

GROUP 21

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Homework 2:

Part 1

(a) Notice that we used 100 epochs which was waste of time and we could have stopped earlier since after about epoch 55 or so, the loss is not getting lower significantly. Modify the above code so that if the change in loss is less than 10%, you exit the iterations.

(b) The above class uses batch gradient descent to find the minimum of the loss function. Modify the original code and use the stochastic gradient descent instead. Iterate over many iterations and see how the RMSE changes. The graph of RMSE for the batch gradient descent is smooth and decreasing as the number of iterations increases. What can you say about the graph of RMSE when the stochastic gradient descent is used?

```
In [27]: import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
from sklearn.model_selection import train_test_split

import warnings
warnings.filterwarnings('ignore')

df = pd.read_csv("/Users/beingrampopuri/Downloads/iris_dataset.csv")
df.head()
```

```
Out[27]:
```

| | sepal_length | sepal_width | petal_length | petal_width | species |
|---|--------------|-------------|--------------|-------------|---------|
| 0 | 5.1 | 3.5 | 1.4 | 0.2 | setosa |
| 1 | 4.9 | 3.0 | 1.4 | 0.2 | setosa |
| 2 | 4.7 | 3.2 | 1.3 | 0.2 | setosa |
| 3 | 4.6 | 3.1 | 1.5 | 0.2 | setosa |
| 4 | 5.0 | 3.6 | 1.4 | 0.2 | setosa |

```
In [28]: np.random.seed(42)
```

```
In [29]: df = df.iloc[:50][["sepal_length", "sepal_width"]]
df.head()
```

```
Out[29]:
```

| | sepal_length | sepal_width |
|---|--------------|-------------|
| 0 | 5.1 | 3.5 |
| 1 | 4.9 | 3.0 |
| 2 | 4.7 | 3.2 |
| 3 | 4.6 | 3.1 |
| 4 | 5.0 | 3.6 |

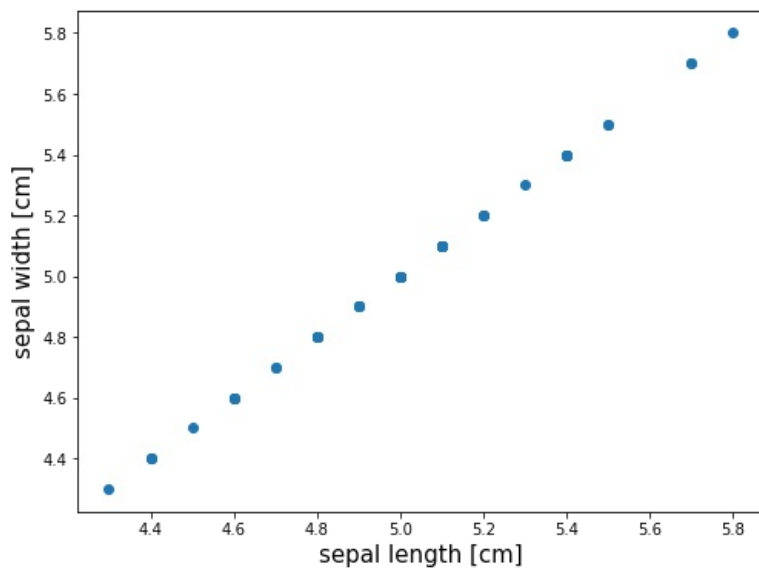
```
In [30]: df.shape
```

```
Out[30]: (50, 2)
```

```
In [31]: # We create the scatter plot

plt.figure(figsize = (8, 6))

plt.scatter(df['sepal_length'], df['sepal_width'])
plt.xlabel("sepal length [cm]", fontsize = 15)
plt.ylabel("sepal width [cm]", fontsize = 15);
```



```
In [32]: # We compute covariance between the two variables
```

```
df.cov()
```

```
Out[32]:
```

| | sepal_length | sepal_width |
|--------------|--------------|-------------|
| sepal_length | 0.124249 | 0.100298 |
| sepal_width | 0.100298 | 0.145180 |

We compute the correlation between the Sepal_length and sepal_width

OBSERVATIONS

- Direction: Positively Correlated
- Strength: Medium

```
In [33]: df.corr()
```

```
Out[33]:
```

| | sepal_length | sepal_width |
|--------------|--------------|-------------|
| sepal_length | 1.00000 | 0.74678 |
| sepal_width | 0.74678 | 1.00000 |

Updated MyLinReg class

```
In [34]: class MyLinReg(object):
    """
    A class used to represent a single artificial neuron for linear regression.

    ...

    Attributes
    -----
    activation_function : function
        The activation function applied to the preactivation linear function.

    theta : numpy.ndarray
        The weights and bias of the single neuron. The last entry being the bias.
        This attribute is created when the fit method is called.

    errors : list
        A list containing the mean squared error computed after each iteration
        of batch gradient descent.

    Methods
    -----
    fit(self, X, y, alpha = 0.001, epochs = 10)
        Iterates the batch gradient descent algorithm through each sample
        a total of epochs number of times with learning rate alpha. The data
        consists of the feature vector X and the associated target y.

    predict(self, X)
        Uses the weights and bias, the feature vector X, and the
        activation_function to make a prediction on each data instance.
    """
    def __init__(self, activation_function):
        self.activation_function = activation_function
```

```

# Initialized a variable that will hold the prev_error (or) error obtained in the prev.iteration
self.prev_errors = 0

def fit(self, X, y, alpha = 0.001, epochs=10):
    self.theta = np.random.rand(X.shape[1] + 1)
    self.errors = []
    n = X.shape[0]

    for idx in range(epochs):
        errors = 0
        sum_1 = 0
        sum_2 = 0
        for xi, yi in zip(X, y):
            sum_1 += (self.predict(xi) - yi)*xi
            sum_2 += (self.predict(xi) - yi)
            errors += ((self.predict(xi) - yi)**2)

        self.theta[:-1] -= 2*alpha*sum_1/n
        self.theta[-1] -= 2*alpha*sum_2/n
        self.errors.append(errors/n)

    print('{} -> {}'.format(self.prev_errors, self.errors[idx]))

    ...

    The below 'if' condition verifies if the difference in error value is < 1 % of the previous.

    LOGIC:

    if the condition is met, then the epochs loop will break and program gets terminated.
    Else,
    prev_error = error in current iteration.

    where error_curr_iteration = self.error[-1] = Last appended element of the errors list.

    ...

    if (abs((self.prev_errors - self.errors[-1]) / self.prev_errors) < 0.01):
        print('Condition met, Breaking the loop.')
        print('\n')

        # Printing the final values of weights and the bias
        print('Weights: \n', self.theta[:-1])
        print('Intercept: ', self.theta[-1])
        break
    else:
        self.prev_errors = self.errors[-1]

    return self

def predict(self, X):
    weighted_sum = np.dot(X, self.theta[:-1]) + self.theta[-1]
    return self.activation_function(weighted_sum)

```

In [35]: `X = df[['sepal_length']].to_numpy()`

In [36]: `X.shape`

Out[36]: (50, 1)

In [37]: `y = df['sepal_width'].to_numpy()`

In [38]: `# We instantiate an instance of MyLinReg class with identity activation function`

```

def identity_function(z):
    return z

model = MyLinReg(identity_function)
model.fit(X, y, alpha = 0.001, epochs = 10)

```

```

0 -> 0.4365926345804007
0.4365926345804007 -> 0.3997479844259732
0.3997479844259732 -> 0.3666602722284338
0.3666602722284338 -> 0.33694641196084896
0.33694641196084896 -> 0.3102623799672599
0.3102623799672599 -> 0.28629923186562545
0.28629923186562545 -> 0.2647795255976996
0.2647795255976996 -> 0.24545410921200236
0.24545410921200236 -> 0.22809923618892428
0.22809923618892428 -> 0.2125139749092665
<__main__.MyLinReg at 0x7fcaf0080070>

```

Out[38]:

From the below code snippet, we are performing mainly three actions:

1. Fitting the values to the model.

2. Printing the Prev_error and current error.
3. Plotting the graphs.

```
In [39]: X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)
```

```
In [40]: model = MyLinReg(identity_function)
print('PREV_ERROR -> CURRENT_ERROR')

'''
    Model Fitting
'''
model.fit(X_train, y_train, alpha = 0.001, epochs = 100)

domain_x = np.linspace(np.min(X_train), np.max(X_train), 2)
domain_y = model.predict(domain_x.reshape(-1, 1))

'''
    Plotting the Linear regression curve
'''

fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(14,7))

ax1.scatter(X_train, y_train)
ax1.plot(domain_x, domain_y, color = "red")
ax1.set_xlabel("sepal length [cm]")
ax1.set_ylabel("sepal width [cm]")
ax1.set_title("Linear Regression", fontsize = 18)

'''
    Plotting the graph of # of iterations Vs error
'''

ax2.plot(range(1, len(model.errors) + 1),
         np.sqrt(model.errors),
         marker = "o")
ax2.set_xlabel("epochs")
ax2.set_ylabel("RMSE")
ax2.set_xticks(range(0, len(model.errors) + 1, 10))
ax2.set_title("RMSE Error at Each Epoch", fontsize = 18);
```

```

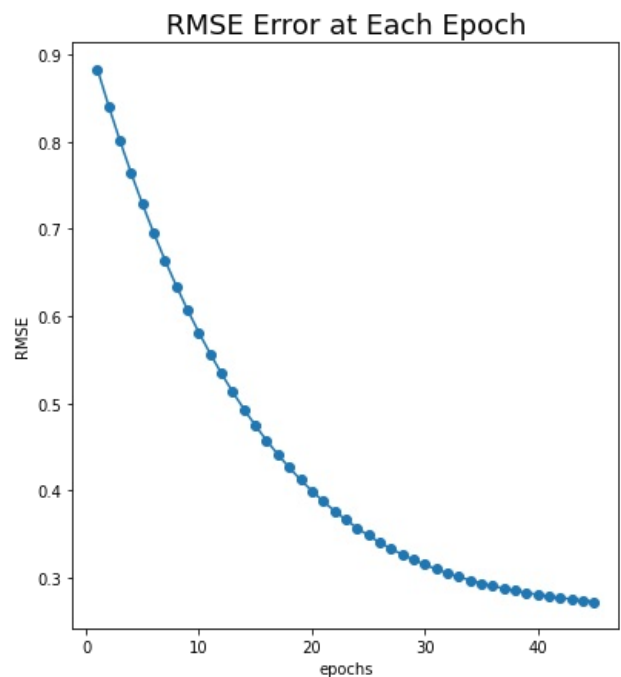
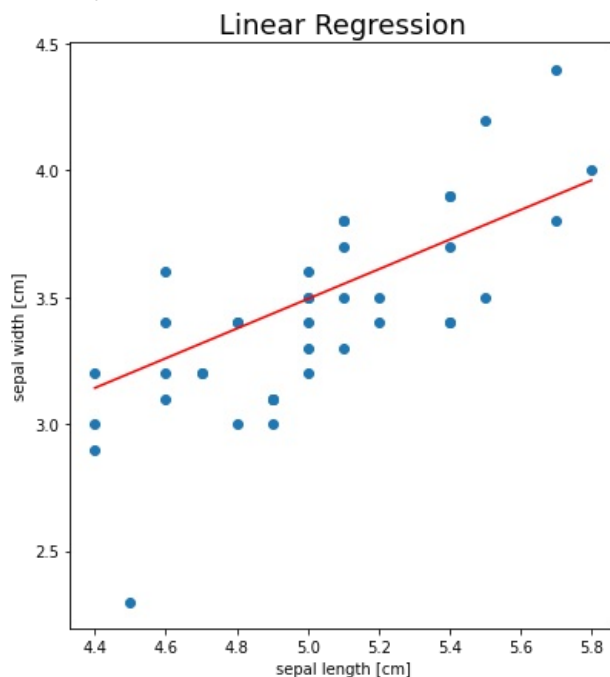
PREV_ERROR -> CURRENT_ERROR
0 -> 0.7800579304377213
0.7800579304377213 -> 0.7072090477573986
0.7072090477573986 -> 0.641816476578103
0.641816476578103 -> 0.583117038712654
0.583117038712654 -> 0.5304256697899958
0.5304256697899958 -> 0.48312742404730724
0.48312742404730724 -> 0.44067029745811537
0.44067029745811537 -> 0.4025587854370364
0.4025587854370364 -> 0.3683480999347778
0.3683480999347778 -> 0.3376389784326265
0.3376389784326265 -> 0.3100730242535547
0.3100730242535547 -> 0.2853285238079173
0.2853285238079173 -> 0.2631166919579189
0.2631166919579189 -> 0.24317830168146135
0.24317830168146135 -> 0.22528065870106592
0.22528065870106592 -> 0.20921488576954922
0.20921488576954922 -> 0.1947934849180601
0.1947934849180601 -> 0.18184814921610365
0.18184814921610365 -> 0.17022779850517966
0.17022779850517966 -> 0.1597968161815918
0.1597968161815918 -> 0.1504334664503752
0.1504334664503752 -> 0.1420284735785327
0.1420284735785327 -> 0.13448374656640746
0.13448374656640746 -> 0.12771123435316636
0.12771123435316636 -> 0.12163189819579567
0.12163189819579567 -> 0.11617478922851239
0.11617478922851239 -> 0.11127622043702337
0.11127622043702337 -> 0.10687902338396604
0.10687902338396604 -> 0.10293188101096004
0.10293188101096004 -> 0.09938872873058177
0.09938872873058177 -> 0.09620821681855751
0.09620821681855751 -> 0.09335322783189952
0.09335322783189952 -> 0.09079044342089197
0.09079044342089197 -> 0.08848995547930816
0.08848995547930816 -> 0.08642491709468916
0.08642491709468916 -> 0.08457122922501911
0.08457122922501911 -> 0.08290725944508127
0.08290725944508127 -> 0.08141358948005523
0.08141358948005523 -> 0.08007278857988782
0.08007278857988782 -> 0.07886921008954803
0.07886921008954803 -> 0.0777888088409898
0.0777888088409898 -> 0.07681897723565315
0.07681897723565315 -> 0.0759483981044641
0.0759483981044641 -> 0.07516691262810073
0.07516691262810073 -> 0.07446540177605879
Condition met, Breaking the loop.

```

```

Weights:
[0.58488473]
Intercept: 0.5689567118846798

```



Predictions using X_test

```
In [41]: predictions = model.predict(X_test)
```

```
In [42]: predictions
```

```
Out[42]: array([3.08396105, 3.55186884, 3.37640342, 3.37640342, 3.55186884,
               3.66884578, 3.49338036, 3.49338036, 3.61035731, 3.55186884])
```

Actual Values of the test dataset.

```
In [43]: y_test
```

```
Out[43]: array([3. , 3.4, 3.1, 3. , 3.5, 3.7, 3.4, 3. , 4.1, 3.8])
```

```
In [44]: result = pd.DataFrame()
result['Actual Value'] = y_test
result['Predicted Value'] = predictions
result['Residue'] = result['Actual Value'] - result['Predicted Value']
result
```

```
Out[44]:
```

| | Actual Value | Predicted Value | Residue |
|---|--------------|-----------------|-----------|
| 0 | 3.0 | 3.083961 | -0.083961 |
| 1 | 3.4 | 3.551869 | -0.151869 |
| 2 | 3.1 | 3.376403 | -0.276403 |
| 3 | 3.0 | 3.376403 | -0.376403 |
| 4 | 3.5 | 3.551869 | -0.051869 |
| 5 | 3.7 | 3.668846 | 0.031154 |
| 6 | 3.4 | 3.493380 | -0.093380 |
| 7 | 3.0 | 3.493380 | -0.493380 |
| 8 | 4.1 | 3.610357 | 0.489643 |
| 9 | 3.8 | 3.551869 | 0.248131 |

Part B of the 1st Question

NOTES

- Unlike in Gradient Descent, Stochastic Gradient Descent updates the parameters after passing through each record.
- The loss function will not be as smooth in case of the Gradient Descent rather in a Zig-Zag manner.
- Relatively faster in convergence when compared to Gradient descent.

```
In [45]: class Stochastic_Grad_Descent(object):

    def __init__(self, activation_function):
        self.activation_function = activation_function

    def fit(self, X, y, alpha = 0.001, epochs=10):
        self.theta = np.random.rand(X.shape[1] + 1)
        self.errors = []
        n = X.shape[0]

        for idx in range(epochs):
            errors = 0
            sum_1 = 0
            sum_2 = 0
            for xi, yi in zip(X, y):
                sum_1 = (self.predict(xi) - yi)*xi
                sum_2 = (self.predict(xi) - yi)
                errors = ((self.predict(xi) - yi)**2)

            self.theta[:-1] -= 2*alpha*sum_1
            self.theta[-1] -= 2*alpha*sum_2
            self.errors.append(errors)

        return self

    def predict(self, X):
        weighted_sum = np.dot(X, self.theta[:-1]) + self.theta[-1]
        return self.activation_function(weighted_sum)
```

```
In [46]: X = df[['sepal_length']].to_numpy()
```

```
In [47]: y = df['sepal_width'].to_numpy()
```

```
In [48]: def identity_function(z):
    return z

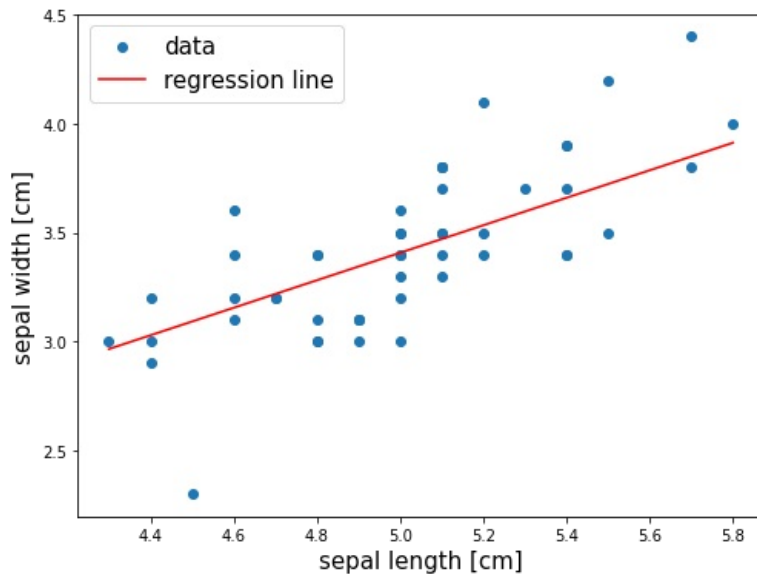
model = Stochastic_Grad_Descent(identity_function)
model.fit(X, y, alpha = 0.001, epochs = 10)
```

```
Out[48]: <__main__.Stochastic_Grad_Descent at 0x7fcdf006dc40>
```

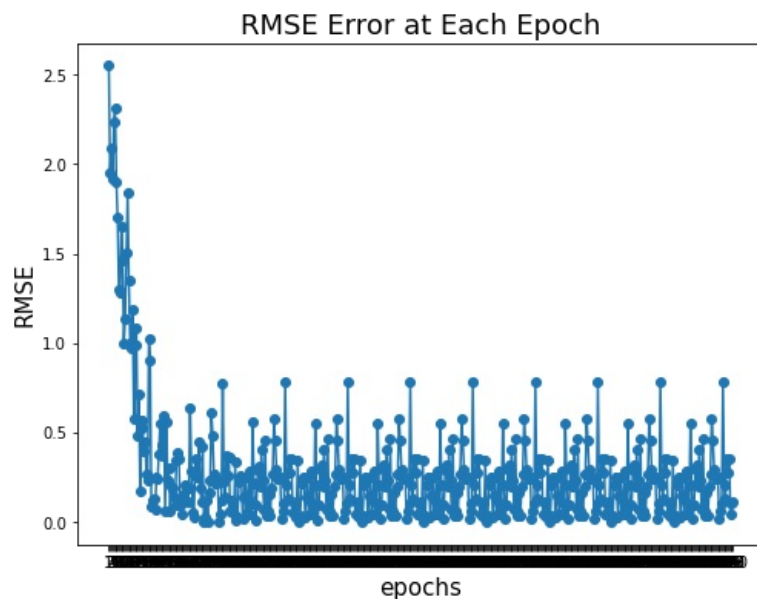
```
In [49]: domain_x.reshape(-1, 1)
```

```
Out[49]: array([[4.4],  
               [5.8]])
```

```
In [50]: domain_x = np.linspace(np.min(X), np.max(X), 5)  
domain_y = model.predict(domain_x.reshape(-1, 1))  
  
plt.figure(figsize = (8, 6))  
  
plt.scatter(X, y, label = "data")  
plt.plot(domain_x, domain_y, color="red", label = "regression line")  
plt.xlabel("sepal length [cm]", fontsize = 15)  
plt.ylabel("sepal width [cm]", fontsize = 15)  
plt.legend(fontsize=15);
```



```
In [51]: plt.figure(figsize = (8, 6))  
  
plt.plot(range(1, len(model.errors) + 1),  
         np.sqrt(model.errors),  
         marker = "o")  
plt.xlabel("epochs", fontsize = 15)  
plt.ylabel("RMSE", fontsize = 15)  
plt.xticks(range(1, len(model.errors) + 1))  
plt.title("RMSE Error at Each Epoch", fontsize = 18);
```



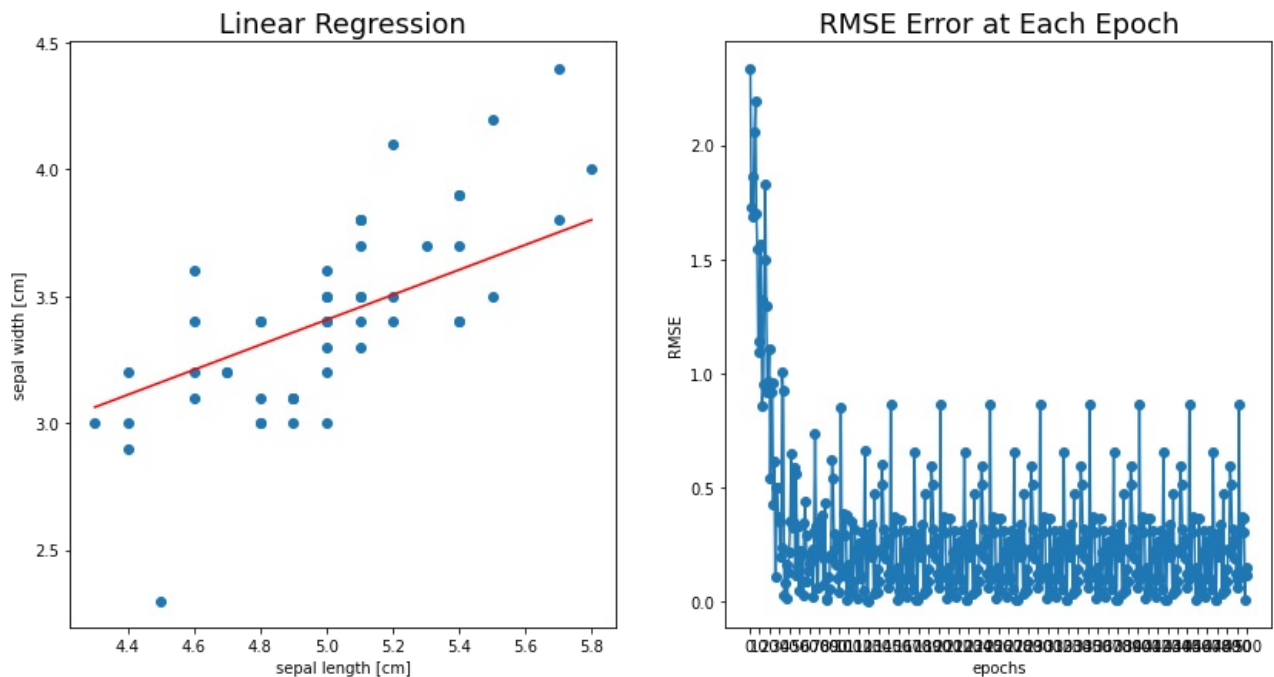
```
In [52]: model = Stochastic_Grad_Descent(identity_function)  
model.fit(X, y, alpha = 0.001, epochs = 10)  
  
domain_x = np.linspace(np.min(X), np.max(X), 2)  
domain_y = model.predict(domain_x.reshape(-1, 1))  
  
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(14,7))  
ax1.scatter(X, y)
```

```

ax1.plot(domain_x, domain_y, color = "red")
ax1.set_xlabel("sepal length [cm]")
ax1.set_ylabel("sepal width [cm]")
ax1.set_title("Linear Regression", fontsize = 18)

ax2.plot(range(1, len(model.errors)+1),
         np.sqrt(model.errors),
         marker = "o")
ax2.set_xlabel("epochs")
ax2.set_ylabel("RMSE")
ax2.set_xticks(range(0, len(model.errors) + 1, 10))
ax2.set_title("RMSE Error at Each Epoch", fontsize = 18);

```



Observations

- The graph resembles a zig-zag pattern rather a smoother decay.
- It is because, the error values keeps on fluctuating up and down.
- On an average, it does converges to the minimal error value after a certain number of iterations.

Processing math: 100%

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Importing Essential Libraries

```
In [65]: import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt

from sklearn.linear_model import SGDRegressor

from sklearn.metrics import mean_squared_error
from sklearn.metrics import mean_absolute_error
from sklearn.model_selection import validation_curve

from sklearn.model_selection import train_test_split

from sklearn.pipeline import Pipeline
from sklearn.preprocessing import MinMaxScaler, StandardScaler

ads_data = pd.read_csv("/Users/beingrampopuri/Downloads/advertising.csv")

import warnings
warnings.filterwarnings('ignore')
```

```
In [66]: np.random.seed(42)
```

Function to detect Outliers

```
In [67]: def outliers(df, feature):

    Q1 = df[feature].quantile(0.25)
    Q3 = df[feature].quantile(0.75)
    IQR = Q3 - Q1

    lowerBound = Q1 - 1.5*IQR
    upperBound = Q3 + 1.5*IQR

    outlier_indices = df.index[(df[feature] < lowerBound) | (df[feature] > upperBound)]

    return outlier_indices
```

Function to replace Outliers with desired Central tendency

```
In [68]: def replace_outliers(df, feature, outlier_list, measure):
    replacement = 0.0
    if measure == 'Mean':
        replacement = df[feature].mean()
    elif measure == 'Median':
        replacement = df[feature].median()
    else:
        replacement = df[feature].mode()
    for idx in outlier_list:
        df[feature][idx] = replacement
    return
```

- No Outliers present in the 'TV' column

```
In [69]: TV_outliers = outliers(ads_data, 'TV')
TV_outliers
```

```
Out[69]: Int64Index([], dtype='int64')
```

- No outliers present in the 'Radio' Column

```
In [70]: Radio_outliers = outliers(ads_data, 'Radio')
Radio_outliers
```

```
Out[70]: Int64Index([], dtype='int64')
```

There are two outliers present in the 'Newspaper' column

- We have to replace this with one of (Mean, Median, Mode)
- Mean is not a good measure when there are outliers present.
- Mode is to be used in case of Categorical variables.
- When there is a skewness observed in the distribution of a feature data, Median best fits the purpose.

```
In [71]: Newspaper_outliers = outliers(ads_data, 'Newspaper')
Newspaper_outliers
```

```
Out[71]: Int64Index([16, 101], dtype='int64')
```

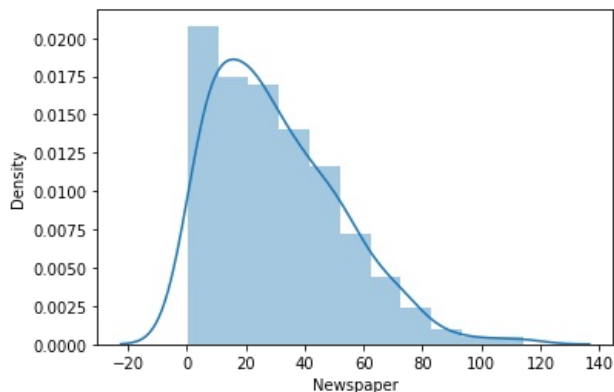
Plotting the 'Newspaper' column data distribution.

Observations

- Data is skewed to the left.
- Since there is a presence of outliers, Median is chosen to replace those values.

```
In [72]: sns.distplot(ads_data['Newspaper'])
```

```
Out[72]: <AxesSubplot:xlabel='Newspaper', ylabel='Density'>
```



Replacing the outliers

```
In [73]: replace_outliers(ads_data, 'Newspaper', Newspaper_outliers, 'Median')
```

```
In [74]: Newspaper_outliers = outliers(ads_data, 'Newspaper')
Newspaper_outliers
```

```
Out[74]: Int64Index([], dtype='int64')
```

```
In [79]: X = ads_data[["TV", "Radio"]]
y = ads_data[["Sales"]]
```

Fitting the data without Scaling

```
In [80]: x_train, x_test, y_train, y_test = train_test_split(X, y, test_size = 0.15, random_state=42)

scaler = MinMaxScaler()
scaler.fit(x_train, y_train)
```

```
Out[80]: MinMaxScaler()
```

```
In [81]: sgd = SGDRegressor(random_state = 42)
sgd.fit(x_train, y_train)
```

```
Out[81]: SGDRegressor(random_state=42)
```

Notice that the weights and the bias are out of context. Very unusual.

```
In [82]: w = sgd.coef_
b = sgd.intercept_
```

```
w, b
```

```
Out[82]: (array([ 1.43645723e+11, -1.66150291e+11]), array([4.321975e+10]))
```

Finding the Mean Absolute Error for training and test set.

```
In [83]: mean_abs_train = mean_absolute_error(y_train, sgd.predict(x_train))
mean_abs_test = mean_absolute_error(y_test, sgd.predict(x_test))

print('Mean Abs Error Train: ', mean_abs_train)
print('Mean Abs Error Test: ', mean_abs_test)
```

```
Mean Abs Error Train: 18279520296188.008
Mean Abs Error Test: 15914415475536.998
```

```
In [84]: mean_squared_train = mean_squared_error(y_train, sgd.predict(x_train))
mean_squared_test = mean_squared_error(y_test, sgd.predict(x_test))

print(f"RMSE on the training data: {np.sqrt(mean_squared_train)}\n")
print(f"RMSE on the test data: {np.sqrt(mean_squared_test)}")
```

```
RMSE on the training data: 21617873498944.027
```

```
RMSE on the test data: 19343184248413.812
```

Note that the mean absolute error is too high in case of without scaling.

- SGD is sensitive to feature scaling.
- In the below code snippet we are trying to add a feature scaling step that reduces the error.

Scaling

```
In [85]: ads_data[['TV', 'Radio']] = Min_Max_Scaler.fit_transform(ads_data[['TV', 'Radio']])
```

```
In [86]: ads_data.head()
```

```
Out[86]:
```

| | TV | Radio | Newspaper | Sales |
|---|----------|----------|-----------|-------|
| 0 | 0.775786 | 0.762097 | 69.2 | 22.1 |
| 1 | 0.148123 | 0.792339 | 45.1 | 10.4 |
| 2 | 0.055800 | 0.925403 | 69.3 | 12.0 |
| 3 | 0.509976 | 0.832661 | 58.5 | 16.5 |
| 4 | 0.609063 | 0.217742 | 58.4 | 17.9 |

```
In [87]: ads_data.head()
```

```
Out[87]:
```

| | TV | Radio | Newspaper | Sales |
|---|----------|----------|-----------|-------|
| 0 | 0.775786 | 0.762097 | 69.2 | 22.1 |
| 1 | 0.148123 | 0.792339 | 45.1 | 10.4 |
| 2 | 0.055800 | 0.925403 | 69.3 | 12.0 |
| 3 | 0.509976 | 0.832661 | 58.5 | 16.5 |
| 4 | 0.609063 | 0.217742 | 58.4 | 17.9 |

```
In [88]: X = ads_data.iloc[:, 0:2]
y = ads_data.iloc[:, 3]
```

```
In [89]: sgd_Regressor = SGDRegressor()
```

```
In [90]: X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.3, random_state = 42)
```

```
In [91]: sgd_Regressor.fit(X_train, y_train)
```

```
Out[91]: SGDRegressor()
```

```
In [92]: w = sgd_Regressor.coef_
b = sgd_Regressor.intercept_
w, b
```

```
Out[92]: (array([14.55611555, 4.94403729]), array([5.82277741]))
```

```
In [93]: print(f"The linear regression model based on the training data is \n")
print(f"predicted_sales = {np.round(w[0],3)} * TV + {np.round(w[1],3)} * Radio + {np.round(b,3)}")
```

The linear regression model based on the training data is

```
predicted_sales = 14.556 * TV + 4.944 * Radio + [5.823]
```

Notice that there is a significant decrease in the error values.

```
In [94]: mse_train = mean_squared_error(y_train, sgd_Regressor.predict(X_train))
mse_test = mean_squared_error(y_test, sgd_Regressor.predict(X_test))

print(f"RMSE on the training data: {np.sqrt(mse_train)}\n")
print(f"RMSE on the test data: {np.sqrt(mse_test)}")
```

RMSE on the training data: 1.736212179500144

RMSE on the test data: 1.6931557303863511

```
In [45]: predictions = sgd_Regressor.predict(X_train)
predictions
```

```
Out[45]: array([20.83092201, 16.98209946, 13.08562483, 10.46983514, 20.49816964,
      8.35172971, 22.99065694,  7.92898758, 12.89250263, 10.94663009,
      11.69469853,  9.74616642, 16.86531641, 17.82863955, 15.65632746,
      18.1560006 , 15.74474228, 19.26238095, 17.16189635, 21.46597565,
      11.22093485, 14.56436992, 11.19878474, 18.05349298, 10.59063009,
      17.13105593, 13.58123738, 23.28181505, 11.83876989, 22.89710391,
      8.03711083, 18.84841078, 23.98459618, 20.8480349 , 18.74516618,
      16.7062133 , 14.10901218, 11.89679293, 19.26639286, 15.41718098,
      15.76495864, 10.73089153, 19.94626756, 12.88860933, 21.03388961,
      12.00610823,  9.63084616, 19.10759099, 15.97239795, 18.33038211,
      10.42885453, 20.87604903, 23.92319895, 18.17587404, 18.63579802,
      15.29954235, 16.50917374,  9.597705 , 17.50382117, 20.72948784,
      17.97639279,  6.48877496,  7.1617715 , 14.75090875, 19.07236223,
      18.70646869, 20.39764391, 22.63501576, 17.99699848, 20.76289973,
      18.73973319, 21.72604023, 12.47680368, 17.13455989, 16.80850305,
      10.49720683, 17.36224435, 17.20179062, 19.37838534, 19.941743 ,
      13.7592767 , 19.63098862, 20.36956403, 10.2576534 , 12.21810734,
      10.64722558, 13.66340531, 17.71731357,  7.35996143, 13.59744183,
      21.01851667, 17.07978772, 15.96039275,  6.43692835, 11.35304443,
      19.09627636, 15.96970787, 19.12563638, 23.57611186, 10.40814313,
      24.28103989, 11.46206967, 16.48494721, 24.21933025, 13.79923027,
      21.5817974 , 10.27725612, 15.93186596, 10.92203184, 21.10266003,
      18.54288271, 12.68336978, 17.97864068, 14.40979783, 16.08400754,
      7.05867431,  9.92881195, 14.39493754, 13.29568373, 18.18377279,
      11.90522841, 20.59119067, 10.57078544,  9.78281628, 12.5874696 ,
      16.7511289 , 16.60111015, 14.0692941 , 15.28767338, 18.74103095,
      8.88982734, 18.06671903, 19.29540996, 21.24305769, 12.62460977,
      8.12902962, 19.11051654, 19.83855126, 14.93838653, 20.57966462])
```

Base Variant of SGDRegressor Pipeline

```
In [95]: sgd_pipeline = Pipeline([("feature scaling", MinMaxScaler()), ("sgd", SGDRegressor())])
sgd_pipeline.fit(X_train, y_train)
```

```
Out[95]: Pipeline(steps=[('feature scaling', MinMaxScaler()), ('sgd', SGDRegressor())])
```

```
In [96]: mse_train = mean_squared_error(y_train, sgd_pipeline.predict(X_train))
mse_test = mean_squared_error(y_test, sgd_pipeline.predict(X_test))

print(f"RMSE on the training data: {np.sqrt(mse_train)}\n")
print(f"RMSE on the test data: {np.sqrt(mse_test)}")
```

RMSE on the training data: 1.7342190466316927

RMSE on the test data: 1.688588328933666

Even after scaling the data, the error is still high as per the industry standards.

- Follow the below steps to minimize the error.
- **STEP 1:** Instantiate the SGDRegressor() with warm_start = true and tolerance = - infinity.
- **STEP 2:** Train SGD Step by Step and record the loss in each step.
- **STEP 3:** Plot the learning curves and see if there are any issues with the data.

```
In [97]: eta0 = 1e-2

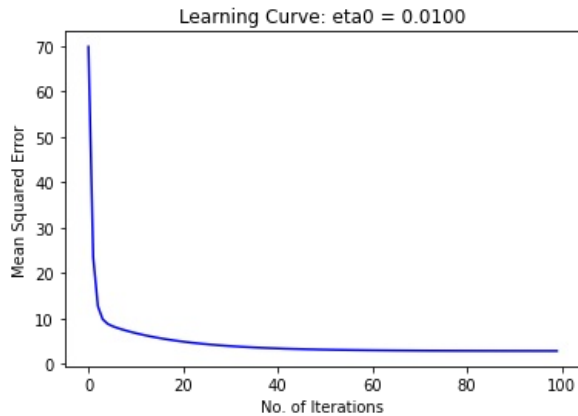
sgd_pipeline = Pipeline([("feature scaling", MinMaxScaler()),
                          ("sgd", SGDRegressor(max_iter = 1, early_stopping = True,
                                                  tol = -np.infty, warm_start = True, random_state=42))])

loss = []

for epoch in range(100):
    sgd_pipeline.fit(X_train, y_train)
    loss.append(mean_squared_error(y_train, sgd_pipeline.predict(X_train)))
```

```
plt.plot(np.arange(len(loss)), loss, 'b-')
plt.xlabel('No. of Iterations')
plt.ylabel('Mean Squared Error')
plt.title(f'Learning Curve: eta0 = {eta0:.4f}')
```

Out[97]: Text(0.5, 1.0, 'Learning Curve: eta0 = 0.0100')



```
In [98]: eta0 = 1e-1

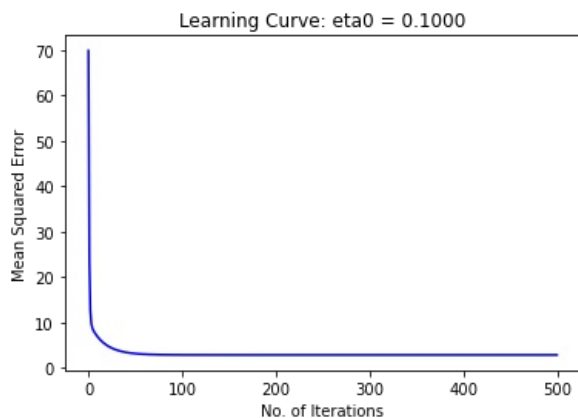
sgd_pipeline = Pipeline([("feature scaling", MinMaxScaler()),
                          ("sgd", SGDRegressor(max_iter = 1, early_stopping = True,
                                                tol = -np.infty, warm_start = True, random_state=42))])

loss = []

for epoch in range(500):
    sgd_pipeline.fit(X_train, y_train)
    loss.append(mean_squared_error(y_train, sgd_pipeline.predict(X_train)))

plt.plot(np.arange(len(loss)), loss, 'b-')
plt.xlabel('No. of Iterations')
plt.ylabel('Mean Squared Error')
plt.title(f'Learning Curve: eta0 = {eta0:.4f}')
```

Out[98]: Text(0.5, 1.0, 'Learning Curve: eta0 = 0.1000')



- This is an ideal learning curve, where the training loss reduces monotonically as the training progresses.

```
In [99]: print('No. of. iterations before reaching the convergence criteria: ', sgd_pipeline[-1].n_iter_)
print('No. of Weight updates: ', sgd_pipeline[-1].t_)
```

No. of. iterations before reaching the convergence criteria: 1
No. of Weight updates: 141.0

```
In [100]: mse_train = mean_squared_error(y_train, sgd_pipeline.predict(X_train))
mse_test = mean_squared_error(y_test, sgd_pipeline.predict(X_test))

print(f"RMSE on the training data: {np.sqrt(mse_train)}\n")
print(f"RMSE on the test data: {np.sqrt(mse_test)}")
```

RMSE on the training data: 1.6937727049258322

RMSE on the test data: 1.539532701664419

Key points

Fixing the learning rates through validation curves

- Step 1: Provide the list of values to be tried for Hyper-parameters

- Step 2: Instantiate an object of validation_curve with estimator, training features and label. Set 'scoring' parameter for relevant scores.
- Step 3: Convert scores to error
- Step 4: Plot Validation curve with Hyper-parameter on X-axis and error on Y-axis
- Step 5: Fix the Hyper-parameter value where the test error is the least.

Predicting the outcomes and residue calculation.

```
In [101...] predictions = sgd_pipeline.predict(X_test)
predictions
```

```
Out[101]: array([16.95471138, 20.3451164 , 23.63569173,  9.28733082, 21.82715094,
        12.52634403, 21.11434896,  8.76614239, 17.26259251, 16.660228 ,
         9.09785065,  8.4966473 , 17.91796638,  8.22794286, 12.63909645,
        14.88669215,  8.14618303, 17.96097387, 11.02904015, 20.54756258,
        20.61212659, 12.2951041 , 11.06064182, 22.20211371,  9.55927491,
         7.97080787, 20.84327574, 13.90043339, 10.80152967,  8.11342759,
        15.95643033, 10.71916393, 20.7090977 , 10.26662047, 21.49754334,
        21.28853446, 12.305904 , 22.65731623, 12.72394709,  6.52152877,
        11.9407911 , 15.40889292,  9.94159103,  9.5349432 , 17.2466592 ,
         7.31652233, 10.33331787, 15.29812504, 11.11814278, 11.82188239,
        13.90486271, 14.7173825 , 10.35597328,  9.28128326,  9.08803035,
        12.46138839, 10.58756895, 24.8986902 ,  7.99163497, 15.92960633])
```

```
In [107...] actual = y_test
```

```
In [108...] result = pd.DataFrame()
result['Actual Value'] = actual
result['Predicted Value'] = predictions
result['Residue'] = actual - predictions
```

```
In [109...] result.head()
```

```
Out[109]:
```

| | Actual Value | Predicted Value | Residue |
|-----|--------------|-----------------|-----------|
| 95 | 16.9 | 16.954711 | -0.054711 |
| 15 | 22.4 | 20.345116 | 2.054884 |
| 30 | 21.4 | 23.635692 | -2.235692 |
| 158 | 7.3 | 9.287331 | -1.987331 |
| 128 | 24.7 | 21.827151 | 2.872849 |

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GROUP 21

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Importing Essential libraries

```
In [258.. import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

from sklearn.model_selection import train_test_split

from sklearn.preprocessing import MinMaxScaler
from sklearn.feature_selection import SelectKBest, f_regression

import warnings
warnings.filterwarnings('ignore')

df = pd.read_csv('/Users/beingrampopuri/Downloads/mtcars.csv')
```

Querying the first 5 records of the data frame.

```
In [259.. df.head()
```

```
Out[259]:
```

| | Unnamed: 0 | mpg | cyl | disp | hp | drat | wt | qsec | vs | am | gear | carb |
|---|-------------------|------|-----|-------|-----|------|-------|-------|----|----|------|------|
| 0 | Mazda RX4 | 21.0 | 6 | 160.0 | 110 | 3.90 | 2.620 | 16.46 | 0 | 1 | 4 | 4 |
| 1 | Mazda RX4 Wag | 21.0 | 6 | 160.0 | 110 | 3.90 | 2.875 | 17.02 | 0 | 1 | 4 | 4 |
| 2 | Datsun 710 | 22.8 | 4 | 108.0 | 93 | 3.85 | 2.320 | 18.61 | 1 | 1 | 4 | 1 |
| 3 | Hornet 4 Drive | 21.4 | 6 | 258.0 | 110 | 3.08 | 3.215 | 19.44 | 1 | 0 | 3 | 1 |
| 4 | Hornet Sportabout | 18.7 | 8 | 360.0 | 175 | 3.15 | 3.440 | 17.02 | 0 | 0 | 3 | 2 |

- It has been observed that one of the columns is not labelled. The below snippet handles such cases.

```
In [260.. df.rename( columns={'Unnamed: 0':'Vehicle Model'}, inplace=True )
```

```
In [261.. df.head(10)
```

```
Out[261]:
```

| | Vehicle Model | mpg | cyl | disp | hp | drat | wt | qsec | vs | am | gear | carb |
|---|-------------------|------|-----|-------|-----|------|-------|-------|----|----|------|------|
| 0 | Mazda RX4 | 21.0 | 6 | 160.0 | 110 | 3.90 | 2.620 | 16.46 | 0 | 1 | 4 | 4 |
| 1 | Mazda RX4 Wag | 21.0 | 6 | 160.0 | 110 | 3.90 | 2.875 | 17.02 | 0 | 1 | 4 | 4 |
| 2 | Datsun 710 | 22.8 | 4 | 108.0 | 93 | 3.85 | 2.320 | 18.61 | 1 | 1 | 4 | 1 |
| 3 | Hornet 4 Drive | 21.4 | 6 | 258.0 | 110 | 3.08 | 3.215 | 19.44 | 1 | 0 | 3 | 1 |
| 4 | Hornet Sportabout | 18.7 | 8 | 360.0 | 175 | 3.15 | 3.440 | 17.02 | 0 | 0 | 3 | 2 |
| 5 | Valiant | 18.1 | 6 | 225.0 | 105 | 2.76 | 3.460 | 20.22 | 1 | 0 | 3 | 1 |
| 6 | Duster 360 | 14.3 | 8 | 360.0 | 245 | 3.21 | 3.570 | 15.84 | 0 | 0 | 3 | 4 |
| 7 | Merc 240D | 24.4 | 4 | 146.7 | 62 | 3.69 | 3.190 | 20.00 | 1 | 0 | 4 | 2 |
| 8 | Merc 230 | 22.8 | 4 | 140.8 | 95 | 3.92 | 3.150 | 22.90 | 1 | 0 | 4 | 2 |
| 9 | Merc 280 | 19.2 | 6 | 167.6 | 123 | 3.92 | 3.440 | 18.30 | 1 | 0 | 4 | 4 |

- Fetching the insights of the data frame as a whole.

```
In [262.. df.info()
```

```

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 32 entries, 0 to 31
Data columns (total 12 columns):
#   Column          Non-Null Count  Dtype
---  -
0   Vehicle Model    32 non-null     object
1   mpg              32 non-null     float64
2   cyl              32 non-null     int64
3   disp             32 non-null     float64
4   hp               32 non-null     int64
5   drat             32 non-null     float64
6   wt               32 non-null     float64
7   qsec             32 non-null     float64
8   vs               32 non-null     int64
9   am               32 non-null     int64
10  gear             32 non-null     int64
11  carb             32 non-null     int64
dtypes: float64(5), int64(6), object(1)
memory usage: 3.1+ KB

```

Obtaining the statistical parameters of each individual feature.

```
In [263]: df.describe()
```

```
Out[263]:
```

| | mpg | cyl | disp | hp | drat | wt | qsec | vs | am | gear | carb |
|-------|-----------|-----------|------------|------------|-----------|-----------|-----------|-----------|-----------|-----------|---------|
| count | 32.000000 | 32.000000 | 32.000000 | 32.000000 | 32.000000 | 32.000000 | 32.000000 | 32.000000 | 32.000000 | 32.000000 | 32.0000 |
| mean | 20.090625 | 6.187500 | 230.721875 | 146.687500 | 3.596563 | 3.217250 | 17.848750 | 0.437500 | 0.406250 | 3.687500 | 2.8125 |
| std | 6.026948 | 1.785922 | 123.938694 | 68.562868 | 0.534679 | 0.978457 | 1.786943 | 0.504016 | 0.498991 | 0.737804 | 1.6152 |
| min | 10.400000 | 4.000000 | 71.100000 | 52.000000 | 2.760000 | 1.513000 | 14.500000 | 0.000000 | 0.000000 | 3.000000 | 1.0000 |
| 25% | 15.425000 | 4.000000 | 120.825000 | 96.500000 | 3.080000 | 2.581250 | 16.892500 | 0.000000 | 0.000000 | 3.000000 | 2.0000 |
| 50% | 19.200000 | 6.000000 | 196.300000 | 123.000000 | 3.695000 | 3.325000 | 17.710000 | 0.000000 | 0.000000 | 4.000000 | 2.0000 |
| 75% | 22.800000 | 8.000000 | 326.000000 | 180.000000 | 3.920000 | 3.610000 | 18.900000 | 1.000000 | 1.000000 | 4.000000 | 4.0000 |
| max | 33.900000 | 8.000000 | 472.000000 | 335.000000 | 4.930000 | 5.424000 | 22.900000 | 1.000000 | 1.000000 | 5.000000 | 8.0000 |

Splitting the data frame into Features and Output Values.

```
In [264]: X = df.iloc[:, 2:]
y = df['mpg']
```

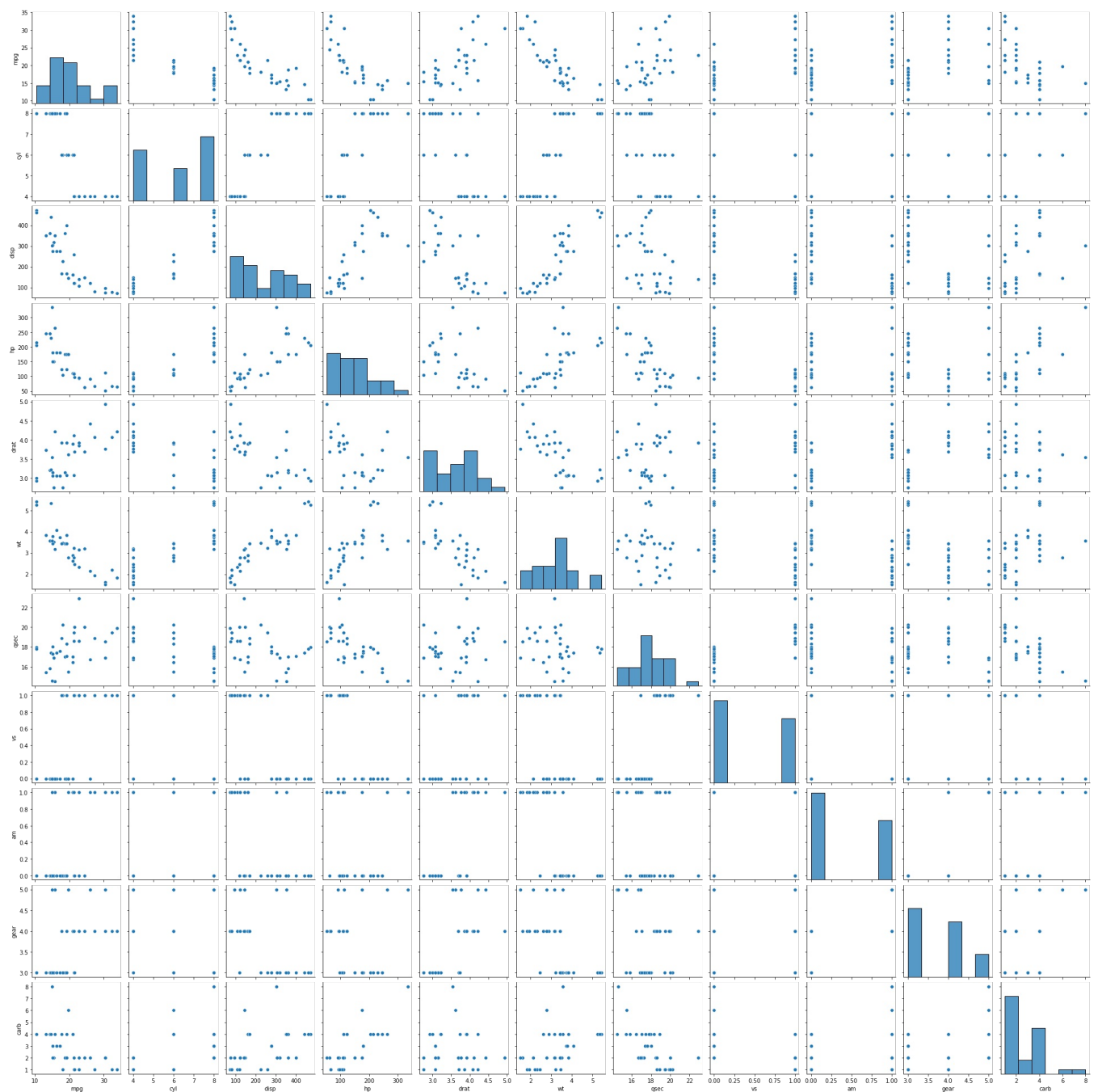
- Creating Test and Training sets with 15% of the data labelled as test set.

```
In [265]: x_train, x_test, y_train, y_test = train_test_split(X, y, test_size = 0.15, random_state=42)
```

- Plotting the relationship between each individual features.

```
In [266]: sns.pairplot(df)
```

```
Out[266]: <seaborn.axisgrid.PairGrid at 0x7ff0582b4850>
```

- Fetching the Correlation matrix that Quantifies the relationship between any two features.
- The below `corr()` function makes use of Pearson's correlation coefficient.
- Greater the magnitude, stronger is the relation between the variables.

- Positive Correlation: As X1 increases, X2 increases.
- Negative Correlation: As X1 increases, X2 decreases.

```
In [267]: '''
To choose 3 most dominant features to carry out multiple linear regression, we need to make use of
CORRELATION COEFFICIENT.

From this below matrix, we need to pick the features that has the strongest relationship with the output variab
which is mpg (miles per gallon).

The direction of the relationship could be either POSITIVE or NEGATIVE.

'''
```

```
Out[267]: '\nTo choose 3 most dominant features to carry out multiple linear regression, we need to make use of \nCORREL
ATION COEFFICIENT. \n\nFrom this below matrix, we need to pick the features that has the strongest relationshi
p with the output variable, \nwhich is mpg (miles per gallon).\n\nThe direction of the relationship could be e
ither POSITIVE or NEGATIVE.\n\n'
```

```
In [268]: df.corr()
```

```
Out[268]:
```

| | mpg | cyl | disp | hp | drat | wt | qsec | vs | am | gear | carb |
|------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| mpg | 1.000000 | -0.852162 | -0.847551 | -0.776168 | 0.681172 | -0.867659 | 0.418684 | 0.664039 | 0.599832 | 0.480285 | -0.550925 |
| cyl | -0.852162 | 1.000000 | 0.902033 | 0.832447 | -0.699938 | 0.782496 | -0.591242 | -0.810812 | -0.522607 | -0.492687 | 0.526988 |
| disp | -0.847551 | 0.902033 | 1.000000 | 0.790949 | -0.710214 | 0.887980 | -0.433698 | -0.710416 | -0.591227 | -0.555569 | 0.394977 |
| hp | -0.776168 | 0.832447 | 0.790949 | 1.000000 | -0.448759 | 0.658748 | -0.708223 | -0.723097 | -0.243204 | -0.125704 | 0.749812 |
| drat | 0.681172 | -0.699938 | -0.710214 | -0.448759 | 1.000000 | -0.712441 | 0.091205 | 0.440278 | 0.712711 | 0.699610 | -0.090790 |
| wt | -0.867659 | 0.782496 | 0.887980 | 0.658748 | -0.712441 | 1.000000 | -0.174716 | -0.554916 | -0.692495 | -0.583287 | 0.427606 |
| qsec | 0.418684 | -0.591242 | -0.433698 | -0.708223 | 0.091205 | -0.174716 | 1.000000 | 0.744535 | -0.229861 | -0.212682 | -0.656249 |
| vs | 0.664039 | -0.810812 | -0.710416 | -0.723097 | 0.440278 | -0.554916 | 0.744535 | 1.000000 | 0.168345 | 0.206023 | -0.569607 |
| am | 0.599832 | -0.522607 | -0.591227 | -0.243204 | 0.712711 | -0.692495 | -0.229861 | 0.168345 | 1.000000 | 0.794059 | 0.057534 |
| gear | 0.480285 | -0.492687 | -0.555569 | -0.125704 | 0.699610 | -0.583287 | -0.212682 | 0.206023 | 0.794059 | 1.000000 | 0.274073 |
| carb | -0.550925 | 0.526988 | 0.394977 | 0.749812 | -0.090790 | 0.427606 | -0.656249 | -0.569607 | 0.057534 | 0.274073 | 1.000000 |

We can make use of the sklearn's - SelectKBest class that automates the feature selection process.

- There are multiple scores that can be used to pick the best 'k' features namely: p-value, F_score, Chi_score etc.
- Here, I am making use of the chi score as a performance metric.

```
In [269]: from sklearn.feature_selection import SelectKBest, chi2, f_regression
```

- Initializing the Object and defining the parameters as required.
- fit_transform(): This method will help us with two things:
 1. Fitting the training set to the regression curve,
 2. Scaling the data and transforming the data.

```
In [270]: x_train_new = SelectKBest(score_func = f_regression,k=3).fit_transform(x_train, y_train)
```

```
In [271]: x_train_new.shape
```

```
Out[271]: (27, 3)
```

Since we have chosen 3 features in our argument, in our resultant array we got the values pertaining to the 3 features from the x_train dataframe.

```
In [272]: x_train_new
```

```
Out[272]: array([[ 6.    , 167.6   ,  3.44  ],
       [ 8.    , 301.   ,  3.57  ],
       [ 4.    ,  79.   ,  1.935 ],
       [ 8.    , 275.8   ,  3.73  ],
       [ 6.    , 160.   ,  2.62  ],
       [ 8.    , 360.   ,  3.44  ],
       [ 8.    , 440.   ,  5.345 ],
       [ 6.    , 225.   ,  3.46  ],
       [ 8.    , 275.8   ,  3.78  ],
       [ 8.    , 275.8   ,  4.07  ],
       [ 8.    , 350.   ,  3.84  ],
       [ 6.    , 160.   ,  2.875 ],
       [ 4.    , 108.   ,  2.32  ],
       [ 4.    , 120.3   ,  2.14  ],
       [ 6.    , 258.   ,  3.215 ],
       [ 8.    , 318.   ,  3.52  ],
       [ 4.    ,  95.1   ,  1.513 ],
       [ 8.    , 304.   ,  3.435 ],
       [ 4.    ,  75.7   ,  1.615 ],
       [ 4.    , 121.   ,  2.78  ],
       [ 4.    , 120.1   ,  2.465 ],
       [ 4.    , 146.7   ,  3.19  ],
       [ 6.    , 167.6   ,  3.44  ],
       [ 8.    , 472.   ,  5.25  ],
       [ 8.    , 351.   ,  3.17  ],
       [ 4.    ,  71.1   ,  1.835 ],
       [ 8.    , 360.   ,  3.57  ]])
```

```
In [273]: x_train_new.shape
```

```
Out[273]: (27, 3)
```

Observation: Though we have secured the feature values, it is a bit ambiguous what those 3 features are.

- We need to compare the values with those in the dataset, to arrive at the desired and the most important features.

```
In [274]: x_train.sort_index()
```

```
Out[274]:
```

| | cyl | disp | hp | drat | wt | qsec | vs | am | gear | carb |
|----|-----|-------|-----|------|-------|-------|----|----|------|------|
| 0 | 6 | 160.0 | 110 | 3.90 | 2.620 | 16.46 | 0 | 1 | 4 | 4 |
| 1 | 6 | 160.0 | 110 | 3.90 | 2.875 | 17.02 | 0 | 1 | 4 | 4 |
| 2 | 4 | 108.0 | 93 | 3.85 | 2.320 | 18.61 | 1 | 1 | 4 | 1 |
| 3 | 6 | 258.0 | 110 | 3.08 | 3.215 | 19.44 | 1 | 0 | 3 | 1 |
| 4 | 8 | 360.0 | 175 | 3.15 | 3.440 | 17.02 | 0 | 0 | 3 | 2 |
| 5 | 6 | 225.0 | 105 | 2.76 | 3.460 | 20.22 | 1 | 0 | 3 | 1 |
| 6 | 8 | 360.0 | 245 | 3.21 | 3.570 | 15.84 | 0 | 0 | 3 | 4 |
| 7 | 4 | 146.7 | 62 | 3.69 | 3.190 | 20.00 | 1 | 0 | 4 | 2 |
| 9 | 6 | 167.6 | 123 | 3.92 | 3.440 | 18.30 | 1 | 0 | 4 | 4 |
| 10 | 6 | 167.6 | 123 | 3.92 | 3.440 | 18.90 | 1 | 0 | 4 | 4 |
| 11 | 8 | 275.8 | 180 | 3.07 | 4.070 | 17.40 | 0 | 0 | 3 | 3 |
| 12 | 8 | 275.8 | 180 | 3.07 | 3.730 | 17.60 | 0 | 0 | 3 | 3 |
| 13 | 8 | 275.8 | 180 | 3.07 | 3.780 | 18.00 | 0 | 0 | 3 | 3 |
| 14 | 8 | 472.0 | 205 | 2.93 | 5.250 | 17.98 | 0 | 0 | 3 | 4 |
| 16 | 8 | 440.0 | 230 | 3.23 | 5.345 | 17.42 | 0 | 0 | 3 | 4 |
| 18 | 4 | 75.7 | 52 | 4.93 | 1.615 | 18.52 | 1 | 1 | 4 | 2 |
| 19 | 4 | 71.1 | 65 | 4.22 | 1.835 | 19.90 | 1 | 1 | 4 | 1 |
| 20 | 4 | 120.1 | 97 | 3.70 | 2.465 | 20.01 | 1 | 0 | 3 | 1 |
| 21 | 8 | 318.0 | 150 | 2.76 | 3.520 | 16.87 | 0 | 0 | 3 | 2 |
| 22 | 8 | 304.0 | 150 | 3.15 | 3.435 | 17.30 | 0 | 0 | 3 | 2 |
| 23 | 8 | 350.0 | 245 | 3.73 | 3.840 | 15.41 | 0 | 0 | 3 | 4 |
| 25 | 4 | 79.0 | 66 | 4.08 | 1.935 | 18.90 | 1 | 1 | 4 | 1 |
| 26 | 4 | 120.3 | 91 | 4.43 | 2.140 | 16.70 | 0 | 1 | 5 | 2 |
| 27 | 4 | 95.1 | 113 | 3.77 | 1.513 | 16.90 | 1 | 1 | 5 | 2 |
| 28 | 8 | 351.0 | 264 | 4.22 | 3.170 | 14.50 | 0 | 1 | 5 | 4 |
| 30 | 8 | 301.0 | 335 | 3.54 | 3.570 | 14.60 | 0 | 1 | 5 | 8 |
| 31 | 4 | 121.0 | 109 | 4.11 | 2.780 | 18.60 | 1 | 1 | 4 | 2 |

Based on the Chi Square Test, we found that disp, hp, carb are the best 3 features.

```
In [275... from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import SGDRegressor
from sklearn.metrics import mean_squared_error

from sklearn.preprocessing import MinMaxScaler

import seaborn as sns
```

```
In [276... X1 = df[['disp', 'hp', 'carb']]
y1 = df['mpg']
```

```
In [277... x_train, x_test, y_train, y_test = train_test_split(X1, y1, test_size=0.3, random_state=42)
```

Fitting the model based on x_train and y_train

```
In [278... regression = LinearRegression()
regression.fit(x_train, y_train)
```

```
Out[278]: LinearRegression()
```

Weights and y-intercept values

```
In [279... w = regression.coef_
b = regression.intercept_
w, b
```

```
Out[279]: (array([-0.03608767, -0.00732251, -0.60961123]), 30.9993068086036)
```

Equation to predict the output : mpg

```
In [280... print(f"The multiple linear regression model based on the training data is \n")
print(f"predicted_mpg = {np.round(w[0],3)} * disp {np.round(w[1], 3)} * hp {np.round(w[2], 3)} * carb + {np.r
```

The multiple linear regression model based on the training data is

predicted_mpg = -0.036 * disp -0.007 * hp -0.61 * carb + 30.999

```
In [281... x_test
```

```
Out[281]:
```

| | disp | hp | carb |
|----|-------|-----|------|
| 29 | 145.0 | 175 | 6 |
| 15 | 460.0 | 215 | 4 |
| 24 | 400.0 | 175 | 2 |
| 17 | 78.7 | 66 | 1 |
| 8 | 140.8 | 95 | 2 |
| 9 | 167.6 | 123 | 4 |
| 30 | 301.0 | 335 | 8 |
| 25 | 79.0 | 66 | 1 |
| 12 | 275.8 | 180 | 3 |
| 0 | 160.0 | 110 | 4 |

```
In [282... y_test
```

```
Out[282]:
```

| | |
|----|------|
| 29 | 19.7 |
| 15 | 10.4 |
| 24 | 19.2 |
| 17 | 32.4 |
| 8 | 22.8 |
| 9 | 19.2 |
| 30 | 15.0 |
| 25 | 27.3 |
| 12 | 17.3 |
| 0 | 21.0 |

Name: mpg, dtype: float64

```
In [283... arr = np.array(x_test[['disp', 'hp', 'carb']])

x_test['Predictions'] = regression.predict(arr)
```

```
x_test
```

```
Out[283]:
```

| | disp | hp | carb | Predictions |
|----|-------|-----|------|-------------|
| 29 | 145.0 | 175 | 6 | 20.827487 |
| 15 | 460.0 | 215 | 4 | 10.386192 |
| 24 | 400.0 | 175 | 2 | 14.063575 |
| 17 | 78.7 | 66 | 1 | 27.066310 |
| 8 | 140.8 | 95 | 2 | 24.003301 |
| 9 | 167.6 | 123 | 4 | 21.611899 |
| 30 | 301.0 | 335 | 8 | 12.806986 |
| 25 | 79.0 | 66 | 1 | 27.055484 |
| 12 | 275.8 | 180 | 3 | 17.899441 |
| 0 | 160.0 | 110 | 4 | 21.981358 |

```
In [284... x_test
```

```
Out[284]:
```

| | disp | hp | carb | Predictions |
|----|-------|-----|------|-------------|
| 29 | 145.0 | 175 | 6 | 20.827487 |
| 15 | 460.0 | 215 | 4 | 10.386192 |
| 24 | 400.0 | 175 | 2 | 14.063575 |
| 17 | 78.7 | 66 | 1 | 27.066310 |
| 8 | 140.8 | 95 | 2 | 24.003301 |
| 9 | 167.6 | 123 | 4 | 21.611899 |
| 30 | 301.0 | 335 | 8 | 12.806986 |
| 25 | 79.0 | 66 | 1 | 27.055484 |
| 12 | 275.8 | 180 | 3 | 17.899441 |
| 0 | 160.0 | 110 | 4 | 21.981358 |

Calculating the difference in predicted values and actual values of mpg

```
In [285... x_test['Actual'] = y_test
x_test['Residue'] = x_test['Predictions'] - x_test['Actual']

x_test
```

```
Out[285]:
```

| | disp | hp | carb | Predictions | Actual | Residue |
|----|-------|-----|------|-------------|--------|-----------|
| 29 | 145.0 | 175 | 6 | 20.827487 | 19.7 | 1.127487 |
| 15 | 460.0 | 215 | 4 | 10.386192 | 10.4 | -0.013808 |
| 24 | 400.0 | 175 | 2 | 14.063575 | 19.2 | -5.136425 |
| 17 | 78.7 | 66 | 1 | 27.066310 | 32.4 | -5.333690 |
| 8 | 140.8 | 95 | 2 | 24.003301 | 22.8 | 1.203301 |
| 9 | 167.6 | 123 | 4 | 21.611899 | 19.2 | 2.411899 |
| 30 | 301.0 | 335 | 8 | 12.806986 | 15.0 | -2.193014 |
| 25 | 79.0 | 66 | 1 | 27.055484 | 27.3 | -0.244516 |
| 12 | 275.8 | 180 | 3 | 17.899441 | 17.3 | 0.599441 |
| 0 | 160.0 | 110 | 4 | 21.981358 | 21.0 | 0.981358 |

Part 4

- Why Mean Squared Error is the best natural algorithm for regression problems?
- Probabilistic Interpretation of Linear Regression

Assumptions of Probabilistic Interpretation of Regression

- There is an error term involved, that handles any neglected values that might have a pertinent role in defining the outcome.
- The error terms are NORMALLY DISTRIBUTED.

Remarks

- The Mean Squared Error derives its inspiration from the Principle of Maximum Likelihood in Probability.
- It states: "From the Normal Equation, we should choose θ so as to make the data as high probability as possible."
- Finding Maximum of $f(x)$ = Minimum of $-f(x)$ which is our loss function in linear regression.

Therefore, the Mean Squared Error best fits the regression problems.

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