

Today's Content

- a) Find Unique element
- b) Search in rotated sorted arr()
- c) Find Sqrt()

When to apply BS

- a) Target b) SearchSpace
- c) Discard SearchSpace

1Q) Every element occurs twice except for 1, find unique element.

Note: Duplicates are adjacent to each other.

Ex:

0 1 2 3 4 5 6 7 8 9 10
arr[] = { 6 6 2 2 7 9 9 4 4 10 10 } ans = 7

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
arr[] = { 3 3 1 1 8 8 10 10 19 6 6 2 2 4 4 } ans = 19

Idea1: 1. Iterate on arr[], check adjacent elements & find unique

2. Iterate on arr[], calculate XOR of all elements

TC: $O(N)$ SC: $O(1)$

Idea2:

Target: Unique element SearchSpace: Entire arr[]

Discard ?

0 1 2 3 4 5 6 7 8 9 10 11 11
arr[] = { 12 12 6 6 21 21 7 9 9 4 4 10 10 }
left unique right unique

0 2 4 6 8 9 11 13
arr[] = { 3 3 7 7 4 4 9 9 24 12 12 14 14 6 6 }
left: 1st occurrence even right: 1st occurrence odd

if mid land on left:
: goto right

Note: if 1st occurrence is even mid is on left

mid if mid land on right:
: goto left

Note: if 1st occurrence is odd mid is on right

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
arr[] =	3	3	1	1	8	8	10 ^{1st} × 10 ^{2nd}	19	6	6	2	2	4	4	
							\uparrow m	\uparrow $m+1$	\uparrow l	\uparrow m		\uparrow m			

if m is 2nd occ: left or right
 $l \quad h \quad m \quad arr[m] == arr[m-1] \quad m \% 2 == 0$
 0 14 7 $m = m - 1 = 6$ $6 \% 2 == 0$ left: goto right $l = m + 2$
 8 14 11 \longrightarrow 11 $11 \% 2 == 1$ right: goto left $h = m - 1$
 8 10 9 \longrightarrow 9 $9 \% 2 == 1$ right: goto left $h = m - 1$
 8 8 8 : unique element: return arr[m];

int unique(int arr[]) { TC: $O(\log N)$ SC: $O(1)$

int n = arr.length;

if (n == 1) { return arr[0]; }

if (arr[0] != arr[1]) { return arr[0]; }

if (arr[n-1] != arr[n-2]) { return arr[n-1]; }

int l = 1, h = n - 2;

while (l < h) {

int m = (l + h) / 2;

if (arr[m] != arr[m-1] && arr[m] != arr[m+1]) {

return arr[m];

if (arr[m] == arr[m-1]) { // m is on 2nd occurrence bring to 1st occurrence.

m = m - 1;

// m is 1st occurrence:

if (m % 2 == 0) { // 1st occurrence even \Rightarrow left: goto right

l = m + 2;

else { // 1st occurrence odd \Rightarrow right: goto left

h = m - 1;

Q: sorted arr[] : { 3 9 14 16 20 28 35 40 49 }

Given k, Can k be present in arr[] or not?

k {min max} check

16 ✓ : $3 \leq 16 \leq 49$ Yes

25 ✓ : $3 \leq 25 \leq 49$ Yes

60 ✗ : $3 \leq 60 \leq 49$ No

1 ✗ : $3 \leq 1 \leq 49$ No

k : { $\min \leq k \leq \max$ }, k can be in given range

Rotate arr[] Revisim

Given arr[N], rotate arr[] Right to left

arr[9] = {
0 1 2 3 4 5 6 7 8
3 6 9 10 11 14 20 23 30}

Rotate 3 times:

Rotate 3 : {
20 23 30 3 6 9 10 11 14}

arr[10] = {
0 1 2 3 4 5 6 7 8 9
2 4 6 8 12 15 19 21 26 30}

Rotate 4 times

{
19 21 26 30 2 4 6 8 12 15}

Note: If we rotate sorted arr[] k times, we get a sorted arrays

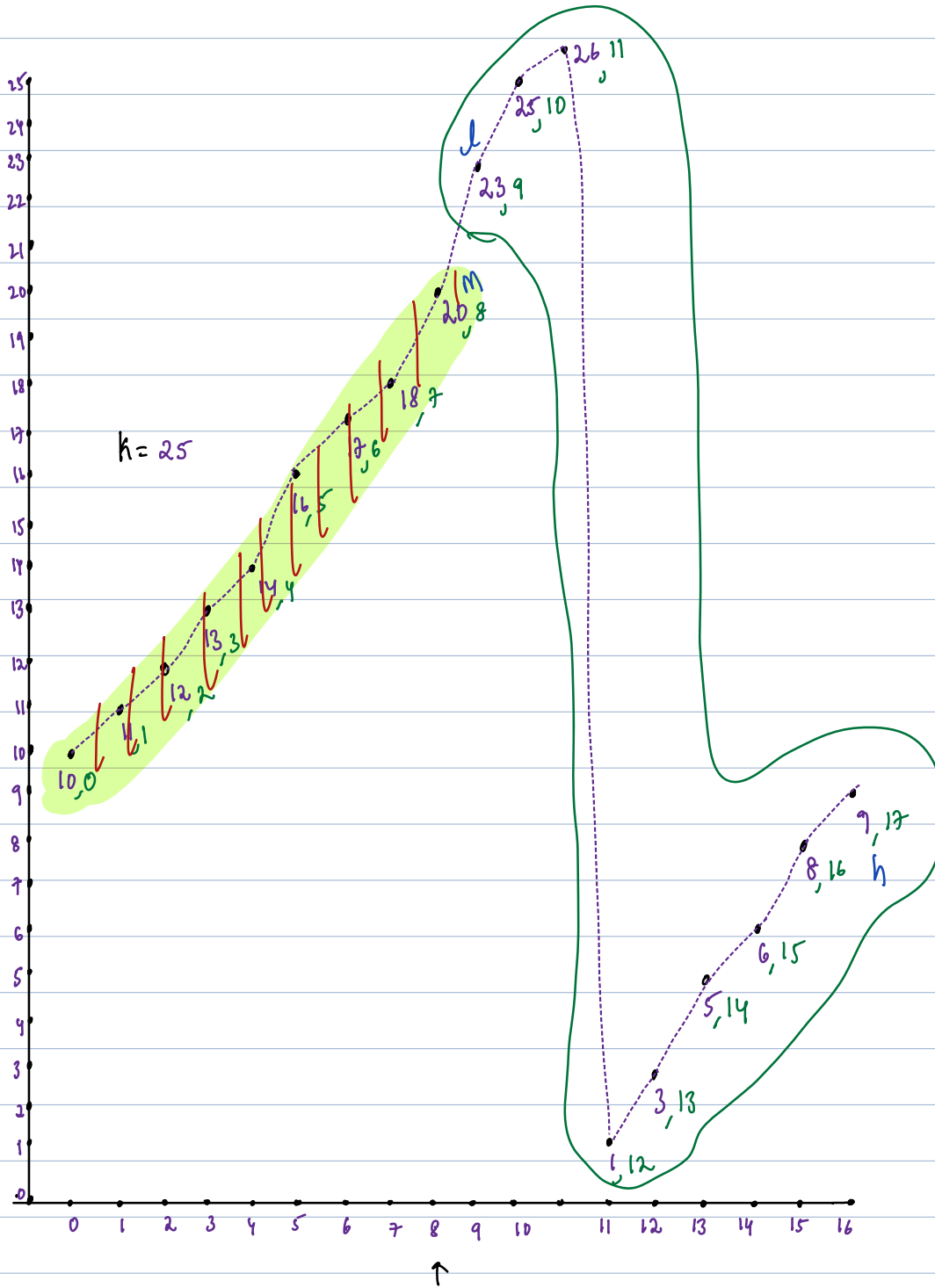
1st sorted arr[] : k elements 2nd sorted arr[] : N-k

2Q)

Given an input arr[], formed by rotating a distinct sorted array right to left by some no. of times.

Search ele & return index in input arr[] If ele is not present return -1

[illegible]

$$\text{arr}[] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 \\ 10 & 11 & 12 & 13 & 14 & 16 & 17 & 18 & 20 & 23 & 25 & 26 & 1 & 3 & 5 & 6 & 8 & 9 \end{matrix} \}$$


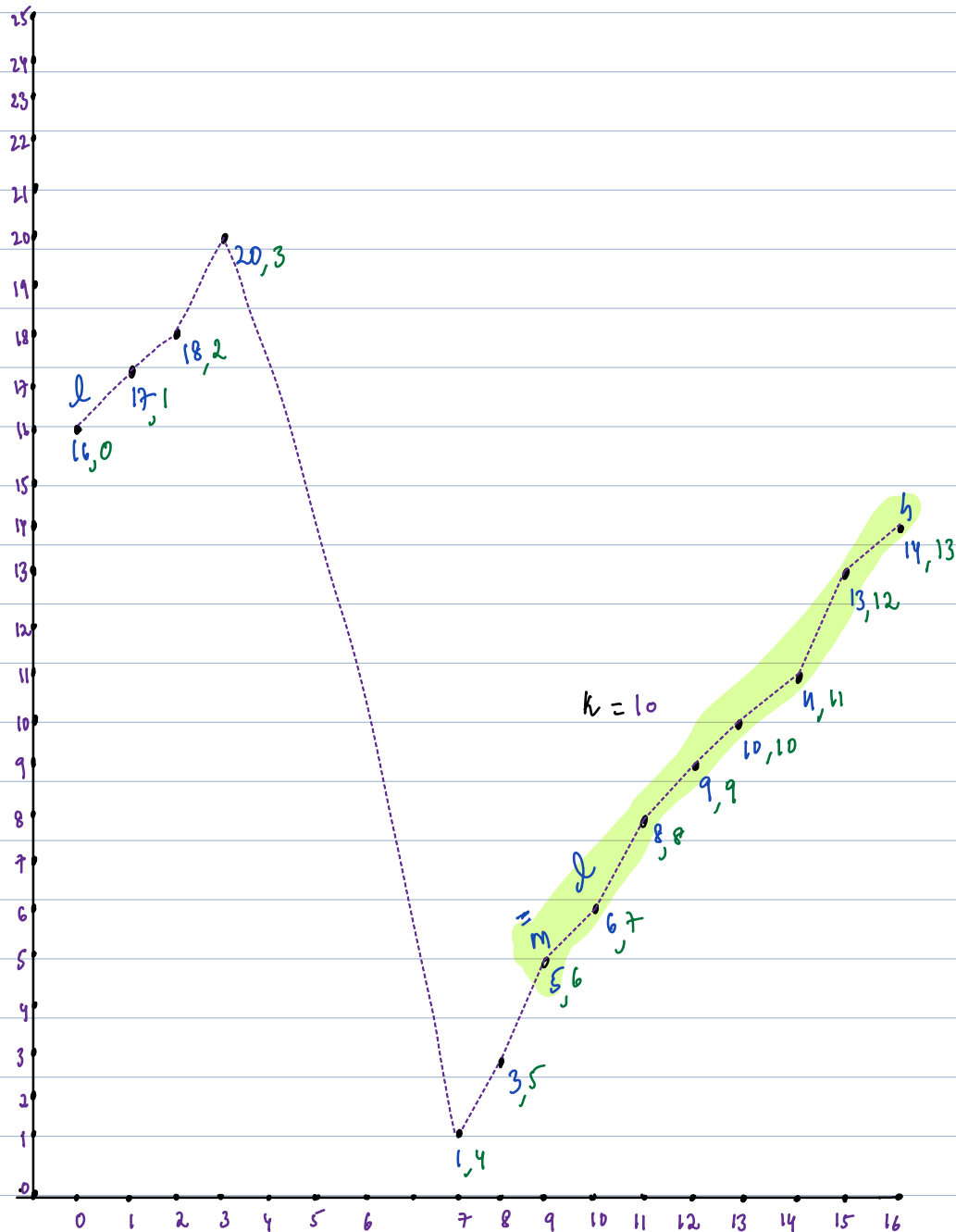
$$E_n: \text{arr}[] = \{ 16 \ 17 \ 18 \ 20 \ 1 \ 3 \ 5 \ 6 \ 8 \ 9 \ 10 \ 11 \ 13 \ 14 \}$$

$k = 10$

l

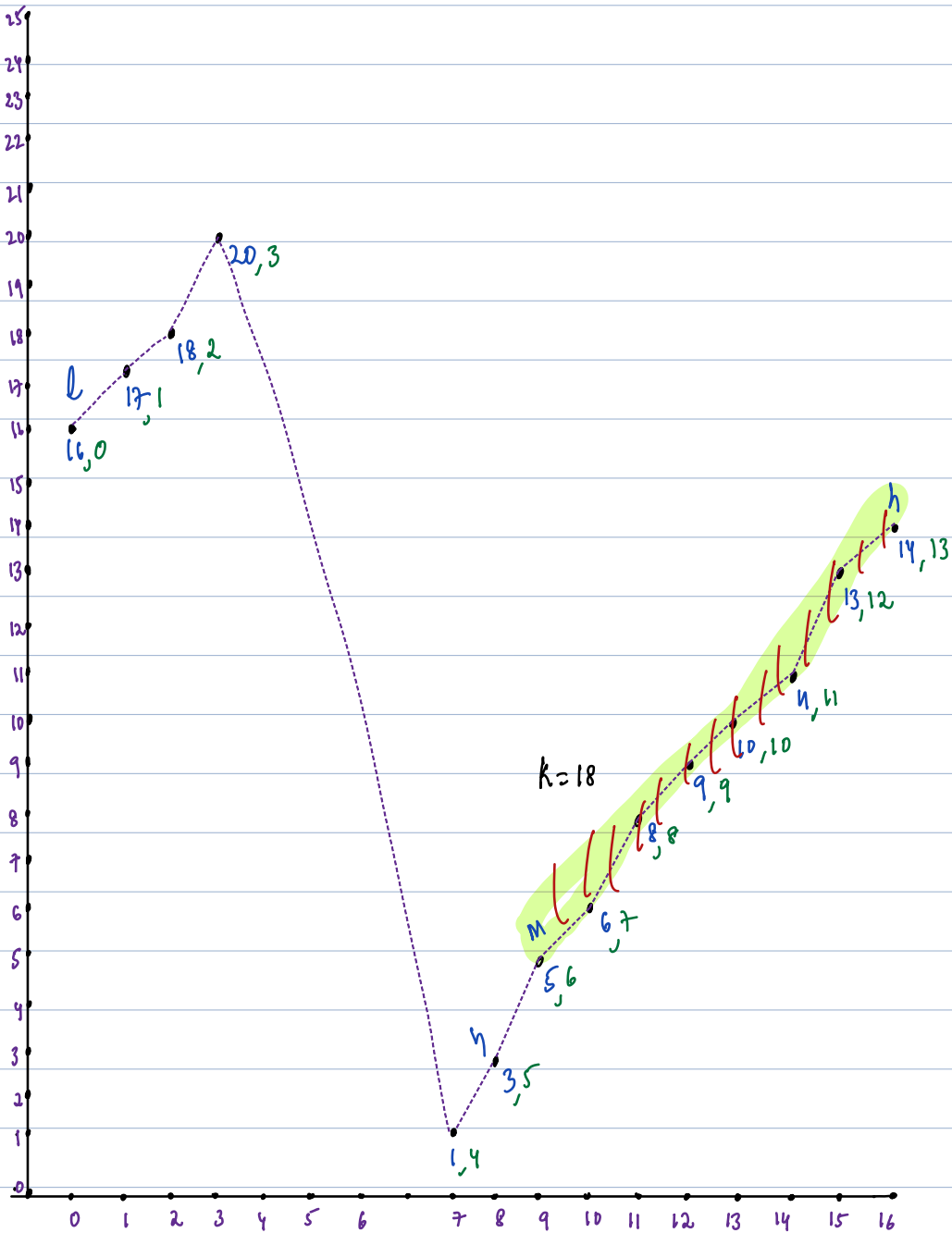
m

h



$$E_n: ar[] = \{ \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 16 & 17 & 18 & 20 & 1 & 3 & 5 & 6 & 8 & 9 & 10 & 11 & 13 & 14 \end{matrix} \}$$

$k=18$ d m h



int searchRotated (int arr[], int k) { TC: $O(\log N)$ SC: $O(1)$

int n = arr.length;

int l = 0, h = n - 1;

while (l <= h) {

int m = (l + h) / 2

if (arr[m] == k) { return m }

if (arr[m] >= arr[l]) { [l..m] sorted

if (arr[l] <= k && k <= arr[m]) { // goto left

h = m - 1;

else { l = m + 1 }

else { [m..h] sorted

if (arr[m] < k && k <= arr[h]) { // goto right

l = m + 1

else {

h = m - 1

return -1;

Q: Given +ve find $\text{Sqrt}(N)$

Find greatest i such that $i^2 \leq N$

$$\text{Sqrt}(25) = 5^2 = 25$$

$$\text{Sqrt}(30) = 6^2 = 36 > 30 \quad 4^2 = 16 < 30 \quad 5^2 = 25 < 30 \quad \text{ans} = 5$$

$N = 30$:

i	$i^2 \leq N$	ans
1	$1^2 = 1 < 30$	ans = 1
2	$2^2 = 4 < 30$	ans = 2
3	$3^2 = 9 < 30$	ans = 3
4	$4^2 = 16 < 30$	ans = 4
5	$5^2 = 25 < 30$	ans = 5
6	$6^2 = 36 > 30$	* return ans = 5
7		No way

int sqrt(int N) { TC: $O(\sqrt{N})$ SC: $O(1)$

int i = 1, ans = 1, i = 1

while ($i^2 \leq N$) { $i^2 \leq N \Rightarrow \text{while}(i \leq \sqrt{N})$

ans = i; // update i++

} i = i + 1;

return ans;

Find $\text{Sqrt}(N)$ using BS

1. Target: greatest ele, such that $ele^2 \leq N$

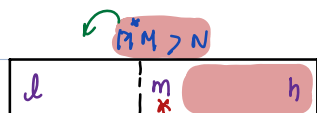
2. SearchSpace: $\text{Sqrt}(N)$ will be in range $[1..N]$

3. Discard



if ($m^2 \leq N$) {

ans = m; $l = m + 1$



if ($m^2 > N$) {

$h = m - 1$

$N=24$

l	h	m	m^2	$m \leq N$	ans
1	24	12	12^2	$12 \leq 24$	12^2 13^2 14^2 ... goto left $h=m-1$
1	11	6	6^2	$6 \leq 24$	6^2 7^2 8^2 ... goto left $h=m-1$
1	5	3	3^2	$3 \leq 24$	1 2 <u>3</u> ans=3 goto right $l=m+1$
4	5	4	4^2	$4 \leq 24$	4 ans=4 goto right $l=m+1$
5	5	5	5^2	$5 \leq 24$	5^2 ... goto left $h=m-1$
5	4	break, return ans = 4			

`int sqrt(int N) { TC: $O(\log N)$ SC: $O(1)$`

```

int l=1, h=N, ans=1;
while(l <= h) {
    int m = (l+h)/2;
    if(m*m <= N) {
        ans = m;
        l = m+1; // l=m;
    } else { // m*m > N
        h = m-1;
    }
}
return ans;
}

```