

# AS Level Probability & Statistics 9709 June 2022

## Paper 51 - Complete Solutions

### Paper Information

- **Subject:** Cambridge International AS Level Mathematics
- **Paper:** 9709/51 - Probability & Statistics 1
- **Session:** May/June 2022
- **Time:** 1 hour 15 minutes
- **Total Marks:** 50

### Grade Boundaries

- A: 38/50
- B: 33/50
- C: 27/50
- D: 21/50
- E: 16/50

### Question 1: Arrangements of Letters (6 marks)

**Context:** Find arrangements of the 8 letters in DECEIVED.

#### 1(a) All three Es together and two Ds together [2 marks]

**Solution:**

The word DECEIVED has letters: D, E, C, E, I, V, E, D (2 Ds, 3 Es, 1 C, 1 I, 1 V)

When all 3 Es are together, treat them as one unit: **(EEE)**

When both Ds are together, treat them as one unit: **(DD)**

Units to arrange: (EEE), (DD), C, I, V → 5 units total

Number of arrangements = **5! = 120**

**Answer: 120**

### 1(b) Three Es are NOT all together [4 marks]

#### Solution:

Total arrangements of DECEIVED:

- Total letters = 8, with 2 Ds and 3 Es repeated
- Total arrangements =  $8!/(2! \times 3!) = 40320/12 = 3360$

Arrangements where all 3 Es ARE together:

- Treat (EEE) as one unit: (EEE), (DD), C, I, V
- 5 units with 2 Ds repeated:  $5!/2! = 60$

Arrangements where Es are NOT all together:

$$= 3360 - 60 = 3300$$

**Answer: 3300**

### Question 2: Committee Selection (6 marks)

**Context:** Book Club with 6 men and 8 women. Committee of 5 members. Mr Lan and Mrs Lan are members.

#### 2(a) Exactly one of Mr Lan or Mrs Lan on committee [2 marks]

##### Solution:

**Case 1:** Mr Lan on committee, Mrs Lan not

- Select Mr Lan + 4 from remaining 12 people
- Ways =  $C(12,4) = 495$

**Case 2:** Mrs Lan on committee, Mr Lan not

- Select Mrs Lan + 4 from remaining 12 people
- Ways =  $C(12,4) = 495$

$$\text{Total ways} = 495 + 495 = 990$$

**Answer: 990**

#### 2(b) Mrs Lan on committee with more women than men [4 marks]

##### Solution:

Mrs Lan selected, choose 4 more from 6 men and 7 women.

Possible compositions (including Mrs Lan):

**Case 1:** 3 women + 2 men

- $C(7,2) \times C(6,2) = 21 \times 15 = 315$

**Case 2:** 4 women + 1 man

- $C(7,3) \times C(6,1) = 35 \times 6 = 210$

**Case 3:** 5 women + 0 men

- $C(7,4) \times C(6,0) = 35 \times 1 = 35$

Total ways =  $315 + 210 + 35 = 560$

**Answer: 560**

### Question 3: Travel Time Statistics (9 marks)

**Data:** 2500 students' travel times

- Intervals: [0,20), [20,30), [30,40), [40,60), [60,90)
- Frequencies: 440, 720, 920, 300, 120
- Given mean = 31.44

#### 3(a) Draw histogram [4 marks]

**Solution:**

Frequency density = Frequency  $\div$  Class width

Interval	Frequency	Width	Frequency Density
0-20	440	20	22
20-30	720	10	72
30-40	920	10	92
40-60	300	20	15
60-90	120	30	4

**Answer: Histogram with frequency densities 22, 72, 92, 15, 4**

#### 3(b) Calculate standard deviation [3 marks]

**Solution:**

Using midpoints: 10, 25, 35, 50, 75

Variance =  $\Sigma f(x - \bar{x})^2 / \Sigma f$

Calculations:

- Class 1:  $440 \times (10 - 31.44)^2 = 202256.38$
- Class 2:  $720 \times (25 - 31.44)^2 = 29860.99$
- Class 3:  $920 \times (35 - 31.44)^2 = 11659.71$
- Class 4:  $300 \times (50 - 31.44)^2 = 103342.08$
- Class 5:  $120 \times (75 - 31.44)^2 = 227696.83$

Total = 574816.00

Variance =  $574816/2500 = 229.93$

Standard deviation =  $\sqrt{229.93} = 15.2$

**Answer: 15.2**

### **3(c) Upper quartile class interval [1 mark]**

**Solution:**

Upper quartile position =  $3/4 \times 2500 = 1875$

Cumulative frequencies:

- 0-20: 440
- 20-30: 1160
- 30-40: 2080
- 40-60: 2380
- 60-90: 2500

1875th value falls between 1160 and 2080.

**Answer: 30-40**

### **3(d) Effect of corrections on standard deviation [1 mark]**

**Solution:**

Original errors: 15 instead of 5, 65 instead of 75

- 15 is closer to mean (31.44) than 5
- 65 is closer to mean (31.44) than 75

Both corrections move values further from the mean, increasing the spread.

**Answer: Increase - values move further from mean**

## **Question 4: Probability Distribution with Coins (10 marks)**

**Context:** Jacob has 4 coins - 1 biased ( $P(\text{Head}) = 7/10$ ), 3 fair

$X$  = number of heads,  $P(X=0) = 3/80$ ,  $P(X=4) = 7/80$

### **4(a) Show $a = 1/5$ and find $b, c$ [4 marks]**

**Solution:**

For  $X = 1$  (exactly 1 head):

- Case 1: Biased H, 3 fair T  $\rightarrow (7/10) \times (1/2)^3 = 7/80$
- Case 2: Biased T, 1 fair H  $\rightarrow (3/10) \times C(3,1) \times (1/2)^3 = 9/80$

$$P(X=1) = 7/80 + 9/80 = 16/80 = \mathbf{1/5} \checkmark$$

For  $X = 2$ :

- Case 1: Biased H, 2 fair H  $\rightarrow (7/10) \times C(3,2) \times (1/2)^3 = 21/80$
- Case 2: Biased T, 2 fair H  $\rightarrow (3/10) \times C(3,2) \times (1/2)^3 = 9/80$
- $P(X=2) = 30/80 = \mathbf{3/8}$

For  $X = 3$ :

- Case 1: Biased H, 3 fair H  $\rightarrow (7/10) \times (1/2)^3 = 7/80$
- Case 2: Biased T, 3 fair H  $\rightarrow (3/10) \times (1/2)^3 = 3/80$
- $P(X=3) = 10/80 = \mathbf{1/8}$

**Answer:  $a = 1/5$ ,  $b = 3/8$ ,  $c = 1/8$**

#### **4(b) Find $E(X)$ [1 mark]**

**Solution:**

$$E(X) = 0 \times (3/80) + 1 \times (16/80) + 2 \times (30/80) + 3 \times (10/80) + 4 \times (7/80)$$

$$E(X) = 0 + 16/80 + 60/80 + 30/80 + 28/80 = 134/80 = \mathbf{1.675}$$

**Answer: 1.675**

#### **4(c) Exactly 1 head on fewer than 3 occasions in 10 throws [3 marks]**

**Solution:**

$Y \sim B(10, 1/5)$  where  $Y$  = number of times exactly 1 head occurs

$$P(Y < 3) = P(Y=0) + P(Y=1) + P(Y=2)$$

- $P(Y=0) = (4/5)^{10} = 0.1074$
- $P(Y=1) = 10 \times (1/5) \times (4/5)^9 = 0.2684$
- $P(Y=2) = 45 \times (1/5)^2 \times (4/5)^8 = 0.3020$

$$P(Y < 3) = 0.1074 + 0.2684 + 0.3020 = \mathbf{0.678}$$

**Answer: 0.678**

#### **4(d) First success on 7th or 8th throw [2 marks]**

**Solution:**

Geometric distribution:

- $P(\text{first success on 7th}) = (4/5)^6 \times (1/5) = 0.0524$
- $P(\text{first success on 8th}) = (4/5)^7 \times (1/5) = 0.0419$

$$\text{Total} = 0.0524 + 0.0419 = \mathbf{0.0944}$$

**Answer: 0.0944**

### Question 5: Normal Distribution of Leaves (11 marks)

**Context:** First type  $N(5.2, 1.5^2)$ . Second type: 500 sample,  $46 < 3\text{cm}$ ,  $95 > 8\text{cm}$ .

#### 5(a) $P(\text{length} < 6)$ for first type [2 marks]

**Solution:**

$$X \sim N(5.2, 1.5^2)$$

$$P(X < 6) = P(Z < (6-5.2)/1.5) = P(Z < 0.533)$$

From tables:  $P(Z < 0.533) \approx \mathbf{0.703}$

**Answer: 0.703**

#### 5(b) Find mean and standard deviation for second type [5 marks]

**Solution:**

Let second type be  $N(\mu, \sigma^2)$

From sample:

- $P(X < 3) = 46/500 = 0.092$
- $P(X > 8) = 95/500 = 0.19$

This gives:

- $(3-\mu)/\sigma = -1.33$  (from  $\Phi^{-1}(0.092)$ )
- $(8-\mu)/\sigma = 0.88$  (from  $\Phi^{-1}(0.81)$ )

Solving simultaneously:

- $3 + 1.33\sigma = 8 - 0.88\sigma$
- $2.21\sigma = 5$
- $\sigma = 2.26$
- $\mu = 6.01$

**Answer:  $\mu = 6.01$ ,  $\sigma = 2.26$**

#### 5(c) Expected leaves $> 1\sigma$ from mean in 2000 sample [4 marks]

**Solution:**

$$P(|X - \mu| > \sigma) = P(Z < -1) + P(Z > 1) = 2 \times 0.1587 = 0.3174$$

$$\text{Expected number} = 2000 \times 0.3174 = 634.8 \approx \mathbf{635}$$

**Answer: 635 leaves**

## Question 6: Computer Game Probability (8 marks)

**Context:** 2-level game, max 2 attempts per level

- Level 1:  $P(\text{success 1st}) = 0.6$ ,  $P(\text{success 2nd} \mid \text{fail 1st}) = 0.3$
- Level 2:  $P(\text{success 1st}) = 0.4$ ,  $P(\text{success 2nd} \mid \text{fail 1st}) = 0.2$

### 6(a) Show $P(\text{moves to level 2}) = 0.72$ [1 mark]

**Solution:**

$$P(\text{completes level 1}) = 0.6 + (0.4 \times 0.3) = 0.6 + 0.12 = \mathbf{0.72} \checkmark$$

### 6(b) Find $P(\text{finishes game})$ [3 marks]

**Solution:**

$$P(\text{completes level 2}) = 0.4 + (0.6 \times 0.2) = 0.52$$

$$P(\text{finishes game}) = 0.72 \times 0.52 = \mathbf{0.3744}$$

**Answer: 0.3744**

### 6(c) $P(\text{fails exactly 1 attempt} \mid \text{finishes game})$ [4 marks]

**Solution:**

Ways to fail exactly 1 attempt AND finish:

1. Fail L1 attempt 1, succeed L1 attempt 2, succeed L2 attempt 1:  $0.4 \times 0.3 \times 0.4 = 0.048$
2. Succeed L1 attempt 1, fail L2 attempt 1, succeed L2 attempt 2:  $0.6 \times 0.6 \times 0.2 = 0.072$

$$P(\text{fail exactly 1 AND finish}) = 0.048 + 0.072 = 0.12$$

$$P(\text{fail exactly 1} \mid \text{finish}) = 0.12 / 0.3744 = \mathbf{0.321}$$

**Answer: 0.321**

## Summary of Final Answers

Question	Part	Answer
1	(a)	120
1	(b)	3300
2	(a)	990
2	(b)	560
3	(a)	Histogram: frequency densities 22, 72, 92, 15, 4
3	(b)	15.2
3	(c)	30-40
3	(d)	Increase

Question	Part	Answer
4	(a)	$a = 1/5, b = 3/8, c = 1/8$
4	(b)	1.675
4	(c)	0.678
4	(d)	0.0944
5	(a)	0.703
5	(b)	$\mu = 6.01, \sigma = 2.26$
5	(c)	635
6	(a)	0.72 (shown)
6	(b)	0.3744
6	(c)	0.321