

## Today's Content

a) Re-arrange array

b) Quick Sort

c) Count Sort

Q1. Given  $arr[N]$ : re-arrange it such that

Bring last element to its correct sorted position

All values  $<$  last ele are continuously on left side of last ele

All values  $\geq$  last ele are continuously on right side of last ele

Ex1:  $arr[8] =$

0	1	2	3	4	5	6	7
9	8	1	6	5	11	4	7

All of them are correct we can return any one of them

0	1	2	3	4	5	6	7
1	6	5	4	7	9	8	11

ans1:

0	1	2	3	4	5	6	7
1	4	6	5	7	8	9	11

ans2:

0	1	2	3	4	5	6	7
1	4	5	6	7	8	9	11

ans3:

0	1	2	3	4	5	6	7
6	5	4	1	7	11	9	8

ans4:

Ex2:  $arr[9] =$

0	1	2	3	4	5	6	7	8
7	10	4	8	2	9	12	3	5

$X = 5$

0	1	2	3	4	5	6	7	8
2	3	4	5	7	8	9	10	12

ans1:

0	1	2	3	4	5	6	7	8
4	3	2	5	8	7	10	12	9

ans2:

0	1	2	3	4	5	6	7	8
7	3	2	5	4	10	12	9	8

ans3:

Idea1: Sort  $arr[]$  using mergeSort

Tc:  $O(N \log N)$  Sc:  $O(N)$

Idea2: Take 2 indices  $i$  &  $j$

$i$ : Used to iterate on  $arr[]$

$j$ : Used to keep an ele on left

$i \rightarrow i \rightarrow i \rightarrow i \rightarrow i \rightarrow i \rightarrow i \rightarrow i$   $i$  at last index stop process  
 0 1 2 3 4 5 6 7 : swap last ele to  $arr[j]$   
 $arr[8] =$  9 8 7 6 5 11 7 7  
 $n = 7$  1 6 5 4 7 11 8 9  
 $j \rightarrow j \rightarrow j \rightarrow j \rightarrow j$

Note: Only ele  $< n$  should come on left

$i \rightarrow i \rightarrow i \rightarrow i \rightarrow i \rightarrow i \rightarrow i \rightarrow i$   $i$   
 0 1 2 3 4 5 6 7 8  
 $arr[9] =$  7 10 7 8 7 9 12 7 5  
 $n = 5$  4 2 3 5 10 9 12 7 8  
 $j \rightarrow j \rightarrow j \rightarrow j$

If ( $arr[i] < n$ )  
 swap  $arr[i]$  &  $arr[j]$   
 $j = j + 1$   
 5  
 Stop: swap  $arr[8] \rightleftharpoons arr[3]$

`int[] rearrange(int arr[]) { TC:  $O(N)$  SC:  $O(1)$`

`int N = arr.length;`

`int n = arr[N-1]`

`int j = 0;`

`for (int i = 0; i < N-1; i++) {`

`if (arr[i] < n) { // arr[i] is relevant, at left`

`int t = arr[i]; arr[i] = arr[j]; arr[j] = t;`

`j++;`

`}`

`swap arr[N-1] & arr[j]`

`return arr`

`}`

	s	n = 18								e
	0	1	2	3	4	5	6	7	8	9
Algo: arr[10] =	3	6	14	11	8	20	27	31	23	18

he-arrangement

s	P-1				P=5	P+1	e			
0	1	2	3	4	5	6	7	8	9	
3	6	14	11	8	18	20	27	31	23	

{S, P-1}

{P+1, e}

s	n = 8				e
0	1	2	3	4	
3	6	14	11	8	

he-arr

s	n = 23				e
6	7	8	9		
20	27	31	23		

he-arr

s	P = 2				e
0	1	2	3	4	
3	6	8	14	11	

s	P = 7				e
6	7	8	9		
20	23	27	31		

{S, P-1}

{P+1, e}

{S, P-1}

{P+1, e}

s	n = 6		e
0	1		
3	6		

he-arr

s	n = 11		e
3	4		
14	11		

he-arr

s	e
6	6
20	

s	n = 31		e
8	9		
27	31		

he-arr

s	P = 1		e
0	1		
3	6		

s	P = 3		e
3	4		
11	14		

s	P = 9		e
8	9		
27	31		

{S, P-1}

{P+1, e}

{S, P-1}

{P+1, e}

{S, P-1}

{P+1, e}

s = e
0 0
3

s > e
2 1
invalid

s > e
3 2
invalid

s e
4 4
14

s e
8 = 8
27

s e
10 > 9

Ass: Given arr[], sort arr[] from [s..e]

void QuickSort(int arr[], int s, int e) {  $\rightarrow T(N)$

if (s >= e) { return }

// Sort arr[]: [s s+1... e-2 e-1 e]  $\rightarrow$  last index

// Step 1: Re-arrange last ele to its correct sorted pos

int x = arr[e];

int j = s;

for (int i = s; i < e; i++) {

if (arr[i] < x) {

swap arr[i] & arr[j]; // write code

j++;

swap arr[j] & arr[e] // correct index of arr[e] = j

int p = j; // no need

QuickSort(arr, s, p-1)  $O : T(N) \rightarrow 1$

sort left

[s s+1... p-1 p p+1... e-2 e-1 e]

sort right

QuickSort(arr, p+1, e)  $N-1 : T(N-1)$

}

Recursive Relation:

To sort N elements assume it take =  $T(N)$

Best Case:

$T(N) = 2T(N/2) + N$   $T(1) = O(1)$

$T(N) = O(N \log N)$

Worst Case:

$T(N) = T(N-1) + N$   $T(1) = O(1)$

$T(N) = O(N^2)$

Research:

90% of time, for most examples  $T: O(N \log N)$

Cons: On a avg  $T: O(N \log N)$

TODO: Inbuilt Sort uses which sorting algo?

## CountSort / FreqSort

Given  $arr[N]$ , all ele in range  $[2-6]$  sort  $arr[]$

	0	1	2	3	4	5	6	7	8	9
$arr[] =$	3	2	4	6	4	2	3	4	3	6

Idea 1: Sort it with mergeSort / QuickSort :  $TC = O(N \log N)$

Idea: Store freq of elements hashmap:

3: 3	} using freq, sort $arr[]$ ?
6: 2	
2: 2	
4: 3	

$i: [2, 6]$	...	0	1	2	3	4	5	6	7	8	9
2: $freq(2) = 2$		<del>X</del>	<del>X</del>	X	<del>X</del>	X	<del>X</del>	<del>X</del>	X	<del>X</del>	<del>X</del>
3: $freq(3) = 3$				2	2	3	3	3	4	4	6
4: $freq(4) = 3$				$j \rightarrow j \rightarrow j \rightarrow j \rightarrow j \rightarrow j \rightarrow j \rightarrow j \rightarrow j \rightarrow j$							

5:  $freq(5) = 0$  Total iterations =  $N + 5$  iterations

6:  $freq(6) = 2$  Total i iterations = 5:  $i = [2, 6] = 6 - 2 + 1$

Total j iterations =  $N$ :  $j = [0, N-1] = N$

Q) Given  $arr[N]$ , sort it, using above idea:

Step 1: Iterate on  $arr[]$  & get min of  $arr[] = a$ , max of  $arr[] = b$

Step 2: Insert all  $arr[]$  ele in hashmap  $\langle \text{Integer}, \text{Integer} \rangle$  hm;

Step 3: Sort  $arr[]$  using hashmap;

```
int j = 0;
```

```
for (int i = a; i <= b; i++) {
```

```
    int f = 0;
```

```
    if (hm.containsKey(i)) { f = hm.get(i);
```

```
    for (int l = 1; l <= f; l++) { // looping f times
```

```
        | arr[j] = i; j++;
```

```
    }
```

SC:  $O(N)$

TC: Total i iterations =  $[a..b] = b - a + 1$  Total j iterations =  $[0..N-1] = N$

TC:  $N + [b - a + 1]$

CountSort TC:

TC:  $N + b - a + 1$  //  $b - a + 1$  is  $\max - \min + 1$  of  $\text{arr}[]$  say  $R$

Assume  $R = b - a + 1$

TC:  $N + R$

Constraints:	CountSort: $N + R$	MergeSort
$N \approx 10^5$ $1 \leq \text{arr}[i] \leq 10^3$	iterations $\approx 10^5 + 10^3$ ✓	✓
$N \approx 10^5$ $1 \leq \text{arr}[i] \leq 10^6$	iterations $\approx 10^5 + 10^6$ ✓	✓
$N \approx 10^5$ $1 \leq \text{arr}[i] \leq 10^9$	iterations $\approx 10^5 + 10^9$ ✗	✓

When to use, which sorting algo?

1. Given  $\text{arr}[N]$  get 3 or 4 largest elements:

a) MergeSort & get last 3/4 elements TC:  $N \log N$

b) In bubble Sort, iterate on  $\text{arr}[]$  3 or 4 times & get required elements TC:  $3N$  SC:  $O(1)$

2. Sorted stream of data, insert 1 ele to make entire sort

a) Insertion idea

3. Given  $\text{arr}[N]$  sort it

Constraints:

$1 \leq N \leq 10^6$   
 $1 \leq \text{arr}[i] \leq 10^2$

} : CountSort

4. Given  $\text{arr}[N]$ , sort it as fast as possible, without help of any temp[]

: QuickSort

5. Sort  $\text{arr}[]$  ele : MergeSort

1. Next monday: Searching 1

1. Next tuesday: Searching 2