ECE/CS 559 Neural Networks, Fall 2019 - Homework #6
Due: 11/05/2019, the end of class.

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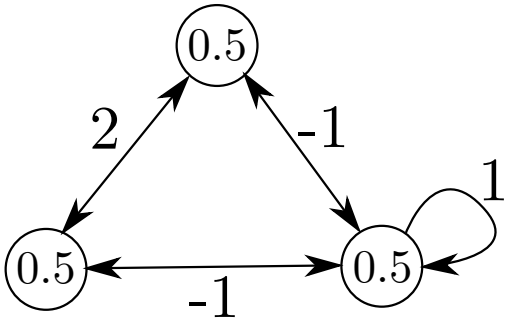
All the notes in the beginning of Homework #1 apply. As usual, please include the computer codes in your report.

1. **(0pts)** Let $\phi(x) = 1$ if $x \geq 0$, and $\phi(x) = -1$ if $x < 0$. Consider some $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \{-1, +1\}^m$. In associative memory, we have designed the synaptic weight matrix as $\mathbf{W} = \sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i^T$. Suppose that there exists $\mathbf{z} \in \mathbb{R}^m$ that satisfies the following properties:

   - $\mathbf{z} = \phi(\mathbf{W}\mathbf{z})$.
   - For any $i \in \{1, \ldots, n\}$, we have $\mathbf{z} \neq \mathbf{x}_i$ and $\mathbf{z} \neq -\mathbf{x}_i$.

   In class, we have called $\mathbf{z}$ a spurious memory pattern. We never gave an example of $\mathbf{x}_1, \ldots, \mathbf{x}_n$ that results in the existence of a spurious pattern. Find such an example and show your work. You may use computer search.

2. **(100pts)** Consider the Hopfield network below. The activation function is, as usual, $\phi(x) = 1$ if $x \geq 0$, and $\phi(x) = -1$ if $x < 0$.



   Draw the state transition diagrams (with energy levels) for both synchronous and asynchronous update rules. Indicate the urstate(s) and the steady state(s) of the network. An urstate is a state that can only occur as an initial state of the network (no predecessors, including itself).