# Functional Dependencies-More Examples

# Exercise #1

- Below is an instance of R(A1,A2,A3,A4). Choose the FD which may hold on R

1. A4->A1
2. A2A3->A4
3. A2A3->A1

| A1 | A2 | A3 | A4 |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 5 |
| 6 | 7 | 8 | 2 |
| 2 | 1 | 3 | 4 |

# Solution #1

1. A4->A1 ???

- **Incorrect:** The 1st and 4th tuple violates it

2. A2A3->A4 ???

- **Incorrect:** The1st and 2nd tuple violates it.

3. A2A3 -> A1 ???

- Correct!

| A1 | A2 | A3 | A4 |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 1 | 2 | 3 | 5 |
| 6 | 7 | 8 | 2 |
| 2 | 1 | 3 | 4 |

# Exercise #2

- Let R(ABCDEFGH) satisfy the following functional dependencies: {A->B, CH->A, B->E, BD->C, EG->H, DE->F}
- Which of the following FD is also guaranteed to be satisfied by R?

1. BFG --> AE
2. ACG --> DH
3. CEG --> AB

**Hint:** Compute the closure of the LHS of each FD that you get as a choice. If the RHS of the candidate FD is contained in the closure, then the candidate follows from the given FDs, otherwise not.

# Solution #2

- FDs: {A->B, CH->A, B->E, BD->C, EG->H, DE->F}

1. BFG --> AE ???
   - **Incorrect:** BFG+ = BFGEH, which includes E, but not A
2. ACG --> DH ???
   - **Incorrect:** ACG+ = ACGBEH, which does not includes D.
3. CEG --> AB ???
   - **Correct:** CEG+ = CEGHAB, which contains AB

# Question #3

- Which of the following could be a key for R(A,B,C,D,E,F,G) with functional dependencies {AB->C, CD->E, EF->G, FG->E, DE->C, and BC->A}

1. BDF
2. ACDF
3. ABDFG
4. BDFG

# Solution #3 - 1

- {AB->C, CD->E, EF->G, FG->E, DE->C, and BC->A}

1. BDF ???

- No. $BDF^+$ = BDF

2. ACDF ???

- No. $ACDF^+$ = ACDFEG (The closure does not include B)

3. ABDFG ???

- No. This choice is a superkey, but it has proper subsets that are also keys (e.g. $BDFG^+$ = BDFGECA)

# Solution #3 - 2

- {AB->C, CD->E, EF->G, FG->E, DE->C, and BC->A}

4. BDFG ???

- $BDFG^+$ = ABCDEFG
- Check if any subset of BDFG is a key:
  - Since B, D, F never appear on the RHS of the FDs, they must form part of the key.
  - $BDF^+$ = BDF ← Not key
  - So, BDFG is the minimal key, hence the candidate key

# Finding Keys using FDs

- **Tricks for finding the key:**
- If an attribute never appears on the *RHS* of any FD, it *must be part of the key*
- If an attribute never appears on the *LHS* of any FD, but appears on the *RHS* of any FD, it *must not be part of any key*

# Exercise #5

Consider R = {A, B, C, D, E, F, G, H} with a set of FDs

$F = \{CD \rightarrow A,\ EC \rightarrow H,\ GHB \rightarrow AB,\ C \rightarrow D,\ EG \rightarrow A,$

$H \rightarrow B,\ BE \rightarrow CD,\ EC \rightarrow B\}$

**Find all the candidate keys of R**

# Solution #5 - 1

F = {CD→A, EC→H, GHB→AB, C→D, EG→A, H→B, BE→CD, EC→B}

- First, we notice that:
  - **EFG** never appear on RHS of any FD. So, **EFG** must be part of ANY key of R
  - **A** never appears on LHS of any FD, but appears on RHS of some FD. So, **A** is not part of ANY key of R
  - We now see if EFG is itself a key…
  - EFG+ = EFGA ≠ R; So, **EFG** alone is not key

# Solution #5 - 2

- Checking by adding single attribute with **EFG** (except **A**):
- BEFG+ = ABCDEFGH = R; it's a key [BE→CD, EG→A, EC→H]
- CEFG+ = ABCDEFGH = R; it's a key [EG→A, EC→H, H→B, BE→CD]
- DEFG+ = ADEFG ≠ R; it's not a key [EG→A]
- EFGH+ = ABCDEFGH = R; it's a key [EG→A, H→B, BE→CD]
- If we add any further attribute(s), they will form the superkey. Therefore, we can stop here searching for candidate key(s).
- Therefore, candidate keys are: {BEFG, CEFG, EFGH}

# Exercise #6

Consider R = {A, B, C, D, E, F, G} with a set of FDs

F = {ABC→DE, AB→D, DE→ABCF, E→C}

**Find all the candidate keys of R**

# Solution #6 - 1

F = {ABC→DE, AB→D, DE→ABCF, E→C}

- First, we notice that:
  - **G** never appears on RHS of any FD. So, **G** must be part of ANY key of R.
  - **F** never appears on LHS of any FD, but appears on RHS of some FD. So, **F** is not part of ANY key of R
  - $G^+ = G \neq R$ So, G alone is not a key!

# Solution #6 - 2

- Now we try to find keys by adding more attributes (except F) to G
  - Add LHS of FDs that have only one attribute (E in E→C):
  - GE+ = GEC ≠ R
  - Add LHS of FDs that have two attributes (AB in AB→D and DE in DE→ABCF):
  - GAB+ = GABD
  - GDE+ = ABCDEFG = R; [DE→ABCF] It's a key!
  - Add LHS of FDs that have three attributes (ABC in ABC→DE), but not taking super set of GDE:
  - GABC+ = ABCDEFG = R; [ABC→DE, DE→ABCF] It's a key!
  - GABE+ = ABCDEFG = R; [AB→D, DE→ABCF] It's a key!
  - If we add any further attribute(s), they will form the superkey. Therefore, we can stop here.
  - The candidate key(s) are {GDE, GABC, GABE}

# Exercise #7

Consider R = {A, B, C, D, E} with a set of FDs F = {$AB \rightarrow DE$, $C \rightarrow E$, $D \rightarrow C$, $E \rightarrow A$}

And we wish to project those FDs onto relation S={A, B, C}

**<u>Give the FDs that hold in S</u>**

- Hint:
- We need to compute the closure of all the subsets of {A, B, C}, except the empty set and ABC.
- Then, we ignore the FDs that are trivial and those that have D or E on the RHS

# Solution #7

R = {A, B, C, D, E}

F = {AB→DE, C→E, D→C, E→A}

S={A, B, C}

- A+ = A
- B+ = B
- C+ = CEA [C→E, E→A]
- AB+ = ABDEC [AB→DE, D→C]
- AC+ = ACE [C→E]
- BC+ = BCEAD [C→E, E→A, AB→DE]
- We ignore D and E.
- So, the FDs that hold in S are:
- {C→A, AB→C, BC→A}
- (Note: BC→A can be ignored because it follows logically from C→A)