AS Level Probability & Statistics 9709/51 - June 2020

Complete Solutions with Marking Scheme

Paper Details:

- Subject: Mathematics 9709/51
- Paper: Probability & Statistics 1
- Session: May/June 2020
- Duration: 1 hour 15 minutes
- Total Marks: 50

Question 1: Geometric Distribution with Dice [5 marks]

Part (a): Show P(score = 4) = 1/12 [1 mark]

Solution:
To get a score of 4 with two fair six-sided dice:

- (1,3): First die = 1, Second die = 3
- (2,2): First die = 2, Second die = 2
- (3,1): First die = 3, Second die = 1

Number of favorable outcomes = 3
Total possible outcomes = 6 × 6 = 36

P(score = 4) = 3/36 = 1/12 ✓

Marking Points:

- **M1:** Identify the 3 ways to get sum of 4 and calculate probability as 3/36 = 1/12

Part (b): Find mean of X [1 mark]

Solution:
X follows a geometric distribution with p = 1/12.

For geometric distribution: E(X) = 1/p
E(X) = 1/(1/12) = **12**

Marking Points:

- **M1:** Apply formula E(X) = 1/p = 12

Part (c): P(first score of 4 on 6th throw) [1 mark]

Solution:
For geometric distribution: $P(X = k) = (1-p)^{(k-1)} \times p$

$P(X = 6) = (1 - 1/12)^{(6-1)} \times (1/12)$
$= (11/12)^5 \times (1/12)$
$=$ **0.0639** (to 3 s.f.)

Marking Points:

- **M1:** Apply geometric probability formula and calculate $(11/12)^5 \times (1/12)$

Part (d): Find P(X < 8) [2 marks]

Solution:
$P(X < 8) = P(X \leq 7) = 1 - P(X \geq 8)$

$P(X \geq 8) = (1-p)^7 = (11/12)^7$

$P(X < 8) = 1 - (11/12)^7 = 1 - 0.4496 =$ **0.550** (to 3 s.f.)

Marking Points:

- **M1:** Use $P(X < 8) = 1 - P(X \geq 8)$
- **A1:** Calculate $1 - (11/12)^7 = 0.550$

Question 2: Permutations of JEWELLERY [6 marks]

Part (a): Es together and Ls together [2 marks]

Solution:
JEWELLERY has: J(1), E(3), W(1), L(2), R(1), Y(1) = 9 letters total

Treat EEE as one unit and LL as one unit:
Units to arrange: [EEE], [LL], J, W, R, Y = 6 units

Number of arrangements = 6! = **720**

Marking Points:

- **M1:** Treat repeated letters as single units
- **A1:** Calculate 6! = 720

Part (b): Ls NOT next to each other [4 marks]

Solution:
Method 1: Total arrangements - Arrangements with Ls together

Total arrangements of JEWELLERY:
$9!/(3!\times2!) = 30{,}240$ (dividing by 3! for Es and 2! for Ls)

Arrangements with Ls together:
Treat LL as one unit: [LL], J, E, E, E, W, R, Y = 8 units
$8!/3! = 6{,}720$ (dividing by 3! for the three Es)

Arrangements with Ls NOT together = 30,240 - 6,720 = **23,520**

Alternative Method:
$7!/3! \times C(8,2) = 840 \times 28 = 23{,}520$ ✓

Marking Points:

- **M1:** Calculate total arrangements: $9!/(3!\times2!) = 30{,}240$
- **M1:** Calculate arrangements with Ls together: $8!/3! = 6{,}720$
- **M1:** Subtract to get 23,520
- **A1:** Final answer: 23,520

Question 3: Hypergeometric and Binomial Distributions [7 marks]

Part (a): Probability distribution table [4 marks]

Solution:
Box contains: 5 jellies, 3 chocolates (8 total)
Choosing 3 sweets randomly.

Let X = number of jellies chosen. $X \in \{0, 1, 2, 3\}$

$P(X = 0) = C(5,0) \times C(3,3) / C(8,3) = 1 \times 1 / 56 =$ **1/56**
$P(X = 1) = C(5,1) \times C(3,2) / C(8,3) = 5 \times 3 / 56 =$ **15/56**
$P(X = 2) = C(5,2) \times C(3,1) / C(8,3) = 10 \times 3 / 56 =$ **30/56**
$P(X = 3) = C(5,3) \times C(3,0) / C(8,3) = 10 \times 1 / 56 =$ **10/56**

X	0	1	2	3
P(X)	1/56	15/56	30/56	10/56

Marking Points:

- **B1:** Correct probability distribution table format
- **M1:** Denominator $8\times7\times6 = 336$, so $C(8,3) = 56$
- **A1:** Any one probability correct

- **A1:** All probabilities correct

Part (b): Binomial probability [3 marks]

Solution:
$X \sim B(10, 0.64)$ where X = number of boxes with more jellies than chocolates

$P(X \leq 7) = 1 - P(X \geq 8) = 1 - [P(X=8) + P(X=9) + P(X=10)]$

$P(X = 8) = C(10,8) \times (0.64)^8 \times (0.36)^2 = 45 \times 0.0281 \times 0.1296 = 0.1642$
$P(X = 9) = C(10,9) \times (0.64)^9 \times (0.36)^1 = 10 \times 0.0180 \times 0.36 = 0.0648$
$P(X = 10) = C(10,10) \times (0.64)^{10} = 0.0115$

$P(X \leq 7) = 1 - (0.1642 + 0.0648 + 0.0115) =$ **0.759**

Marking Points:

- **M1:** Use 1 - P(8,9,10) approach
- **M1:** Calculate the three individual probabilities
- **A1:** Final answer: 0.759

Question 4: Combinatorial Selection [4 marks]

Solution:
Musicians: 8 pianists (P), 4 guitarists (G), 6 violinists (V)
Constraints: $P \geq 2$, $G \geq 1$, $V > G$, $P + G + V = 7$

Valid combinations:

- Case 1: P=2, G=1, V=4 (4>1 ✓)
- Case 2: P=2, G=2, V=3 (3>2 ✓)
- Case 3: P=3, G=1, V=3 (3>1 ✓)
- Case 4: P=4, G=1, V=2 (2>1 ✓)

Calculations:

- Case 1: $C(8,2) \times C(4,1) \times C(6,4) = 28 \times 4 \times 15 = 1{,}680$
- Case 2: $C(8,2) \times C(4,2) \times C(6,3) = 28 \times 6 \times 20 = 3{,}360$
- Case 3: $C(8,3) \times C(4,1) \times C(6,3) = 56 \times 4 \times 20 = 4{,}480$
- Case 4: $C(8,4) \times C(4,1) \times C(6,2) = 70 \times 4 \times 15 = 4{,}200$

Total = 1,680 + 3,360 + 4,480 + 4,200 = 13,720

Question 5: Conditional Probability and Tree Diagrams [8 marks]

Part (a): Tree diagram [2 marks]

Solution:

```
                Fruit (0.3)
Pizza (0.35) ──────────────────────
                No Fruit (0.7)

                Fruit (0.8)
Burger (0.44) ─────────────────────
                No Fruit (0.2)

                Fruit (0)
Curry (0.21) ──────────────────────
                No Fruit (1)
```

Marking Points:

- **B1:** Correct meal probabilities on first branches
- **B1:** Correct conditional fruit probabilities on second branches

Part (b): P(Fruit) [2 marks]

Solution:
$P(\text{Fruit}) = P(\text{Pizza})\times P(\text{Fruit}|\text{Pizza}) + P(\text{Burger})\times P(\text{Fruit}|\text{Burger}) + P(\text{Curry})\times P(\text{Fruit}|\text{Curry})$
$= 0.35 \times 0.3 + 0.44 \times 0.8 + 0.21 \times 0$
$= 0.105 + 0.352 + 0 =$ **0.457**

Marking Points:

- **M1:** Apply law of total probability
- **A1:** Calculate $0.35\times0.3 + 0.44\times0.8 = 0.457$

Part (c): P(not burger | no fruit) [4 marks]

Solution:
$P(B'|F') = P(B' \cap F') / P(F')$

$P(F') = 1 - 0.457 = 0.543$

$P(B' \cap F') = P(\text{Pizza} \cap \text{No Fruit}) + P(\text{Curry} \cap \text{No Fruit})$
$= 0.35 \times 0.7 + 0.21 \times 1 = 0.245 + 0.21 = 0.455$

$P(B'|F') = 0.455 / 0.543 =$ **0.838**

Marking Points:

- **M1:** Set up conditional probability formula

- **M1:** Calculate $P(B' \cap F') = 0.455$
- **M1:** Use $P(F') = 1 - \text{their(b)} = 0.543$
- **A1:** Final answer: 0.838

Question 6: Normal Distribution [9 marks]

Part (a): P(50 < X < 60) for female snakes [4 marks]

Solution:
$X \sim N(54, 6.1^2)$

Standardize: $P(50 < X < 60) = P((50-54)/6.1 < Z < (60-54)/6.1)$
$= P(-0.6557 < Z < 0.9836)$
$= \Phi(0.9836) - \Phi(-0.6557)$
$= \Phi(0.9836) + \Phi(0.6557) - 1$
$= 0.8375 + 0.7441 - 1 =$ **0.582**

Marking Points:

- **M1:** Standardize: $Z = (50-54)/6.1$ and $Z = (60-54)/6.1$
- **A1:** Both z-values correct: -0.6557 and 0.9836
- **M1:** Use $\Phi(0.9836) + \Phi(0.6557) - 1$
- **A1:** Final answer: 0.582

Part (b): Estimate μ and σ for male snakes [5 marks]

Solution:
Given: n=200, 32 have length < 45cm, 17 have length > 56cm

$P(X < 45) = 32/200 = 0.16 \rightarrow P(X \geq 45) = 0.84$
$P(X > 56) = 17/200 = 0.085 \rightarrow P(X \leq 56) = 0.915$

From normal tables:

- For $P(Z \geq z) = 0.84$: $z = -0.994$
- For $P(Z \leq z) = 0.915$: $z = 1.372$

Simultaneous equations:
$(45 - \mu)/\sigma = -0.994 \rightarrow 45 - \mu = -0.994\sigma$
$(56 - \mu)/\sigma = 1.372 \rightarrow 56 - \mu = 1.372\sigma$

Subtracting: $11 = 2.366\sigma$
$\sigma = 4.65$

Substituting: **$\mu = 49.6$**

Marking Points:

- **B1:** $(45 - \mu)/\sigma = -0.994$
- **B1:** $(56 - \mu)/\sigma = 1.372$
- **M1:** One appropriate standardisation equation
- **M1:** Correct algebraic elimination: $11 = 2.366\sigma$
- **A1:** $\sigma = 4.65$, $\mu = 49.6$

Question 7: Histogram and Statistical Measures [11 marks]

Part (a): Draw histogram [5 marks]

Solution:
Calculate frequency densities (frequency ÷ class width):

Class	Frequency	Width	Frequency Density
1-10	18	10	1.8
11-15	24	5	4.8
16-30	30	15	2.0
31-50	20	20	1.0
51-60	8	10	0.8

Draw histogram with frequency density on y-axis, chocolate bars on x-axis.

Part (b): Maximum interquartile range [2 marks]

Solution:
Total frequency = 100
Q1 position = 25th value → in class 11-15
Q3 position = 75th value → in class 31-50

Maximum IQR = 50 - 11 = **39**

Part (c): Mean and standard deviation estimates [4 marks]

Solution:
Using class midpoints:

Class	Midpoint	Frequency	$f \times x$	$f \times x^2$
1-10	5.5	18	99	544.5
11-15	13	24	312	4056
16-30	23	30	690	15870
31-50	40.5	20	810	32805

Class	Midpoint	Frequency	f×x	f×x²
51-60	55.5	8	444	24642
Total		**100**	**2355**	**77918**

Mean = 2355/100 = **23.6**

Variance = 77918/100 - $(23.55)^2$ = 779.18 - 554.6 = 224.6
Standard deviation = $\sqrt{224.6}$ = **15.0**

Marking Points:

- **M1:** Use midpoints correctly
- **A1:** Calculate mean = 23.6
- **M1:** Calculate $\Sigma(fx^2)$ and apply variance formula
- **A1:** Standard deviation = 15.0

Summary

This paper tested key concepts in:

- **Geometric distributions** (dice problems)
- **Permutations** with restrictions
- **Hypergeometric and binomial** distributions
- **Combinatorics** with multiple constraints
- **Conditional probability** and tree diagrams
- **Normal distribution** applications and parameter estimation
- **Histograms** and descriptive statistics

Total Marks: 50
Key Skills: Probability calculations, statistical inference, data representation, distribution applications