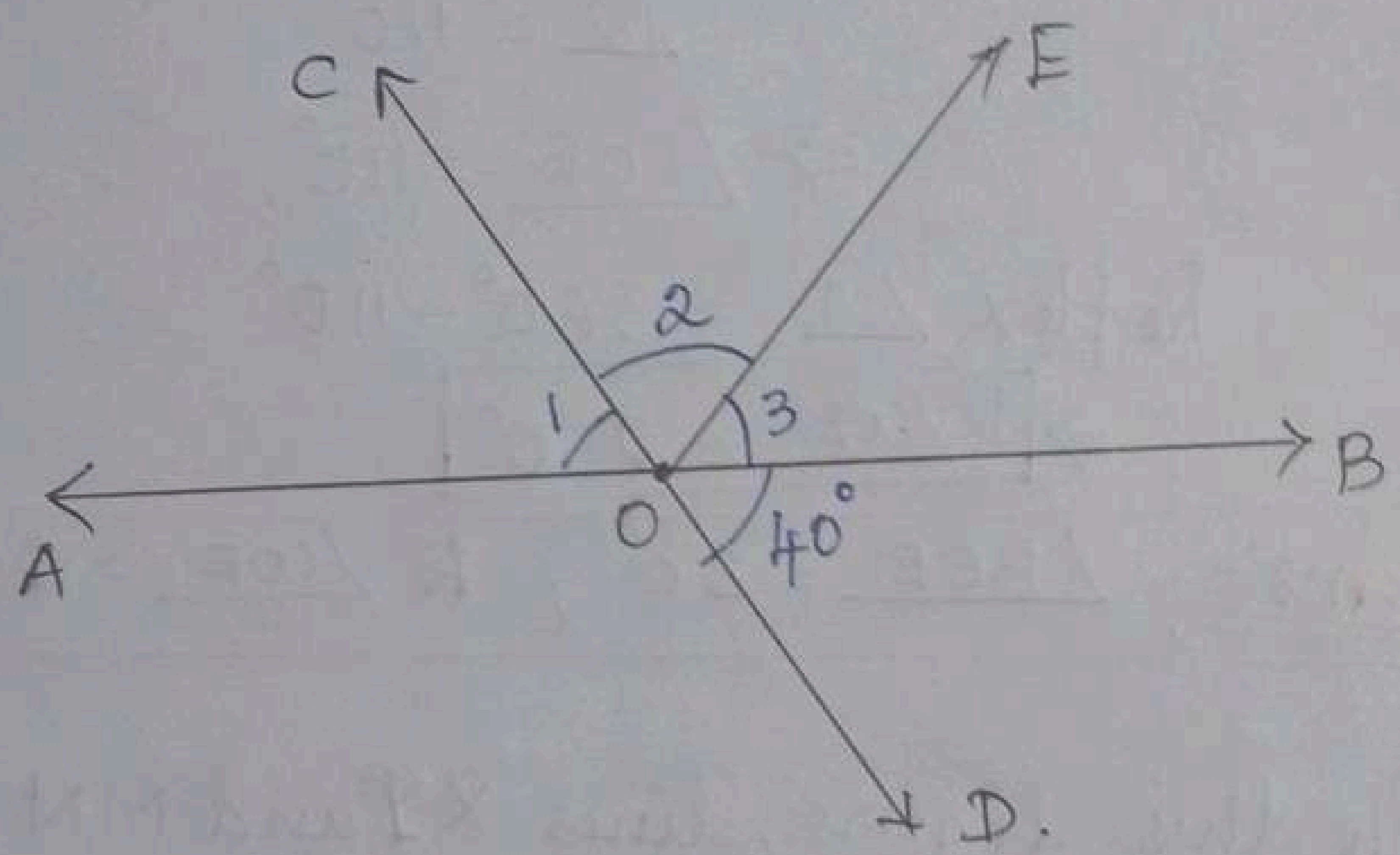
Chapter - 6
Lines And Angles.

Exercise 6.1

1) In the below figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and angle $\angle BOD = 40^\circ$. Find $\angle BOE$ and reflex $\angle COE$.



Solution:-

Given: $\angle AOC + \angle BOE = 70^\circ$

$\angle BOD = 40^\circ$.

Let $\angle AOC = \angle 1$, $\angle BOE = \angle 3$, $\angle COE = \angle 2$

To find: $\angle 3$ ($\angle BOE$) and reflex $\angle COE$

$\Rightarrow \angle 1 = \angle BOD = 40^\circ$ [$\because$ Vertically Opposite Angles]

$\because \angle 1 + \angle 3 = 70^\circ$

$40^\circ + \angle 3 = 70^\circ$

$\angle 3 = 70^\circ - 40^\circ$.

$$\angle 3 = 30^\circ.$$

$$\Rightarrow \boxed{\angle BOE = 30^\circ.}$$

[$\because$ AOB is a straight line],

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

$$(\angle 1 + \angle 3) + \angle 2 = 180^\circ$$

$$\angle 2 + 70^\circ = 180^\circ$$

$$\angle 2 = 180^\circ - 70^\circ$$

$$\angle 2 = 110^\circ$$

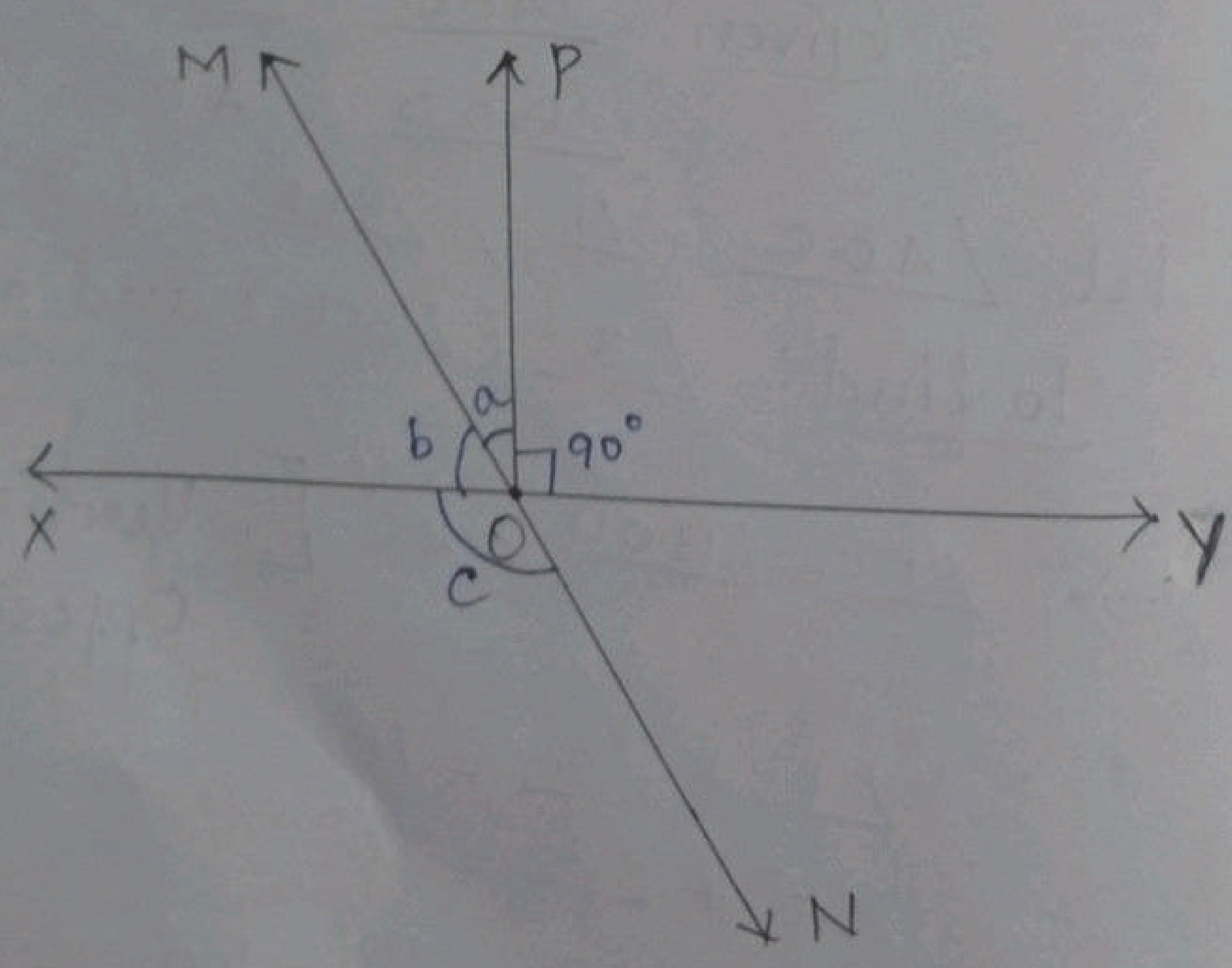
$$\Rightarrow \angle COE = 110^\circ.$$

$$\text{Reflex } \angle 2 = 360^\circ - 110^\circ$$

$$\Rightarrow \boxed{R\angle COE = 250^\circ}$$

Ans:- $\angle BOE = 30^\circ$; $R\angle COE = 250^\circ$

2) In this figure, lines XY and MN intersect at O. If $\angle POY = 90^\circ$ and $a : b = 2 : 3$, find c



Solution :-

Given $\angle POY = 90^\circ$

$a : b = 2 : 3$

To find c

$\Rightarrow$ let angles 'a' and 'b' be $2x$, $3x$.

[$\because$ XY is a line],

$\angle XOP + \angle POY = 180^\circ$ [Linear Pair Angles]

$\angle XOP = 180^\circ - \angle POY$

$= 180^\circ - 90^\circ$

$\Rightarrow \angle XOP = 90^\circ$

$\angle XOP = a + b$ [$\because \angle XOP = \angle XOM + \angle MOP$]

$90^\circ = 2x + 3x \Rightarrow 2x + 3x = 90^\circ$

$5x = 90^\circ$

$x = \frac{90^\circ}{5}$

$\Rightarrow x = 18^\circ$.

$\therefore$ $a = 2x = 2 \times 18^\circ = 36^\circ$

$b = 3x = 3 \times 18^\circ = 54^\circ$.

[$\because$ MN is a line],

$\angle MOX + \angle XON = 180^\circ$ [Linear Pair Angles]

$b + c = 180^\circ$ [from the figure].

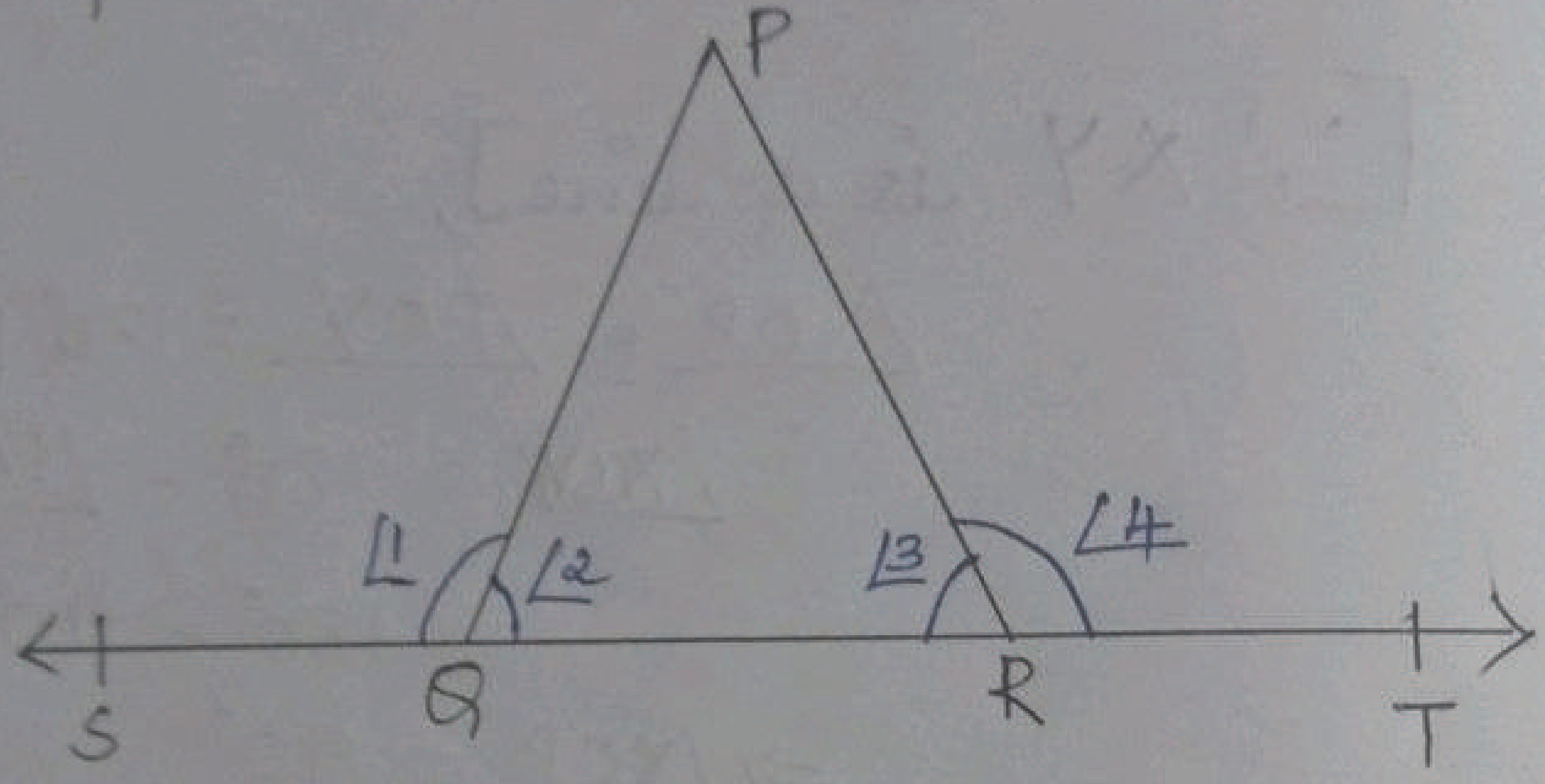
$54^\circ + c = 180^\circ$

$c = 180^\circ - 54^\circ$.

$\Rightarrow \boxed{C = 126^\circ.}$

$\boxed{\text{Ans:- } C = 126^\circ}$

3) In the below figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Solution:-

Let $\angle PQR = \angle 2$, $\angle PQS = \angle 1$,
$\angle PRQ = \angle 3$, $\angle PRT = \angle 4$.

Given $\angle 2 = \angle 3$.

To prove $\angle 1 = \angle 4$.

$\Rightarrow$ [$\because$ ST is a straight line],

$\left.\begin{array}{l} \angle 1 + \angle 2 = 180^\circ \\ \angle 3 + \angle 4 = 180^\circ \end{array}\right\}$ [Linear Pair Angles]

$\therefore \angle 1 + \angle 2 = \angle 3 + \angle 4$ [$\because$ Both sums $= 180^\circ$].

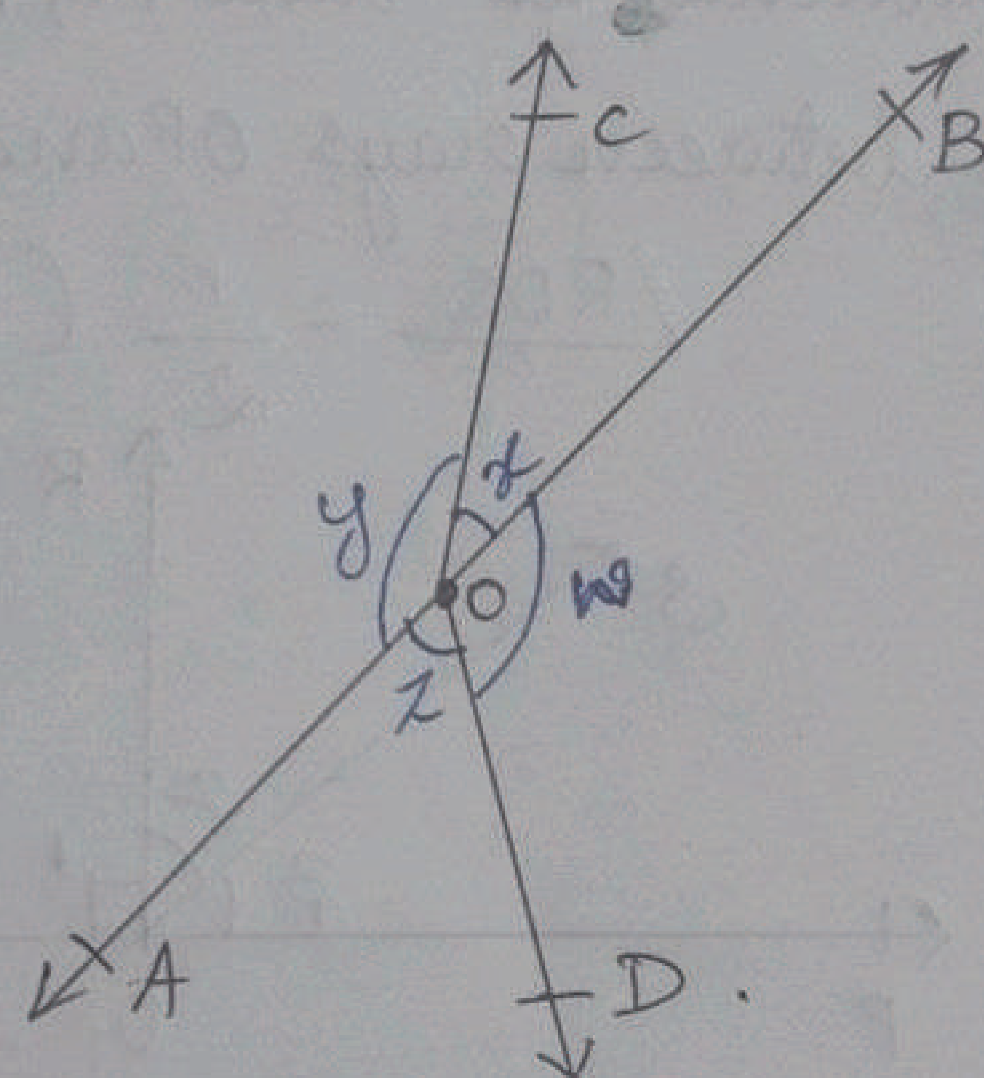
$\angle 1 + \cancel{\angle 3} = \cancel{\angle 3} + \angle 4$ [$\because$ Given $\angle 2 = \angle 3$]

$$\angle 1 = \angle 4$$

$$\Rightarrow \boxed{\angle PQS = \angle PRT}$$

Hence Proved.

4) In the below figure, if $x+y = w+z$, then prove that AOB is a line.



Solution :-

from the figure,

$$\angle x + \angle y + \angle w + \angle z = 360^\circ \text{ [Complete Angle]} \quad \text{—} (1)$$

Also, given $x + y = w + z$

$$\therefore (1) \Rightarrow x+y+x+y = 360^\circ$$

$$2x + 2y = 360^\circ.$$

$$2(x+y) = 360^\circ.$$

$$x+y = \frac{360^\circ}{2}$$

$$\Rightarrow \boxed{x+y=180^\circ.}$$

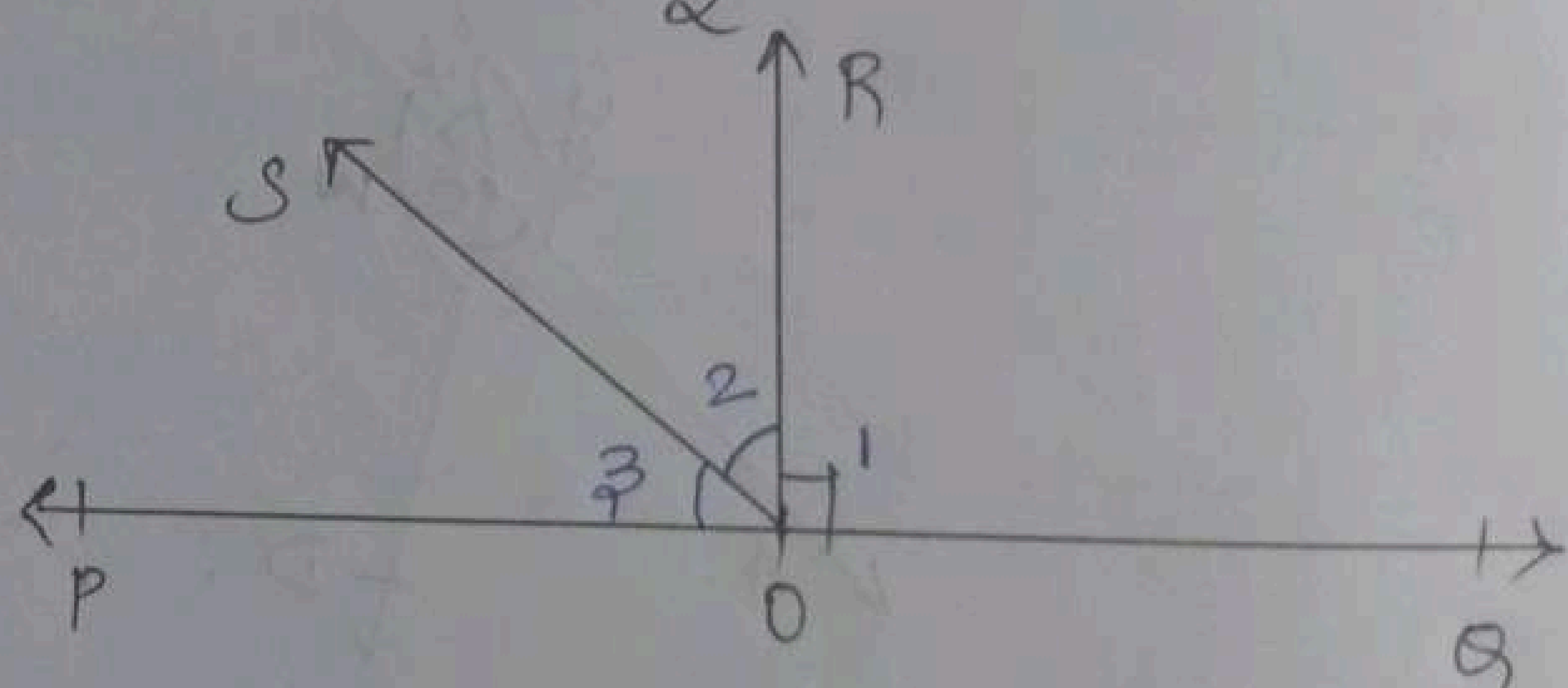
$\Rightarrow$ This means that x and y forms a linear pair of angles

$\therefore$ $\boxed{AOB \text{ is a line}}$.

Hence Proved.

5) In the below figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS).$$



Solution :-

Given POQ is a line.

OR $\perp$ PQ $\Rightarrow \angle ROQ = 90^\circ$.

To prove $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.

$\Rightarrow \angle ROQ + \angle ROP = 180^\circ$ [Linear Pair Angles]

Let $\angle ROQ = \angle 1$, $\angle ROS = \angle 2$, $\angle SOP = \angle 3$.

$\therefore \angle 1 + (\angle 2 + \angle 3) = 180^\circ$ [$\because \angle ROS + \angle SOP = \angle ROP$]

$\angle 2 + \angle 3 = 180^\circ - \angle 1$

$\angle 2 + \angle 3 = 180^\circ - 90^\circ$ [$\because \angle 1 = \angle ROQ = 90^\circ$]

$\angle 2 + \angle 3 = 90^\circ$.

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$$\Rightarrow \angle 2 = 90^\circ - \angle 3.$$

This can also be written as,

$$\angle 2 = \angle 1 - \angle 3 \quad [\because \angle 1 = 90^\circ]. \quad \text{— (1)}$$

Now, consider RHS,

$$\frac{1}{2}(\angle QOS - \angle POS)$$

$$= \frac{1}{2}(\angle 1 + \angle 2 - \angle 3) \quad [\because \angle QOS = \angle QOR + \angle ROS]$$

$$= \frac{1}{2}(\angle 1 - \angle 3 + \angle 2)$$

$$= \frac{1}{2}(\angle 2 + \angle 2) \quad [\text{From (1)}].$$

$$= \frac{1}{2}(2\angle 2)$$

$$= \angle 2$$

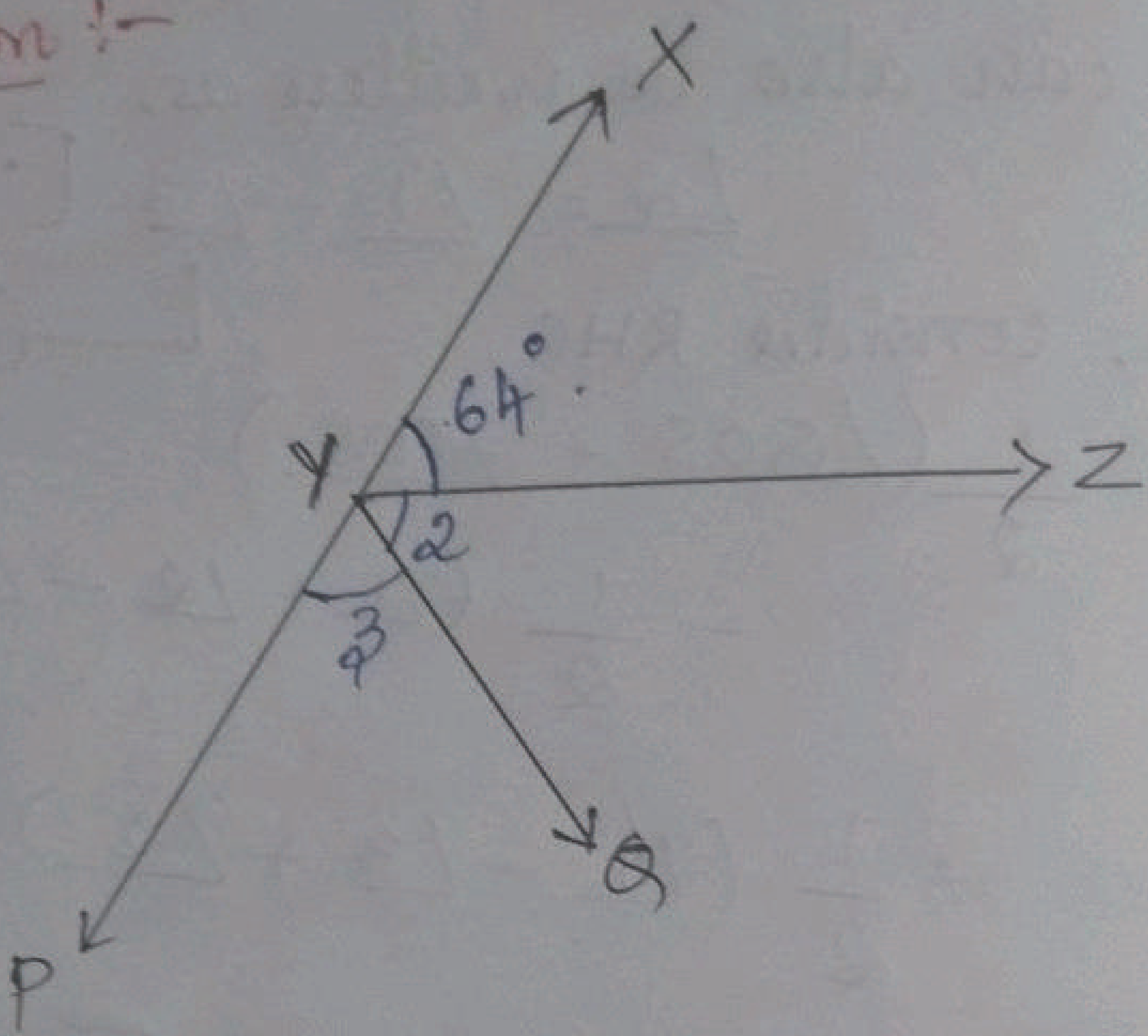
$$= \angle ROS$$

$$\Rightarrow \boxed{\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)}$$

Hence Proved.

6) It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Solution :-



From the given data, the above figure is drawn.

Given $\angle XYZ = 64°$.

[$\because$ XP is a line],

$\angle XYZ + \angle ZYP = 180°$ [Linear Pair axiom] —— ①

Also, Given ray YQ bisects $\angle ZYP$,

$\angle ZYQ = \angle YQP$.

Let $\angle ZYQ = \angle 2$, $\angle QYP = \angle 3$.

$\Rightarrow \angle 2 = \angle 3$ —— ②.

① $\Rightarrow \angle XYZ + \angle 2 + \angle 3 = 180°$ [$\because \angle ZYP = \angle ZYQ + \angle YQP$]

$64° + \angle 2 + \angle 3 = 180°$

$64° + 2\angle 2 = 180°$ [from ②]

$2\angle 2 = 180° - 64°$

$2\angle 2 = 116°$

$\angle 2 = \frac{116°}{2}$

$$\angle 2 = 58^\circ.$$

$$\angle 2 = \angle 3 = 58^\circ.$$

$$\therefore \angle XYQ = \angle XYZ + \angle ZYQ$$

$$= \angle XYZ + \angle 2$$

$$= 64^\circ + 58^\circ$$

$$\Rightarrow \boxed{\angle XYQ = 122^\circ}$$

$$\text{Reflex } \angle QYP = R\angle 3$$

$$= 360^\circ - \angle 3$$

$$= 360^\circ - 58^\circ$$

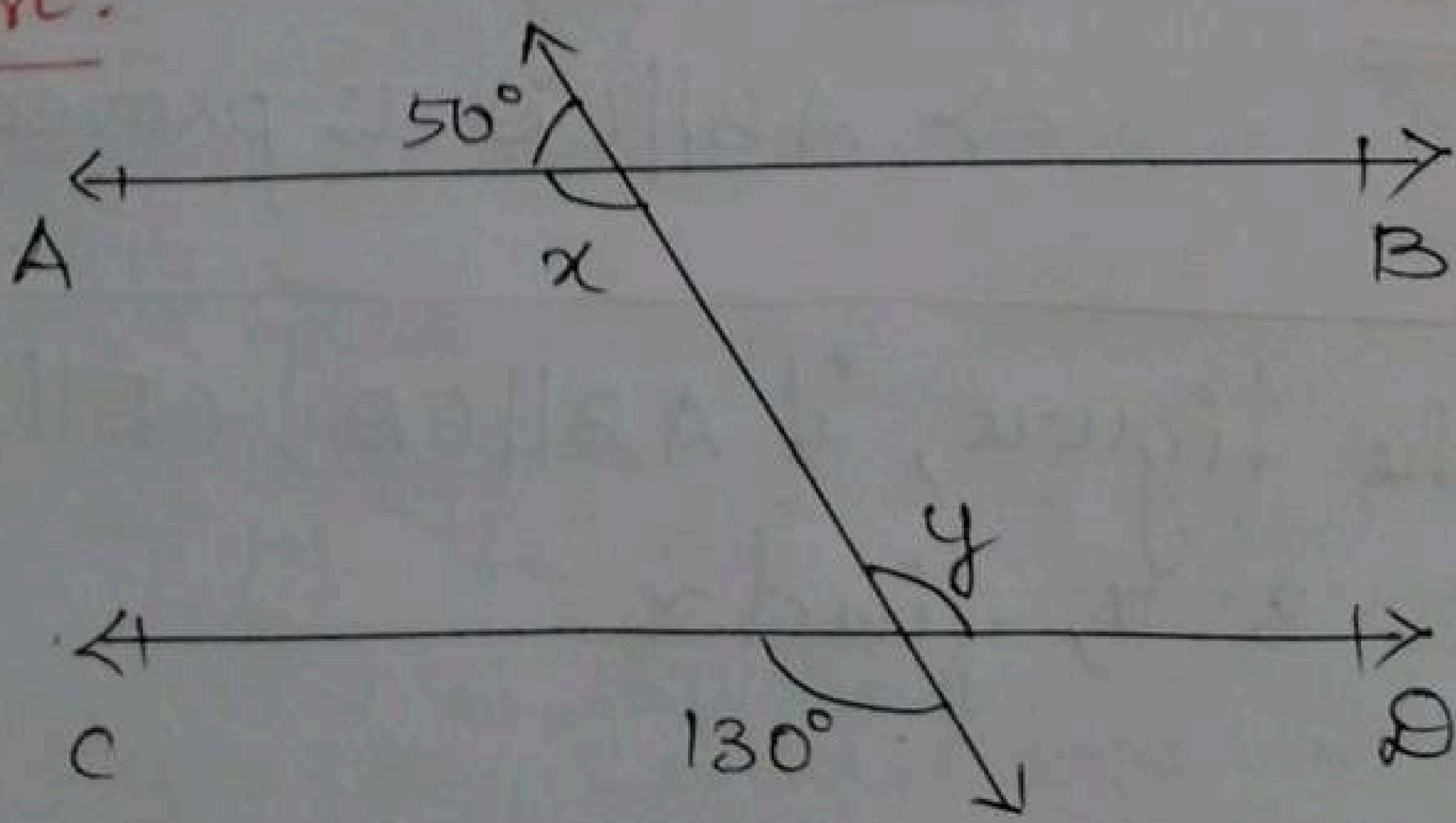
$$\Rightarrow \boxed{R\angle QYP = 302^\circ}.$$

Ans:- $\angle XYQ = 122^\circ$, $R\angle QYP = 302^\circ$.

Exercise 6.2

2 In the figure, find the values of x and y and then show that $AB \parallel CD$.

Solution:-



Given :-

* The figure.

To find :-

* values of x and y
* Prove that $AB \parallel CD$.

$\Rightarrow$ The transversal intersects 2 lines AB and CD such that,

$x + 50^\circ = 180^\circ$. [$\because$ Linear Pair Axiom]

$\Rightarrow x = 180^\circ - 50^\circ$

$x = 130^\circ$

$\therefore y = 130^\circ$ [$\because$ Vertically Opposite Angles].

$\Rightarrow \angle x = \angle y = 130^\circ$ [Alternate Interior Angles].

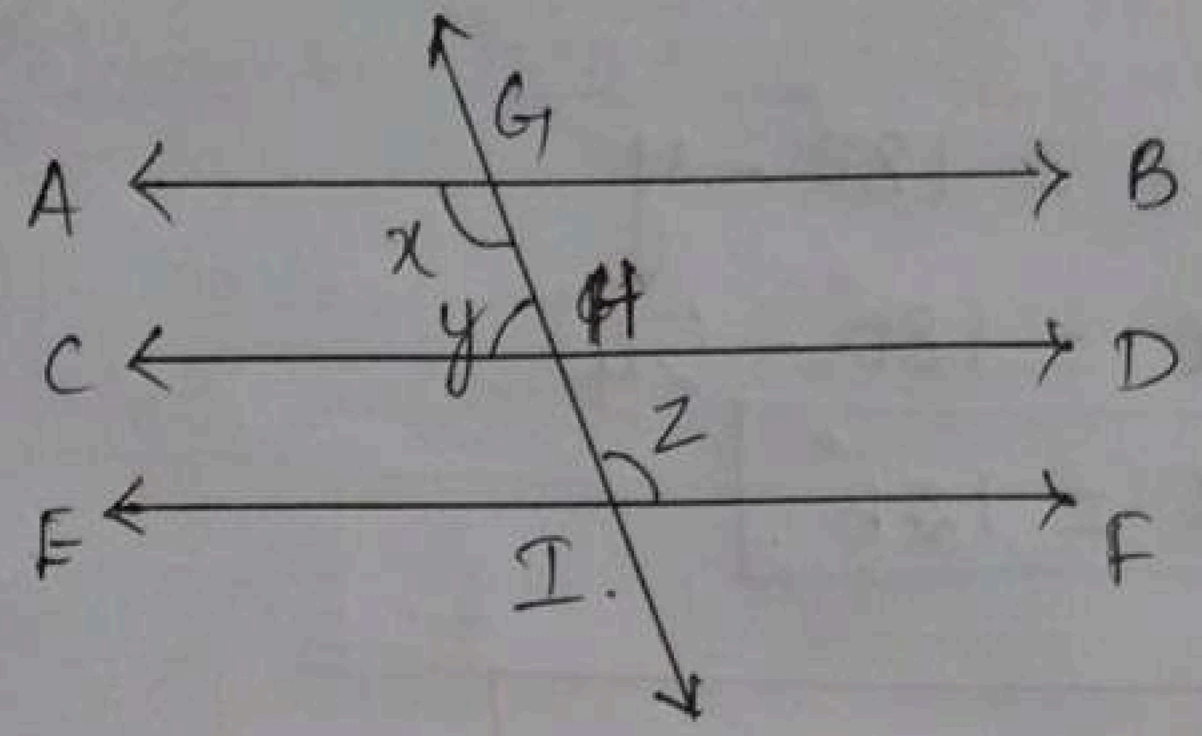
When a transversal intersects 2 lines, and alternate interior angles are equal, then the 2 lines will be parallel.

$\Rightarrow \therefore AB \parallel CD$.

Ans :- $\angle x = \angle y = 130^\circ$

$\Rightarrow AB \parallel CD$ is proved.

2) In the figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x.



Solution:-

Given:- $AB \parallel CD$

$CD \parallel EF$

$y : z = 3 : 7$

To find:- x.

⇒ Let the transversal meet at G, H, I the lines AB, CD, EF respectively.

Let $y = 3a$, $z = 7a$.

$\because \angle CHG = y \Rightarrow \angle DHI = y$ [Vertically Opposite Angles]

$\angle DHI + \angle FIH = 180°$ [Angles on the same side of the transversal]

$\therefore y + z = 180°$ [$\angle DHI = y$, $\angle FIH = z$]

$3a + 7a = 180°$

$10a = 180°$

$\boxed{a = 18°}$

$\therefore y = 3a = 3 \times 18° = 54°$

$z = 7a = 7 \times 18° = 126°$

$\Rightarrow \boxed{y = 54°, z = 126°}$

$x + y = 180°$ [Angles on the same side of the transversal]

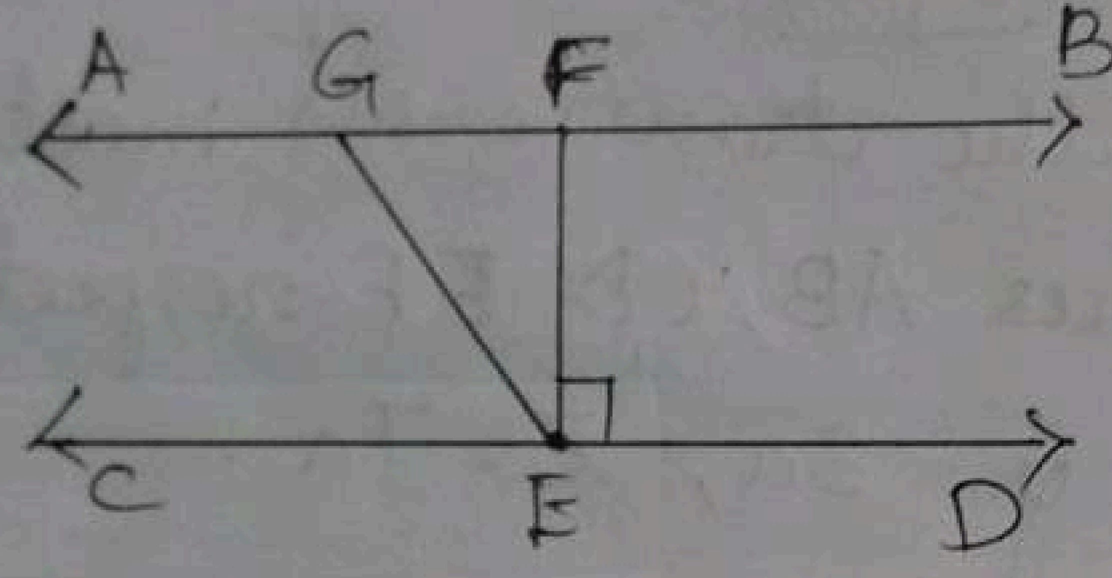
$x = 180° - y$

$x = 180° - 54°$

$\boxed{x = 126°}$

$\boxed{\text{Ans:- } x = 126°}$

3) In the figure, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126°$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.



Solution :-

Given $AB \parallel CD$

$EF \perp CD$.

$\angle GED = 126°$.

To find $\angle AGE$, $\angle GEF$, $\angle FGE$.

$\Rightarrow \angle AGE = \angle GED$ [Alternate Angles]

$\therefore \boxed{\angle AGE = 126°}$

Also, $\angle GED = \angle GEF + \angle FED$

$126° = \angle GEF + 90°$ [$\because \angle FED = 90°$].

$\angle GEF = 126° - 90°$

$\boxed{\angle GEF = 36°.}$

Here, $\angle AGE + \angle FGE = 180^\circ$ [Linear Pair Axiom]. (21)

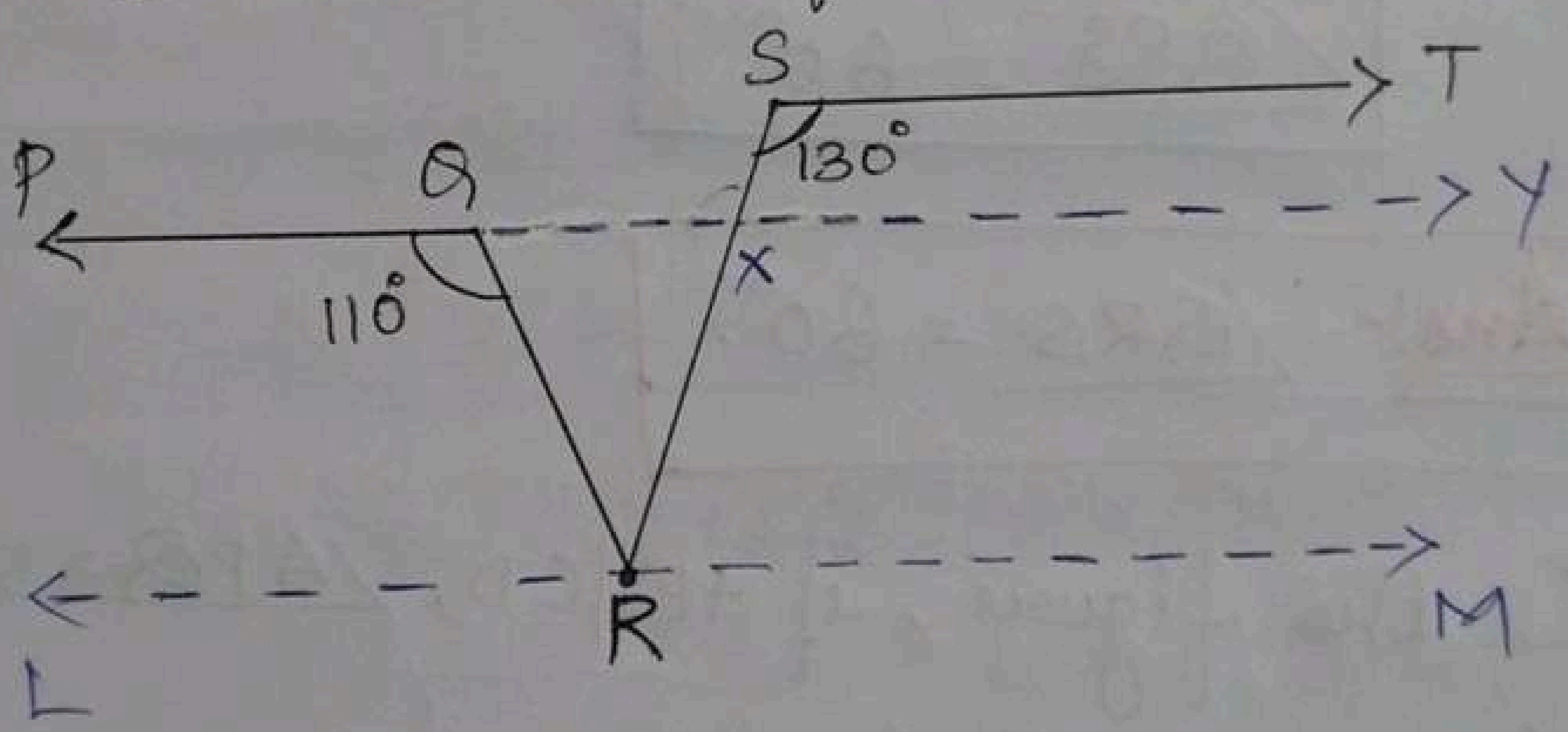
$$\angle FGE = 180^\circ - \angle AGE$$

$$= 180^\circ - 126^\circ$$

$$\boxed{\angle FGE = 54^\circ}$$

Ans:- $\angle AGE = 126^\circ$, $\angle GEF = 36^\circ$, $\angle FGE = 54^\circ$

4) In the figure, if $PQ \parallel ST$, $\angle PQR = 110^\circ$, and $\angle RST = 130^\circ$, find $\angle QRS$.



Solution:-

Extend PQ to Y and $LM \parallel ST$ through R.

$\Rightarrow \angle TSX = \angle QXS$ [$\because$ Alternate Angles]

$\therefore \angle QXS = 130^\circ$.

Also, $\angle QXS + \angle RXQ = 180^\circ$ [$\because$ Linear Pair Angles]

$\Rightarrow \angle RXQ = 180^\circ - 130^\circ$

$\angle RXQ = 50^\circ$ —— (1)

Here, $\angle PQR = \angle QRM$ [Alternate Angles]

$\boxed{\angle QRM = 110^\circ.}$ —— (2)

And, $\angle RXQ = \angle XRM$ [Alternate Angles]

$\therefore \boxed{\angle XRM = 50^\circ}$ [from (1)]. —— (3).

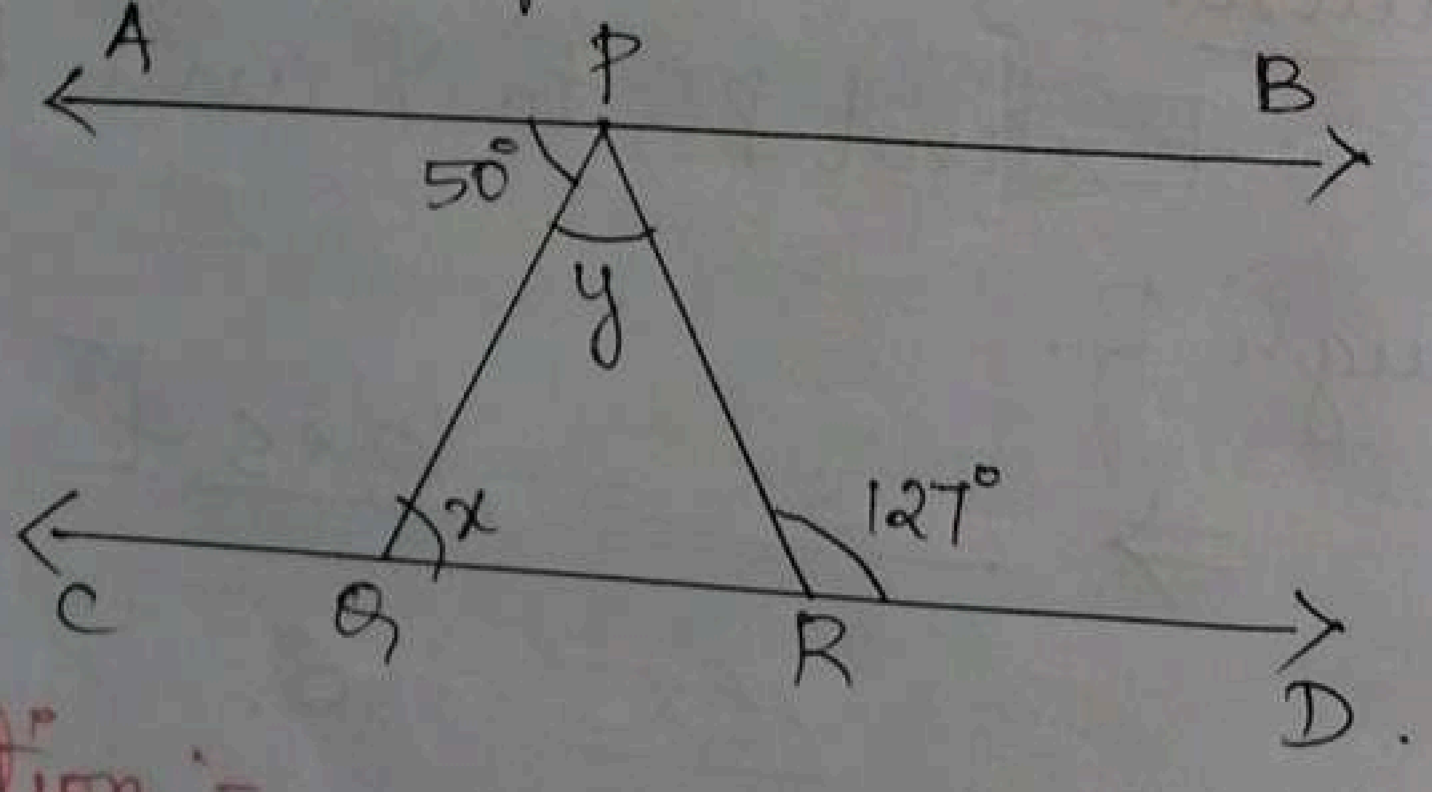
$\therefore \angle QRS = \angle QRM - \angle XRM$

$= 110^\circ - 50^\circ$ [from (2) and (3)]

$\boxed{\angle QRS = 60^\circ}$

Ans:- $\angle QRS = 60^\circ$.

5) In the figure, if $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y.



Solution:-

Given $AB \parallel CD$, $\angle APQ = 50^\circ$, $\angle PRD = 127^\circ$

To find x and y.

$\Rightarrow \angle APQ + \angle PQC = 180^\circ$ [Pair of consecutive interior angles $= 180^\circ$].

$50^\circ + \angle PQC = 180^\circ$ [Given $\angle APQ = 50^\circ$]

$\angle PQC = 180^\circ - 50^\circ$

$\boxed{\angle PQC = 130^\circ.}$

$\angle PQC + \angle PQR = 180^\circ$ [Linear Pair Axiom]

$\angle PQR = 180^\circ - \angle PQC$

$\angle PQR = 180^\circ - 130^\circ$ [$\angle PQC = 130^\circ$]

$\Rightarrow \angle PQR = 50^\circ$

$\Rightarrow \boxed{x = 50^\circ}$

Here, $x + y = 127^\circ$ [Exterior Angle of a triangle = sum of 2 interior opposite angles]

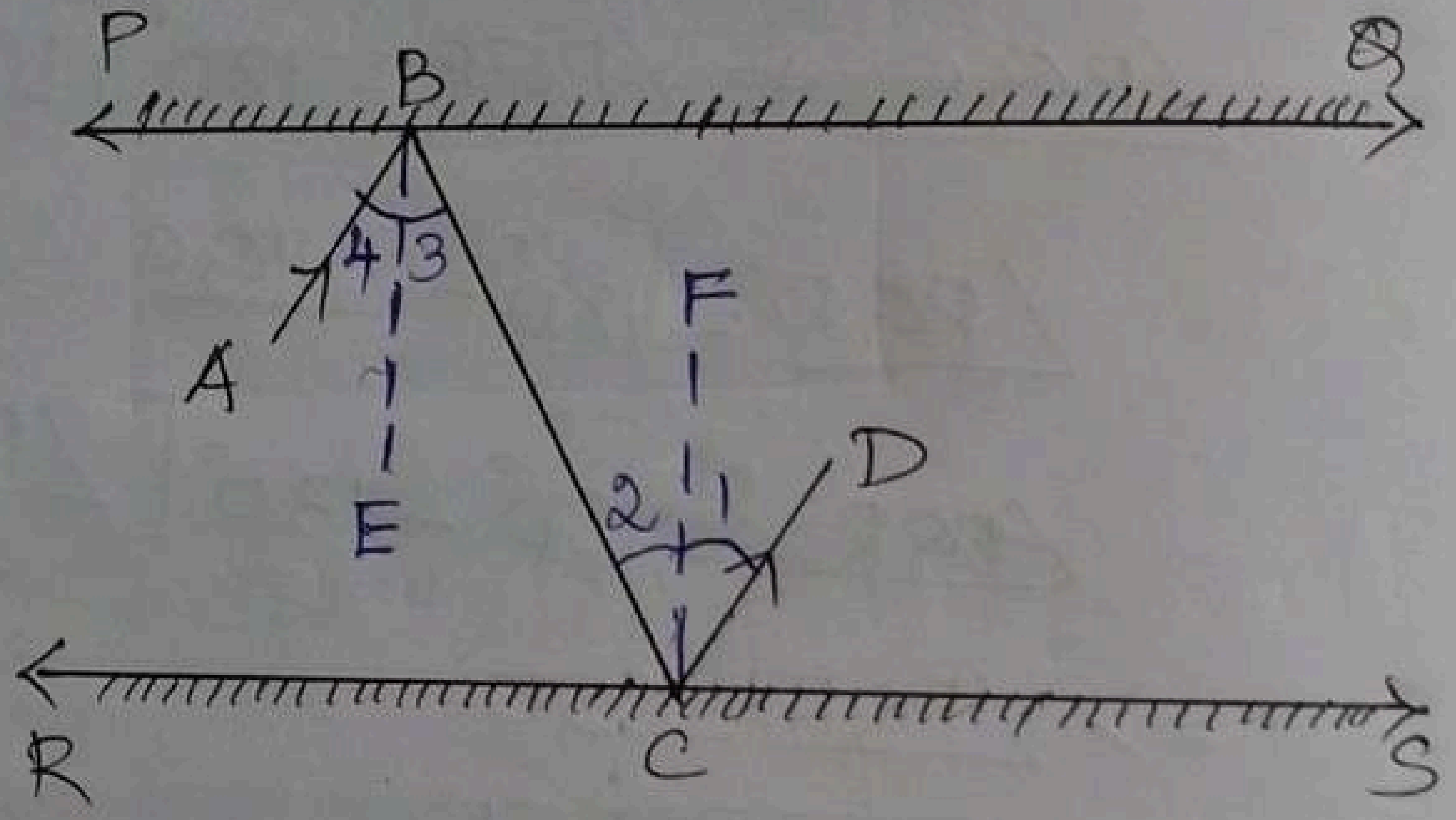
$y = 127^\circ - x$

$y = 127^\circ - 50^\circ$ [$x = 50^\circ$].

$\Rightarrow \boxed{y = 77^\circ}$

Ans:- $x = 50^\circ$ and $y = 77^\circ$

6) In the figure, PQ and RS are 2 mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



Solution:-

At point B, draw BE ⊥ PQ and at point C, draw CF ⊥ RS.

$\angle 1 = \angle 2 \rightarrow (1)$
$\angle 3 = \angle 4 \rightarrow (2)$ [Angle of incidence = Angle of reflection]

$\angle 2 = \angle 3 \rightarrow (3)$ [Alternate Angles]

from (1), (2), (3),

$\angle 1 = \angle 4 \rightarrow (4)$

Add 3 and 4,

$$\angle 1 + \angle 2 = \angle 3 + \angle 4.$$

$$\therefore \angle ABC = \angle BCD.$$

Since Alternate Angles are equal,

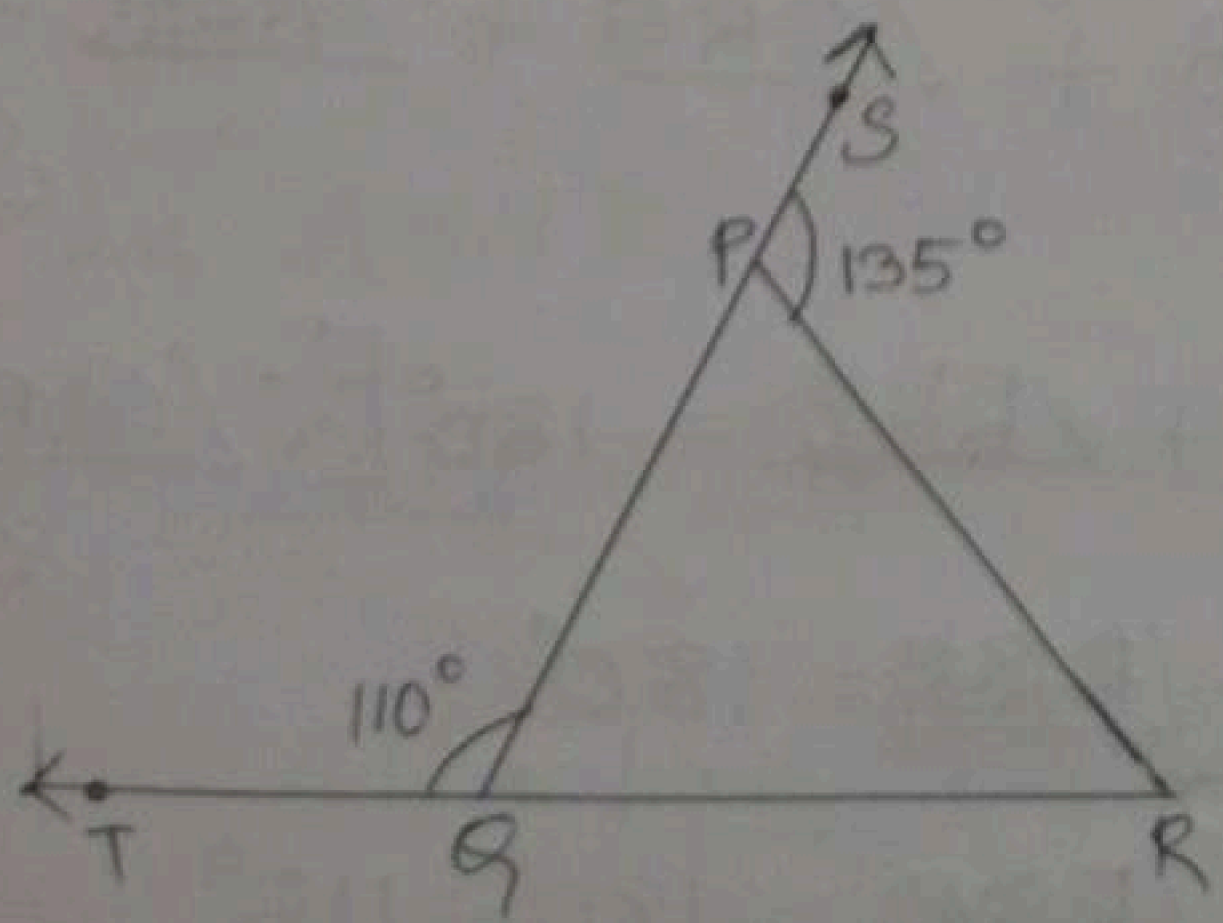
$$\boxed{AB \parallel CD.}$$

Ans:- $AB \parallel CD$

Hence Proved.

Exercise 6.3

1) In the figure, sides QP and RQ of a ΔPQR. are produced to points S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, find $\angle PRQ$



Solution:-

Given $\angle SPR = 135°$, $\angle PQT = 110°$

To find $\angle PRQ$.

Here, $\angle SPR + \angle QPR = 180°$ [Linear Pair Axiom]

$135° + \angle QPR = 180°$ [$\because$ Given $\angle SPR = 135°$]

$\angle QPR = 180° - 135°$

$\boxed{\angle QPR = 45°}$

Also, $\angle PQT + \angle PQR = 180°$ [Linear Pair Axiom]

$110° + \angle PQR = 180°$ [$\because$ Given $\angle PQT = 110°$]

$\angle PQR = 180° - 110°$

$\boxed{\angle PQR = 70°}$

Now, In triangle PQR,

$\angle QPR + \angle PRQ + \angle PQR = 180°$ [Angle Sum Property]

$45° + 70° + \angle PRQ = 180°$ [$\because$ $\angle QPR = 45°$ and $\angle PQR = 70°$]

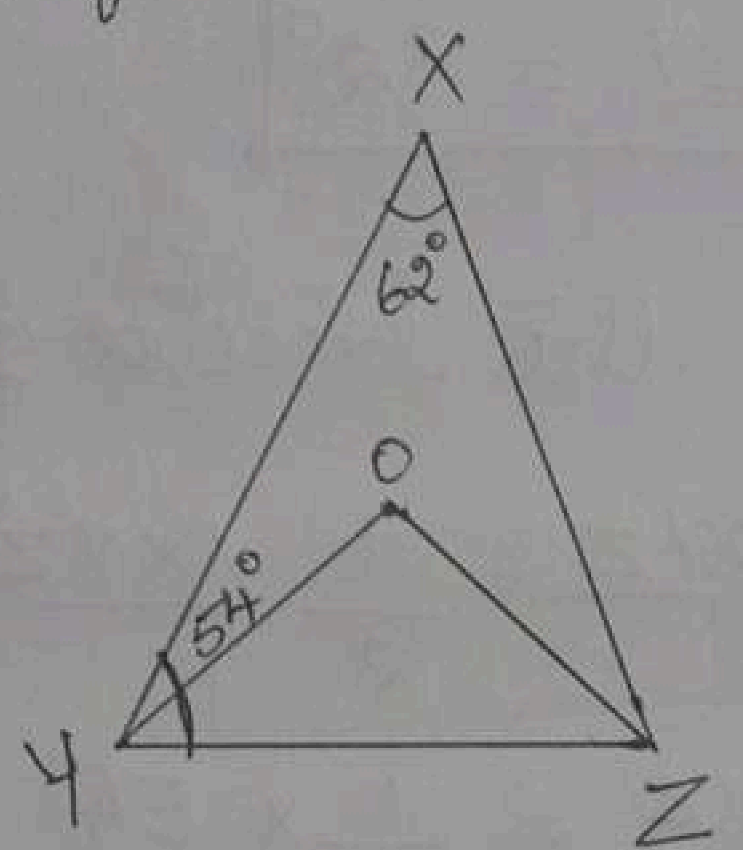
$115° + \angle PRQ = 180°$

$\angle PRQ = 180° - 115°$

$\Rightarrow \boxed{\angle PRQ = 65^\circ}$

Ans:- $\angle PRQ = 65^\circ$

2) In the figure, $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$. If YO and ZO are bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.



Solution:-

Given $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$.

YO and ZO are bisectors of $\angle XYZ$ and $\angle XZY$.

To find $\angle OZY$, $\angle YOZ$

$\Rightarrow$ In ΔXYZ,

$\angle XYZ + \angle YZX + \angle ZXY = 180^\circ$ [Angle Sum Property]

$54^\circ + \angle YZX + 62^\circ = 180^\circ$ [Given $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$]

$\angle YZX + 116^\circ = 180^\circ$

$$\angle YZX = 180^\circ - 116^\circ$$

$$\Rightarrow \angle YZX = 64^\circ.$$

Since ZO is the bisector of $\angle XZY$,

$$\angle OZY = \frac{1}{2} \angle XZY \quad [\because \angle YZX = 64^\circ]$$

$$\angle OZY = \frac{1}{2} \times 64^\circ$$

$$\Rightarrow \boxed{\angle OZY = 32^\circ}$$

Also, YO is the bisector of $\angle XYZ$

$$\therefore \angle OYZ = \frac{1}{2} \angle XYZ$$

$$\angle OYZ = \frac{1}{2} \times 54^\circ \quad [\text{Given } \angle XYZ = 54^\circ]$$

$$\Rightarrow \boxed{\angle OYZ = 27^\circ}$$

In ΔOYZ,

$$\angle YOZ + \angle OZY + \angle ZYO = 180^\circ \quad [\text{Angle Sum Property}]$$

$$\angle YOZ + 32^\circ + 27^\circ = 180^\circ \quad \left[\because \angle OZY = 32^\circ, \ \angle OYZ = 27^\circ\right]$$

$$\angle YOZ + 59^\circ = 180^\circ$$

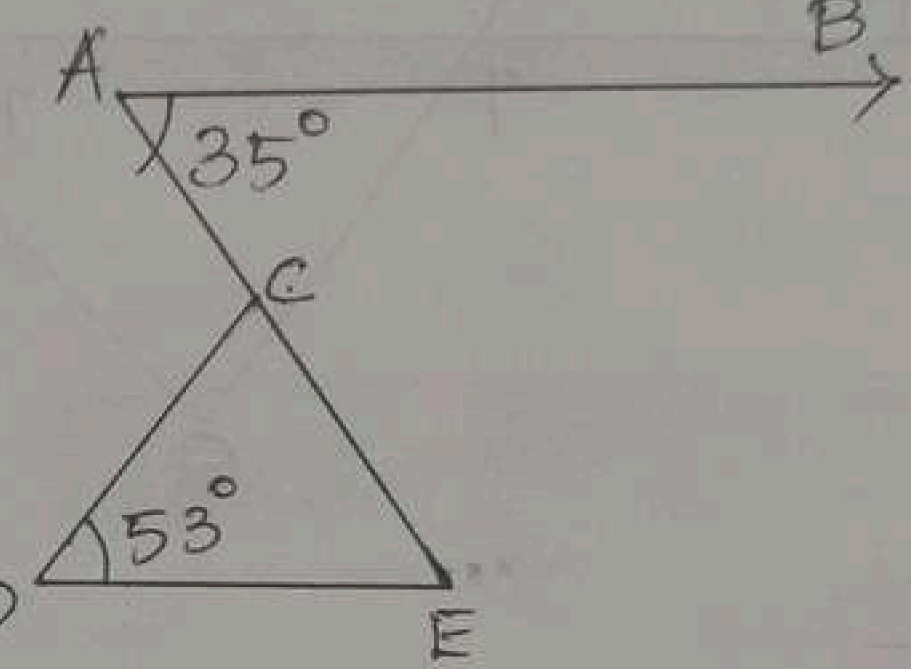
$$\angle YOZ = 180^\circ - 59^\circ$$

$$\Rightarrow \boxed{\angle YOZ = 121^\circ}$$

Ans:- $\angle OZY = 32^\circ$ and $\angle YOZ = 121^\circ$

3) In the figure, if $AB \parallel DE$, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, find $\angle DCE$.

Solution:-



Given $\angle BAC = 35^\circ$, $\angle CDE = 53^\circ$, $AB \parallel DE$

To find $\angle DCE$

$\Rightarrow \angle BAC = \angle CED = 35^\circ$ [Alternate Angles].

In $\triangle CDE$,

$\angle CDE + \angle DEC + \angle ECD = 180^\circ$ [Angle Sum Property]

$53^\circ + 35^\circ + \angle ECD = 180^\circ$ [$\because \angle CDE = 53^\circ$, $\angle CED = 35^\circ$]

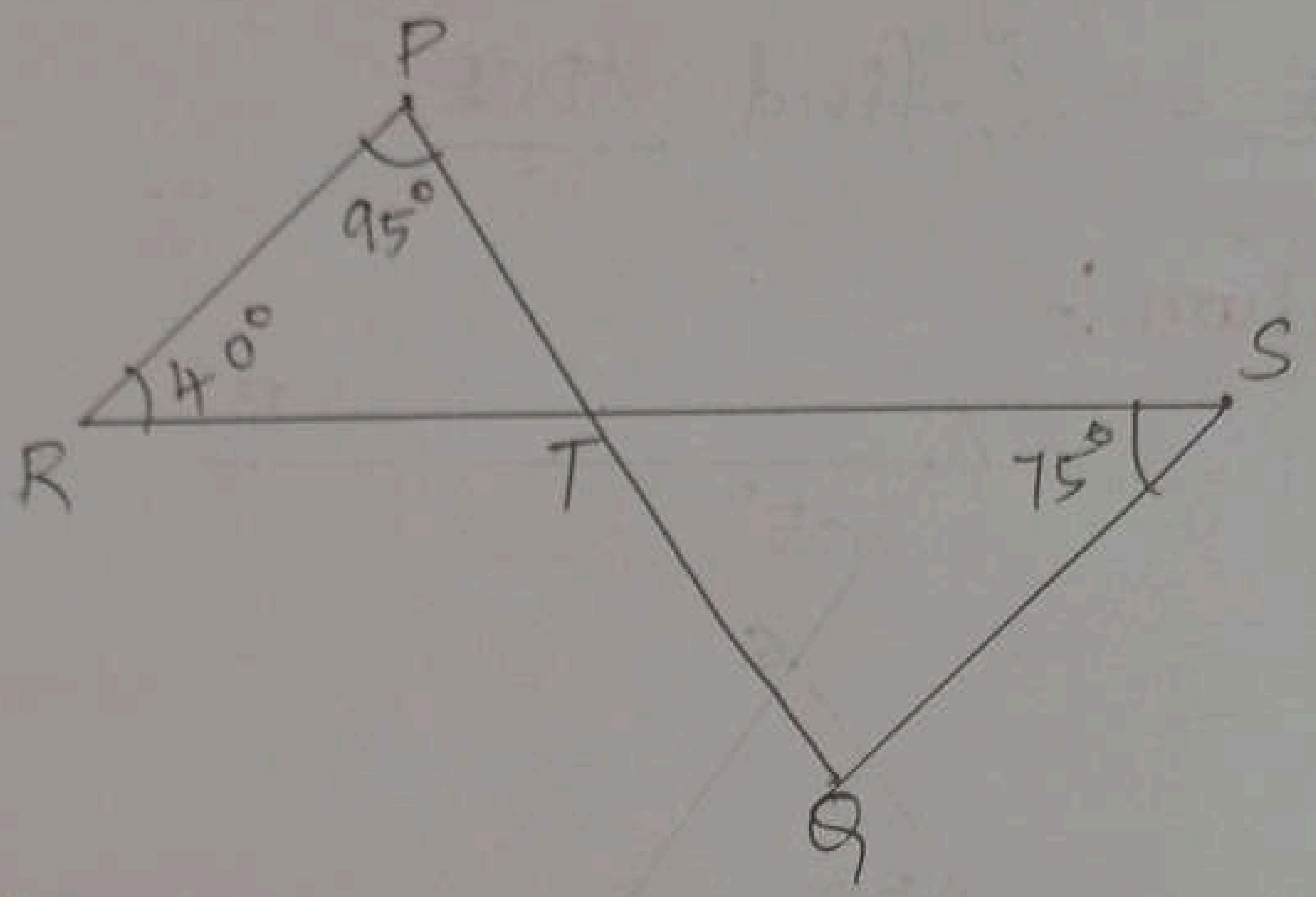
$\angle ECD + 88^\circ = 180^\circ$

$\angle ECD = 180^\circ - 88^\circ$

$\Rightarrow \angle ECD = 92^\circ$

Ans:- $\angle DCE = 92^\circ$.

4) In the figure, if lines PQ and RS intersect at point T such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$, find $\angle SQT$.



Solution:-

Given $\angle PRT = 40°$, $\angle RPT = 95°$
$\angle TSQ = 75°$

To find $\angle SQT$

$\Rightarrow$ In ΔPTR,

$\angle RPT + \angle PRT + \angle PTR = 180°$ [Angle Sum Property]

$95° + 40° + \angle PTR = 180°$ [$\because$ Given $\angle PRT = 40°$, $\angle RPT = 95°$]

$\angle PTR + 135° = 180°$

$\angle PTR = 180° - 135°$

$\Rightarrow \angle PTR = 45°$

Since, $\angle PTR = \angle STQ$, [Vertically Opposite Angles]

$\Rightarrow \angle STQ = 45°$ —— (1)

In ΔSTQ,

$\angle STQ + \angle TSQ + \angle SQT = 180^\circ$ [Angle Sum Property]

$45^\circ + 75^\circ + \angle SQT = 180^\circ$ [from ① ⇒ $\angle STQ = 45^\circ$ and Given $\angle TSQ = 75^\circ$]

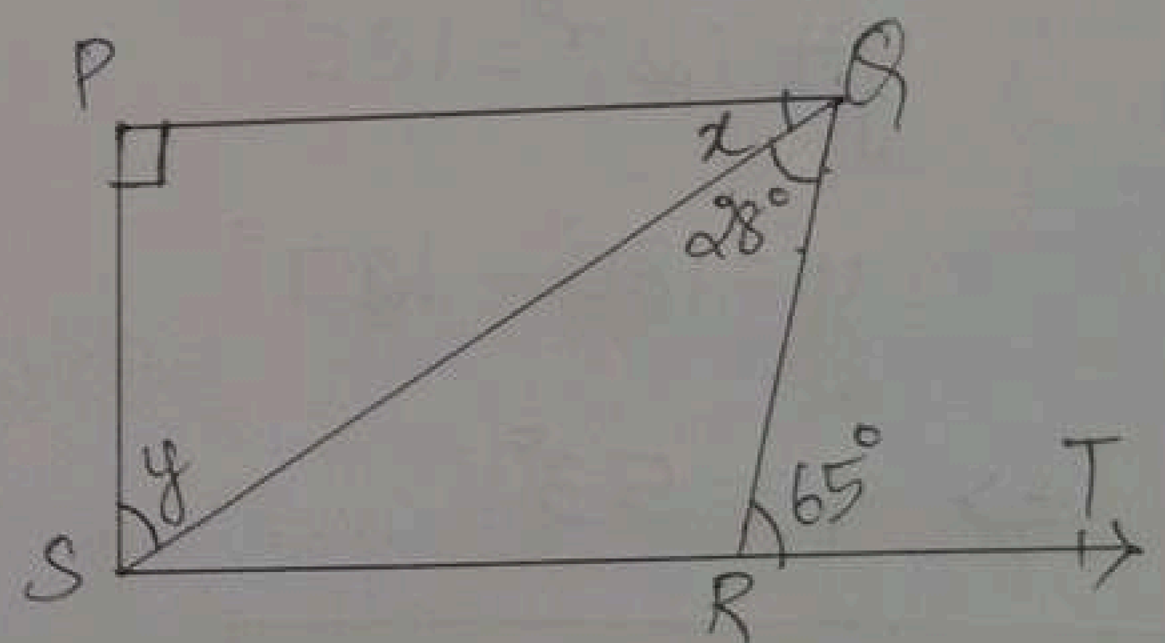
$\angle SQT + 120^\circ = 180^\circ$

$\angle SQT = 180^\circ - 120^\circ$

$\boxed{\angle SQT = 60^\circ}$

Ans:- $\angle SQT = 60^\circ$

5) In the figure, if $PT \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y.



Solution:-

Given $PT \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$,

$\angle QRT = 65^\circ$

To find $\angle x$, $\angle y$.

(32)

$\Rightarrow \angle PQR = \angle QRT$ [Since $PQ \parallel SR$, $\angle PQR$, $\angle QRT$ are alternate angles]

$\angle PQR = 65°$ [$\because \angle QRT = 65°$].

$\angle PQR = \angle PQS + \angle SQR$

$65° = x + 28°$. [Given $\angle PQR = 65°$, $\angle SQR = 28°$]

$x = 65° - 28°$

$\Rightarrow x = 37°$

In ΔPQS,

$\angle SPQ + \angle PQS + \angle QSP = 180°$ [Angle Sum Property]

$90° + x + y = 180°$ [$\angle SPQ = 90°$].

$90° + 37° + y = 180°$ [$\because x = 37°$]

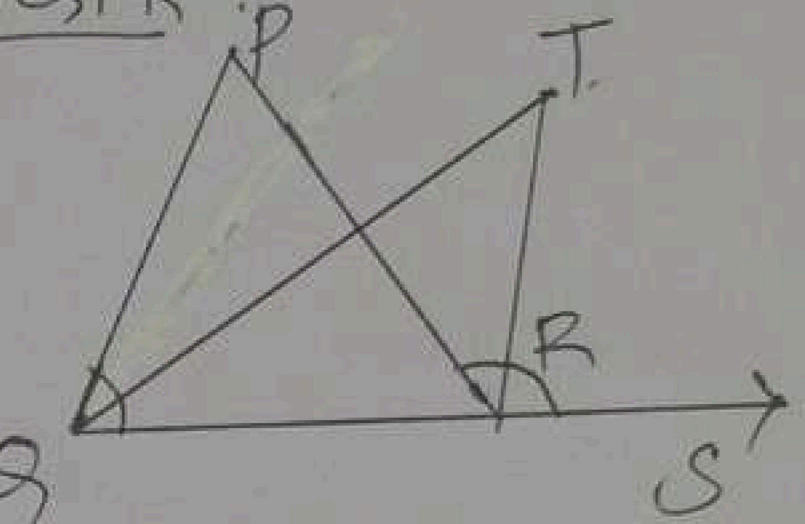
$y + 127° = 180°$

$y = 180° - 127°$

$\Rightarrow y = 53°$

Ans:- $\boxed{x = 37°, y = 53°}$.

6) In the figure, side QR of ΔPQR is produced to a points S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$



Solution :-

$\angle PRS = \angle PQR + \angle QPR$ [Exterior Angle Property].

$\therefore \frac{1}{2}\angle PRS = \frac{1}{2}\angle PQR + \frac{1}{2}\angle QPR$.

$\angle TRS = \angle TQR + \frac{1}{2}\angle QPR$ —(1) [RT is the bisector of $\angle PRS$ and QT is the bisector of $\angle PQR$].

In ΔQTR,

$\angle TRS = \angle TQR + \angle QTR$ —(2) [Exterior Angle Property].

from (1) and (2),

$\cancel{\angle TQR} + \frac{1}{2}\angle QPR = \cancel{\angle TQR} + \angle QTR$

$\Rightarrow \boxed{\angle QTR = \frac{1}{2}\angle QPR}$

Hence Proved