

Todays Content:

- a) Number System Basics
- b) Binary to Decimal & Viceversa
- c) Adding 2 Binary numbers
- d) Bit Wise operations
 - i) Basic Properties
 - ii) Basic Problems

Decimal Number System → Each Digit: [0 : 0 1 2 3 4 5 6 7 8 9]

$$\underline{10^3} \quad \underline{10^2} \quad \underline{10^1} \quad \underline{10^0}$$

Each power: [10]

$$3 \ 4 \ 2 = 3 \cdot 10^2 + 4 \cdot 10^1 + 2 \cdot 10^0$$

$$2 \ 5 \ 6 \ 3 = 2 \cdot 10^3 + 5 \cdot 10^2 + 6 \cdot 10^1 + 3 \cdot 10^0$$

Binary Number System → Each Digit: [0 1]

Each power: [2]

$$\begin{array}{r} 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 \ 0 \ 0 \ 1 \ 0 \ 1 \end{array} \Rightarrow 2^5 + 0 + 0 + 2^2 + 0 + 2^0 = 32 + 4 + 1 = 37$$

$$\begin{array}{r} 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 \ 0 \ 0 \ 1 \ 1 \end{array} \Rightarrow 2^4 + 0 + 0 + 2^1 + 2^0 = 19$$

$$\left[\begin{array}{r} 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 \ 1 \ 0 \ 0 \ 1 \end{array} \right] \Rightarrow 2^4 + 2^3 + 0 + 0 + 2^0 = 16 + 8 + 1 = 25$$

$$\begin{array}{r} 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 0 \ 0 \ 1 \ 0 \ 1 \ 0 \end{array} \Rightarrow 2^5 + 2^1 = 32 + 2 = 10$$

Note: Adding 0 at start won't change value

Decimal to Binary

$$\begin{array}{r} \text{Rem} \\ \hline 2 | 37 : -1 \\ 2 | 18 : -0 \\ 2 | 9 : -1 \\ 2 | 4 : -0 \\ 2 | 2 : -0 \\ 2 | 1 : -1 \\ 0 \end{array}$$

= 100101

$$\begin{array}{r} \text{Rem} \\ \hline 2 | 25 : 1 \\ 2 | 12 : 0 \\ 2 | 6 : 0 \\ 2 | 3 : 1 \\ 2 | 1 : 1 \\ 0 \end{array}$$

= 11001

$$\begin{array}{r} \text{Rem} \\ \hline 2 | 27 : 1 \\ 2 | 13 : 1 \\ 2 | 6 : 0 \\ 2 | 3 : 1 \\ 2 | 1 : 1 \\ 0 \end{array}$$

= $\begin{array}{r} 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 \ 1 \ 0 \ 1 \ 1 \end{array} = 2^4 + 2^3 + 2^1 + 2^0 = 27$

Note: Keep dividing by 2 until Quotient becomes 0

Add 2 decimal numbers: $d = 59.10$ $c = 5/10$: 10 decimal number system

$$\begin{array}{r} c = 1 \quad 1 \\ \text{Ex1: } \begin{array}{r} 7 \quad 8 \quad 9 \\ 0 \quad 1 \quad 4 \quad 2 \\ \hline 9 \quad 13 \quad 11 \\ d = \underline{0 \quad 9 \quad 3 \quad 1} \end{array} \end{array}$$

$$\begin{array}{r} c = 1 \quad 0 \quad 1 \\ \text{Ex2: } \begin{array}{r} 17/10 \quad 8/10 \quad 13/10 \\ 1/10 \quad \begin{array}{r} 1 \quad 0 \quad 1 \\ 7 \quad 8 \quad 3 \quad 9 \\ 3 \quad 9 \quad 4 \quad 8 \\ \hline 11/10 \quad 17/10 \quad 8/10 \quad 17/10 \\ d = \underline{1 \quad 1 \quad 7 \quad 8 \quad 7} \end{array} \end{array} \end{array}$$

Add 2 Binary Numbers $d = 5/2$ $c = 5/2$: $2 \rightarrow$ Binary numbers

$$\begin{array}{r} c = 1/2 \quad 3/2 \quad 2/2 \quad 1/2 \\ \begin{array}{r} 0 \quad 1 \quad 1 \quad 0 \\ 1 \quad 0 \quad 1 \quad 1 \quad 0 \end{array} \\ \begin{array}{r} 1/2 \\ \begin{array}{r} 0 \quad 0 \quad 1 \quad 1 \quad 1 \\ \hline 1/2 \quad 1/2 \quad 3/2 \quad 2/2 \quad 1/2 \\ d = \underline{0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1} \end{array} \end{array} \end{array}$$

$$\begin{array}{r} c = 2/2 \quad 1/2 \quad 2/2 \quad 1/2 \\ \begin{array}{r} 1 \quad 0 \quad 1 \quad 0 \\ 1 \quad 1 \quad 0 \quad 1 \quad 1 \end{array} \\ \begin{array}{r} 3/2 \\ \begin{array}{r} 1 \quad 1 \quad 0 \quad 1 \quad 0 \\ \hline 3/2 \quad 2/2 \quad 1/2 \quad 2/2 \quad 1/2 \\ d = \underline{1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1} \end{array} \end{array} \end{array}$$

Why Binary?

In Electronics current passes

if voltage

$$\begin{array}{l} > \text{Certain limit : 1} \\ < \text{Certain limit : 0} \end{array} \xrightarrow{\text{TODD}}$$

int $n = 25;$

In System data stored in Binary

$$n = \boxed{11001}$$

Decimal to Binary

These internal conversions are internally done by your System.

We don't have to worry.

Binary to Decimal

$$\text{print}(n) = 25$$

Bitwise operations : { AND, OR, XOR, Invert, leftshift, rightshift }
 & | ^ ~ << At End >>

A	B	Both 1	one 1	diff 1	A & B	$\sim A$
0	0	0	0	0	0	1
0	1	0	1	1	0	1
1	0	0	1	1	1	0
1	1	1	1	0	0	0

: Same Same puppy same

// a = 29, b = 19

$$\begin{array}{cccccc} 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \underline{1} & \underline{1} & \underline{1} & \underline{0} & \underline{1} \end{array}$$

$$\begin{array}{cccccc} a: & \underline{1} & \underline{1} & \underline{1} & \underline{0} & \underline{1} \\ b: & \underline{1} & \underline{0} & \underline{0} & \underline{1} & \underline{1} \end{array}$$

$$\text{print}(a \& b): \underline{0} \underline{1} \underline{1} \underline{1} \underline{0} \rightarrow 2^3 + 2^2 + 2^1 = 14$$

$$\text{print}(a \vee b): \underline{1} \underline{0} \underline{0} \underline{0} \underline{1} \rightarrow 2^4 + 2^0 = 16 + 1 = 17$$

$$\text{print}(a \oplus b): \underline{1} \underline{1} \underline{1} \underline{1} \underline{1} \rightarrow 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 31$$

// a = 13, b = 10

$$\begin{array}{cccccc} 2^3 & 2^2 & 2^1 & 2^0 \\ \underline{1} & \underline{1} & \underline{0} & \underline{1} \end{array}$$

$$a: \underline{1} \underline{1} \underline{0} \underline{1}$$

$$b: \underline{1} \underline{0} \underline{1} \underline{0}$$

Decimal

$$\text{print}(a \& b): \underline{1} \underline{0} \underline{0} \underline{0} \rightarrow 2^3 = 8$$

$$\text{print}(a \vee b): \underline{1} \underline{1} \underline{1} \underline{1} \rightarrow 2^3 + 2^2 + 2^1 + 2^0 = 15.$$

$$\text{print}(a \oplus b): \underline{0} \underline{1} \underline{1} \underline{1} \rightarrow 2^2 + 2^1 + 2^0 = 7.$$

Bitwise Properties

$$\begin{array}{r}
 \begin{array}{c} 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 & 0 & 1 & 0 \end{array} \\
 \begin{array}{r} 1 : 0 \ 0 \ 0 \ 1 \\ \hline A \& 1 : 0 \ 0 \ 0 \ 0 \end{array} \text{ ans} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{c} 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 & 1 & 1 & 0 \end{array} \\
 \begin{array}{r} 1 : 0 \ 0 \ 0 \ 1 \\ \hline A \& 1 : 0 \ 0 \ 0 \ 0 \end{array} \text{ ans} = 0
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{c} 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 & 0 & 1 & 1 \end{array} \\
 \begin{array}{r} 1 : 0 \ 0 \ 0 \ 1 \\ \hline A \& 1 : 0 \ 0 \ 0 \ 1 \end{array} \text{ ans}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{c} 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 & 1 & 0 & 1 \end{array} \\
 \begin{array}{r} 1 : 0 \ 0 \ 0 \ 1 \\ \hline A \& 1 : 0 \ 0 \ 0 \ 1 \end{array} \text{ ans} = 1
 \end{array}$$

obs: $A \& 1$

- $\rightarrow 0: \text{In } A, 0^{\text{th}} \text{ bit} = 0 \Rightarrow A \text{ is even number}$
- $\rightarrow 1: \text{In } A, 0^{\text{th}} \text{ bit} = 1 \Rightarrow A \text{ is odd number}$

Ex: 86 & 87

$$\begin{array}{r}
 \begin{array}{c} 128 \ 64 \ 32 \ 16 \ 8 \ 4 \ 2 \ 1 \\ \hline 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{array} \\
 \begin{array}{r} 2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{array}
 \end{array}$$

$$A = 2^7 + 2^5 + 2^4 + 2^1 = \text{even}$$

$$\begin{array}{r}
 \begin{array}{c} 128 \ 64 \ 32 \ 16 \ 8 \ 4 \ 2 \ 1 \\ \hline 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{array} \\
 \begin{array}{r} 2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{array}
 \end{array}$$

$$A = \frac{2^7 + 2^5 + 2^4 + 2^1}{\text{even}} + 2^0 = \text{odd}$$

Few More Interesting Properties

1. $A \& 1 \rightarrow A : 0 : A \text{ is even} \quad 1 : A \text{ is odd}$

2. $A \& 0 \rightarrow A : 1 0 \mid 1 : 0$

$$\begin{array}{r} 0 : 0000 \\ \hline A \& 0 : 0000 \Rightarrow 0 \end{array}$$

3. $A \& A \rightarrow A : 1 0 \mid 0 : A$

$$\begin{array}{r} A : 1 0 \mid 0 \\ \hline A \& A : 1 0 \mid 0 = A \end{array}$$

4. $A \mid 0 \rightarrow A : 1 0 \mid 0 : A$

$$\begin{array}{r} 0 : 0000 \\ \hline A \mid 0 : 1 0 \mid 0 = \cdot \end{array}$$

5. $A \mid A \rightarrow A : 1 0 \mid 0 : A$

$$\begin{array}{r} A : 1 0 \mid 0 \\ \hline \end{array}$$

$$\begin{array}{r} A \mid A : 1 0 \mid 0 \\ \hline \end{array}$$

$A : \text{if } A \text{ is odd}$

6. $A \mid 1 \rightarrow \text{TODO} \quad \begin{cases} A+1 : \text{if } A \text{ is even} \end{cases}$

7. $A^A 0 \rightarrow A : 1 0 \mid 0 : A$

$$\begin{array}{r} 0 : 0000 \\ \hline A^A 0 : 1 0 \mid 0 \end{array}$$

8. $A^A A \rightarrow A : 1 0 \mid 0 : 0$

$$\begin{array}{r} A : 1 0 \mid 0 \quad 1 : \text{Same:0 Diff:1} \\ \hline A^A A : 0000 : 0 \end{array}$$

$A+1 : \text{if } A \text{ is even}$

9. $A^A 1 \rightarrow \text{TODO} \quad \begin{cases} A-1 : \text{if } A \text{ is odd} \end{cases}$

Commutative Property :

True/False

1. $A \wedge B = B \wedge A$ = True

2. $A \vee B = B \vee A$ = True

3. $A \wedge B = B \wedge A$ = True

4. $A \wedge B \wedge C = B \wedge C \wedge A = C \wedge B \wedge A$ = order won't matter

5. $A \vee B \vee C = B \vee C \vee A = C \vee B \vee A$ = order won't matter

6. $A \wedge B \wedge C = B \wedge C \wedge A = C \wedge B \wedge A$ = order won't matter

XOR Calculations

1. $a^a b^b a^a d^d b^b = a^a a^a b^b b^b d^d = 0^0 0^0 d^d = 0^0 d^d = d$

2. $d^d e^e f^f a^a a^a d^d f^f = d^d d^d a^a a^a f^f f^f e^e = e$

3. $\overrightarrow{1} \overrightarrow{3} \overrightarrow{5} \overrightarrow{3} \overrightarrow{2} \overrightarrow{1} \overrightarrow{5} = 1^1 1^1 3^3 3^3 5^5 5^5 2^2 = 2$

1: 0 0 1

$$\begin{array}{r} 3: 0 1 1 \\ \hline 1 \wedge 3 : 0 1 0 \end{array}$$

$$\begin{array}{r} 5: 1 0 1 \\ \hline 1^1 3^3 5^5 : 1 1 1 \end{array}$$

$$\begin{array}{r} 3 : 0 1 1 \\ \hline 1^1 3^3 5^5 3^3 : 1 0 0 \end{array}$$

$$\begin{array}{r} 2 : 0 1 0 \\ \hline 1^1 3^3 5^5 3^3 2^2 : 1 1 0 \end{array}$$

$$\begin{array}{r} 1 : 0 0 1 \\ \hline 1^1 3^3 5^5 3^3 2^2 1^1 : 1 1 1 \end{array}$$

$$\begin{array}{r} 5 : 1 0 1 \\ \hline 1^1 3^3 5^5 3^3 2^2 1^1 5^5 : 0^{2^2} 1^{2^1} 0^{2^0} = 2 \end{array}$$

(Q) Given $\text{arr}[N]$, where every element repeats twice except for 1 element, which occurs once, find that unique element.

Constraints:

$$1 \leq N \leq 10^5$$

$$1 \leq \text{arr}[i] \leq 10^9$$

$$\text{Ex1: } \text{arr}[5] = \frac{\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array}}{6^{10} \times 9^{10} \times 6^{10} \times 10^{10} \times 9^{10}} = \underline{\underline{10}}$$

$$\text{Ex2: } \text{arr}[7] = \frac{\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array}}{2^6 \times 3^5 \times 5^4 \times 6^3 \times 10^3 \times 9^2 \times 2^1} = \underline{\underline{5}}$$

Idea: Calculate XOR of all elements

`int unique(int arr[]){ TC: O(N) SC: O(1)`

```

int N = arr.length;
int val = 0; // 0 ^ n = n : 0 won't effect your ans
for(int i=0; i<N; i++) {
    val = val ^ arr[i];
}
return val;

```

$$\text{Ex1: } \text{arr}[5] = \frac{\begin{array}{ccccc} \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \end{array}}{6^{10} \times 9^{10} \times 6^{10} \times 10^{10} \times 9^{10}} = \underline{\underline{10}}$$

val	$i: val = val \wedge arr[i]$	val
0	$0: val = val \wedge 6$	$0 \wedge 6 = 6$
6	$1: val = val \wedge 9$	$6 \wedge 9 = 15$
15	$2: val = val \wedge 6$	$15 \wedge 6 = 9$
9	$3: val = val \wedge 10$	$9 \wedge 10 = 3$
3	$4: val = val \wedge 9$	$3 \wedge 9 = 10$

$10 \rightarrow$ unique element

$$\begin{array}{r}
 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\
 6: \quad 0 \quad 1 \quad 1 \quad 0 \\
 9: \quad 1 \quad 0 \quad 0 \quad 1 \\
 \hline
 15: \quad 1 \quad 1 \quad 1 \quad 1 \\
 6: \quad 0 \quad 1 \quad 1 \quad 0 \\
 \hline
 9: \quad 1 \quad 0 \quad 0 \quad 1 = 9 \\
 10: \quad 1 \quad 0 \quad 1 \quad 0 \\
 \hline
 3: \quad 0 \quad 0 \quad 1 \quad 1 = 3 \\
 9: \quad 1 \quad 0 \quad 0 \quad 1 \\
 \hline
 10 \quad 1 \quad 0 = \underline{\underline{10}}
 \end{array}$$

leftShift <<

Say a is 8 bit number? At max it can store '8 bits

Observation

$$\begin{array}{r}
 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0 \\
 \hline
 a = 25 : 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1 \\
 \downarrow \quad \downarrow \\
 dcl \quad 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0 \\
 \downarrow \quad \downarrow \\
 dcl \quad 0\ 1\ 1\ 0\ 0\ 1\ 0\ 0 \\
 \downarrow \quad \downarrow \\
 dcl \quad 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0 \\
 \downarrow \quad \downarrow \\
 des \quad 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0
 \end{array}$$

$$\begin{aligned}
 a = 25 &= 16 + 8 + 1 = 25 = 25 * 1 = 25 * 2^0 \\
 a \ll 1 &= 32 + 16 + 2 = 50 = 25 * 2 = 25 * 2^1 \\
 a \ll 2 &= 64 + 32 + 4 = 100 = 25 * 4 = 25 * 2^2 \\
 a \ll 3 &= 128 + 64 + 8 = 200 = 25 * 8 = 25 * 2^3 \\
 a \ll 4 &= 128 + 16 = 144 = 25 * 2^4 = 25 * 16 = 400?
 \end{aligned}$$

Here: Data is getting overflowed,
We are exceeding limit of storage

Obs: $a \ll n = a * 2^n$: if no overflow

$$1 \ll 0 = 1 * 2^0 = 2^0$$

$$5 \ll 3 = 5 * 2^3 = 5 * 8 = 40$$

When overflow occurs: Learn in Bit Manipulations 2

Rightshift >>

Say a is 8 bit number.

$$2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$$

Observation

$$a = 25 : \begin{array}{ccccccccc} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{array} = 16 + 8 + 1 = 25 = 25$$

$$a_{771} : \begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{array} = 8 + 4 = 12 = 25/2$$

$$a_{772} : \begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array} = 4 + 2 = 6 = 25/2^2$$

$$a_{773} : \begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array} = 2 + 1 = 3 = 25/2^3$$

$$a_{774} : \begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} = 2^0 = 1 = 25/2^4$$

$$a_{775} : \begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} = 0 = 0 = 25/2^5 = 25/32 = 0$$

$$a_{776} : \begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} = 0 = 0 = 25/2^6 = 25/64 = 0$$

obs: $a_{771} = a/2^1$

$$40773 = 40/3 = 40/8 = 5$$