

**N**

①

$$N/8 + N/4 + N/2$$

②

$$N/8 \cdot 1/8 + N/8 \cdot 1/4 + N/8 \cdot 1/2 + \frac{N}{4} \cdot 1/8 + \frac{N}{4} \cdot 1/4 + \frac{N}{4} \cdot 1/2 + \frac{N}{2} \cdot 1/8 + \frac{N}{2} \cdot 1/4 + \frac{N}{2} \cdot 1/2$$

At step - ① - depth of the tree =  $\frac{7N}{8}$

At step - ② -  $= \left(\frac{7}{8}\right)^2 N$

At step - ③  $= \left(\frac{7}{8}\right)^3 N$

⋮

At  $x$  step - depth of tree =  $\left(\frac{7}{8}\right)^x N$

The height of the tree at  $x^{\text{th}}$  step :

$$x = \log N \quad [\because \text{As the base is not counted}]$$

$$T(N) = T(N/8) + T(N/4) + T(N/2)$$

$$\Rightarrow \frac{7}{8}N + \left(\frac{7}{8}\right)^2 N + \left(\frac{7}{8}\right)^3 N + \dots + \left(\frac{7}{8}\right)^{\log N} N$$

$$\left(\frac{7}{8}\right)N \left[ 1 + \frac{7}{8} + \left(\frac{7}{8}\right)^2 + \dots + \left(\frac{7}{8}\right)^{\log N - 1} \right]$$

$$\text{Geometric progression sum} = \frac{1}{1-r} = \frac{1}{1-7/8} = 8$$

$$\left(\frac{7}{8}\right)N [8]$$

$$\boxed{7N} = O(n)$$

$\therefore$  Time complexity of recursion relation is Linear complexity