CAMBRIDGE
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

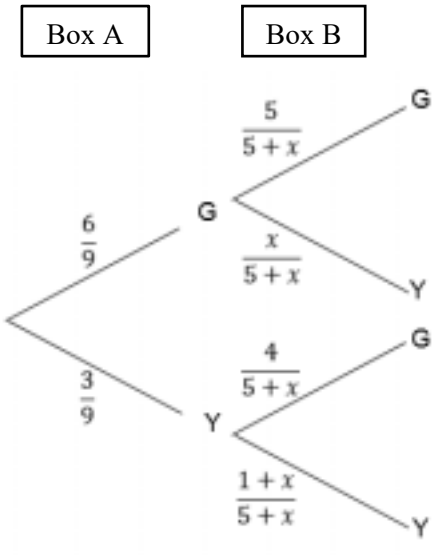
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

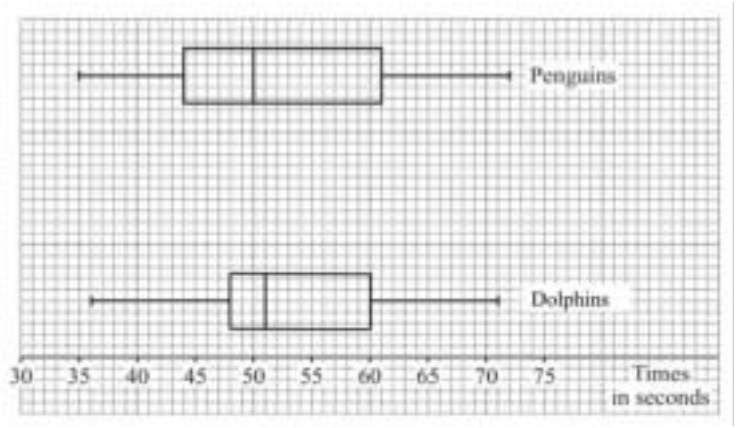
Question	Answer	Marks	Guidance
1(a)	x | 2 | 3 | 4 | 5 | 6 $P(X = x)$ | $\frac{1}{36}$ | $\frac{4}{36}$ | $\frac{10}{36}$ | $\frac{12}{36}$ | $\frac{9}{36}$ | $\frac{1}{36}$ | $\frac{1}{9}$ | $\frac{5}{18}$ | $\frac{1}{3}$ | $\frac{1}{4}$ Decimal equivalent 3sf: 0.0278, 0.111, 0.278, 0.333, 0.25	**B1**	Table with correct X values and at least one correct probability associated with the correct X value. Values need not be in order, lines may not be drawn, may be vertical, X and $P(X)$ may be omitted. Condone any additional X values if probability stated as 0.
		B1	Three other probabilities associated with correct x values, need not be in table, accept unsimplified.
		B1	Five correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3sf. **SC B1** for five probabilities summing to 1 placed in a probability distribution table with the correct x values.
		3	

Question	Answer	Marks	Guidance
1(b)	$\mathrm{E}(X) = \dfrac{1\times2+4\times3+10\times4+12\times5+9\times6}{36} =$ $\dfrac{2}{36}+\dfrac{12}{36}+\dfrac{40}{36}+\dfrac{60}{36}+\dfrac{54}{36}\left[=\dfrac{14}{3}\text{ or }4.67\right]$	**M1**	Accept unsimplified expression or sum of fractions seen. May be calculated in variance. FT *their* table with five probabilities summing to $0.999 \leqslant total \leqslant 1$ $(0 < p < 1)$.
	$\mathrm{Var}(X) = \dfrac{1\times2^2+4\times3^2+10\times4^2+12\times5^2+9\times6^2}{36} - \left(\dfrac{14}{3}\right)^2$	**M1**	Appropriate variance formula using *their* $(\mathrm{E}(X))^2$ value. FT *their* table with 4 or more probabilities. $(0 < p < 1)$ which need not sum to 1. Note: If table is correct, then $\left(\dfrac{824}{36}\text{ or }\dfrac{206}{9}\text{ or }22.89\right) - \left(\dfrac{196}{9}\text{ or }21.78\text{ or }\left(\dfrac{14}{3}\right)^2\right)$ implies M1.
	$\left[=\dfrac{824}{36}-\dfrac{196}{9}=22.89-21.78\right] = \dfrac{10}{9}$	**A1**	$1\dfrac{1}{9}$, $1.11[1]$, $1.\dot{1}$
		3	
1(c)	$\mathrm{P}(X \text{ even} \mid X > 3) = \dfrac{\frac{10}{36}+\frac{9}{36}}{\frac{31}{36}}$	**M1**	$\dfrac{their\,\mathrm{P}(4)+their\,\mathrm{P}(6)}{their\,\mathrm{P}(4)+\mathrm{P}(5)+\mathrm{P}(6)}$, all probabilities $(0 < p < 1)$. If sample space seen in any part of the question, then M1 $\dfrac{their\,19}{their\,31}$.
	$= \dfrac{19}{31}$	**A1**	0.613
		2	

Question	Answer	Marks	Guidance
2(a)	$P\left(Z > \frac{1.93-1.64}{0.25}\right) = P(Z > 1.16)$	**M1**	Using ± standardisation formula with 1.93, 1.64 and 0.25 substituted, not σ^2, not $\sqrt{\sigma}$, no continuity correction.
	$1 - 0.8770$	**M1**	Appropriate area Φ resulting from a standardisation, from final process, must be probability. Note: the appropriate probability answer implies this M1.
	0.123	**A1**	$0.123 \leqslant p < 0.12303$ If M0 M0, **SC B1** if no standardisation shown.
		3	
2(b)	$\frac{1.56-\mu}{\sigma} = -0.44$ $\frac{1.86-\mu}{\sigma} = 0.674$	**B1**	$-0.441 < z_1 < -0.439$ or $0.439 < z_1 < 0.441$ seen.
		B1	$z_2 = 0.674$ or $z_2 = -0.674$ seen, CAO, critical value.
		M1	Use of the ±standardisation formula once with μ, σ equating to a z-value (not 0.33, 0.67. 0.25, 0.75, 0.6293, 0.5987, 0.7486, 0.7734, (1 – 0.44), (1 – 0.674)). Condone continuity correct ±0.005, not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ $\mu = 1.68, \quad \sigma = 0.269$	**M1**	Solve two equations in μ and σ using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ.
		A1	AWRT $\mu = 1.68, \ \sigma = 0.269$. If one or both of the M marks have not been awarded, **SC B1** for both correct.
		5	

Question	Answer	Marks	Guidance
3(a)		**B1**	Correct structure and probabilities for Box A branches.
		B1	Completely correct structure and one correct probability for a Box B branch including label for G or Y.
		B1	Completely correct structure and second correct probability on a Box B branch including label G or Y.
		B1	Completely correct structure and remaining two probabilities correct on Box B branches, including labels for G or Y. **SC B1** if correct shape diagram but only four correct algebraic probs for GG, GY, YG and YY.
		4	
3(b)	P(same colour) = $\frac{6}{9}\times\frac{5}{5+x}+\frac{3}{9}\times\frac{1+x}{5+x}$	**M1**	P(GG) + P(YY) = $\left(\frac{6}{9}\text{ or }\frac{3}{9}\right)\times \textit{their}\left(\frac{5}{5+x}\right)+\left(\frac{3}{9}\text{ or }\frac{6}{9}\right)\times \textit{their}\left(\frac{1+x}{5+x}\right)$
	$\frac{6}{9}\times\frac{5}{5+x}+\frac{3}{9}\times\frac{1+x}{5+x}=\frac{8}{15}$ and arrange as a linear equation	**M1**	$15(x+11)=24(x+5)$ OE Accept sum of their products equated to $\frac{8}{15}$ and rearranged to form a linear equation.
	Solve: $x=5$	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	Penguins | Dolphins 9 5 | 3 | 6 8 5 5 4 2 | 4 | 1 3 8 9 9 9 8 6 0 | 5 | 0 1 4 6 6 8 6 1 | 6 | 0 1 4 2 | 7 | 1 Key: 2|4|1 means 42 seconds for Penguins and 41 seconds for Dolphins	**B1**	Correct stem, ignore extra values (not in reverse, not split). If a split stem-and-leaf plot is used (i.e. stem values are repeated), the remaining B marks are available.
		B1	Correct Penguins labelled on left, leaves in order from right to left and lined up vertically (less than halfway to next column), no commas or other punctuation.
		B1	Correct Dolphins labelled on same diagram, leaves in order and lined up vertically (less than halfway to next column), no commas or other punctuation. If the correct data for Penguins & Dolphins is transposed, treat as a single error in Penguins and condone in Dolphins.
		B1	Correct key for their diagram, need both clubs labelled and 'sec' or 's' stated at least once here, or in leaf headings or title. If two separate diagrams drawn: **SC B1** if both keys meet these criteria.
		4	

Question	Answer	Marks	Guidance
4(b)			
	For Dolphins, median is 51	**B1**	Plotted on box.
	LQ = 48, UQ = 60	**B1**	Plotted on box.
	Correct end points for whiskers and diagram labelled Dolphins	**B1**	Correct end points of whiskers (36 and 71). Whiskers not through box, not drawn at corners of boxes, diagram labelled.
		3	
4(c)	Dolphins have more consistent times than Penguins or Penguins are faster (have faster times) than Dolphins	**B1**	Reason given in context. Can be reference to either the central tendency or spread.
		1	

Question	Answer	Marks	Guidance
5(a)	$\left[(0.35)^4 \times 0.65 =\right]$ 0.00975	**B1**	AWRT
		1	
5(b)	$(0.35)^3 \times (0.65)^2 \times 4$	**M1**	$(0.35)^3 \times (0.65)^2 \times k$, where k is an integer. $1 \leqslant k \leqslant 5$ no + or −
	= 0.0725	**A1**	
		2	
5(c)	**Method 1**		
	[1 – P(5, 6, 7) =] $1 - ({}^7C_5\, 0.65^5\, 0.35^2 + {}^7C_6\, 0.65^6 0.35^1 + 0.65^7)$ [= 1 – (0.29848 + 0.18478 + 0.049022)]	**M1**	One term ${}^7C_x (p)^x (1-p)^{7-x}$, $0 < p < 1, 0 < x < 7$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer
	= 0.468	**B1**	AWRT
	Method 2		
	[P(0, 1, 2, 3, 4) =] $0.35^7 + {}^7C_1\, 0.65^1\, 0.35^6 + {}^7C_2\, 0.65^2\, 0.35^5 + {}^7C_3\, 0.65^3\, 0.35^4 + {}^7C_4\, 0.65^4$ 0.35^3 [0.00064 + 0.00836 + 0.04660 + 0.14424 + 0.26787]	**(M1)**	One term ${}^7C_x (p)^x (1-p)^{7-x}$, $0 < p < 1, 0 < x < 7$.
		(A1)	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.468	**(B1)**	AWRT
		3	

Question	Answer	Marks	Guidance
5(d)	[Mean =] $84\times0.65=54.6$ [Var =] $84\times0.65\times0.35=19.11$	**B1**	54.6 and 19.11 seen, allow unsimplified. May be seen in the standardisation formula ([σ =] 4.371 or $\frac{7\sqrt{39}}{10}$ implies correct variance). Incorrect notation is penalised, but condone use of values in standardisation formula.
	$\mathrm{P}(X>50)=\mathrm{P}\left(Z>\frac{50.5-54.6}{\sqrt{19.11}}\right)$	**M1**	Substituting *their* μ and *their* positive σ into the ±standardising formula (any number), not *their* σ^2, or $\sqrt{\textit{their}\,\sigma}$.
		M1	Use continuity correction 49.5 or 50.5 in *their* standardisation formula. Note: $\frac{\pm4.1}{\sqrt{19.11}}$ or $\pm\frac{4.1}{4.371}$ seen gains M2 BOD.
	$\mathrm{P}(Z>-0.9379)=\Phi(0.9379)$	**M1**	Appropriate area Φ, from final process, must be a probability.
	0.826	**A1**	$0.8255 < p \leq 0.826$
		5	

Question	Answer	Marks	Guidance
6(a)	$\left[\frac{9!}{2!3!}=\right]$ 30240	**B1**	
		1	
6(b)	**Method 1: Number of arrangements with E at each end – Number of arrangements with E at each end and the three Rs together**		
	$\frac{7!}{3!}-5!=$	**B1**	$\frac{7!}{3!}-e$, ${}^{7}\mathrm{P}_{4}-e$, e a positive integer.
		M1	$f-\frac{5!}{r!}, f>120, r=1, 2$
	720	**A1**	If no marks scored **SC B1** for 840 – 120 = 720.
	Method 2: Number of arrangements with E at each end and no Rs together + No of arrangements with E at each end and two Rs together		
	${}^{5}\mathrm{C}_{3}\times 4!+{}^{4}\mathrm{C}_{1}\times 5!$ or $\frac{{}^{5}\mathrm{P}_{3}}{3!}\times 4!+{}^{5}\mathrm{P}_{2}\times 4!$	**(B1)**	One of ${}^{5}\mathrm{C}_{3}\times 4!$, $\frac{{}^{5}\mathrm{P}_{3}}{3!}\times 4!$, ${}^{4}\mathrm{C}_{1}\times 5!$ or ${}^{5}\mathrm{P}_{2}\times 4!$ seen.
		(M1)	$a\times 4!+b\times 5!$ where a and b are integers between 1 and 10 inclusive, or $c\times 4!+d\times 4!$ where c and d are integers between 1 and 20 inclusive.
	240 + 480 = 720	**(A1)**	
		3	

Question	Answer	Marks	Guidance
6(c)	**Method 1**		
	Group of 5 3 Rs 2 Es = 1 3 Rs 1 E = $^2C_1 \times {}^4C_1$ = 8 3 Rs 0 Es = $^4C_2$ = 6 Group of 4 3 Rs 1 E = $^2C_1$ = 2 3 Rs 0 Es = $^4C_1$ = 4	**B1**	Correct no of ways for two correct identified scenarios other than three Rs two Es.
	[Total =] 21	**M1**	No of ways for five correct identified scenarios added or correct.
	[Number of ways of splitting into the two groups =] $^9C_5$ (= 126) seen as a denominator	**M1**	Accept evaluated, accept $^9C_4$.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	**A1**	

Question	Answer	Marks	Guidance
6(c)	**Method 2**		
	3Rs in Group of 5 = ${}^6C_2$ = 15 3Rs in Group of 4 = ${}^6C_1$ = 6	**(B1)**	One correct case evaluated accurately and linked with correct scenario.
	[Total =] 21	**(M1)**	No of ways for two correct scenarios added or correct.
	[Number of ways of splitting into the two groups =] ${}^9C_5$ (= 126) seen as a denominator	**(M1)**	Accept evaluated, accept ${}^9C_4$.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	**(A1)**	
	Method 3: Considering the possible positions of R within the groups		
	3Rs in Group of 5 $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{15}{126}$ 3Rs in Group of 4 $\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{6}{126}$	**(B1)** **(M1)**	For one correct product unsimplified and linked with correct scenario. For second correct product.
	$\frac{15}{126} + \frac{6}{126}$	**(M1)**	For adding probabilities of two correct scenarios or correct.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	**(A1)**	

Question	Answer	Marks	Guidance
6(c)	**Method 4: Probability method**		
	Group of 5 2Es $\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\times\frac{2}{6}\times\frac{1}{5}\times\frac{5!}{3!2!}=\frac{1}{126}$ 1E $\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}\times\frac{2}{5}\times\frac{5!}{3!}=\frac{8}{126}$ 0E $\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\times\frac{4}{6}\times\frac{3}{5}\times\frac{5!}{3!2!}=\frac{6}{126}$ Group of 4 $\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\times\frac{6}{6}\times\frac{4!}{3!}=\frac{6}{126}$	**(B1)** **(M1)**	Two correct probabilities linked with correct scenarios, accept unsimplified. Four probabilities with denominators including a factor of $9\times8\times7\times6\times n$, where n is 1 or 5.
	$\frac{1+8+6+6}{126}$	**(M1)**	Probabilities of four correct scenarios added or correct.
	Probability = $\frac{21}{126}=\frac{1}{6}$ (0.167)	**(A1)**	
		4	