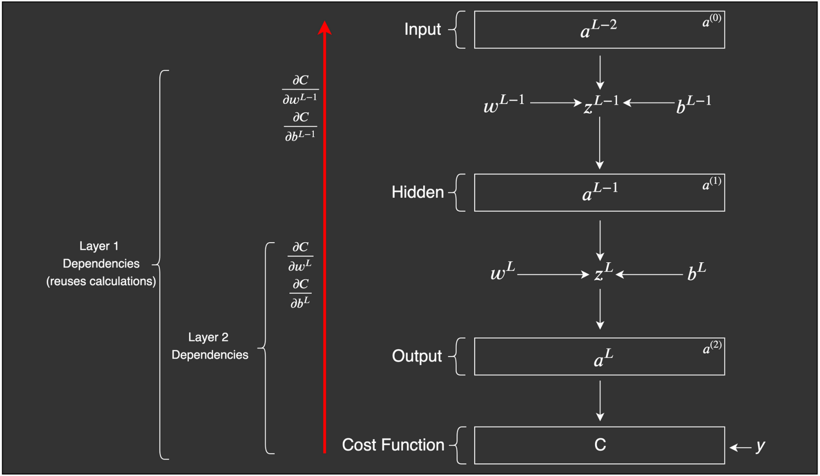
Its not just CNN...



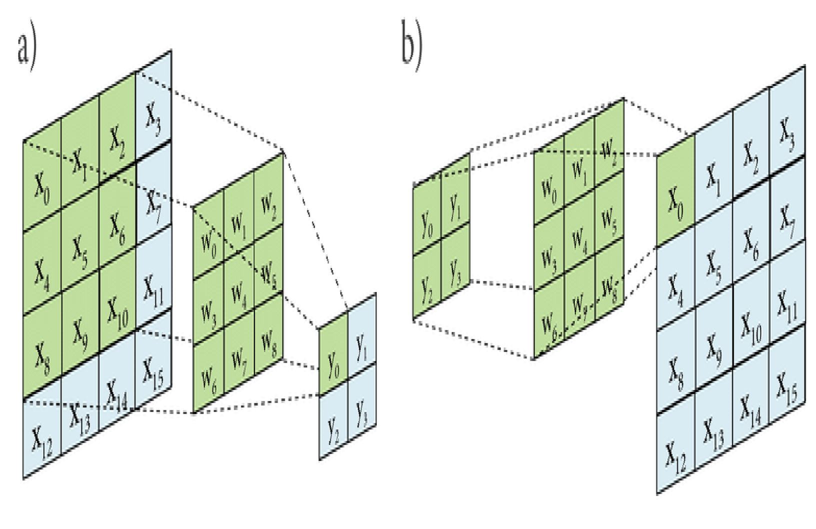
Perceptron: The Artificial Neuron



Backpropagation in Perceptron



A Convolution



Backprop Maths: A simple example

Input:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$b$$

Output:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

More math...

Forward pass — convolution with one filter w, stride=1, padding=0

$$y_1 = w_1 x_1 = w_2 x_2 + b$$

$$y_2 = w_1 x_2 = w_2 x_3 + b$$

$$y_3 = w_1 x_3 = w_2 x_4 + b$$

And more ...

Gradient of loss function w.r.t y:

$$dy = \frac{\partial L}{\partial y}$$

Jacobian notation:

$$dy = \begin{bmatrix} \frac{\partial L}{\partial y_1} & \frac{\partial L}{\partial y_2} & \frac{\partial L}{\partial y_3} \end{bmatrix}$$

$$dy = \begin{bmatrix} dy_1 & dy_2 & dy_3 \end{bmatrix}$$

We want:

$$dx = \frac{\partial L}{\partial x} \qquad dw = \frac{\partial L}{\partial w} \qquad db = \frac{\partial L}{\partial b}$$

Some more...

db:

$$db = \frac{\partial L}{\partial b} = \frac{\partial L}{\partial y}.\frac{\partial y}{\partial b} = dy.\frac{\partial y}{\partial b}$$

$$= \begin{bmatrix} dy_1 & dy_2 & dy_3 \end{bmatrix} . \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$db = dy_1 + dy_2 + dy_3$$

dw:

$$dw = \frac{\partial L}{\partial w} = \frac{\partial L}{\partial y}.\frac{\partial y}{\partial w} = dy.\frac{\partial y}{\partial w}$$

$$\frac{\partial y}{\partial w} = \begin{bmatrix} \frac{\partial y_1}{\partial w_1} & \frac{\partial y_1}{\partial w_2} \\ \frac{\partial y_2}{\partial w_1} & \frac{\partial y_2}{\partial w_2} \\ \frac{\partial y_3}{\partial w_1} & \frac{\partial y_3}{\partial w_2} \end{bmatrix}$$

$$\frac{\partial y}{\partial w} = \begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \\ x_3 & x_4 \end{bmatrix}$$

$$dw = \begin{bmatrix} dy_1 & dy_2 & dy_3 \end{bmatrix} . \begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \\ x_3 & x_4 \end{bmatrix}$$

$$dw_1 = x_1 dy_1 + x_2 dy_2 + x_3 dy_3$$

$$dw_2 = x_2 dy_2 + x_3 dy_3 + x_4 dy_4$$

$$dw = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} * \begin{bmatrix} dy_1 \\ dy_2 \\ dy_3 \end{bmatrix}$$

It is a convolution operation

dx

$$dx = \frac{\partial L}{\partial x} = \frac{\partial L}{\partial y}.\frac{\partial y}{\partial x} = dy.\frac{\partial y}{\partial x}$$

$$\frac{\partial y}{\partial x} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \frac{\partial y_1}{\partial x_3} & \frac{\partial y_1}{\partial x_4} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \frac{\partial y_2}{\partial x_3} & \frac{\partial y_1}{\partial x_4} \\ \frac{\partial y_3}{\partial x_1} & \frac{\partial y_3}{\partial x_2} & \frac{\partial y_3}{\partial x_3} & \frac{\partial y_1}{\partial x_4} \end{bmatrix}$$

$$= \frac{\partial y}{\partial x} = \begin{bmatrix} w_1 & w_2 & 0 & 0 \\ 0 & w_1 & w_2 & 0 \\ 0 & 0 & w_1 & w_2 \end{bmatrix}$$

$$dx_1 = w_1 dy_1$$

$$dx_2 = w_2 dy_1 + w_1 dy_w$$

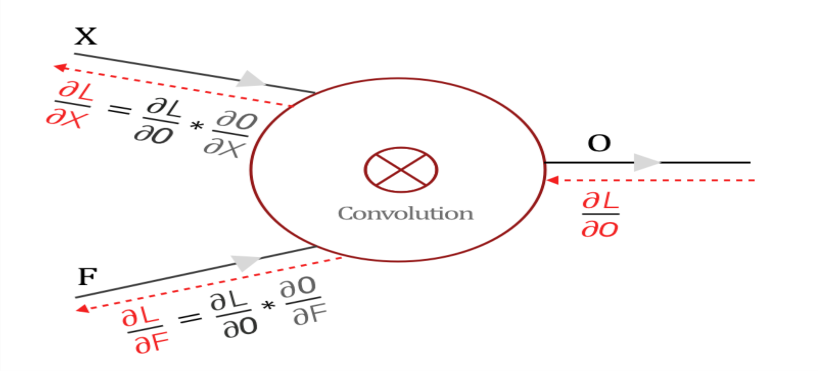
$$dx_3 = w_2 dy_2 + w_1 dy_3$$

$$dx_4 = w_2 dy_3$$

$$dw = \begin{bmatrix} 0 \\ dy_1 \\ dy_2 \\ dy_3 \\ 0 \end{bmatrix} * \begin{bmatrix} w_2 \\ w_1 \end{bmatrix}$$

It is a convolution operation

Back pass



$\frac{\partial O}{\partial X}$ & $\frac{\partial O}{\partial F}$ *are local gradients*

$\frac{\partial L}{\partial z}$ *is the loss from the previous layer which has to be backpropagated to other layers*

Code time!!!

Summary

Backpropagation in a Convolutional Layer of a CNN

Finding the gradients:

$$\frac{\partial L}{\partial F} = \text{Convolution}\left(\text{Input } X \text{ , Loss gradient } \frac{\partial L}{\partial O}\right)$$

$$\frac{\partial L}{\partial X} = \begin{matrix}\text{Full}\\ \text{Convolution}\end{matrix}\left(\begin{matrix}180^{\circ}\text{rotated}\\ \text{Filter } F\end{matrix} , \begin{matrix}\text{Loss}\\ \text{Gradient}\end{matrix} \frac{\partial L}{\partial O}\right)$$

La Fin!