Advanced Control Technology Assignment

MAGNETIC LEVITATION SYSTEM
Assignment 2

Submitted By

Group 25

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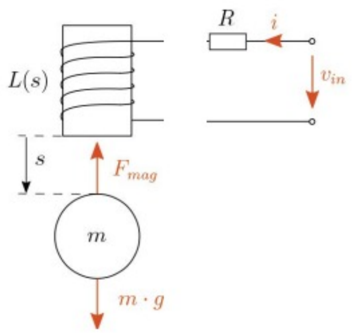
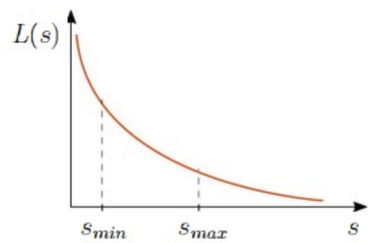
1.1 Problem Statement

Magnetic Levitation System

Consider the depicted model of a magnetic levitation system, which consists of an electromagnet and an iron ball of mass m.





The expression $L(s) = \frac{k}{s}$ with the constant k approximates the inductance of the electromagnet with the ball in its magnetic field in a range of distances $s \in [s_{min}, s_{max}]$. The magnetic force towards the magnet can be calculated with

$$F_{mag} = -\frac{\partial W^*_{mag}(i,s)}{\partial s} \qquad \text{where} \qquad W^*_{mag}(i,s) = \frac{1}{2}L(s)i^2$$

is the magnetic coenergy "stored" in the system. With the states and the input

$x_1 = s$	distance of the ball,	$x_2 = \dot{s}$	velocity of the ball,
$x_3 = i$	current through the electromagnet,	$u = v_{in}$	input voltage,

the nonlinear state differential equations can be written as

$$\dot{\boldsymbol{x}} = \begin{bmatrix} x_2 \\ g - \frac{k}{2m}\frac{x_3^2}{x_1^2} \\ -\frac{R}{k}x_1x_3 + \frac{x_2x_3}{x_1} + \frac{1}{k}x_1u \end{bmatrix}, \qquad y = x_1.$$

1.1.1 Parameters

1. Mass of ball (m) = 1.1Kg
2. Gravitational constant (g) = 1.1 m/s²
3. Inductance (k) = 1.1 Hm
4. Resistance (R) = 1.1 ohm
5. Equilibrium distance (x1*) = 1.1 m

1.1.2 Assignment Tasks

- To find out the eigenvalues of the linearized system and discuss its asymptotic stability.
- To design a state space controller and observer using either linear quadratic or pole placement approach.
- Find and design the Lyapunov function for stability based on the Direct Lyapunov method.
- Design a controller using any one of the methods. (Feedback Linearization, Model Predictive Control, or Rule-Based Fuzzy Systems).
- Analyze the controller design and record the observations.

1.2 Stability of the system

The stability of a system can be defined as its ability to return to an equilibrium state after being subjected to disturbances. The stability of a system can be analyzed with the help of eigenvalues of the matrix. Eigenvalue has two components, a real part and an imaginary part. For a system to be stable all real parts of eigenvalues should be negative or in all other cases the system can be considered as unstable [1]. Eigenvalues can be calculated by

$$|\lambda I - A| = 0$$

For ease of calculation, it was done with the help of Matlab and the obtained eigenvalues are:

$$Eigenvalues = \begin{bmatrix} 1.0182 + 0.0000i \\ -1.0591 + 1.0173i \\ -1.0591 - 1.0173i \end{bmatrix}$$

Asymptotic stability can be defined as the ability of a system if the system to eventually return to its equilibrium state. Asymptotic stability can be described similar to that of stability itself, that is real part of eigenvalues must be negative plane for a system to be asymptotic stable. The positive real part of the first element of the value indicates an unstable system. The negative real part of the next two eigenvalues indicate the stability of the system and the imaginary part indicates oscillatory behavior. The positive real part of the first eigenvalue dominates the other eigenvalues, so it can be concluded that the system is not system **is not asymptotically stable**. [1]

1.3 Lyapunov function for the system

The Lyapunov method is a commonly used method to analyze the stability of a system. It was developed by the Russian scientist Aleksandr Mikhailovich Lyapunov. He proposed two methods, the Linearization method and the Direct method, which is based on the energy concept. In this assignment, the Direct method or the second method is used to analyze the system's stability. [2]
Lyapunov equation is:

$$A^T P + PA = -Q$$

The Lyapunov method uses a candidate function to check the stability of a system. Common candidate functions are quadratic and cubic functions. The stability conditions for a candidate function $V(x)$ are:

1. $V(0) = 0$
2. $V(x) > 0 \quad \forall x \neq 0$
3. $\dot{V}(x) \leq 0 \quad \forall x \neq 0$

1.3.1 Quadratic Function

At first, a Lyapunov quadratic equation was used to check the stability. It can be represented as,

$$V(x) = x^T P x$$

Taking a positive definite function,

$$V(x) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \tag{1}$$

From assignment 1 the equilibrium are:

- $x_1(\text{distance}) = 1.1\,\text{m}$
- $x_2(\text{velocity}) = 0\,\text{m/s}$
- $x_3(\text{current}) = 1.63\,\text{Amp}$
- $u(\text{voltage}) = 1.793\,\text{V}$

Checking stability condition 1 for equation (1).

$$V(0) = c_1 \cdot 0^2 + c_2 \cdot 0^2 + c_3 \cdot 0^2 = 0$$

Stability condition 2,

$$V(x) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) > 0$$

For checking condition 3 the partial derivative of equation(1) has to be calculated,

$$\dot{V}(x) = \frac{\partial V}{\partial x_1}\dot{x}_1 + \frac{\partial V}{\partial x_2}\dot{x}_2 + \frac{\partial V}{\partial x_3}\dot{x}_3 \tag{2}$$

According to the state space representation,

$$\dot{x}_1 = x_2, \tag{3}$$

$$\dot{x}_2 = g - \frac{kx_3^2}{2mx_1^2}, \tag{4}$$

$$\dot{x}_3 = -\frac{Rx_1x_3}{k} + \frac{x_2x_3}{x_1} + \frac{x_1u}{k}. \tag{5}$$

$$\frac{\partial V}{\partial x_1} = 2x_1, \quad \frac{\partial V}{\partial x_2} = 2x_2, \quad \frac{\partial V}{\partial x_3} = 2x_3$$

Substituting state space values and partial derivative values in equation (2),

$$\dot{V}(x) = x_1x_2 + x_2g - \frac{1}{2m}x_1^2kx_2x_3^2 - kRx_1x_3^2 + x_1x_2x_3^2 + kx_1x_3u \tag{6}$$

Due to the dependency on constants and sign, it has to be concluded that equation (6) do not satisfy condition 3.

1.3.2 Cubic Function

Since quadratic function was non compliant with the Lyapunov stability condition, a cubic equation was considered.

$$V(x) = \frac{1}{3}(x_1^3 + x_2^3 + x_3^3) \tag{7}$$

Checking condition 1 and condition 2 on equation (7), V(0) = 0 and V(x) is greater than zero for all non zero values of x. For condition 3 taking the time derivative,

$$\dot{V}(x) = \frac{\partial x_1}{\partial V}\dot{x}_1 + \frac{\partial x_2}{\partial V}\dot{x}_2 + \frac{\partial x_3}{\partial V}\dot{x}_3 \tag{8}$$

$$\frac{\partial V}{\partial x_1} = 3x_1^2, \quad \frac{\partial 3x_2^2}{\partial V} = 3x_2^2, \quad \frac{\partial x_3}{\partial V} = 3x_3^2$$

Substituting state variable values and partial derivatives in equation (8),

$$\dot{V}(x) = c_1x_1^2x_2 + c_2x_2^2g - \frac{c_2}{2m}x_2^2x_1^2kx_3^2 - c_3x_3^3kRx_1 + c_3x_3^3x_1x_2 + c_3x_3^2kx_1u \tag{9}$$

As in the quadratic function, the cubic function is also non compliant to condition 3. Since both equations satisfy the first two conditions but not the third condition, it can be concluded that the system is not asymptotically stable. [3,4]

1.4 State Space Controller and Observer using Pole Placement Approach

The function of a controller is to manipulate the system input to achieve a desired output behavior. They help to improve stability, and performance and achieve the desired effect by reducing errors. Observer is used to monitor the state variables of the system during the operation. It helps to understand how the parameters change during the process and helps to fine-tune the controller to be more efficient. [4,5]

Pole placement method is an approach used to design a controller and observer for a system. This method involves placing the eigenvalues or poles at desired locations to obtain a suitable controller gain matrix (K) and observer gain matrix (L). The controller gain feedback matrix (K) indicates how each system variable will affect control input. The poles should be chosen such that they ensure ideal performance and at the same time should not make the system too sensitive or cause problems like actuator saturation. [4,5]

The controller gain matrix (K) is used to regulate the system input based on the state of the system. The magnitude of the values in the matrix indicates the influence it has on control input and the sign of each gain value indicates the direction of the control action corresponding to the state variable. A higher K matrix means more control effort is required.[4,5]

The controller and observer were designed with MATLAB and the pseudo-code is given below,

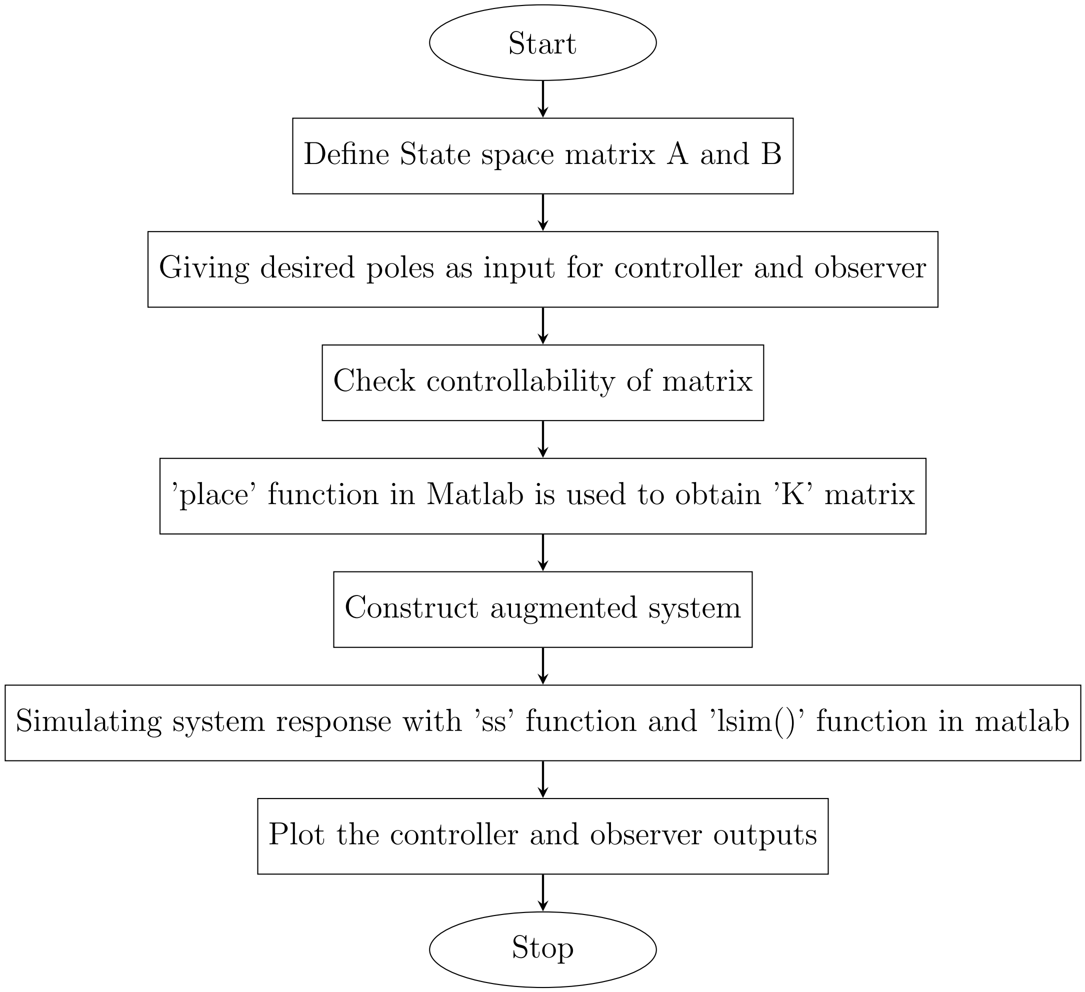


Figure 1: Pseudo code for controller and observer.

1.4.1 Using positive real parts

In this approach, all the dominant poles are placed in the positive or right-hand side of the quadrant.

- **Eigen value from assignment 1**

The eigenvalues obtained from assignment 1 are taken as desired poles,
1.0182
-1.0591+1.0173i
-1.0591-1.0173i

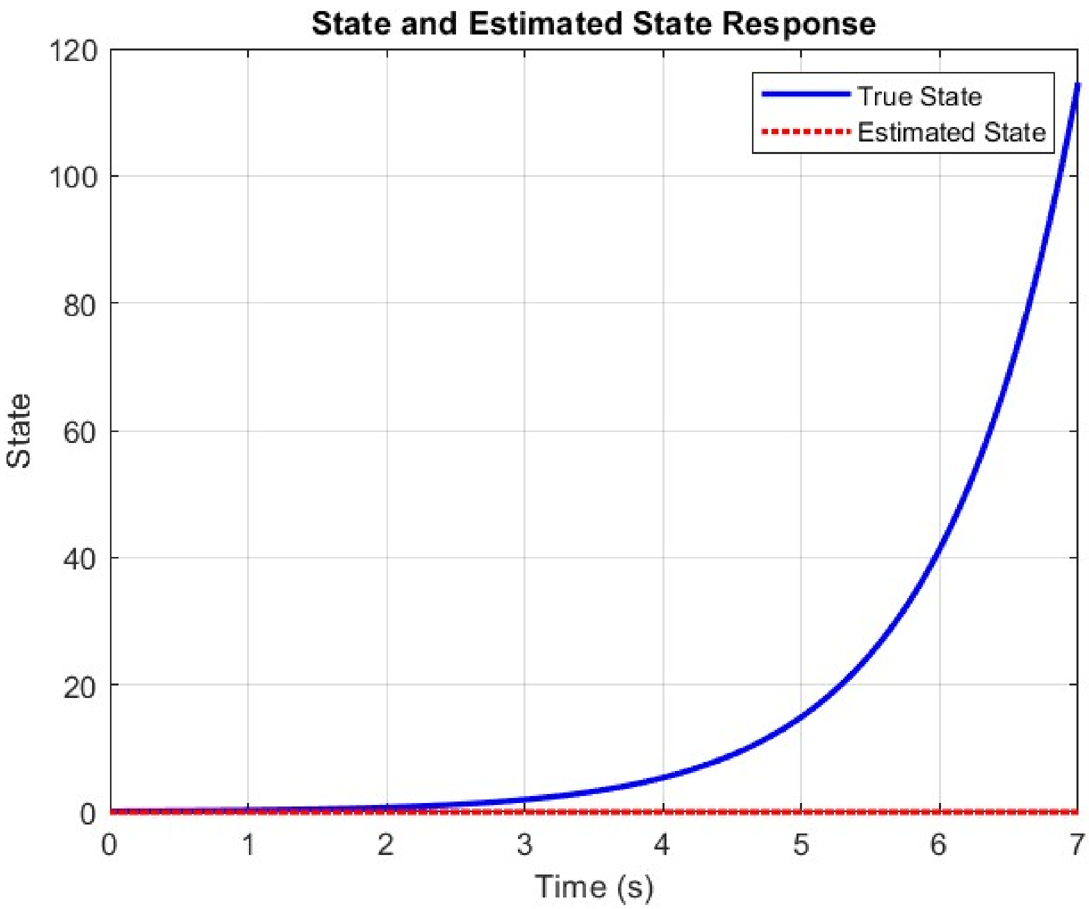


Figure 2: Controller output with eigenvalues from assignment 1.

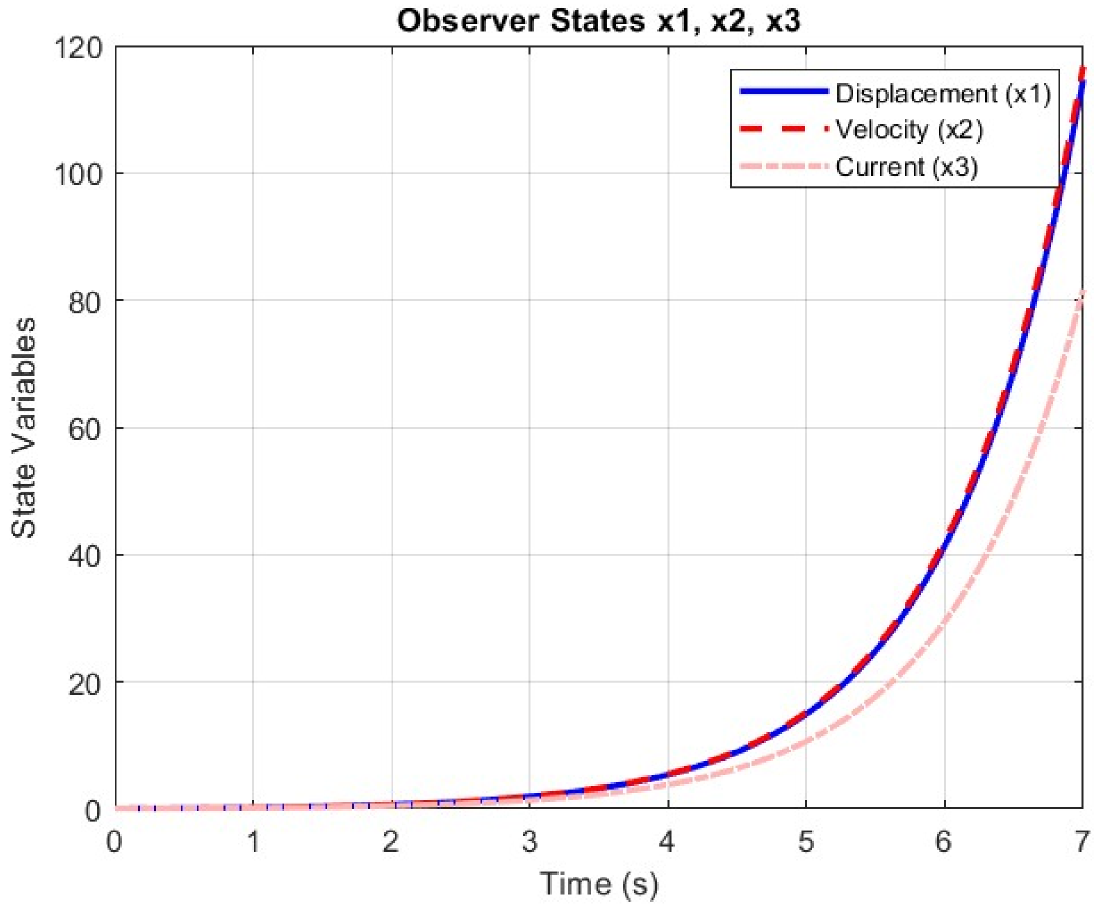


Figure 3: Observer output with eigenvalues from assignment 1.

- **Three times the eigenvalue from assignment 1**

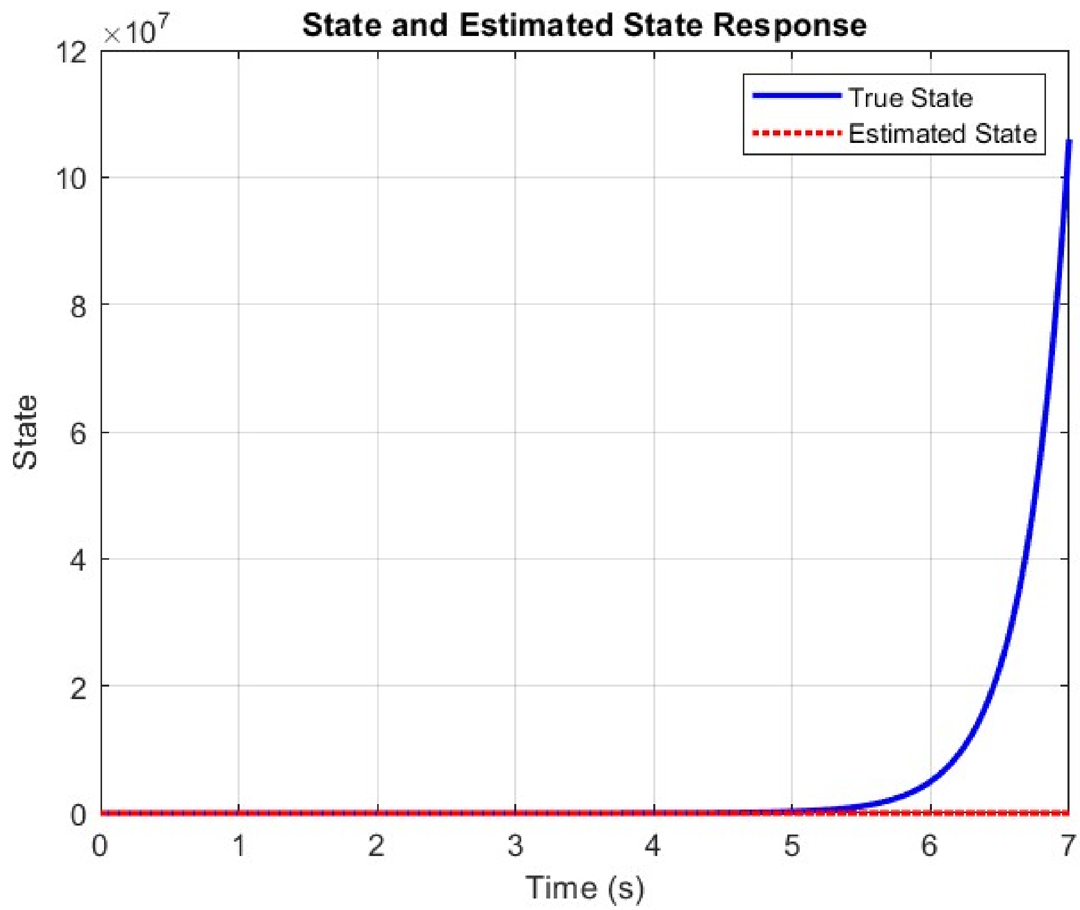


Figure 4: Controller output with three times the eigenvalues.

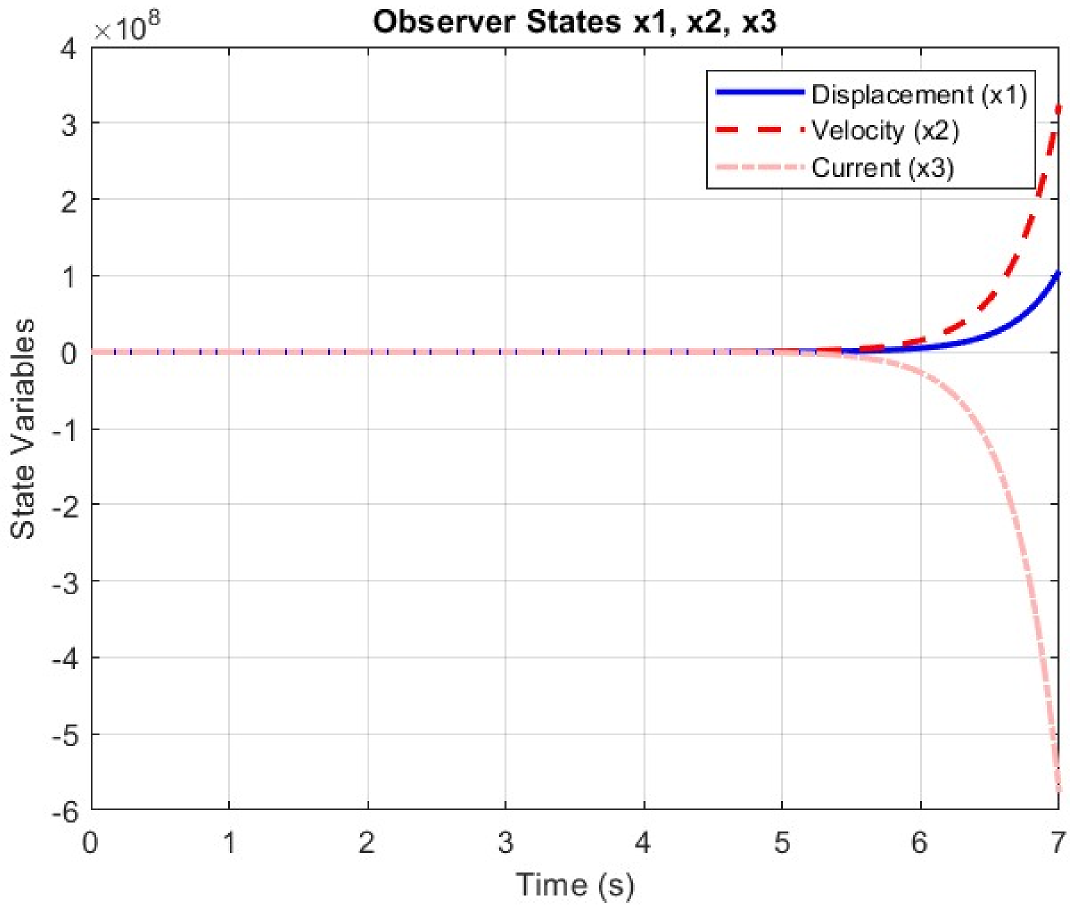


Figure 5: Observer output with three times the eigenvalues.

- **five times the eigenvalue from assignment 1**

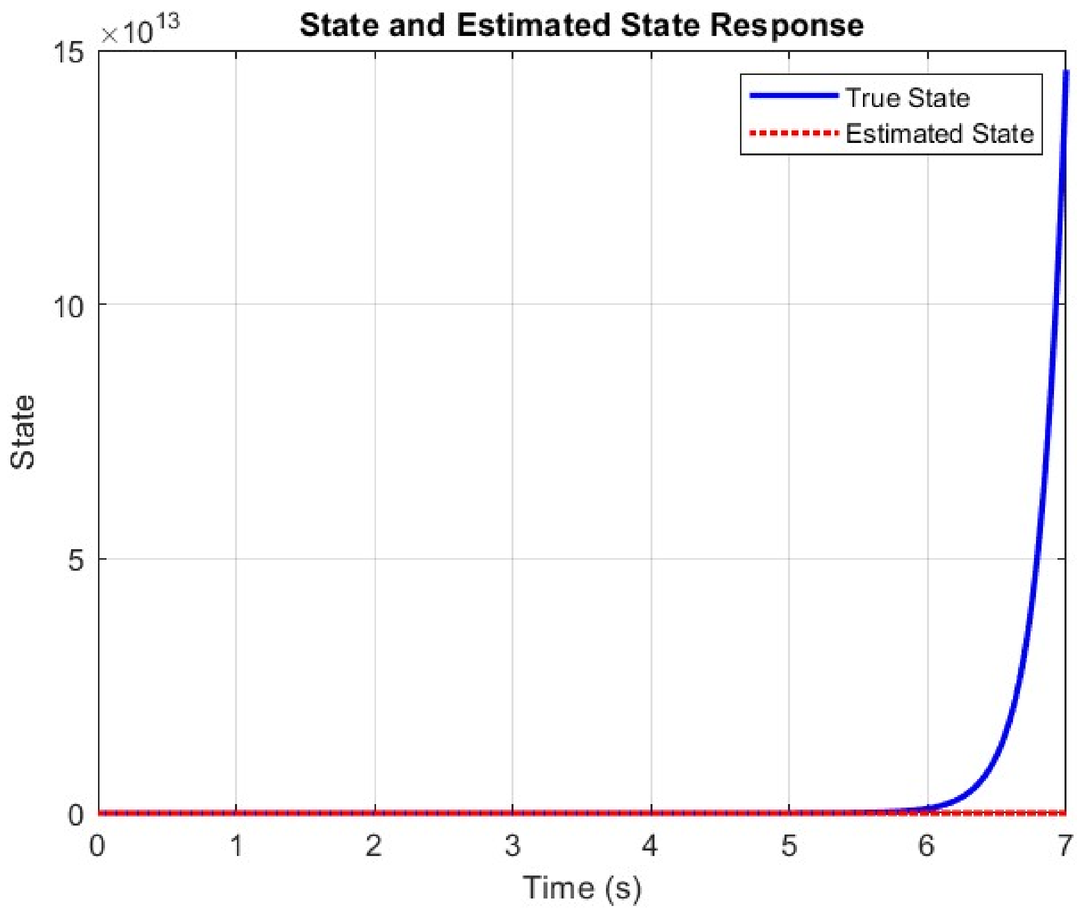


Figure 6: Controller output with five times the eigenvalues.

From the output graph of both the controller and observer, it is observable that if the dominant pole is positive the system will not attain stability. So, in order to make a system stable all the pole values must be in the negative plane while choosing the desired poles.

1.4.2 Using negative pole values

In this approach, the pole values are given negative values.

- **Eigenvalues from assignment 1 in negative**

-1.0182
-1.0591+1.0173i
-1.0591-1.0173i

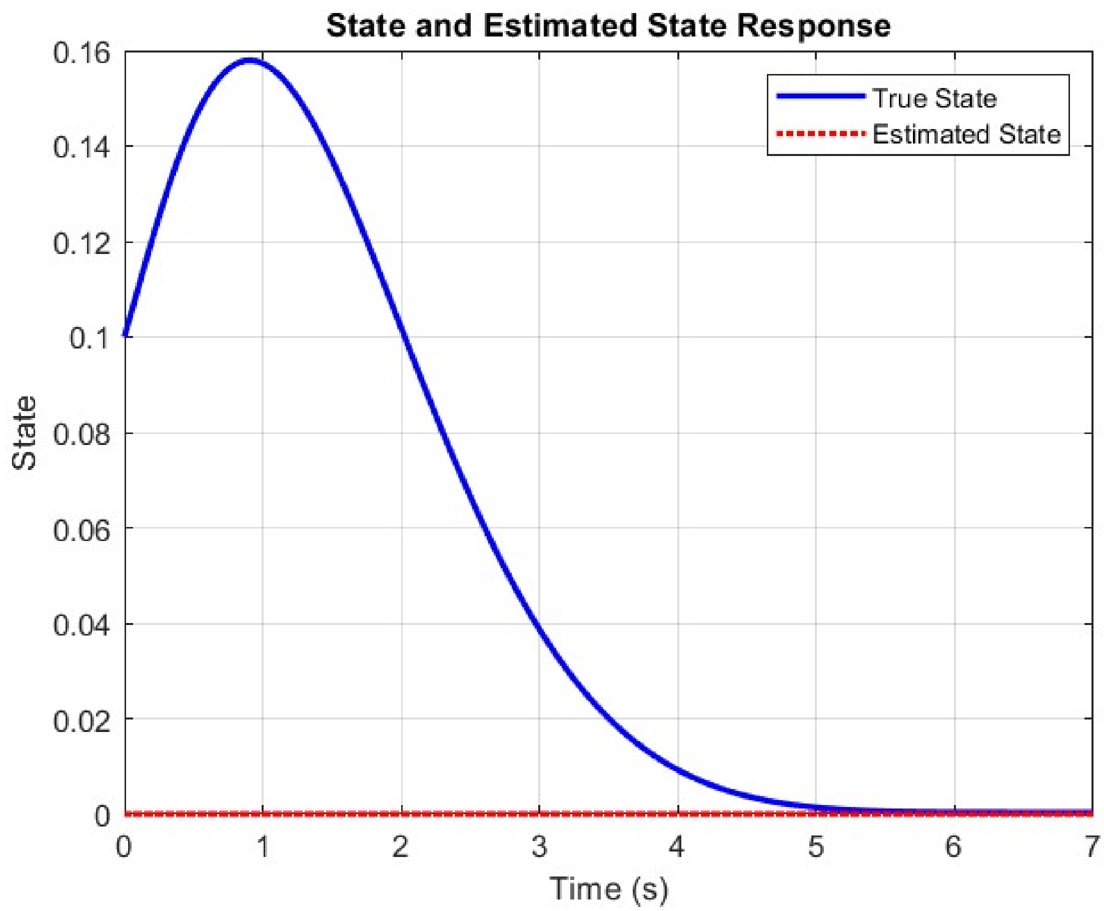


Figure 7: Controller output with negative eigenvalues.

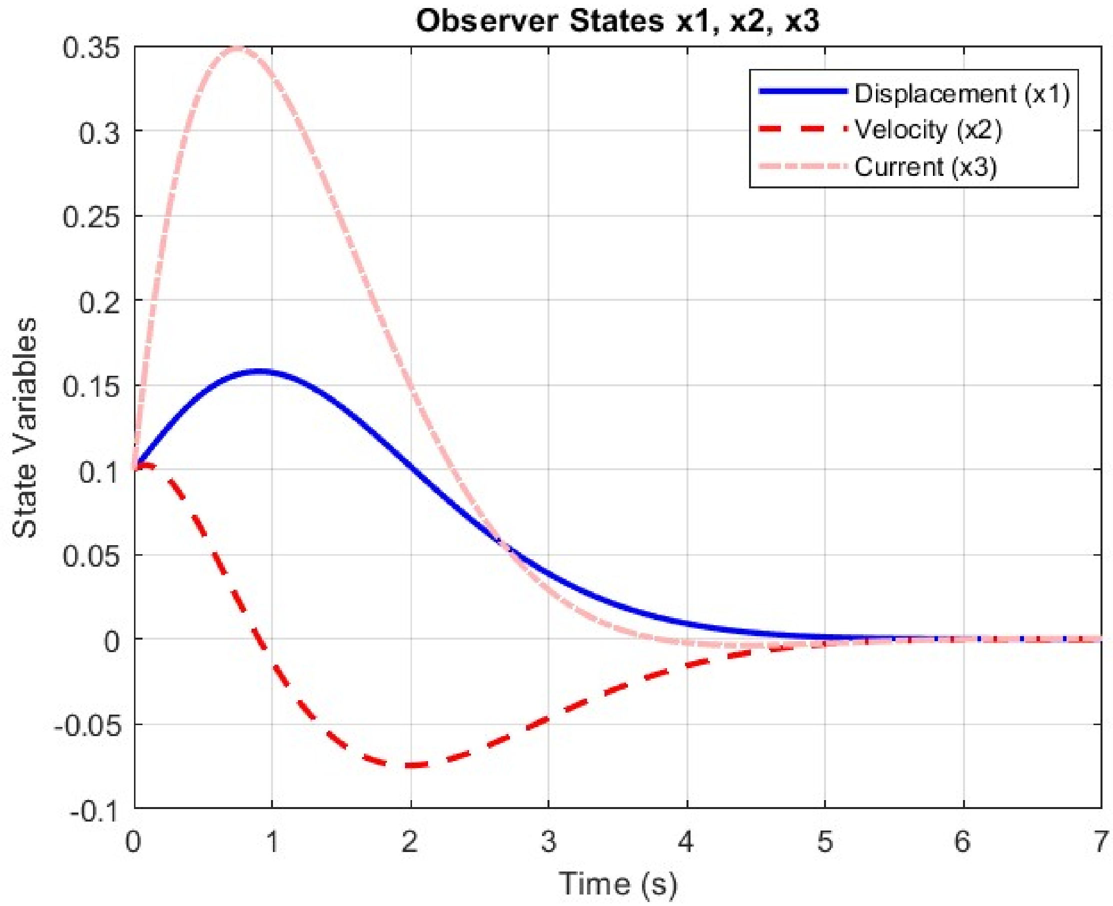


Figure 8: Observer output with negative eigenvalues.

From the controller and observer ouput graphs it is evident that system is stabilized at around 5.5 seconds. The gain matrix of the controller and observer of the above pole is,

$$K = \begin{bmatrix} -6.2777 & -3.2020 & 2.0364 \end{bmatrix}$$

$$L = \begin{bmatrix} 14.5820 \\ 91.7935 \\ -108.8232 \end{bmatrix}$$

- **Three times the eigenvalue in negative**

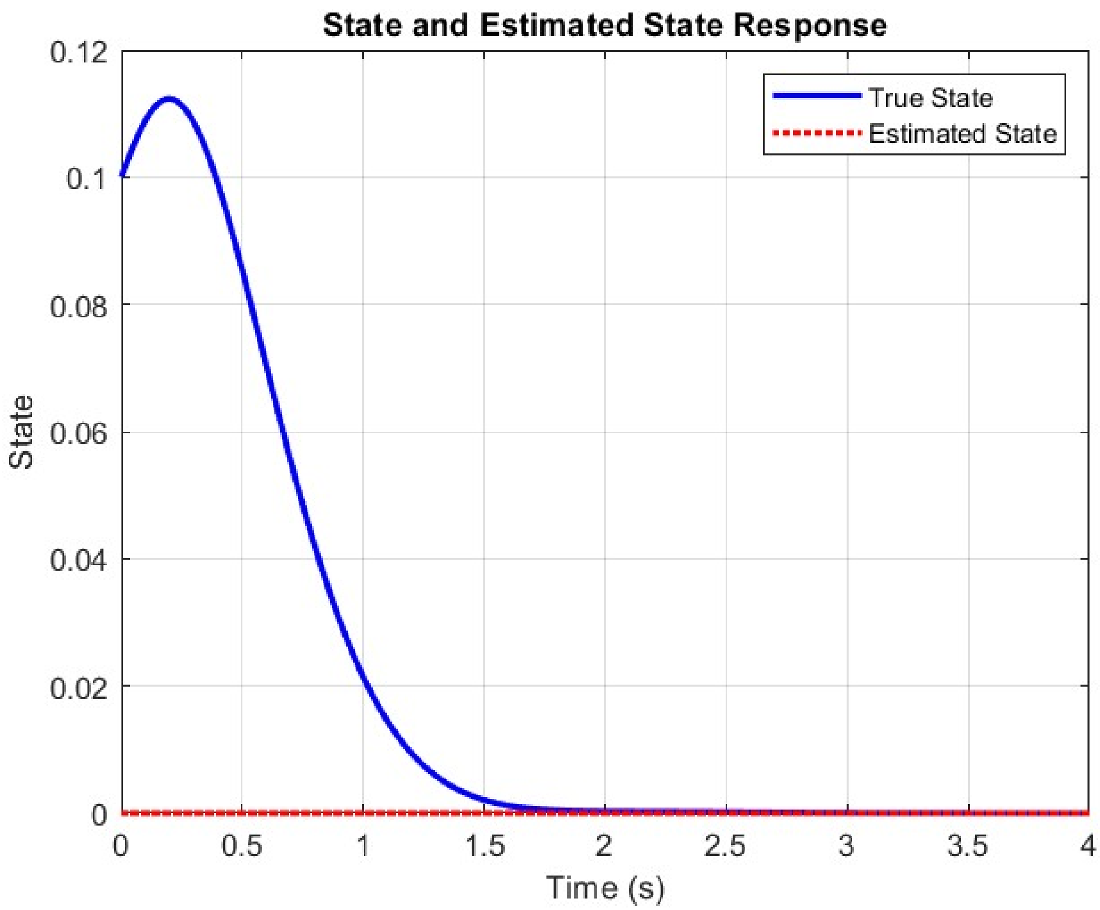


Figure 9: Controller output with three times negative eigenvalues.

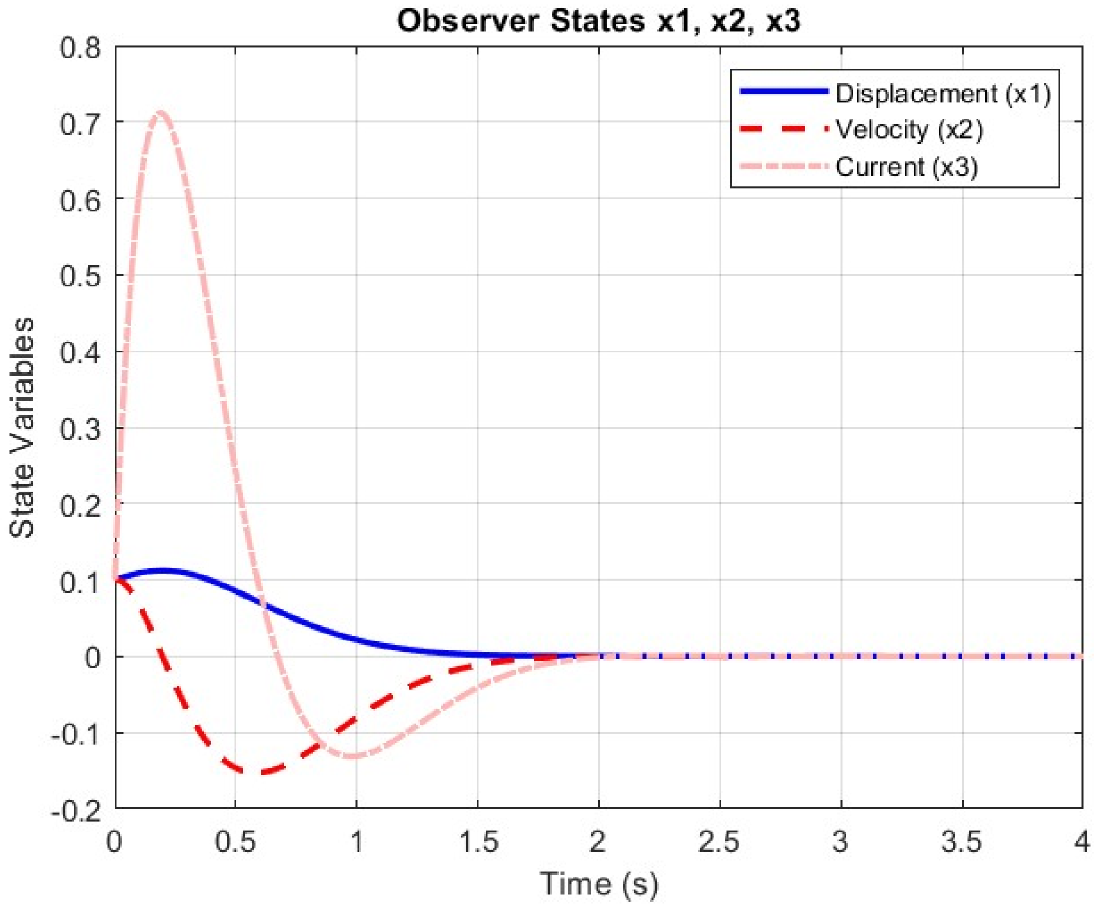


Figure 10: Observer output with three times negative eigenvalues.

System stabilizing around 1.8 seconds and gain matrices are,

$$L = \begin{bmatrix} -57.9544 & -28.8176 & 8.3092 \end{bmatrix}$$

$$L = \begin{bmatrix} 45.9 \\ 920.0 \\ -4683.8 \end{bmatrix}$$

- **Five times the eigenvalue in negative**

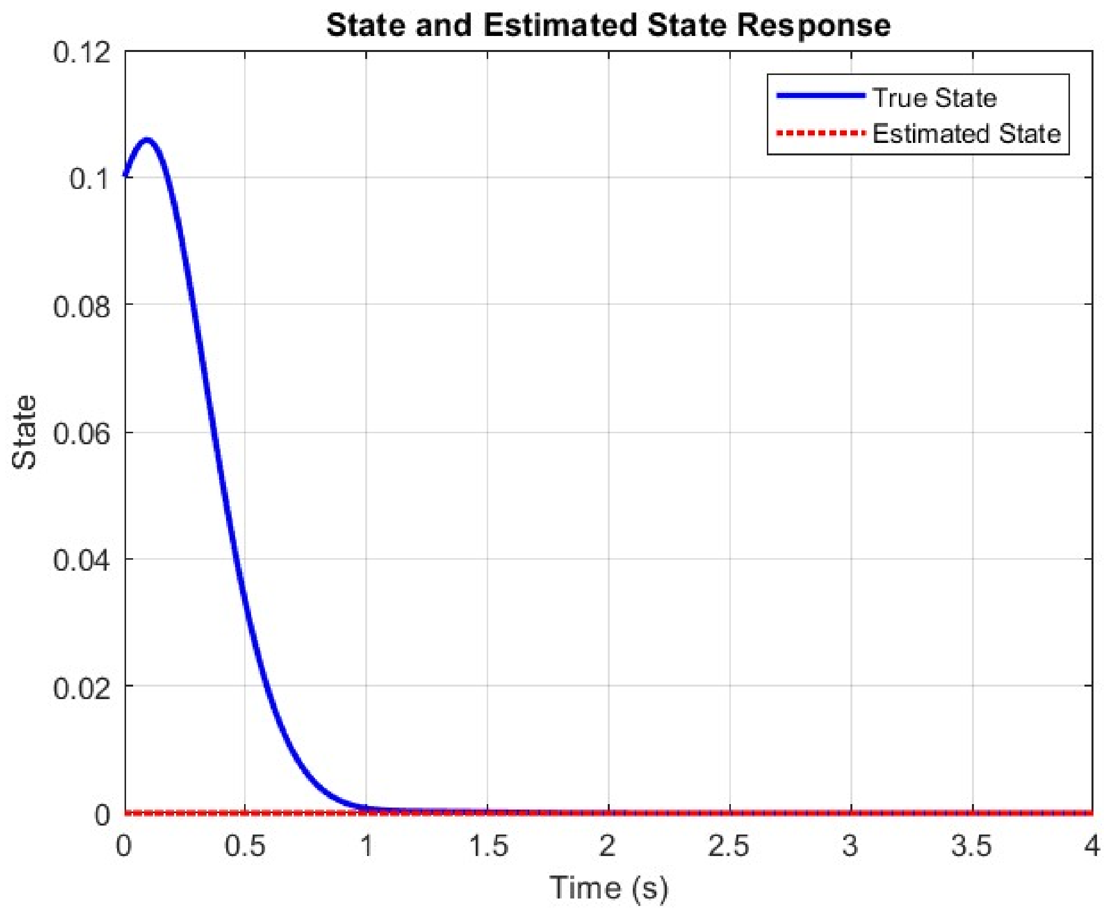


Figure 11: Controller output with five times negative eigenvalues.

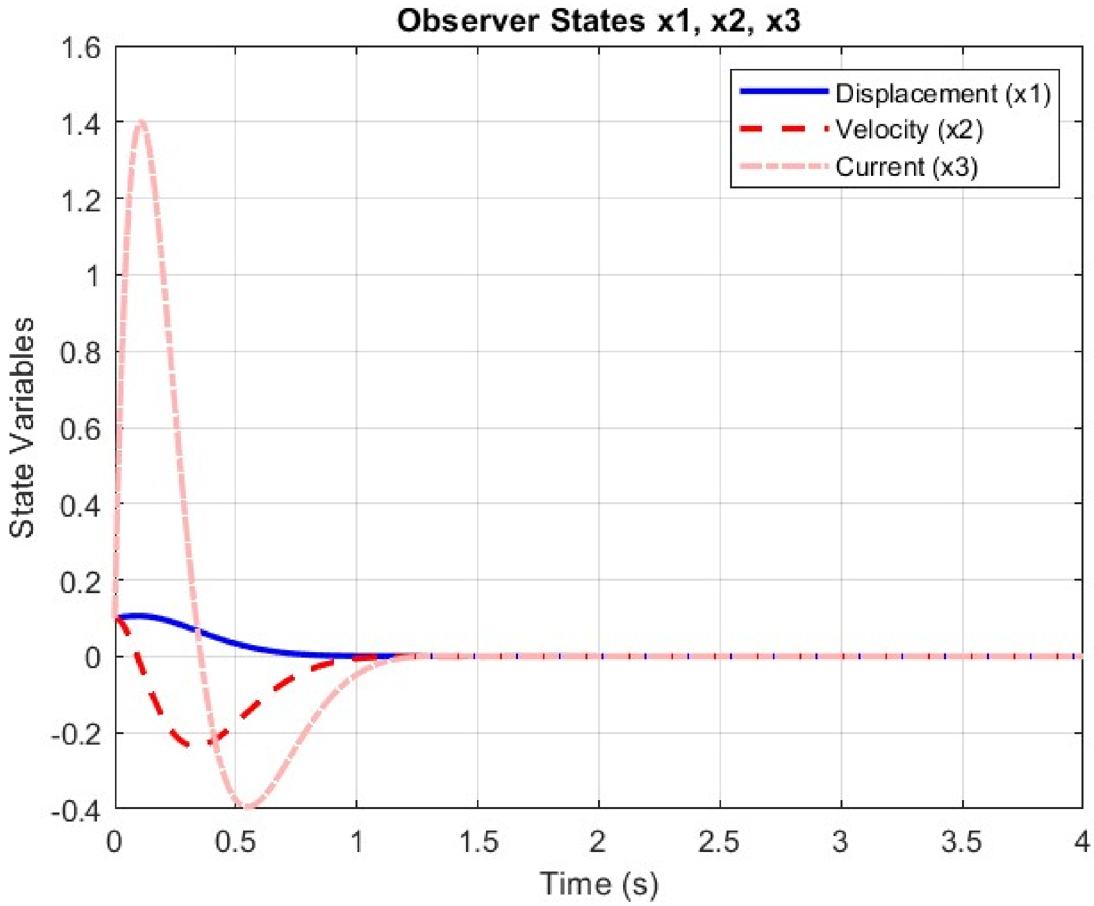


Figure 12: Observer output with five times negative eigenvalues.

The system is stabilizing around 1.2 seconds. Gain matrices are,

$$K = \begin{bmatrix} -226.9948 & -80.0487 & 14.5820 \end{bmatrix}$$

$$L = \begin{bmatrix} 77.0 \\ 2611 \\ -23225 \end{bmatrix}$$

- **Seven times the eigenvalue in negative**

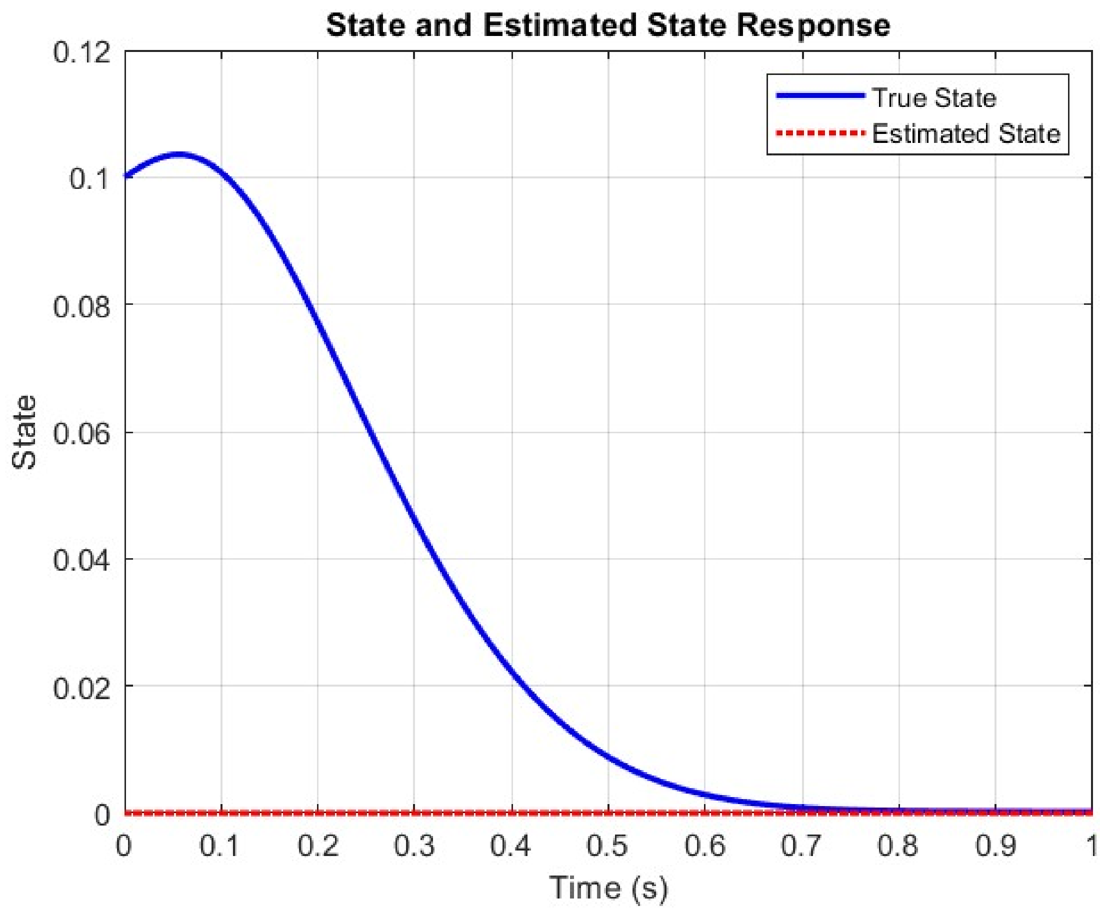


Figure 13: Controller output with seven times negative eigenvalues.

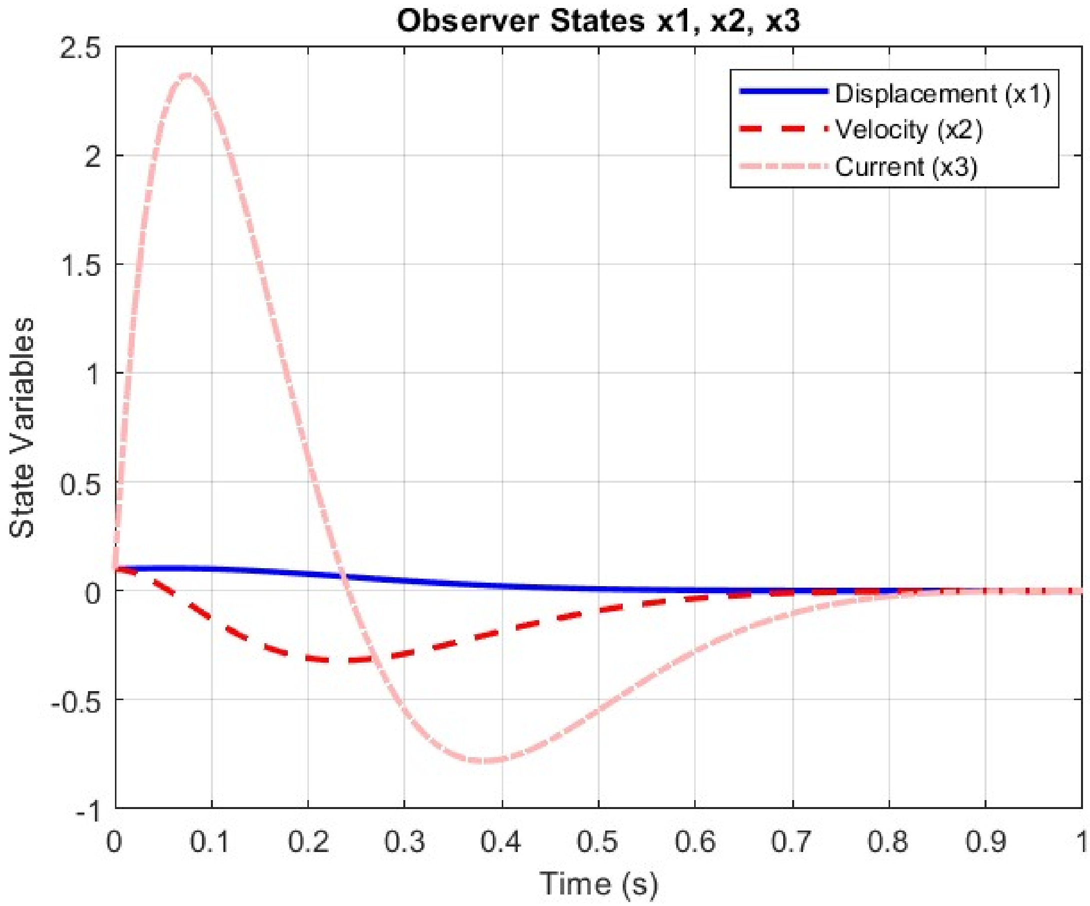


Figure 14: Observer output with seven times negative eigenvalues.

The system is stabilizing around 0.8 seconds. Gain matrices are,

$$K = \begin{bmatrix} -592.0485 & -157.0034 & 20.8648 \end{bmatrix}$$

$$L = \begin{bmatrix} 78 \\ 2657 \\ -23645 \end{bmatrix}$$

- **Ten times the eigenvalue in negative**

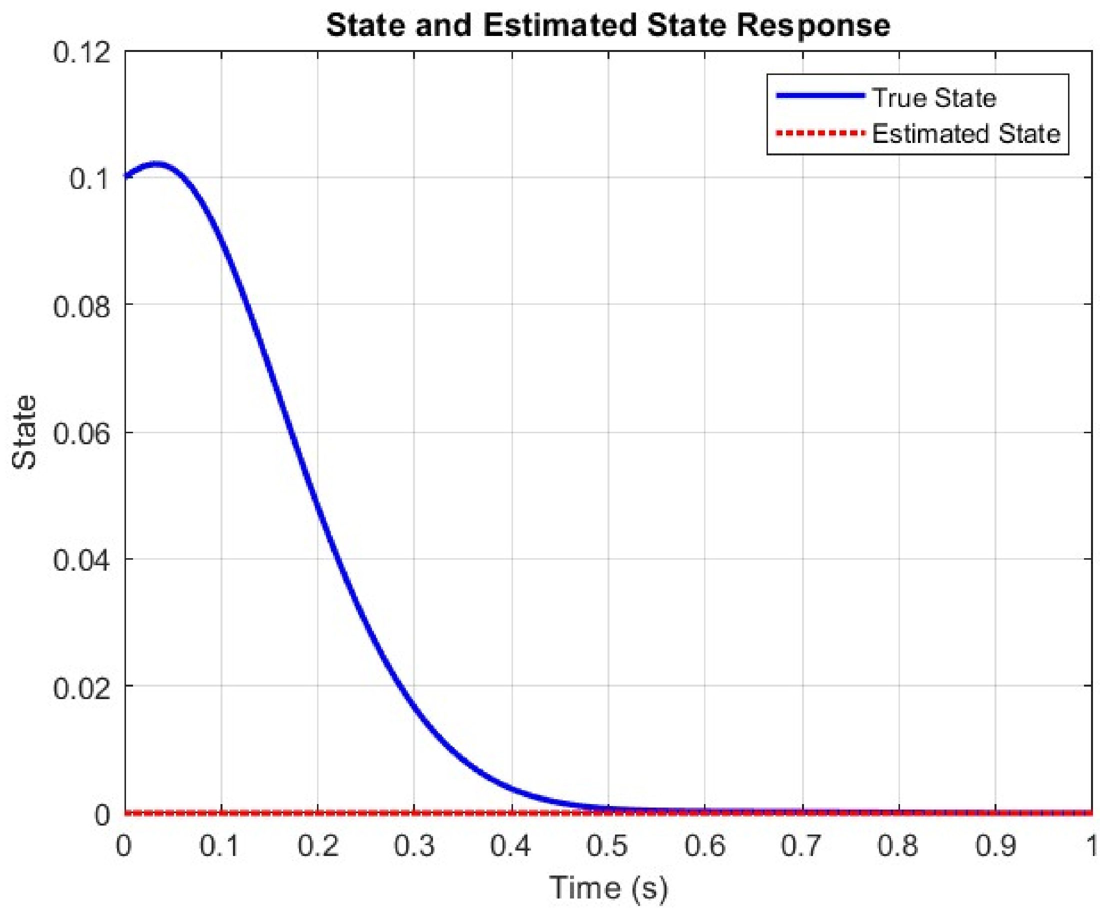


Figure 15: Controller output with ten times negative eigenvalues.

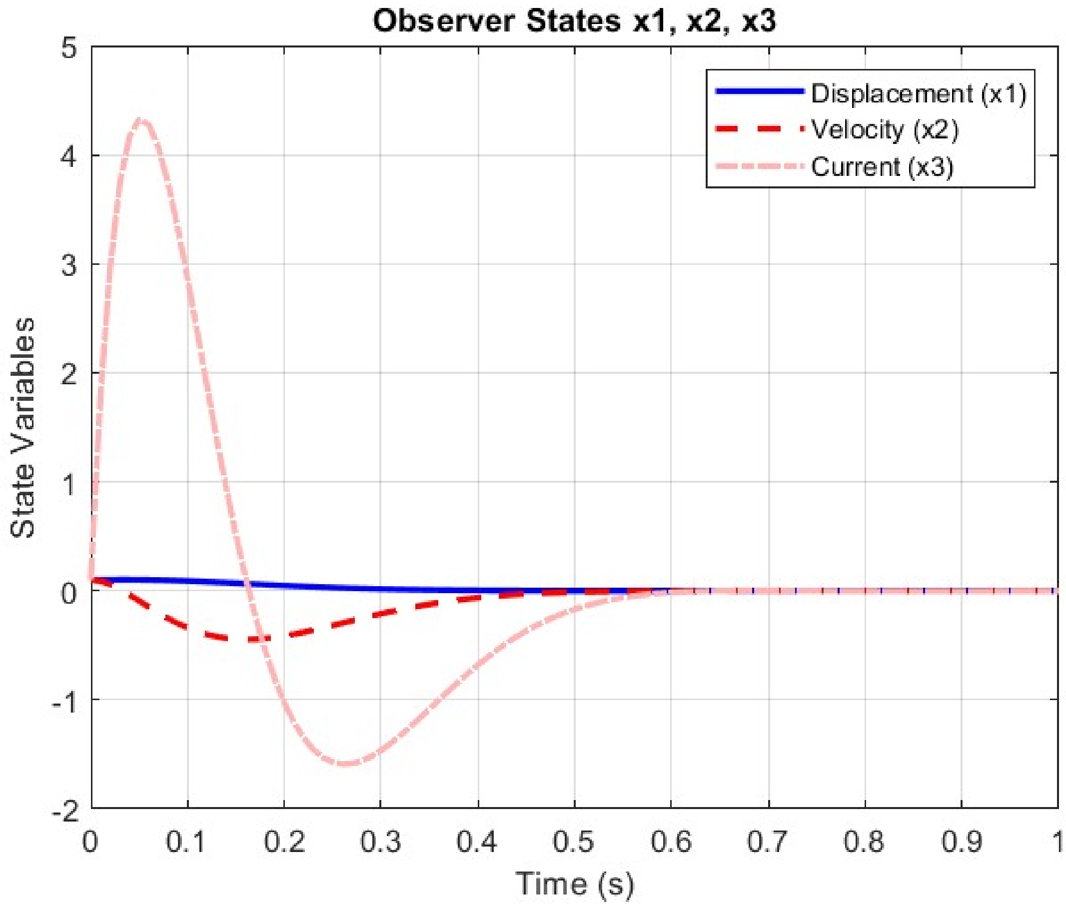


Figure 16: Observer output with seven times negative eigenvalues.

The system is stabilizing around 0.6 seconds. Gain matrices are,

$$K = \begin{bmatrix} -1676.5 & -320.2 & 030.3 \end{bmatrix}$$

$$L = \begin{bmatrix} 160 \\ 10610 \\ -194860 \end{bmatrix}$$

1.4.3 Selecting suitable poles

From the above graphs, it is clear that when the poles are placed further left of the negative plane the system takes less time to achieve stability. But while selecting the ideal pole values the K matrix should also be considered. [6]

It is observed that when poles are placed at ten times the eigenvalue in the negative plane, the system stabilizes at around 0.6 seconds and for seven times the value system takes around 0.8 seconds. When considering the stabilizing time, the former is a good option.

But when analyzing the K matrix, the former has elements two times higher than the latter. It means selecting the poles at 10 times the value in the negative plane can result in good stabilizing and response time but can create problems like high control effort, actuator saturation, and sensitivity towards noise. So, the ideal pole placement for the system is

$$poles = \begin{bmatrix} -7.1274 \\ -7.4187 + 7.1211i \\ -7.4187 - 7.1211i \end{bmatrix}$$

1.5 Controller Using Feedback linearization

Feedback linearization is a powerful technique used in nonlinear control systems to transform a nonlinear system into an equivalent linear system through a change of variables and a suitable control input. The main goal is to simplify the control of complex nonlinear systems by leveraging well-established linear control methods. [7]

To apply feedback linearization, consider a non-linear system

$$\begin{aligned} \dot{x} &= a(x) + b(x)u \\ y &= x_1 \end{aligned}$$

In this system:

- $\dot{x}$ is the time derivative of the state vector x.
- $a(x)$ captures the nonlinear dynamics.
- $b(x)$ represents the input-output relationship.
- u is the control input.
- y denotes the output variable.

where,

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ g - \frac{k}{2m}\left(\frac{x_3}{x_1}\right)^2 \\ -\frac{R}{k}x_1x_3 + \frac{x_2x_3}{x_1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{x_1}{k} \end{bmatrix} U$$

Using Lie derivatives, we can calculate the Lie derivative of h(x) with respect to f(x):

$$L_g h(x) = \frac{\partial h(x)}{\partial x} f(x) = \nabla^T h(x) \cdot f(x)$$

In this case, applying the Lie derivatives to the output function c,

$$L_a c(x) = \frac{\partial c(x)}{\partial x} a(x)$$

- **Hence, if we derivate the output y, we obtain:**

$$\dot{y} = \frac{dc(x)}{dt} = \frac{\partial c(x)}{\partial x_1}\dot{x}_1 + \ldots + \frac{\partial c(x)}{\partial x_n}\dot{x}_n = \frac{\partial c(x)}{\partial x}\dot{x}$$

- **Which is**

$$\dot{y} = \frac{\partial c(x)}{\partial x} a(x) + \frac{\partial c(x)}{\partial x} b(x) \cdot u$$

- **Or using Lie derivative**

$$\dot{y} = Ł_a c(x) + L_b c(x) \cdot u \tag{10}$$

1.5.1 First Lie Derivative

Given:

$$c(x) = x_1$$

$$a(x) = \begin{pmatrix} x_2 \\ g - \frac{kx_3^2}{2mx_1^2} \\ -\frac{Rx_1x_3}{k} + \frac{x_2x_3}{x_1} \end{pmatrix}$$

The first Lie derivative $L_a c(x)$ is:

$$L_a c(x) = \frac{\partial c(x)}{\partial x_1} a_1(x) + \frac{\partial c(x)}{\partial x_2} a_2(x) + \frac{\partial c(x)}{\partial x_3} a_3(x)$$

Since $c(x) = x_1$:

$$\frac{\partial x_1}{\partial x_1} = 1, \quad \frac{\partial x_1}{\partial x_2} = 0, \quad \frac{\partial x_1}{\partial x_3} = 0$$

Thus:

$$L_a c(x) = 1 \cdot x_2 + 0 \cdot \left(g - \frac{kx_3^2}{2mx_1^2}\right) + 0 \cdot \left(-\frac{Rx_1x_3}{k} + \frac{x_2x_3}{x_1}\right) = x_2$$

Therefore:

$$L_a c(x) = x_2 \tag{11}$$

To find $L_b c(x)$:
Given:

$$b(x) = \begin{pmatrix} 0 \\ 0 \\ \frac{x_1}{k} \end{pmatrix}$$

The Lie derivative $L_b c(x)$ is:

$$L_b c(x) = \frac{\partial c(x)}{\partial x_1} b_1(x) + \frac{\partial c(x)}{\partial x_2} b_2(x) + \frac{\partial c(x)}{\partial x_3} b_3(x)$$

Since $c(x) = x_1$:

$$\frac{\partial x_1}{\partial x_1} = 1, \quad \frac{\partial x_1}{\partial x_2} = 0, \quad \frac{\partial x_1}{\partial x_3} = 0$$

Thus:

$$L_b c(x) = 1 \cdot 0 + 0 \cdot 0 + 0 \cdot \frac{x_1}{k} = 0$$

Therefore:

$$L_b c(x) = 0 \tag{12}$$

Now substituting equations (11) and (12) into the formula for $\dot{y}$:

$$\dot{y} = L_a c(x) = x_2$$

Since there is no control function u in the output function $\dot{y}$, we proceed to the second Lie derivative.

1.5.2 Second Lie Derivative

The second Lie derivative $\ddot{y}$ is given by:

$$\ddot{y} = L_a^2 c(x) + L_b L_a c(x) \cdot u \tag{13}$$

First, compute $L_a^2 c(x)$:

$$L_a^2 c(x) = L_a[L_a c(x)]$$

From equation (11), $L_a c(x) = x_2$:

$$L_a^2 c(x) = L_a x_2$$

Using the definition of L_a on x_2:

$$L_a x_2 = \frac{\partial x_2}{\partial x_1} a_1(x) + \frac{\partial x_2}{\partial x_2} a_2(x) + \frac{\partial x_2}{\partial x_3} a_3(x)$$

Given:

$$\frac{\partial x_2}{\partial x_1} = 0, \quad \frac{\partial x_2}{\partial x_2} = 1, \quad \frac{\partial x_2}{\partial x_3} = 0$$

Therefore:

$$L_a x_2 = 0 \cdot x_2 + 1 \cdot \left(g - \frac{kx_3^2}{2mx_1^2} \right) + 0 \cdot \left(-\frac{Rx_1x_3}{k} + \frac{x_2x_3}{x_1} \right) = g - \frac{kx_3^2}{2mx_1^2}$$

So:

$$L_a^2 c(x) = g - \frac{kx_3^2}{2mx_1^2} \tag{14}$$

Next, compute $L_b L_a c(x)$:

$$L_b L_a c(x) = L_b[L_a c(x)]$$

From equations (11) and (12):

$$L_a c(x) = x_2, \quad L_b c(x) = 0$$

Thus:

$$L_b L_a c(x) = L_b x_2$$

Using the definition of L_b on x_2:

$$L_b x_2 = \frac{\partial x_2}{\partial x_1} b_1(x) + \frac{\partial x_2}{\partial x_2} b_2(x) + \frac{\partial x_2}{\partial x_3} b_3(x)$$

Given:

$$\frac{\partial x_2}{\partial x_1} = 0, \quad \frac{\partial x_2}{\partial x_2} = 1, \quad \frac{\partial x_2}{\partial x_3} = 0$$

Therefore:

$$L_b x_2 = 0 \cdot 0 + 1 \cdot 0 + 0 \cdot \frac{x_1}{k} = 0$$

Thus:

$$L_b L_a c(x) = 0 \tag{15}$$

Now substituting equations (14) and (15) into equation (13):

$$\ddot{y} = g - \frac{kx_3^2}{2mx_1^2}$$

Since there is no control function u in the output function $\ddot{y}$, we move on to compute the third Lie derivative.

1.5.3 Third Lie Derivative

The second Lie derivative $\dddot{y}$ is given by:

$$\dddot{y} = L_a^3 c(x) + L_b L_a^2 c(x) \cdot u \tag{16}$$

Compute $L_a^3 c(x)$:

$$L_a^3 c(x) = L_a[L_a^2 c(x)]$$

From equation (14), $L_a^2 c(x) = g - \frac{kx_3^2}{2mx_1^2}$:

Therefore,

$$L_a^3 c(x) = L_a[g - \frac{kx_3^2}{2mx_1^2}] \cdot a(x)$$

Using the definition of L_a:

$$L_a \left(g - \frac{kx_3^2}{2mx_1^2} \right) = \frac{\partial}{\partial x_1} \left(g - \frac{kx_3^2}{2mx_1^2} \right) a_1(x) + \frac{\partial}{\partial x_2} \left(g - \frac{kx_3^2}{2mx_1^2} \right) a_2(x) + \frac{\partial}{\partial x_3} \left(g - \frac{kx_3^2}{2mx_1^2} \right) a_3(x)$$

Calculating partial derivatives:

1. For x_1:

$$\frac{\partial}{\partial x_1} \left(g - \frac{kx_3^2}{2mx_1^2} \right) = \frac{kx_3^2}{mx_1^3}$$

2. For x_2:

$$\frac{\partial}{\partial x_2} \left(g - \frac{kx_3^2}{2mx_1^2} \right) = 0$$

3. For x_3:

$$\frac{\partial}{\partial x_3} \left(g - \frac{kx_3^2}{2mx_1^2} \right) = -\frac{kx_3}{mx_1^2}$$

Therefore:

$$L_a \left(g - \frac{kx_3^2}{2mx_1^2} \right) = \left(\frac{kx_3^2}{mx_1^3} \right) x_2 + 0 + \left(-\frac{kx_3}{mx_1^2} \right) \left(-\frac{Rx_1x_3}{k} + \frac{x_2x_3}{x_1} \right)$$

Simplifying:

$$L_a \left(g - \frac{kx_3^2}{2mx_1^2} \right) = \frac{kx_3^2x_2}{mx_1^3} + \frac{Rx_3^2}{mx_1} - \frac{kx_2x_3^2}{mx_1^3}$$

So:

$$L_a^3 c(x) = \frac{Rx_3^2}{mx_1} \tag{17}$$

To compute $L_b L_a^2 c(x)$:
Since $L_a^2 c(x) = g - \frac{kx_3^2}{2mx_1^2}$ and $L_b c(x) = 0$,

$$L_b L_a^2 c(x) = L_b[g - \frac{kx_3^2}{2mx_1^2}]$$

Calculate $L_b[g - \frac{kx_3^2}{2mx_1^2}]$:

$$L_b[g - \frac{kx_3^2}{2mx_1^2}] = \frac{\partial}{\partial x_1}[g - \frac{kx_3^2}{2mx_1^2}]b_1(x) + \frac{\partial}{\partial x_2}[g - \frac{kx_3^2}{2mx_1^2}]b_2(x) + \frac{\partial}{\partial x_3}[g - \frac{kx_3^2}{2mx_1^2}]b_3(x)$$

Given:

$$\frac{\partial}{\partial x_1}[g - \frac{kx_3^2}{2mx_1^2}] = -\frac{kx_3^2}{mx_1^3}, \quad \frac{\partial}{\partial x_2}[g - \frac{kx_3^2}{2mx_1^2}] = 0, \quad \frac{\partial}{\partial x_3}[g - \frac{kx_3^2}{2mx_1^2}] = -\frac{kx_3}{mx_1^2}$$

Multiply by $b(x)$:

$$L_b[g - \frac{kx_3^2}{2mx_1^2}] = -\frac{kx_3^2}{mx_1^3} \cdot 0 + 0 \cdot 0 + \left(-\frac{kx_3}{mx_1^2}\right) \cdot \frac{x_1}{k}$$

$$L_b[g - \frac{kx_3^2}{2mx_1^2}] = -\frac{x_3}{mx_1}$$

Therefore,

$$L_b L_a^2 c(x) = -\frac{x_3}{mx_1} \tag{18}$$

Now substituting equations (17) and (18) into equation (16):

$$\dddot{y} = \frac{Rx_3^2}{mx_1} - \frac{x_3}{mx_1} \cdot u$$

As controllable function u is in the output function $\dddot{y}$, the system is now feedback linearized.

1.5.4 Controller Design

The system can be controlled by control law:

$$u(x, w) = -r(x) + v(x) \cdot w \tag{19}$$

where,

$$r(x) = \frac{L_a^3 c(x) + a_2 L_a^2 c(x) + a_1 L_a c(x) + a_0 c(x)}{L_b L_a^2 c(x)} \tag{20}$$

$$v(x) = \frac{v}{L_b L_a^2 c(x)} \tag{21}$$

To calculate the values of a_2, a_1, and a_0, we start by considering the characteristic equation:

$$(S - \mu_0)(S - \mu_1)(S - \mu_2) = 0 \tag{22}$$

Given the values:

$$\mu_0 = -7.1274$$
$$\mu_1 = -7.4187 + 7.1211i$$
$$\mu_2 = -7.4187 - 7.1211i$$

Substituting these values into the characteristic equation, we get:

$$(S + 7.1274)\,[(S + 7.4187 - 7.1211i)(S + 7.4187 + 7.1211i)] = 0$$

Simplifying the quadratic term:

$$(S + 7.4187)^2 + 50.706 = S^2 + 14.8374S + 105.7436$$

Multiplying this with $(S + 7.1274)$:

$$(S + 7.1274)(S^2 + 14.8374S + 105.7436)$$

Expanding the product:

$$S^3 + 21.9648S^2 + 211.4872S + 753.86$$

Comparing this with the general equation:

$$S^3 + a_2S^2 + a_1S + a_0 = 0$$

We can deduce the values of a_2, a_1, and a_0 as follows:

$$\begin{aligned} a_2 &= 21.9648 \\ a_1 &= 211.4872 \\ a_0 &= 753.86 \end{aligned}$$

The expressions for $r(x)$ and $v(x)$ are:

$$r(x) = -\left(x_1x_3 + \frac{24.16128x_1}{x_3} - \frac{10.9824x_3}{x_1} + \frac{211.4872x_1x_2}{x_3} + \frac{753.86x_1^2}{x_3}\right)$$

$$v(x) = -\frac{1.1x_1}{x_3}$$

Substituting $r(x)$ and $v(x)$ into Eq.19, we get:

$$u(x, w) = -\left(x_1x_3 + \frac{24.16128x_1}{x_3} - \frac{10.9824x_3}{x_1} + \frac{211.4872x_1x_2}{x_3} + \frac{753.86x_1^2}{x_3}\right) - \frac{1.1x_1}{x_3} \cdot w$$

In the given system dynamics, the state variable x_1 appears in the denominator of the third equation. This means that when x_1 reaches zero, it leads to a singularity in the system. During the integration process, at the initial time (usually $t = 0.0$), the initial conditions of the state variables are set. If the initial condition for x_1 is set to zero, it immediately causes a singularity in the system.

1.6 Controller Using Rule-Based Fuzzy System

Fuzzy logic was introduced by Lotfi Zadeh in 1965. In fuzzy logic, instead of being strictly true or false, varying degrees of truth are accepted to make a decision as output. This capability is beneficial to overcome the rigidity of binary systems. Fuzzy rules are expressed in the 'if-then' form with the help of 'AND' or 'OR' operators. The steps involved in creating a fuzzy controller are, defining fuzzy variables (input and output variables), creating membership functions, and defining fuzzy rules. [8]

For creating a controller based on rule rule-based fuzzy system for the maglev system two approach is considered. In the first approach, seven fuzzy rules were defined to create the fuzzy controller and in the second approach a set of fifteen rules were used. In both approaches Mamdani-type fuzzy inference system is used and a triangular membership function is used. [8]

1.6.1 Fuzzy system 1

Seven fuzzy rules were defined in this system.

Table 1: Fuzzy Inference Rules

Displacement	**Velocity**	**Control Signal**
Negative Large (NL)	Negative (N)	Low (L)
Negative Large (NL)	Positive (P)	Medium (M)
Negative Medium (NM)	Negative (N)	Low (L)
Negative Medium (NM)	Positive (P)	High (H)
Zero (Z)	Negative (N)	Low (L)
Zero (Z)	Positive (P)	High (H)
Positive Large (PL)	Negative (N)	Low (L)

A set of three values are used to test the fuzzy controller system,

- Rule 1
  Displacement = -1.8
  Velocity = 1.0

- Rule 2
  Displacement = 0.2
  Velocity = -1.0

- Rule 3
  Displacement = 2.5
  Velocity = 3.5

- Rule 1

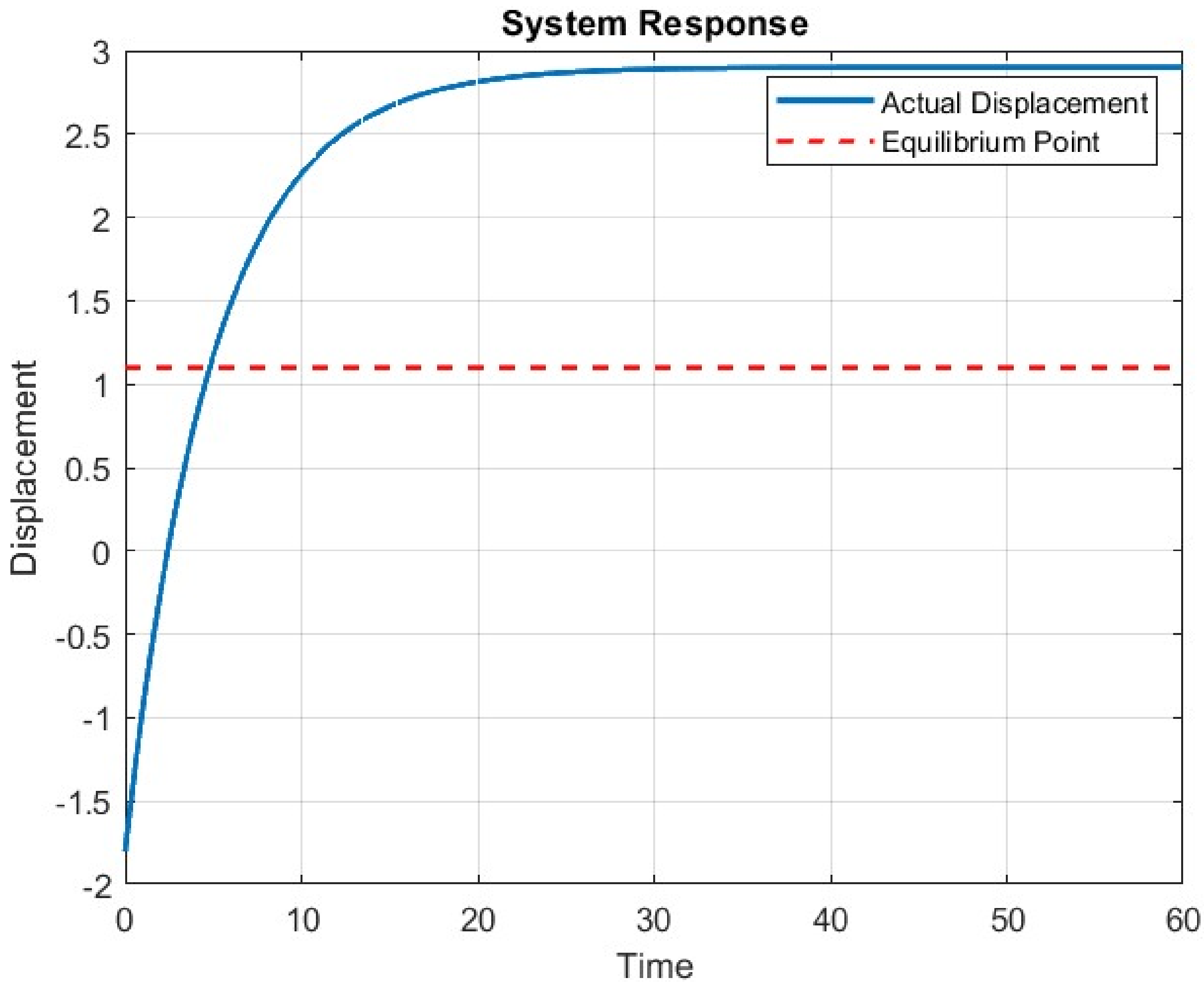


Figure 17: System response with rule 1.

- Rule 2

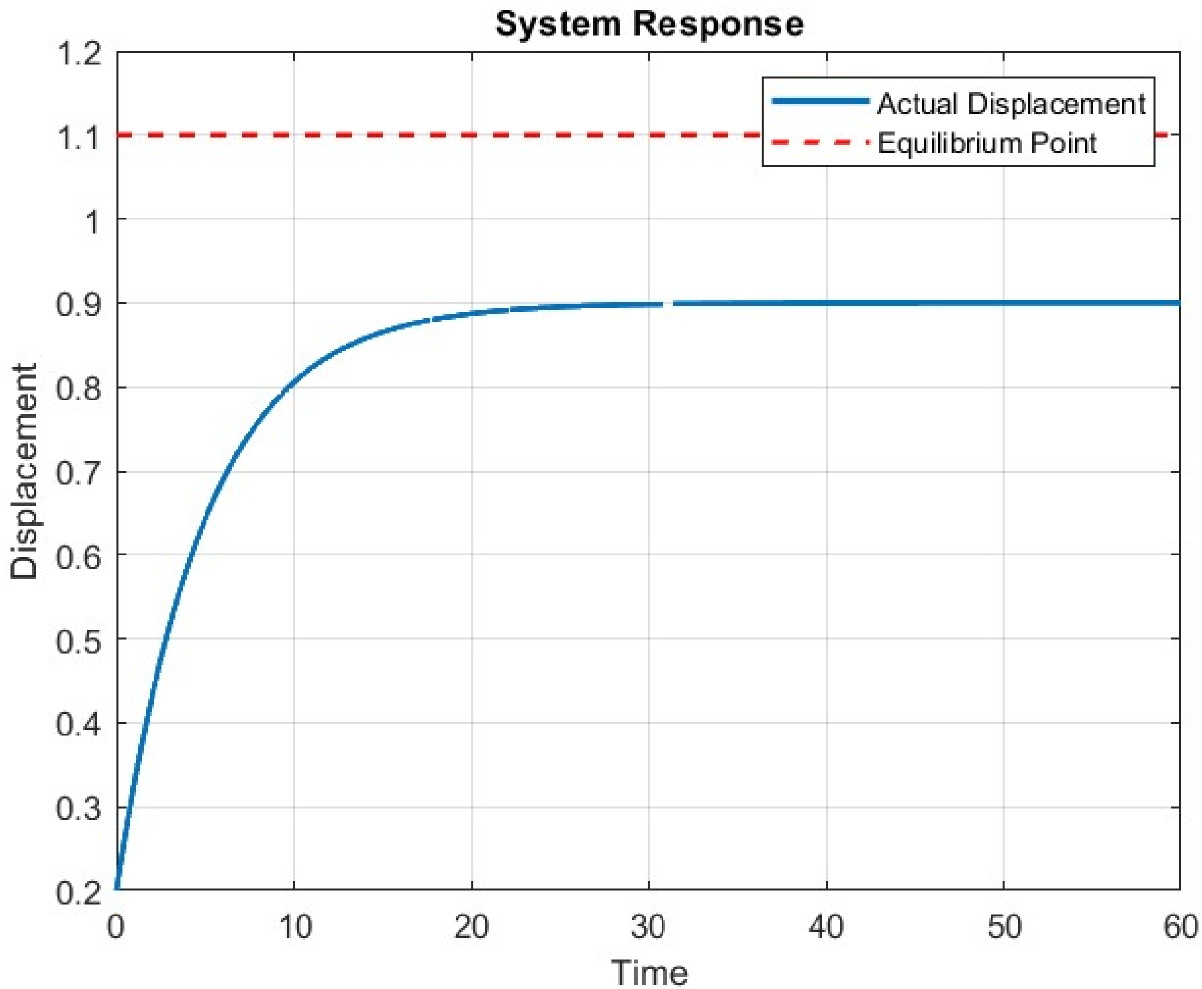


Figure 18: System response with rule 2.

- Rule 3

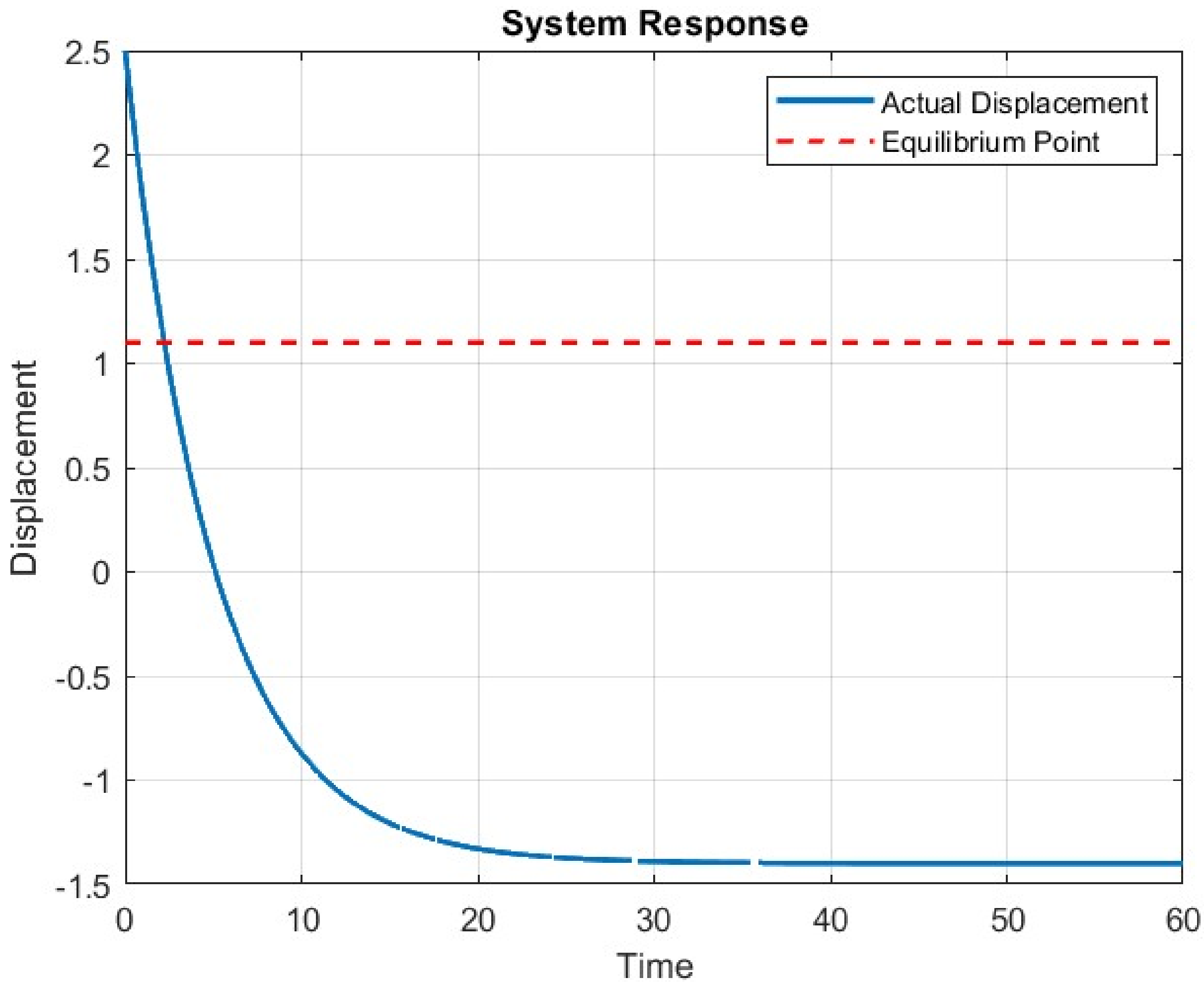


Figure 19: State response with rule 3.

Observing the state response graph of the system, it is evident that the controller with seven fuzzy rules is not sufficient to stabilize the system. So, more rules need to be defined to make the controller able to stabilize the system.

1.6.2 Fuzzy system 2

Fifteen rules were defined in this system.

Table 2: Fuzzy Inference System Rules

Displacement	**Velocity**	**Control Signal**
NL	N	L
NL	Z	L
NL	P	M
NM	N	L
NM	Z	M
NM	P	H
Z	N	L
Z	Z	M
Z	P	H
PM	N	L
PM	Z	M
PM	P	H
PL	N	L
PL	Z	M
PL	P	H

A set of three values are used to test the fuzzy controller system,

- Rule 1
  Displacement = -1.5
  Velocity = 0.5

- Rule 2
  Displacement = 0.3
  Velocity = -1.8

- Rule 3
  Displacement = 3
  Velocity = 2.5

- Rule 1

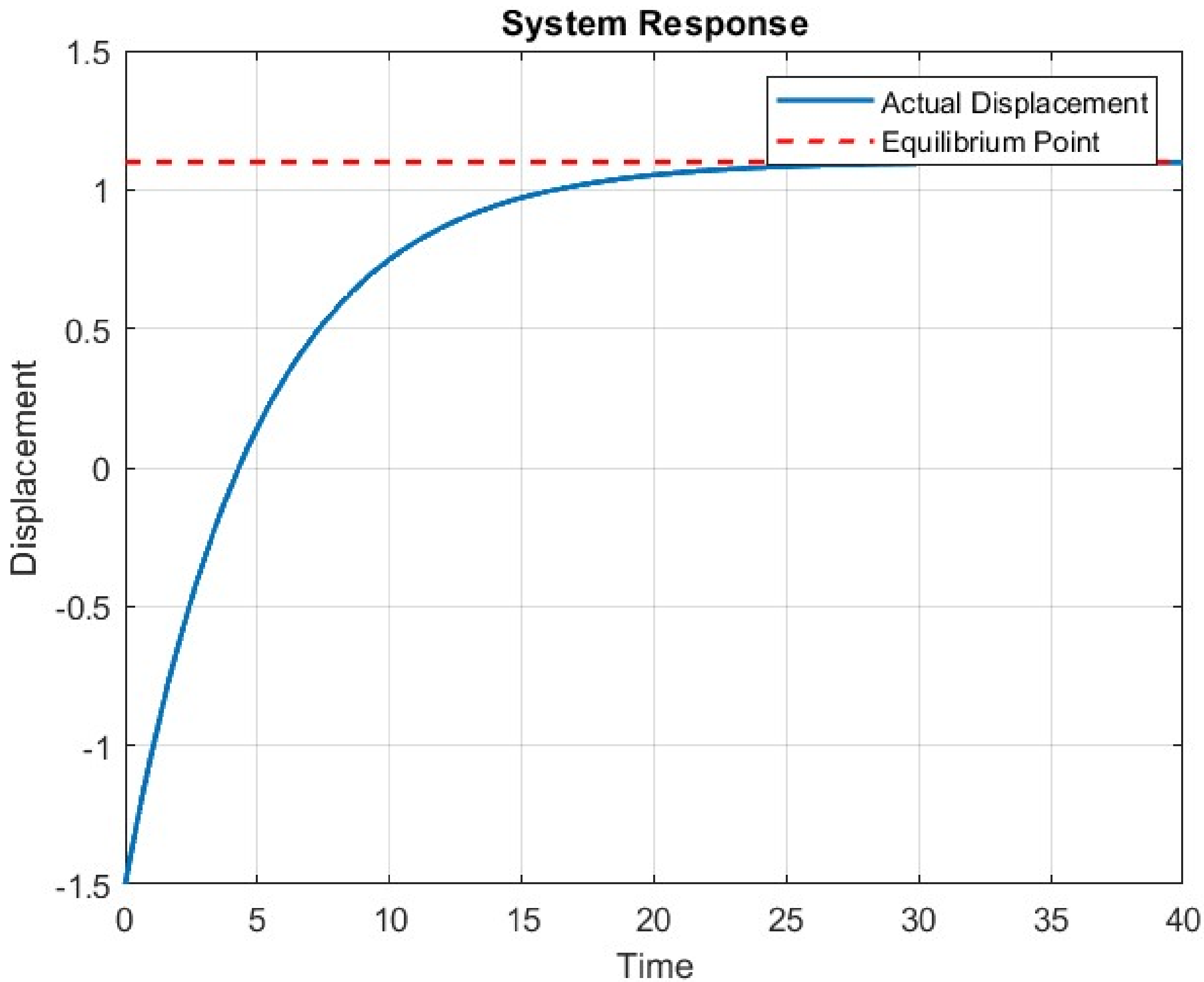


Figure 20: State response with rule 1.

- Rule 2

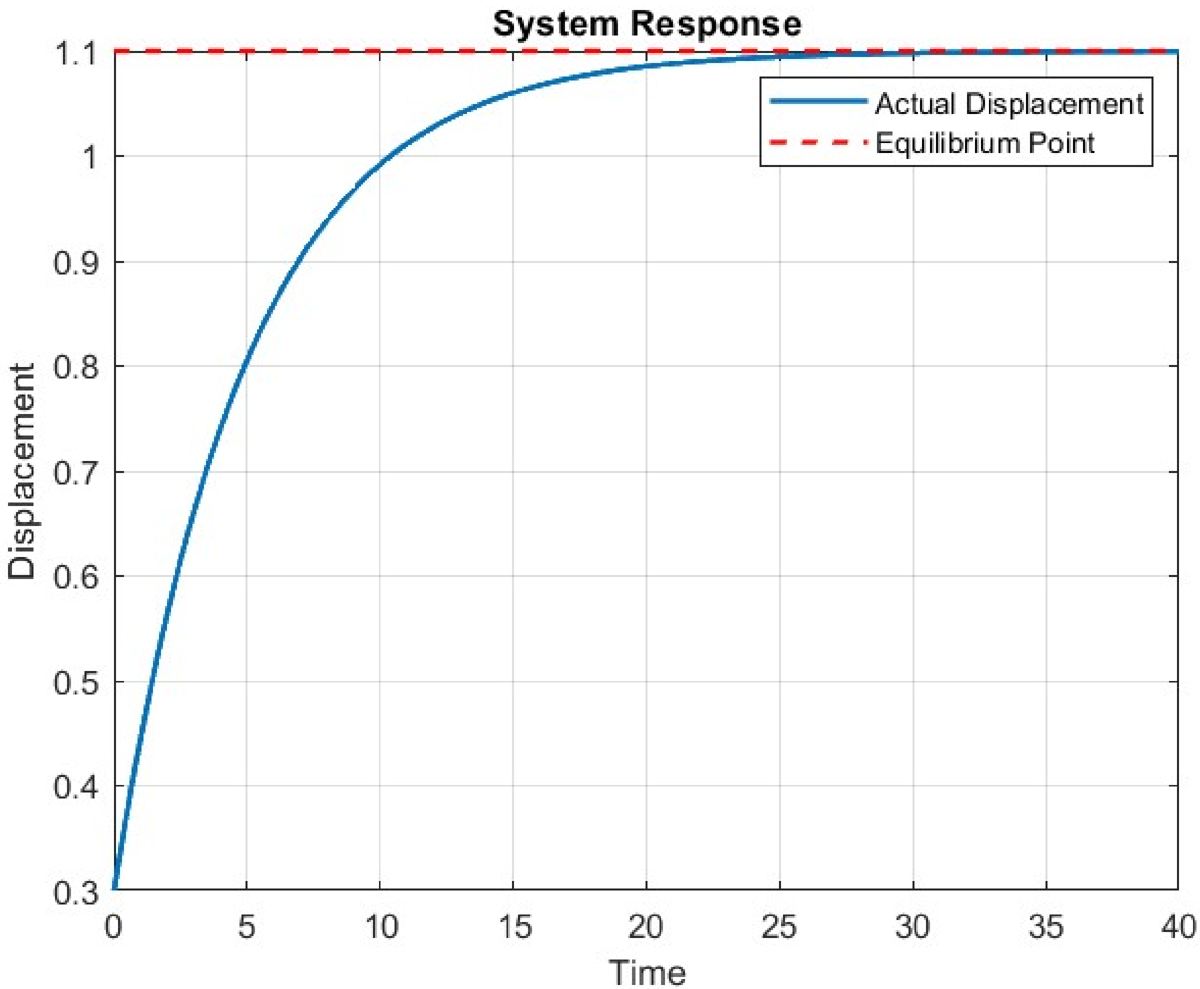


Figure 21: State response with rule 2.

- Rule 3

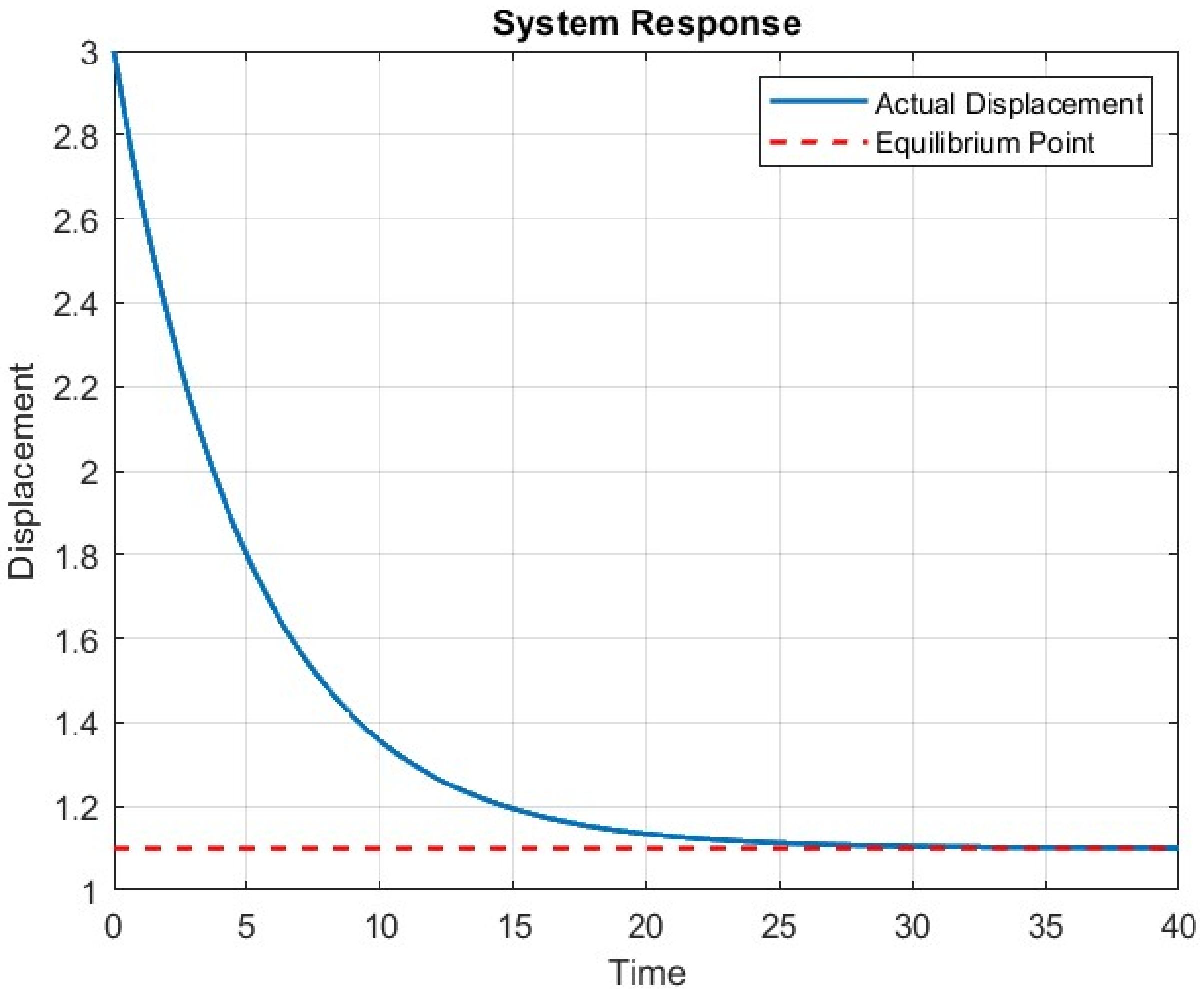


Figure 22: State response with rule 3.

From the above state response graphs it is observed that with a fuzzy controller of fifteen rules, the system is stabilizing. However, the long stabilizing time indicates the scope for the addition of more fuzzy rules. The range of input signal was also defined between a range of zero and ten for easy observation. For future analysis, the range can be increased to obtain deep understanding about the system behavior. [4]

1.7 Controller Design Analysis

1.7.1 State Space Controller Using Pole Placement

The state space controller was designed using pole placement aimed to position the poles in locations that would ensure stability and desired performance. Different pole values were tested, including positive and negative values. When the poles were placed in the positive plane, the system was not stabilizing. Only when the poles were placed in the negative plane the system showed stabilizing behaviour. As the poles were placed more deep into the negative plane it was stabilizing at faster rate, but when choosing an ideal pole the K matrix should also be considered to avoid excess control effort. It can be concluded that selecting suitable poles was critical to achieving the desired system response and stability.

1.7.2 Feedback Linearization

While attempting feedback linearization, a singularity error was encountered. This issue arises when the system's control input cannot be uniquely determined due to the nonlinear dynamics of the system, leading to singularity in the Lie derivatives. This indicates that feedback linearization may not be suitable for this particular system due to the presence of singularities in its model, making it challenging to achieve a well-defined control law.

1.7.3 5. Rule-Based Fuzzy System

A fuzzy logic controller was designed using a set of rules to handle the system's non-linearities. The fuzzy system provided a means to handle uncertainties and approximate reasoning for control actions. Two fuzzy systems were designed and compared, with different rule sets. The performance of the fuzzy controllers was evaluated based on their ability to maintain stability and respond to disturbances. The fuzzy system with more rules was stabilizing the system while the one with fewer rules failed to stabilize the system.

1.8 Observations

1.8.1 Stability

The state space controller demonstrated the potential for achieving stability with appropriate pole placement. However, the feedback linearization approach was hindered by singularity issues, preventing it from being a viable solution. The fuzzy logic controller successfully maintained stability by effectively handling nonlinearities and uncertainties.

1.8.2 Robustness

The fuzzy logic controller showed better robustness due to its adaptability to system uncertainties and nonlinearities. The rule-based approach provided flexibility and consistent performance under varying conditions. However, the fuzzy controller has more room for improvement by adding more rules and trying different membership functions. The state space controller, while effective for linear systems, was less robust to parameter variations and external disturbances compared to the fuzzy logic controller.

1.8.3 Response to Disturbances

The state space controller exhibited good performance in maintaining desired system behavior, but its effectiveness was highly dependent on accurate model parameters and system dynamics. The fuzzy controller was more resilient to disturbances and model inaccuracies, providing smoother and more reliable control actions under varying conditions.

1.9 Conclusion

Considering the singularity issues with feedback linearization, the fuzzy logic controller emerges as the most reliable and practical solution for stabilizing the system. Its ability to handle nonlinearities and uncertainties, coupled with robust performance under disturbances, makes it the preferred choice for practical implementations and it also allows scope for further improvements in controller design.

1.10 References

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