Diffusion Posterior Sampling for MRI Reconstruction without clean data

Akash P, Sai Teja Kuchi
IISc
{pakash,kuchisai}@iisc.ac.in

Abstract

Diffusion models have demonstrated impressive results in various tasks like data generation, inverse problems, etc. However, training them usually requires large amounts of clean data and in fields like medical imaging, access to un-corrupted and clean samples is impossible or expensive to acquire. It has been shown in previous works that image-denoising without clean image data can be addressed by finding the mode of the posterior distribution and Tweedie's formula offers an explicit solution through the score function (i.e. the gradient of log likelihood). In this work, we train an score-based generative model of the clean distribution, from noisy training data by leveraging Stein's unbiased risk estimate to jointly denoise the data and learn the score function via denoising score matching. Finally, we show the results of models trained with access only to Fourier subsampled multi-coil MRI measurements at various acceleration factors (R= 2, 4, 8, 16).

1 Introduction

Reducing scan time is an important goal in magnetic resonance imaging, making it an inverse problem where the image is recovered from under sampled measurements. Traditionally, the problem is posed as a regularized minimization problem where the regularization function enforces a certain prior on the images, including sparsity[8] and smoothness[9]. With the advent of deep learning, more sophisticated priors were used to solve the reconstruction problem[10]. However, with deep learned priors, it is difficult to obtain a functional form for the minimization problem, making interpretability and stability an issue [11].

The integration of neural networks into MRI reconstruction faces two main challenges. Firstly, the diverse set of forward models used in MRI systems make supervised training difficult. Secondly, obtaining high-resolution ground truth data is often impractical due to long scan times. To integrate neural networks into the imaging pipeline, it is necessary to decouple the network from the measurement model and train them in an unsupervised manner using noisy measurements.

Recent advances in score-based (diffusion) generative modeling [4], [5] have helped substantially improve the capabilities of solving ill-posed imaging inverse problems using fewer measurements and with higher reconstruction fidelity in various domains such as medical imaging and more. However, learning high-quality score-based generative models for distributions over real-world signals currently assumes a large database of fully-sampled and noise-free training samples are available. In many application domains like medical fields etc, acquiring such a training set is impossible in practice, because noise is inherently present in the sensors used to acquire measurements. Training Score functions from noisy data turns out to be a Bayesian deconvolution problem which is often intractable.

In our work, we explore the link between the score function and image denoising using Tweedie's formula to come up with a framework for unsupervised training of the score function. We train the score function by passing denoised images to the training of score functions. The denoiser is in turn represented using the score function with the help of Tweedie's formula, which is illustrated

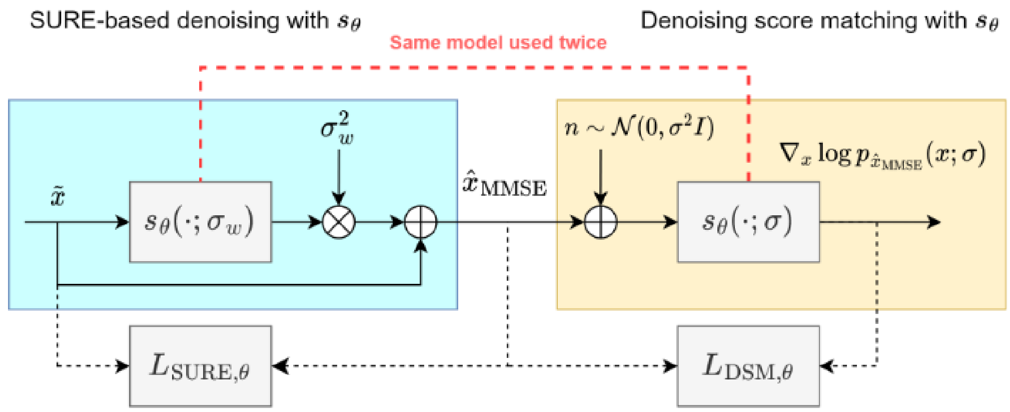


Figure 1: Overall flow of training[7]

in the figure 1. We also integrate Psuedo-Inverse guided sampling pipeline to the baseline langevin sampling method to improve conditioning and show improvements in PSNR.

2 Background

2.1 MRI Reconstruction

Magnetic resonance Imaging reconstruction is an inverse problem where the signal $x \in \mathbb{C}^N$ is recovered from noisy measurements $y_i \in \mathbb{C}^M$ and they are related as,

$$y_i = N_f * S_i(x) + n \qquad \forall i \in 1, 2, ..N_c \tag{1}$$

where N_f is the under sampled Fourier operator, S_i is the i^{th} Coil Sensitivity Map(CSM) which is a diagonal matrix containing the intensity profile of the received data and N_c is the number of coils. The operator N_f reduces to a undersampled version of the Fourier transform $P * F$ where P is a diagonal matrix called as the undersampling mask when the acquisition is on a cartesian grid. In our case, N_f is treated as a general Fourier operator taking care of both cartesian and non-cartesian acquisitions.

In cartesian MRI, P can be a random diagonal matrix and can vary based on the acceleration factor R. The operator N_f can turn into an NuFFT operator[12] in case of Non-cartesian MRI, where there can be spiral, radial acquisitions to name a few. In general, the forward operator N_f varies from scan to scan making supervised learning difficult.

The MRI inverse problem can be posed as the following optimization problem

$$x^* = argmin_x \sum_i^{N_c} \|y_i - N_f * S_i(x)\|_2^2 + \lambda R(x) \tag{2}$$

where $R(x)$ is a regularization prior on images and λ is the regularization parameter. The CSM's are not known and must be estimated. Another way to look at the same method is through the Bayesian outlook, where the objective is to find the maximum aposteriori estimate (MAP)

$$x^* = argmax_x p(x|y) \tag{3}$$

2.2 Diffusion Models for Inverse problems

With diffusion models, we can train a score function to learn the underlying prior distribution p(x) which can then be used to sample new data unconditionally. However, when it comes to inverse problems, we need to sample from the posterior P(x | y) which requires paired data x, y for training and has the same drawbacks as supervised learning techniques.

By making use of Bayes rule on the score function of posterior distribution, we have

$$\nabla_{x_t} logp_{X_t|y}(x_t|y) = \nabla_{x_t} log(\frac{p_{Y|X_t}(y|x_t)p_{X_t}(x_t)}{p_Y(y)}) = \nabla_{x_t} log(p_{Y|X_t}(y|x_t)) + \nabla_{x_t} logp_{X_t}(x_t) \tag{4}$$

$\nabla_{x_t} logp_{X_t}(x_t)$ is the unconditional score-function is learned and p(y|x) from $\nabla_{x_t} log(p_{Y|X_t}(y|x_t))$ is the measurement distribution which is Gaussian. It can be approximated from Langevin dynamics with the below equation during the sampling, where A is forward operator.

$$\nabla_{x_t} log(p_{Y|X_t}(y|x_t)) = \frac{A^H(y - Ax_t)}{\sigma^2} \tag{5}$$

The above equation is slow to converge, hence in practice many use annealed Langevin Dynamics, where γ_t is the annealing factor,

$$\nabla_{x_t} log(p_{Y|X_t}(y|x_t)) = \frac{A^H(y - Ax_t)}{\gamma_t^2 + \sigma^2} \tag{6}$$

3 Psuedo Inverse Guided Sampling with an Unsupervised Score

In this section, we describe our method in two parts: Training an Unsupervised Score function, Psuedo Inverse Guided Sampling for solving the reconstruction problem.

3.1 Training an unsupervised score function

As our data is complex-valued, we need a score function that can take in real and imaginary parts as input. While a significant portion of our work was focused on training a two channel score function, we understood that phase distortion was a major reason why we could not train a two channel score function. As an alternative, inspired by [13], we proceeded with training a score function with a single channel input of normalized magnitude images. Training with single channel images proved to be a simpler task because of the availability of multiple pretrained diffusion models. We used NCSNv2 as a pretrained model to begin training on brain images.

$$x^* = y + \sigma^2 \nabla log(s(y)) \tag{7}$$

As shown in 1, we use a tweedie based image denoiser in the first stage. We obtain a composite loss function written as the sum of denoising score matching loss and SURE loss.

$$L = L_{DSM} + \lambda L_{SURE} \tag{8}$$

$$L_{SURE-SCORE}(\tilde{x}; \sigma, \sigma_w) = \sigma^2 \|s_\theta(g_\theta(\tilde{x}; \sigma_w) + z; \sigma) + \frac{z}{\sigma^2}\|_2^2 + \lambda\|\tilde{x} - g_\theta(\tilde{x}; \sigma_w)\|_2^2 + 2\lambda\sigma_w^2 \cdot div_{\tilde{x}}(g_\theta(\tilde{x}; \sigma_w)) \tag{9}$$

where s_θ is the score function, $z \sim N(0, \sigma^2 I)$, g_θ is derived from Tweedie formula as,

$$g_\theta(\tilde{x}; \sigma_w) = \tilde{x} + \sigma_w^2 s_\theta(\tilde{x}; \sigma_w) \tag{10}$$

and divergence term fo the vector field is approximated using Monte Carlo approximation with n = N(0,I) for small ϵ as follows,

$$div_x(g_\phi(x)) \approx n^T(\frac{g_\phi(x+\epsilon n) - g_\phi(x)}{\epsilon}) \tag{11}$$

3.2 Psuedo-Inverse Guided Sampling

As shown in section 2, langevin sampling steps can be split using Bayes' rule and we can solve inverse problems using langevin sampling. To strengthen the model further, we can solve the inverse problem in the image space which gives rise to the gradient step in langevin sampler with the psuedo-inverse of the forward model as shown in the 2. With this approach, we ensure strengthening of the conditioning of the score model.

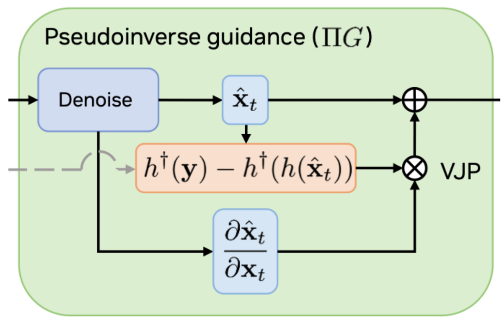


Figure 2: Psuedo Inverse Guided Langevin Sampling [14]

4 Results

Figure 3 shows the power of using Tweedie denoising on images corrupted using gaussian noise. When corrupted clean image with gaussian noise, it's able to beat state of art supervised image denoiser (DRUNet) by a significant amount for different acceleration factors (R).

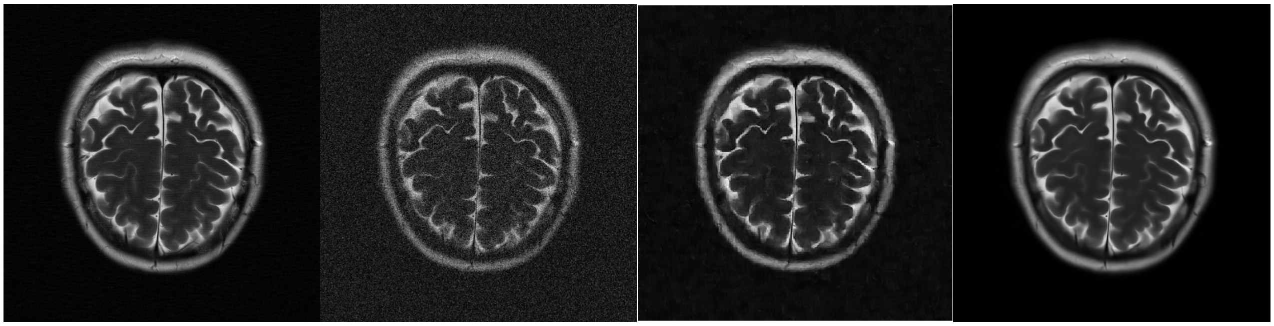
Acceleration Factor (R)	Langevin Sampling with Supervised Score (PSNR dB)	PID Sampling with Supervised Score (PSNR dB)	PID Sampling with Unsupervised Score (Ours) (PSNR dB)
2	28.64	**30.32**	25.68
4	27.66	**29.23**	22.82
8	25.43	**27.32**	20.20
16	23.22	**26.73**	16.23

Table 1: Results for different Acceleration Factors

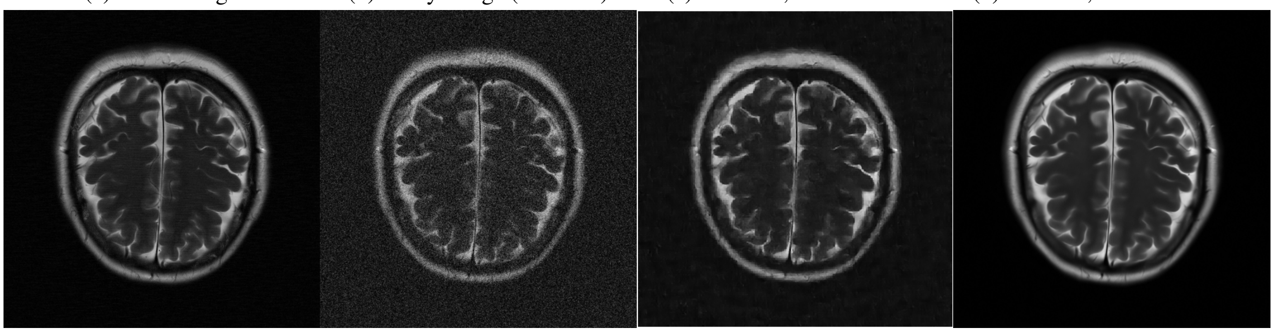
Figure 4, shows the results obtained by making use of PseudoInverse Diffusion on supervised and unsupervised models.

We observe that while PseudoInverse Diffusion is able to improve the PSNR in the case of supervised model, it's creating artifacts in case of unsupervised models which can be seen in figure 5 along with having low PSNR values for different acceleration factors which is provided in the table 1.

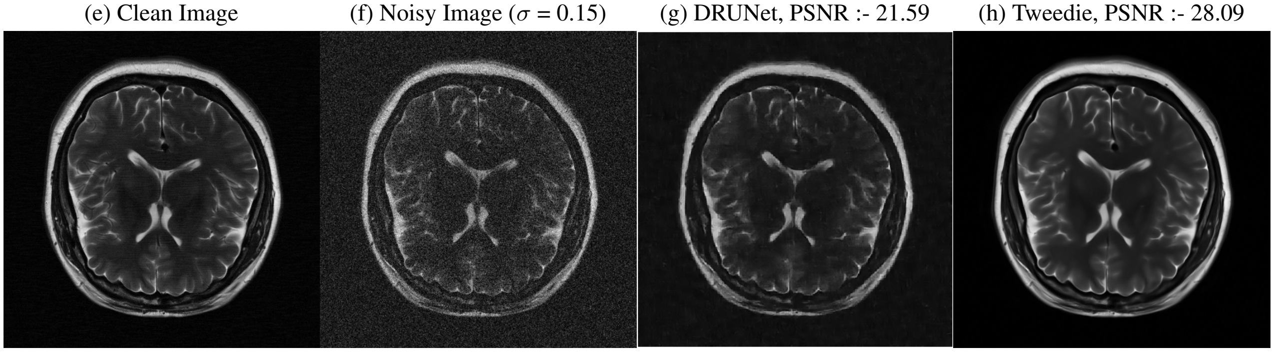
We assume this has to do with the training of the model with sure-score loss as only single channel data is used for training in our case, while most models that train on MRI data make use of both real and complex data parts.



(a) Clean Image (b) Noisy Image (σ = 0.15) (c) DRUNet, PSNR :- 23.8 (d) Tweedie, PSNR :- 31.858



(e) Clean Image (f) Noisy Image (σ = 0.15) (g) DRUNet, PSNR :- 21.59 (h) Tweedie, PSNR :- 28.09



(i) Clean Image (j) Noisy Image (σ = 0.15) (k) DRUNet, PSNR :- 22.54 (l) Tweedie, PSNR :- 31.55

Figure 3: Comparison of Tweedie Denoising with SOTA Supervised Image Denoiser

5 Conclusion

In this project, we trained a score-based generative model along with PseudoInverse diffusion sampling, which is used for denoising MRI data in the absence of clean data samples. While the results for unsupervised model were not satisfactory, we were able to show that making use of PseudoInverse diffusion sampling on supervisied model was able to improve the overall PSNR value across different acceleration factors.

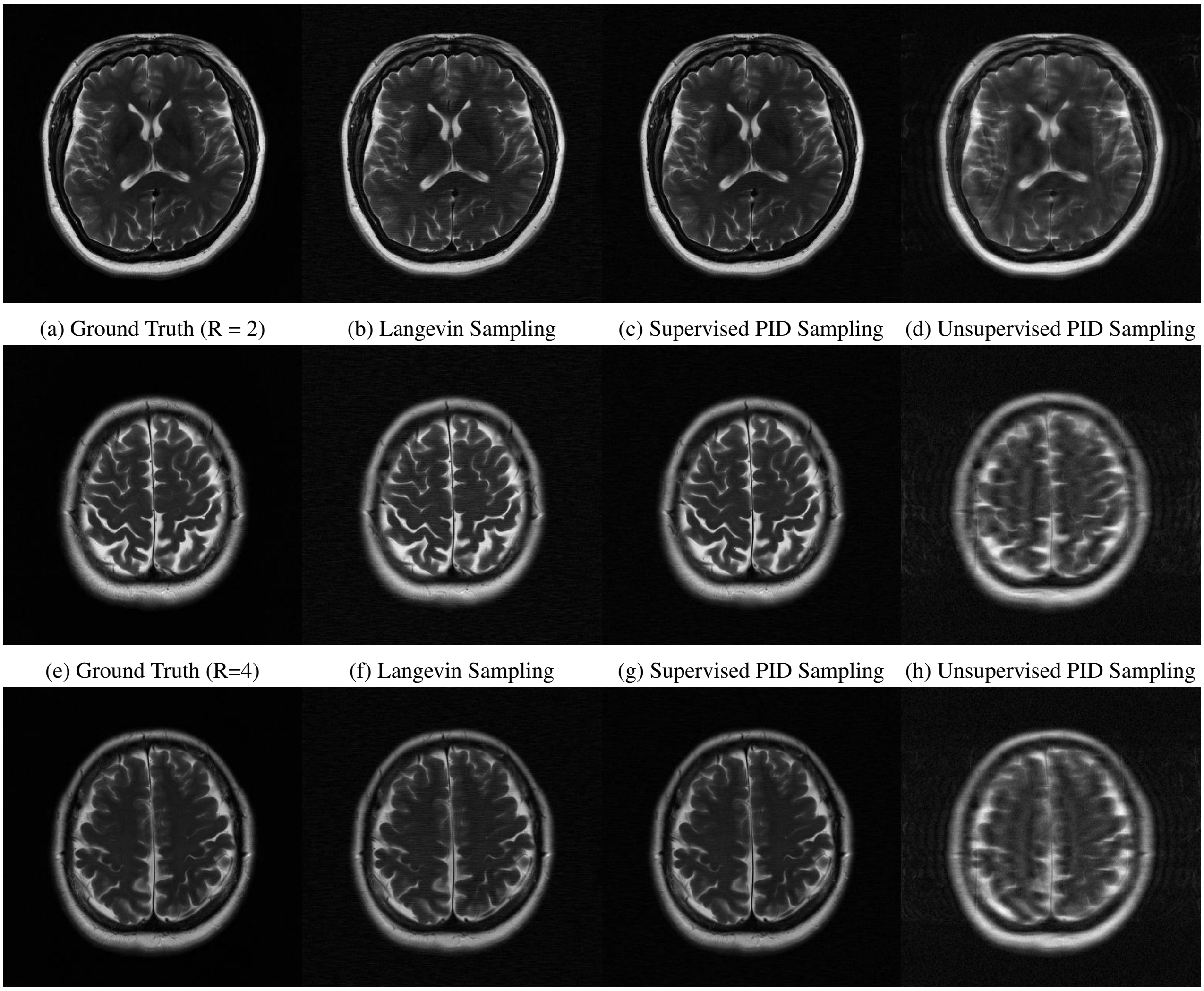


Figure 4: Results of PseudoInverse Diffusion sampling

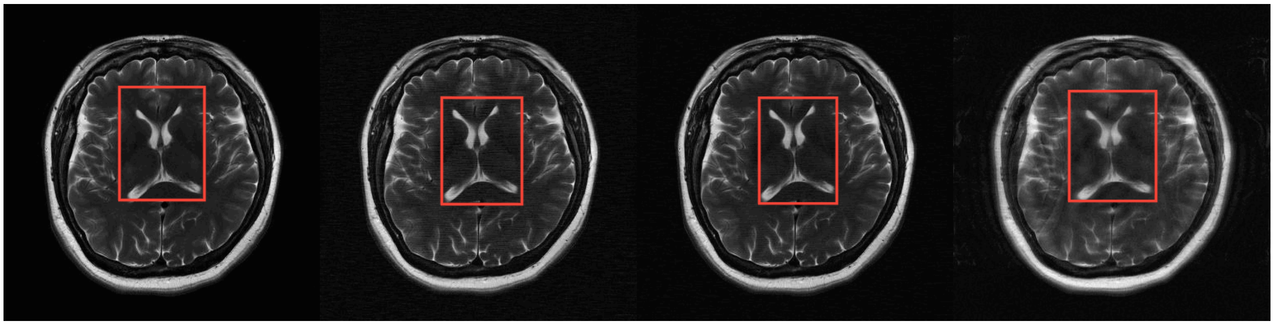
We leveraged the structure in MRI data by expressing the gradient step of sampling in the image space which provided improvement in the SNR values. We showed a consistent improvement across all acceleration factors in Pseudo inverse guided diffusion. We believe diffusion methods must get significantly accelerated to use in real time environments, but this was an effort towards using high representative power of score functions in reconstruction problems.

In showing superior denoising performance through Tweedie reparameterization, we also provided further scope to use score functions to solve inverse problems traditionally using plug and play methods.

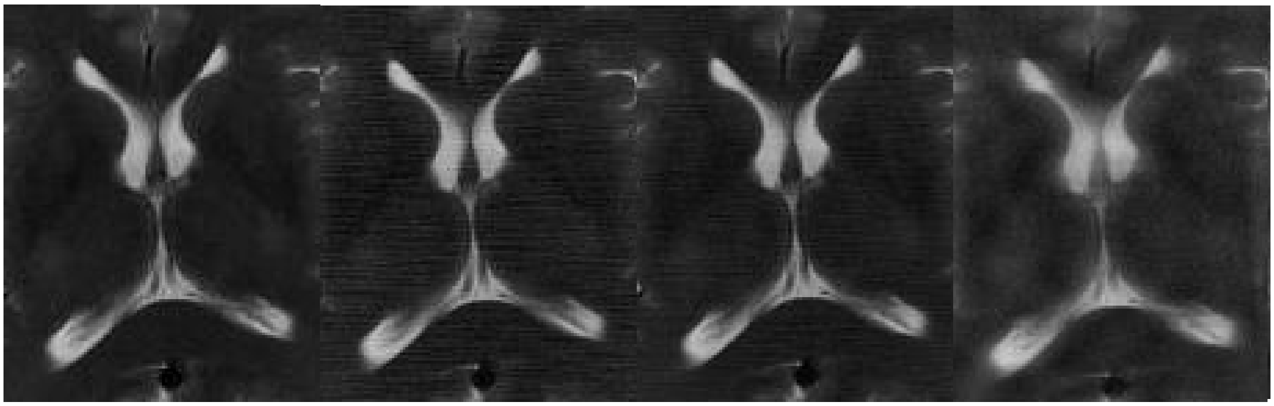
References

[1] Dabov, Kostadin, Alessandro Foi, Vladimir Katkovnik, and Karen Egiazarian. "Image denoising with block-matching and 3D filtering." In Image processing: algorithms and systems, neural networks, and machine learning, vol. 6064, pp. 354-365. SPIE, 2006.

[2] Sathish Ramani, Thierry Blu, and Michael Unser. Monte-carlo sure: A black-box optimization of regularization parameters for general denoising algorithms. IEEE Transactions on image processing, 17(9):1540–1554, 2008.



(a) Ground Truth (b) Langevin Sampling (c) PID Langevin Sampling (d) Unsupervised PID Sampling



(e) PSNR Value (f) 28.73 (g) 29.04 (h) 25.2

Figure 5: Results for Acceleration Factor (R=2)

[3] Kai Zhang, Wangmeng Zuo, Yunjin Chen, Deyu Meng, and Lei Zhang. Beyond a gaussian denoiser: Residual learning of deep cnn for image denoising. IEEE transactions on image processing, 26(7):3142–3155, 2017.

[4] Y. Song and S. Ermon, “Generative modeling by estimating gradients of the data distribution,” Advances in neural information processing systems, vol. 32, 2019.

[5] J. Ho, A. Jain, and P. Abbeel, “Denoising diffusion probabilistic models,” Advances in Neural Information Processing Systems, vol. 33, pp. 6840– 6851, 2020.

[6] Bradley Efron. Tweedie’s formula and selection bias. Journal of the American Statistical Association, 106(496):1602–1614, 2011.

[7] Aali, Asad, Marius Arvinte, Sidharth Kumar, and Jonathan I. Tamir. "Solving inverse problems with score-based generative priors learned from noisy data." arXiv preprint arXiv:2305.01166 (2023).

[8] Michael Lustig, David Donoho, and John M Pauly. Sparse mri: The application of compressed sensing for rapid mr imaging. Magnetic Resonance in Medicine: An Official Journal of the International Society for Magnetic Resonance in Medicine, 58(6):1182– 1195, 2007.

[9] Klaas P Pruessmann, Markus Weiger, Peter Börnert, and Peter Boesiger. Advances in sensitivity encoding with arbitrary k-space trajectories. Magnetic Resonance in Medicine: An Official Journal of the International Society for Magnetic Resonance in Medicine, 46(4):638–651, 2001.

[10] Hemant K Aggarwal, Merry P Mani, and Mathews Jacob. Modl: Model-based deep learning architecture for inverse prob- lems. IEEE transactions on medical imaging, 38(2):394–405, 2018.

[11] Vegard Antun, Francesco Renna, Clarice Poon, Ben Adcock, and Anders C Hansen. On instabilities of deep learning in image reconstruction and the potential costs of ai. Proceedings of the National Academy of Sciences, 117(48):30088–30095, 2020.

[12] Jeffrey A Fessler and Bradley P Sutton. Nonuniform fast fourier transforms using min-max interpolation. IEEE transactions on signal processing, 51(2):560–574, 2003.

[13] Chung, Hyungjin, and Jong Chul Ye. "Score-based diffusion models for accelerated MRI." Medical image analysis 80 (2022): 102479.

[14] Song, Jiaming, Arash Vahdat, Morteza Mardani, and Jan Kautz. "Pseudoinverse-guided diffusion models for inverse problems." In International Conference on Learning Representations. 2022.