Lab 10: Simple Pendulum
San Diego State University
Department of Physics
Physics 182A/195L

TA:
Lab partner 1:
Lab partner 2:
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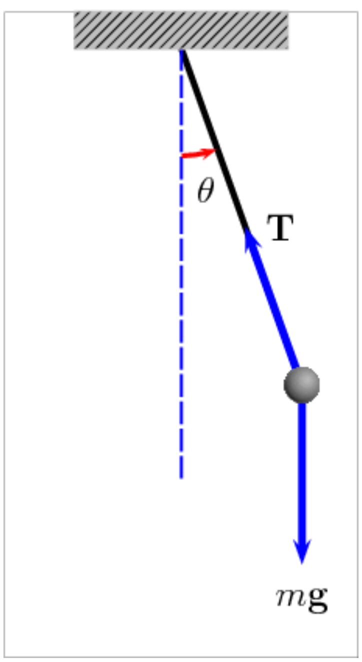
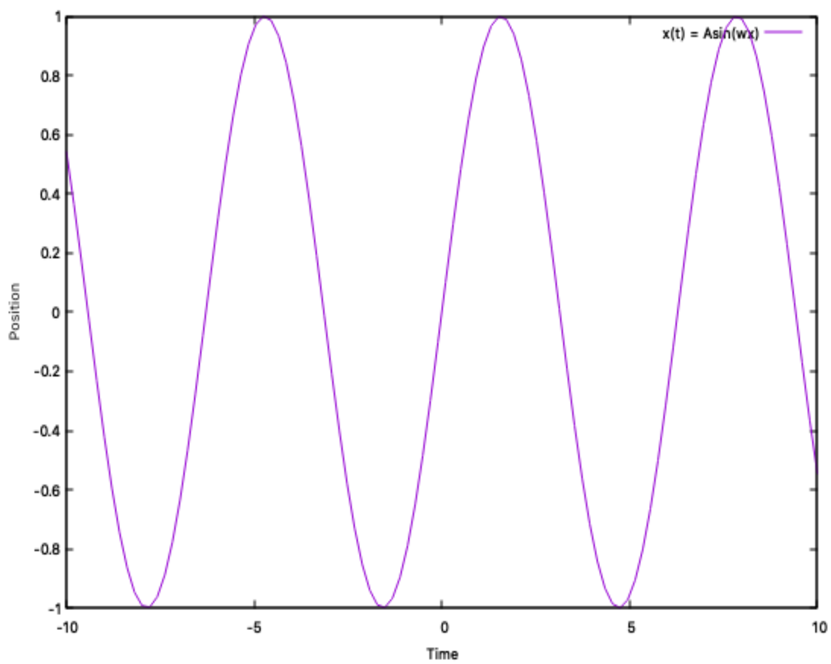
Theory

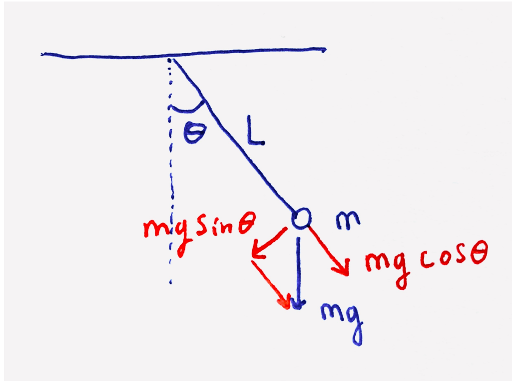
Oscillations can occur whenever a system is pulled away from a stable equilibrium. For example, Hooke's law $F = -kx$ describes a spring system with an equilibrium at $x = 0$. A spring will oscillate if you pull it away from that equilibrium and then release it. In this lab we will investigate the oscillations of the simple pendulum system: a mass m hangs from a fixed point by a string of length L and negligible mass. We will see that this system exhibits *simple harmonic motion*, a kind of oscillation that can be described by a simple sine or cosine function.





Force diagram

To figure out how to describe this motion, we start by writing down the forces acting on the pendulum mass m . Looking at the free-body diagram above, we can see that there are two forces acting on the mass: there is gravity pulling downward on the mass with a force $\vec{f_1} = -mg\hat{y}$ (mg in the negative y-direction), and there is the force of the pendulum arm creating a tension $\vec{f_2} = Te\vec{n}sion$ keeping the mass a fixed length from the pivot.



Newton's second law for this system reads $\vec{f_1} + \vec{f_2} = m\vec{a}$, a vector equation. Let's decompose each vector into its components parallel to and perpendicular to the direction of the pendulum arm. Along the direction parallel to the pendulum arm:

$$Tension - mg\cos(\theta) = ma_{parallel}$$

The cosine function comes from the fact that the gravitational force is shifted θ degrees away from being parallel to the pendulum arm. Something convenient happens here: since we know the length of the pendulum arm will not change, we can safely assume that the tension in the string and the force $-mg\cos(\theta)$ will cancel, leaving $a_{parallel} = 0$.

Along the direction perpendicular to the pendulum arm, or in other words, along the direction of motion:

$$-mg\sin(\theta) = ma_{motion}$$

Again, the sine function appears here due to the right-triangle geometry depicted above. Since only a_{motion} is non-zero, we will just call it 'a' from here on.

The position equation

We will briefly describe some math that you may have seen in your more advanced calculus courses. If you don't want to see any calculus, skip ahead to the calculus free conclusion.

The equation of motion above is really a differential equation for the displacement $x(t)$ of the pendulum mass:

$$-mg\sin(\theta) = ma = m\frac{d^2x(t)}{dt^2}$$

$$\frac{d^2x(t)}{dt^2} = -g\sin(\theta)$$

This says that the second derivative of $x(t)$ is related to $-g\sin(\theta(t))$. In general, this is a difficult equation to solve. In fact, it can't be solved with normal calculus methods. We can solve this equation with an approximation**.** We will apply the **small angle approximation**, which says that as long as the angle θ is small,

$$\sin(\theta(t)) \approx \theta(t).$$

By applying this small angle approximation, our equation for $x(t)$ becomes much simpler:

$$\frac{d^2x(t)}{dt^2} = -g\theta(t).$$

This will make more sense if we replace θ with something that depends on x and t. If we remember from geometry that arc length is related to the radius and angle, we can see for this situation that:

$$x(t) = L\theta(t)$$

$$\theta(t) = \frac{x(t)}{L}$$

If we plug this into the equation above:

$$\frac{d^2x(t)}{dt^2} = -\frac{g}{L}x(t)$$

Notice that this looks a lot like Hooke's law! The force (ma) is equal to a negative constant times displacement: $F = -(gm/L)x$. A function that satisfies this equation is

$$\boxed{x(t) = A\sin(\sqrt{\frac{g}{L}}t)}$$

Calculus free conclusion: *a simple pendulum will oscillate in a way according to a sine function, as long as the angle it swings to is small.*

Period of motion

There's more that we can learn given the boxed equation for x(t). One question we might like to answer is: how long will it take the pendulum to swing back and forth? In other words, what is the period of the motion? Recalling our knowledge of sine waves, we know that they repeat every 2π radians, and if we write the sine wave in the form:

$$x(t) = A\sin(\frac{2\pi}{T}t)$$

Then we can identify the constant T as the period of the motion. If we set these two forms of $x(t)$ equal to each other:

$$x(t) = A\sin(\frac{2\pi}{T}t) = A\sin(\sqrt{\frac{g}{L}}t),$$

We can recognize that

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

The period of motion T (the time it takes to complete one cycle of oscillation) of a simple pendulum is given by:

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Procedure

In this procedure we will measure the period of motion of a simple pendulum. We will try to answer questions about the period T, to see how it is affected by different properties of the pendulum system such as the length of the pendulum arm L, the mass of the pendulum bob m, the acceleration due to gravity g, and the amplitude of the motion A. You can probably guess from the equation for T which of these will have an impact and which will not.

Setup

1. Run the string through the hole in the brass cylinder and put the ends of the string on the inner and outer clips of the clamp (**Figure 1**), so the string forms a 'V' shape as it hangs.

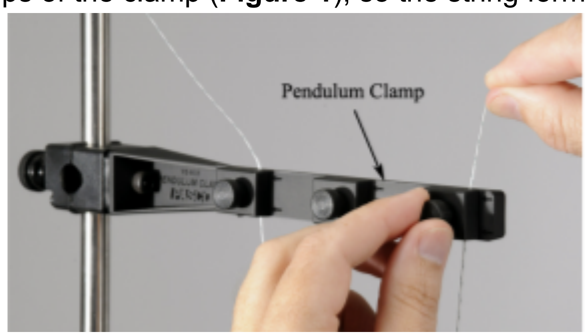


Figure 1: Pull the string between the inner and outer clips of the ramp

2. Adjust the string so the vertical distance from the bottom edge of the Pendulum Clamp the middle of the pendulum bob is 0.70 m.
3. Position the Motion Sensor in front of the pendulum so the brass colored disk is vertical and facing the bob, aimed along the direction that the pendulum will swing (**Figure 2**).
4. Make sure that the range switch on the Motion Sensor is positioned on top. Adjust the Motion Sensor up or down so that it is at the same height as the pendulum bob.
5. Adjust the position of the Rod Base so that the Motion Sensor is 0.25 m from the pendulum bob.

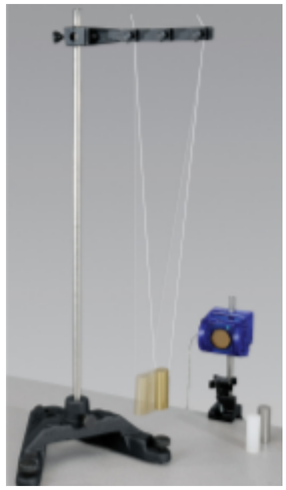


Figure 2: Suspend the bob in a “V” shape, with the motion sensor facing the bob.

Measuring the Period

There are three methods available for measuring the period of motion. Ask your TA which method or methods to use.

Method 1: The most straightforward way to measure the period of oscillation is to set the pendulum into motion and to simply time N complete cycles. If the pendulum swings back and forth, returning to its original position N times in t seconds, then the period is $T = t/N$. It's important to choose N>10 so that random errors from the timing method will average out.

The other two methods use PASCO Capstone™ software to find your period. Start by recording trials of your oscillations:

- Click on Record to start recording data, data should appear on the graph. After 15 seconds, stop recording. Make sure the amplitude of the swing is relatively small (such as 0.05 m). If readings are not being recorded, make sure the objects are more than 0.15 m away from the sensor.
- If the resulting data is not a smooth sine curve, you can try changing the position of the range switch on top of the Motion Sensor. In general, the position with the cart icon is for smaller, closer objects and the position with the person icon is for larger objects further away. You should position the switch for whichever works the best.
- Select the rate that data is recorded by changing the frequency of the motion sensor at the bottom of the screen. Make sure there are enough points to create a smooth sine wave. A rate of 50 Hz will work well for this experiment.

Once these steps have been performed, you can find your period with one of the two following methods:

Method 2: PASCO Capstone™ can find the period using a recording of the position versus time of the pendulum bob. Begin by using the Coordinates Tool (from the Graph Tool palette), select a peak in your data recovered from oscillating a pendulum. Right click on the graph box and select Show Delta. Find an adjacent peak where ∆x ~ 0m to view the period.

Method 3: Another method involves using various Curve Fit functions available. Select the Sine Fit from the Graph Tool palette. The general form of the sine wave is: $Asin(\frac{2\pi}{T}t + \phi) + C$, where T is the period. The values from your Curve Fit can be used to calculate the period of the oscillation. (Note: ω in the Sine Fit option is equal to $2\pi/T$).

Part A: Period of Oscillation as a Function of Length

We will test how changing L, the length of the pendulum, affects the period of motion.

1. Adjust the length of the pendulum to be 0.7 meters by lengthening or shortening the string. Measure from the bottom edge of the pendulum clamp down to the middle of the cylinder.
2. Pull the pendulum bob about 0.05 m away from equilibrium and then release. Allow the bob to oscillate for a few seconds until the oscillations are smooth.
3. Measure the period of oscillation using the methods from **Measuring the Period**.
4. Record your measured period in Table A.1.
5. Stop the pendulum from swinging and then shorten the pendulum length by 0.05 m.
6. Repeat steps 2-5, recording data until you have a range of lengths from 0.70 m to 0.15 m according to Table A.1.

Part B: Period of Oscillation as a Function of Amplitude

1. Keep the pendulum length fixed at 0.35 m from the bottom of the clamp to the middle of the bob (last length used from **Part A**).
2. Pull the pendulum bob about 0.10 m away from equilibrium and then release. Allow the bob to oscillate for a few seconds until the oscillations are smooth.
3. Measure the period of oscillation using the methods from **Measuring the Period**.
4. Record the measured period in Table B.1.
5. Stop the pendulum from swinging.
6. Repeat steps 2-5 for amplitudes of 0.09 m, 0.08 m, 0.07 m, 0.06 m and 0.05 m, recording each measured period in Table B.1.

Part C: Period of Oscillation as a Function of Mass

1. Remove the brass cylinder from the pendulum and measure the mass of the brass, aluminum, and plastic cylinders. Record these values in Table C.1.
2. Reattach the brass cylinder to the pendulum system.
3. Adjust the length of the pendulum so that the distance from the bottom edge of the pendulum clamp to the middle of the cylinder is 0.60 m.
4. Pull the pendulum bob about 0.06 m away from equilibrium and then release. Allow the bob to oscillate for a few seconds until the oscillations are smooth.
5. Measure the period using one of the methods from the **Measuring the Period**.
6. Record the measured period in Table C.1.
7. Stop the pendulum from swinging.
8. Repeat steps 3-6 for both the aluminum and plastic cylinder. Make sure to keep the length of the pendulum, and the maximum amplitude, the same.

Data

Table A.1: Period and Varying Length

Length (m)	Cycles observed	Time, if using Method 1 (s)	Period (s)
0.70			
0.65			
0.55			
0.45			
0.35			
0.25			
0.20			
0.15			

Table B.1: Period and Varying Amplitude

Amplitude (m)	Cycles observed	Time, if using Method 1 (1)	Period (s)
0.10			
0.09			
0.08			
0.07			
0.06			
0.05			

Table C.1: Period and Varying Mass

Bob (type)	Bob Mass (kg)	Cycles observed	Time, if using Method 1 (s)	Period (s)
Brass				
Aluminum				
Plastic				

Analysis

Part A: Period of Oscillation as a Function of Length

1. What happens to the period as you adjust the length of the pendulum?

2. We can solve for g, the acceleration due to gravity, using the length and period. Beginning from the equation of motion found in the **Theory** section, we can solve for g:
$$g = \frac{4\pi^2 L}{T^2}$$
Find the value of g for each of the lengths, using Table A.1 as reference:

Length (m)	**g (m/s²)**
0.70	
0.65	
0.55	
0.45	
0.35	
0.25	
0.20	
0.15	

3. Do your values for g change significantly for different values of the pendulum length? Does this make sense? Why or why not?

Part B: Period of Oscillation as a Function of Amplitude

What happens to the period as you adjust the amplitude of the pendulum?

Part C: Period of Oscillation as a Function of Mass

1. What happens to the period as you adjust the mass of the pendulum?

2. The amplitude of a pendulum's oscillation is usually measured by the angle through which it swings, not its horizontal displacement. In **Part C** the length of the pendulum

was about 60 cm and the maximum amplitude was about 6cm. Using right-triangle trigonometry, calculate the angle of the amplitude. Show your work.

3. The period of a pendulum is independent of amplitude only if the angle is small (remember the small angle approximation we made?). Was this the case?

Conclusion

Which variables (g, L, m, A) affect the period of motion T of the simple pendulum, and which do not?

Questions

1. Must a spring obey Hooke's Law in order to oscillate? Explain why or why not.

2. When a body is oscillating in simple harmonic motion, is its acceleration zero at any point? If so, where and why?

3. You are trying to build a metronome (a time keeping device) using a simple pendulum. Assume that your pendulum bob weighs 1 kg and $g=9.8 m/s^2$. How long should you make the pendulum arm so that the metronome ticks with a frequency of 100 BPM (beats per minute)? (Hint: frequency = 1/T.) Show your work.

4. What is the frequency of a pendulum whose normal period is T when it is in an elevator in free fall? (Hint: what is the apparent value of g in this situation?)

5. We usually assume that the mass of the pendulum arm is negligible compared to the mass hung from it. But if not negligible, does the mass of the string increase or decrease the period of motion?