**Introduction:**

Electrical circuits are fundamental components of various electronic devices. Among the simplest yet most important circuits are the Resistor-Capacitor (RC) and Resistor-Inductor (RL) circuits. These circuits exhibit transient behaviours that can be modelled mathematically, providing insights into their dynamic responses to voltage changes over time. Understanding these models is crucial for designing and analyzing electronic systems.

**RC Circuit Model**

**Circuit Description:**

An RC circuit consists of a resistor (R) and a capacitor (C) connected in series or parallel with a voltage source V0​. The behaviour of this circuit is governed by the charging and discharging of the capacitor.

**Governing Equations:**

When a voltage V0​ is applied to the circuit, the capacitor begins to charge. The voltage across the capacitor VC(t) and the resistor VR​(t) can be described using Kirchhoff's voltage law:

V0 = VC(t) + VR(t)

According to Ohm’s law, the voltage across the resistor is:

VR(t) = I(t)R

The current I(t) through the capacitor can be defined as:

I(t) = C [ dVC​(t)**​/**dt]

**Differential Equation:**

Substituting these relationships into Kirchhoff’s equation yields the following first-order linear differential equation:

V0 = VC(t) + RC [dVC(t)**/**dt]​

Rearranging gives:

RC [ dVC(t)**/**dt] + VC(t) = V0 ​

**Solution:**

To solve this equation, we can apply the method of separation of variables. The general solution for the voltage across the capacitor during the charging process is:

VC(t) = V0 (1− e-1/RC)

This equation shows that as time increases, VC(t) approaches V0​, indicating that the capacitor is fully charged.

**Current through the Circuit:**

The current I(t) can be derived by differentiating the voltage across the capacitor:

I(t) = C [dVC(t)**/**dt] = [V0 /R] e−1/RC

This equation indicates that the current decreases exponentially as the capacitor charges, ultimately approaching zero.

**Discharging Phase:**

When the capacitor discharges, the voltage VC(t) can be described as:

VC(t) = [V0] e−1/RC

The current during discharging is given by:

I(t) = [−V0 /R] e−1/RC

**RL Circuit Model**

**Circuit Description:**

An RL circuit consists of a resistor (R) and an inductor (L) connected in series with a voltage source V0​. This circuit is characterized by its inductance, which opposes changes in current.

**Governing Equations:**

When the voltage V0​ is applied, the current I(t) through the inductor begins to increase. The voltage across the inductor VL(t)and the resistor VR(t) is given by:

V0 = VL(t) + VR(t)

Substituting the expressions for VR(t) and VL(t):

VR(t) = I(t)R

VL(t) = L[dI(t)/dt​]

**Differential Equation:**

This leads to the following differential equation for the current:

V0 = L [dI(t)/dt] + I(t)R

Rearranging gives:

L [dI(t)/dt] + RI(t)=V0 ​

**Solution:**

The solution for the current I(t) during the charging phase is given by:

I(t) = V0R (1 − e−Rt/L)

This equation indicates that the current asymptotically approaches V0/R​​,the steady-state current.

**Voltage across the Inductor:**

The voltage across the inductor during the charging phase can be expressed as:

VL(t) = V0 − I(t)R

**Summary of Key Equations:**

* **RC Circuit**:
* Charging Voltage:

VC(t) = V0(1 − e−1/RC)

* Current:

I(t) = [V0/R] e−1/RC

* **RL Circuit**:
  + Current:

I(t) = [V0 /R] (1 − e−Rt/L)

* + Voltage across Inductor:

VL(t) = V0 − I(t)R

**Conclusion:**

Understanding the mathematical models of RC and RL circuits is essential for analyzing their transient responses to applied voltages. These models provide insights into the behaviour of electrical components in various applications, from simple filters to complex signal processing systems. By analyzing these circuits, engineers can design more efficient and reliable electronic devices.