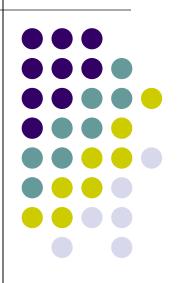
Functional Dependencies-More Examples



Exercise #1



- Below is an instance of R(A1,A2,A3,A4).
 Choose the FD which may hold on R
- 1. A4->A1
- 2. A2A3->A4
- 3. A2A3->A1

A1	A2	A3	A4
1	2	3	4
1	2	3	5
6	7	8	2
2	1	3	4

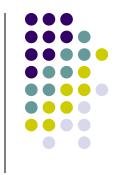
Solution #1



- 1. A4->A1???
- Incorrect: The 1st and 4th tuple violates it
- 2. A2A3->A4???
- Incorrect: The1st and 2nd tuple violates it.
- 3. A2A3 -> A1???
- Correct!

A1	A2	A3	A4
1	2	3	4
1	2	3	5
6	7	8	2
2	1	3	4

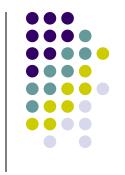
Exercise #2



- Let R(ABCDEFGH) satisfy the following functional dependencies: {A->B, CH->A, B->E, BD->C, EG->H, DE->F}
- Which of the following FD is also guaranteed to be satisfied by R?
- 1. BFG --> AE
- 2. ACG --> DH
- 3. CEG --> AB

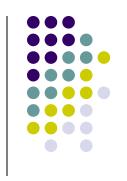
Hint: Compute the closure of the LHS of each FD that you get as a choice. If the RHS of the candidate FD is contained in the closure, then the candidate follows from the given FDs, otherwise not.

Solution #2



- FDs: {A->B, CH->A, B->E, BD->C, EG->H, DE->F}
- 1. BFG --> AE ???
 - Incorrect: BFG+ = BFGEH, which includes E, but not A
- 2. ACG --> DH ???
 - Incorrect: ACG+ = ACGBEH, which does not includes D.
- 3. CEG --> AB ???
 - Correct: CEG+ = CEGHAB, which contains AB

Question #3



- Which of the following could be a key for R(A,B,C,D,E,F,G) with functional dependencies {AB->C, CD->E, EF->G, FG->E, DE->C, and BC->A}
- 1. BDF
- 2. ACDF
- 3. ABDFG
- 4. BDFG





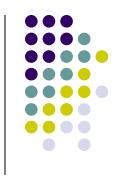
- {AB->C, CD->E, EF->G, FG->E, DE->C, and BC->A}
- 1. BDF ???
- No. BDF+ = BDF
- 2. ACDF ???
- No. ACDF⁺ = ACDFEG (The closure does not include B)
- 3. ABDFG ???
- No. This choice is a superkey, but it has proper subsets that are also keys (e.g. BDFG+ = BDFGECA)

Solution #3 - 2



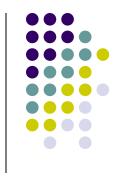
- {AB->C, CD->E, EF->G, FG->E, DE->C, and BC->A}
- 4. BDFG ???
- BDFG⁺ = ABCDEFG
- Check if any subset of BDFG is a key:
 - Since B, D, F never appear on the RHS of the FDs, they must form part of the key.
 - BDF⁺ = BDF ← Not key
 - So, BDFG is the minimal key, hence the candidate key





- Tricks for finding the key:
- If an attribute never appears on the RHS of any FD, it must be part of the key
- If an attribute never appears on the LHS of any FD, but appears on the RHS of any FD, it must not be part of any key





Consider R = {A, B, C, D, E, F, G, H} with a set of FDs

 $F = \{CD \rightarrow A, EC \rightarrow H, GHB \rightarrow AB, C \rightarrow D, EG \rightarrow A, H \rightarrow B, BE \rightarrow CD, EC \rightarrow B\}$

Find all the candidate keys of R

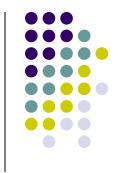
Solution #5 - 1



$$F = \{CD \rightarrow A, EC \rightarrow H, GHB \rightarrow AB, C \rightarrow D, EG \rightarrow A, H \rightarrow B, BE \rightarrow CD, EC \rightarrow B\}$$

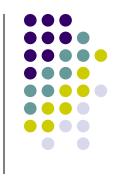
- First, we notice that:
 - EFG never appear on RHS of any FD. So, EFG must be part of ANY key of R
 - A never appears on LHS of any FD, but appears on RHS of some FD. So, A is not part of ANY key of R
 - We now see if EFG is itself a key...
 - EFG+ = EFGA ≠ R; So, EFG alone is not key

Solution #5 - 2



- Checking by adding single attribute with EFG (except A):
- BEFG+ = ABCDEFGH = R; it's a key [BE→CD, EG→A, EC→H]
- CEFG+ = ABCDEFGH = R; it's a key [EG→A, EC→H, H→B, BE→CD]
- DEFG+ = ADEFG ≠ R; it's not a key [EG→A]
- EFGH+ = ABCDEFGH = R; it's a key [EG→A, H→B, BE→CD]
- If we add any further attribute(s), they will form the superkey. Therefore, we can stop here searching for candidate key(s).
- Therefore, candidate keys are: {BEFG, CEFG, EFGH}





Consider R = {A, B, C, D, E, F, G} with a set of FDs

 $F = \{ABC \rightarrow DE, AB \rightarrow D, DE \rightarrow ABCF, E \rightarrow C\}$ Find all the candidate keys of R

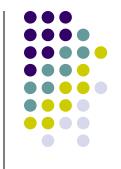
Solution #6 - 1



$$F = \{ABC \rightarrow DE, AB \rightarrow D, DE \rightarrow ABCF, E \rightarrow C\}$$

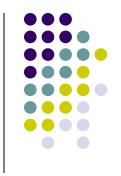
- First, we notice that:
 - G never appears on RHS of any FD. So, G must be part of ANY key of R.
 - F never appears on LHS of any FD, but appears on RHS of some FD. So, F is not part of ANY key of R
 - $G+=G \neq R$ So, G alone is not a key!

Solution #6 - 2



- Now we try to find keys by adding more attributes (except F) to G
 - Add LHS of FDs that have only one attribute (E in E→C):
 - GE+ = GEC ≠ R
 - Add LHS of FDs that have two attributes (AB in AB→D and DE in DE→ABCF):
 - GAB+ = GABD
 - GDE+ = ABCDEFG = R; [DE→ABCF] It's a key!
 - Add LHS of FDs that have three attributes (ABC in ABC→DE), but not taking super set of GDE:
 - GABC+ = ABCDEFG = R; [ABC→DE, DE→ABCF] It's a key!
 - GABE+ = ABCDEFG = R; [AB→D, DE→ABCF] It's a key!
 - If we add any further attribute(s), they will form the superkey.
 Therefore, we can stop here.
 - The candidate key(s) are {GDE, GABC, GABE}

Exercise #7



Consider R = {A, B, C, D, E} with a set of FDs F = $\{AB \rightarrow DE, C \rightarrow E, D \rightarrow C, E \rightarrow A\}$

And we wish to project those FDs onto relation S={A, B, C}

Give the FDs that hold in S

- Hint:
- We need to compute the closure of all the subsets of {A, B, C}, except the empty set and ABC.
- Then, we ignore the FDs that are trivial and those that have D or E on the RHS

Solution #7

$$R = \{A, B, C, D, E\}$$

$$F = \{AB \rightarrow DE, C \rightarrow E, D \rightarrow C, E \rightarrow A\}$$

$$S=\{A, B, C\}$$

- \bullet A+ = A
- B+ = B
- C+ = CEA [C \rightarrow E, E \rightarrow A]
- AB+ = ABDEC [AB \rightarrow DE, D \rightarrow C]
- AC+ = ACE $[C \rightarrow E]$
- BC+ = BCEAD [C \rightarrow E, E \rightarrow A, AB \rightarrow DE]
- We ignore D and E.
- So, the FDs that hold in S are:
- $\{C \rightarrow A, AB \rightarrow C, BC \rightarrow A\}$
- (Note: BC→A can be ignored because it follows logically 17 from C→A)