

**EXERCISES**  
**on**  
**Functional Dependencies, Normalization, Lossless Join**

**QUESTIONS**

**Question 1:** Consider relation  $R=(A,B,C,D)$  with the following FDs:

$AB \rightarrow C, C \rightarrow D, \text{ and } D \rightarrow A$

- a. List all candidate keys of  $R$ .
- b. Is  $R$  in 3NF? BCNF??

**Question 2:** Consider relation  $S=(A,B,C,D)$  with the following FDs:

$A \rightarrow B, B \rightarrow C, C \rightarrow D, \text{ and } D \rightarrow A$

- a. List all candidate keys of  $S$ .
- b. Is  $S$  in 3NF? in BCNF?

**Question 3:** Given relation  $R=(A,B,C,D)$ , find if  $R$  is in 3NF or BCNF with respect to the following FDs (each Roman-numeral question is separate):

- i.  $B \rightarrow C, C \rightarrow A, C \rightarrow D$
- ii.  $ABC \rightarrow D, D \rightarrow A$
- iii.  $A \rightarrow C, B \rightarrow D$

**Question 4:** Consider a relation  $R=(A,B,C,D,E,F)$  that satisfies the following four FDs:

$AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B$

Does  $AB \rightarrow D$  hold? If so, show a formal proof; otherwise, give a counterexample.

**Question 5:** Consider a relation  $R=(A,B,C)$ . For each of the following rules/implications, determine whether it holds (i.e., yes or no). If yes, provide a formal proof; otherwise, give a counterexample.

- i. If  $AB \rightarrow C$ , then  $A \rightarrow C$ ?
- ii. If  $AB \rightarrow C$ , then  $B \rightarrow C$ ?
- iii. If  $AB \rightarrow C$ , then  $(A \rightarrow C) \text{ or } (B \rightarrow C)$ ?

## ANSWERS

### Question 1:

- a. 3 candidate keys for R: (A,B), (B, C), (B, D)
- b. R is in **3NF, but not in BCNF**

### Question 2:

- a. 4 candidate keys for S: A, B, C, D
- b. S is in **BCNF**

### Question 3:

- i. B is the candidate key and R is not in 3NF
- ii. (A,B,C), (B,C,D) are candidate keys and R is in 3NF, but not in BCNF
- iii. (A,B) is the key and R is not in 3NF

### Question 4:

Yes,  $AB \rightarrow D$  holds. Here is a proof:

- 1.  $AB \rightarrow B$                       reflexivity
- 2.  $AB \rightarrow BC$                       union: 1 and FD1
- 3.  $AB \rightarrow AD$                       transitivity: 2 and FD2
- 4.  $AB \rightarrow D$                       decomposition: 3

**Or you can compute {A, B}+**

### Question 5:

i. No, counterexample:

A	B	C
1	1	1
1	2	2

ii. No, counterexample:

A	B	C
1	1	1
2	1	2

iii. No, counterexample:

A	B	C
1	1	1
1	2	2
2	1	2