

Todays Content:

- a) Quizes on Big O ✓
- b) Comparing 2 Algos ✓
 - a) Using execution time ✓
 - b) Using Iterations & graphs ✓
- c) Why Big O needed ? ✓
 - a) Why only higher order terms considered ✓
 - b) Why constant coefficient terms neglected ✓
 - c) Issues in Big O ,
 - i) Working for large inputs ✓
 - ii) Same Big O ✓
 - d) Worst Case Iterations ✓
- a) TLE : Time limit Exceeded
 - a) Why TLE occurs & Working of Online Editors
 - b) Importance of Constraints

Quizes on TC:

Big O:

a) Calculate Iterations ✓

Why Big O? ✓

b) Only higher Order Term ✓

c) Neglect Constant Coefficient ✓

Higher Order Term Big O

$$f(n) = \underset{\sim}{3N^2} + 6N + 10^3 = 3N^2 \longrightarrow O(N^2)$$

$$f(n) = 3N^2 + \underset{\sim}{6N^3} + 10^3 = 6N^3 \longrightarrow O(N^3)$$

$$f(n) = 3N + 6N\log N + 10^3 = 6N\log N \longrightarrow O(N\log N)$$

Q) Given $N = 10^4$ elements sort them in increasing order?

$$ar[5] = \{3 \ 2 \ 6 \ 8 \ 1\} \rightarrow \{1 \ 2 \ 3 \ 6 \ 8\}$$

→ 2 People Submitted their Algos: Both are executed on same Input

Execution Time : Time taken for system to run program

Algo1: Chandini

(15sec) : WindowsXP

↓
(8sec) Macbook M1

↓
8sec : C++ lang

↓
8sec : Very fast

↓
6sec Very cool breezy

Algo2: Suraj

(10sec) : Macbook M1

↓
(10sec) : Python

↓
(6sec) : C++

↓
(6sec) : Very cool breezy

↓
(6sec) : Very cool breezy

Issues with Execution Time : It depends on lot of external factors

Hence not a correct way to compare 2 algorithms.

void func(int ar[]){

 int n = ar.length

 int s = 0

 for (int i=0; i < n; i++) { → iterations: $i : \{0..N-1\} = N - 0 + 1 = N$.

 |
 s = s + ar[i]

}

Iterations: It doesn't depend on external factors, hence we calculate & compare performance of 2 algos.

Comparing Iterations:

Q: Say for a Question following codes are submitted.

Nirupum (Algo1) Blue Deeksha (Algo2) green

Iterations : $100 \log_2 N$ $N/10$ Faster Code

$N < 3550$: Nirupum iterations > Deeksha iterations : Deeksha Code

$N > 3550$: Deeksha iterations > Nirupum iterations : Nirupum Code

In Real World: Data will keep increases

a) Ind vs Pak : 2cr

b) Google Search : million

c) YouTube : 100cr

Obs: For larger inputs
Nirupum better.

Same Comparison with Big O

Iterations:	$100 \log_2 N$	$N/10$	Iterations	Code
Big O :	$O(\log_2 N)$	$O(N)$	N	$\log_2 N$

Asymptotic Analysis of Algorithms:

Analyzing performance of algorithms for very large inputs

3 Notations:

1. Big O

2. Omega

3. Theta

: TODO Resources

How to Calculate Big O

1. Calculate no: of iterations

2. Consider only higher order Term & Neglect Constant Coefficients

Why Consider only higher Order Terms?

Say Numpy Code : $N^2 + 10N$ / Iterations lower order : 10N

<u>Input Size</u>	<u>Total Iterations</u>	<u>% lower order terms Iterations in Total</u>
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$$N=10 \quad f(10) : 10^2 + 10^1 \cdot 10 = 200 \rightarrow \frac{100}{200} * 100\% = 50\%$$

$$N=100 \quad f(100) : 100^2 + 10^1 \cdot 100 = \frac{10^3}{10^4 + 10^3} * 100\% = \frac{10^5}{10^4 + 10^3} \approx 10\%, \rightarrow$$

$$N=10^4 \quad f(10^4) : [10^4]^2 + 10 \cdot [10^4] = \frac{10^5 + 100\%}{10^8 + 10^5} = \frac{10^7}{10^8 + 10^5} \approx 0.1\%$$

Obs: As Input increases, contribution of lower order term less, hence neglect

Neglect Constant Coefficients: Comp happens for very large Inputs

	Alg01	Alg02	Iterations	Code Factor
Q1:	$10 \log_2 N$	N	N	$10 \log_2 N$
Q2	$100 \log_2 N$	N		$100 \log_2 N$
Q3	$10^3 \log_2 N$	$N/10$		$1000 \log_2 N$
Q4	$10N$	$N^2/10$		$10N$

Obs2: Constant coefficients won't effect your comparisons for large Inputs

Issues in Big O

Big O :	Algo 1	Algo 2	Iterations	Comparisons
	$O(N)$	$O(N^2)$		
	$10^3 N$	N^2	Algo 2	Algo 1 is faster than Algo 2 for all N ? *
			Algo 2	Algo 1 is faster than Algo 2 for, after a certain input value / large value / ✓
	Algo 1	Algo 2		
N	$10^3 N$	N^2		
10	$10^3 \times 10 = 10^4$	10^2		Algo 2 faster
10^2	$10^3 \times 10^2 = 10^5$	10^4		Algo 2 faster
10^3	$10^3 \times 10^3 = 10^6$	$[10^3]^2 = 10^6$		Same
10^4	$10^3 \times 10^4 = 10^7$	$[10^4]^2 = 10^8$	Algo 1 faster	if $N > 10^3$: Algo 1 faster
10^5			Algo 1 faster	

Issues in Big O I:

Algo 1	Algo 2	More iterations	Faster
$O(N^2)$	$O(2N^2 + 5N)$	Algo 1	Algo 2
Big O : $O(N^2)$	$O(N^2)$		

Note 1: If we have same, Big O, then cannot compare.

Note 2: If Big O same compare Iterations

Imp: Code: Search for k in arr[]

```
bool Search(int arr[], int k) {
    int N = arr.length
    for(int i=0; i<N; i++) {
        if(arr[i] == k) { return True }
    }
    return False
}
```

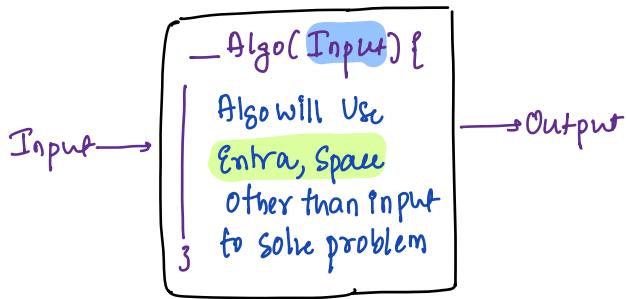
Best Worst
Iterations: $arr[0] == k$ $arr[0..N-1] == k$ or Not present
i, iteration N iterations

Note 3: While calculating Big O we consider worst case iterations to calculate it.

Space Complexity

Code → Time Complexity : Calculate iterations & get Big O
Space Complexity : Calculate Space Code takes & get Big O

Space Complexity Algo:



Note: When we calculate SC Space Complexity for code,
 Don't take input memory for calculating SC for code.
 Consider only Extra Memory taken by Algo.
 Space algo creates to solve: Only that Considered

Examples:

→ Input Note: int: 4 B long: 8 B

```

void func(int n){
    int x = n → 4 B
    int y = x*x → 4 B
    long z = x+y → 8 B
}
  
```

Created by your Algo

$\text{Space} = 16 \text{ Bytes} \rightarrow O(1)$
 It is fixed/Constant

Big O

void func(int N) { Input

```

int n = N → 4B
int y = n² → 4B
long p = n+y → 8B
int arr[] = new int[N] → int arr[] of N :
N int ele → 4N

```

Total Memory → Big O
 $16 + 4N$ $O(N)$

void func(int N) { Input

```

int n = N → 4B
int y = n² → 4B
long p = n+y → 8B
int arr[] = new int[N] → 4N
int mat[][] = new int[N][N] → int mat[][] of N*N
N*N int ele → 4N²

```

Total memory → Big O
 $16 + 4N + 4N^2$ $O(N^2)$

Q) Given a arr[N] write a program to get max of arr[]

→ given / Input Space
int maxarr(int arr[]) {

```

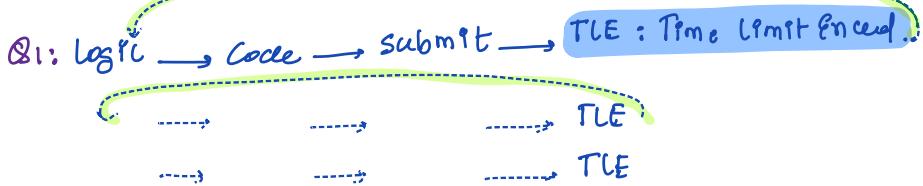
int N = arr.length → 4B
int ans = Integer.MIN_VALUE → 4B
for(int i=0; i<N; i++) { → 4B
    if(ans < arr[i]) {
        ans = arr[i]
    }
}

```

Total memory → Big O
 $12B$ $O(1)$

TLE : Time Limit Exceeded

Shukoor (Microsoft) → Tiring Challenge Gomin



Idea: Without writing logic can get TLE or not?

Working of Online Editors:

Code (Submit) → Run → Online Servers → Process Speed → $1 \text{ GHz} = 10^9 \text{ Instruction/sec}$

In general Time limit in Online Server = 1 sec

Obs: Code can have at most 10^9 instructions

What 1 instruction

a. operators, +, -, /, %

b. function call

c. declaring variable

```

bool countFactors(int n) {
    int c = 0;
    for (int i = 1; i <= n; i++) {
        if (n % i == 0) {
            c++;
        }
    }
    return c;
}
  
```

Iterations: $i = [1..N]$: N iterations

Instructions: $1 + 1 + 1 + GN$

Inside loop: 1 iteration = 6 instructions

Total instructions = $6N + 3 \approx 6N$ instructions.

For very code calculating iterations Tough.

Approach: Say in our Code 1 iteration = 10^9 instructions

↳ Code Instructions / sec = 10^9 instructions

$$= 10^8 \times \frac{10^9 \text{ instructions}}{1 \text{ iteration}}$$

10^8 Iterations

Code Can have 10^8 iterations.

Approach: Say in our Code 1 iteration = 10^2 instructions

↳ Code Instructions / sec = 10^9 instructions = $10^7 \times 10^2$ instructions

$$= 10^7 \times 1 \text{ iteration} = 10^7 \text{ iterations}$$

Code Can have 10^7 iterations

Con: Code Can have $10^7 \approx 10^8$ iterations

Question Structure

Question Desc: Given $\text{arr}[N]$... do something

Constraints: $1 \leq N \leq 10^5$] Input Range:
 $1 \leq \text{arr}[i] \leq 10^9$ $1 \leq N \leq 10^5$: min $N = 1$ max $N = 10^5$

Input format:

$\text{arr}[i] \leq 10^9$: min $\text{arr}[i] = 1$
max $\text{arr}[i] = 10^9$

Output format:

Question:

Example Input:

a) Logic: Say Our logic Nested loop?

Example Output:

TC: Based on Input Size: $O(N^2)$ // Idea \rightarrow TC practice

Explanation:

Max $N = 10^5 \rightarrow O(N^2) \Rightarrow [10^5]^2 = 10^{10}$ iterations > 10^8 TLE

No need to code

Ex1: a) Given $\text{arr}[N]$.. do something

Constraints:

$$1 \leq N \leq 10^3$$

$$1 \leq \text{arr}[i] \leq 10^9$$

Idea: \rightarrow TC: $O(N^2)$

$$\text{Max } N = 10^3 \rightarrow O(N^2) = [10^3]^2 = [10^3 \times 10^3] = 10^6 \ll 10^8 \text{ work}$$

Go to Code.

Ex2: a) Given $\text{arr}[N]$.. do something

Constraints:

$$1 \leq N \leq 10^4$$

$$1 \leq \text{arr}[i] \leq 10^9$$

Idea: \rightarrow TC: $O(N^2)$

$$\text{Max } N = 10^4 \rightarrow O(N^2) = [10^4]^2 = [10^4 \times 10^4] = 10^8 \text{ on border}$$

Touch 480

Code & Check

Q: Given $ar[N]$ calculate & return sum of $ar[i]$ etc

Constraints: $1 \leq N \leq 10^5$
 $1 \leq ar[i] \leq 10^9$

Sum($\text{int } ar[1]$) $\notin T.C: O(N)$

~~int~~ $s = 0;$ $N = 10^5$
long
 $10^5 < 2 \cdot 10^8$

```
int N = ar.length;  
for(int i=0; i<N; i++) {  
    s = s + ar[i];  
}  
return s;
```

MinSum:

$$ar[1] = \{1\}$$

$$\min S = 1$$

MaxSum:

$$ar[10^5] = \frac{0 \quad 1 \quad 2 \quad \dots}{10^9 \quad 10^9 \quad 10^9 \quad \dots \quad 10^9}$$

$$\max S = 10^9 + 10^5 = 10^{14}$$

Sum Range: $1 \dots 10^{14} \times \text{Cannot store in int}$

int Range: $-2 \cdot 10^9 \dots 2 \cdot 10^9$

long Range: $-8 \cdot 10^{18} \dots 8 \cdot 10^{18}$

Constraints

- Will tell whether idea gives TLE or not
- Data types we need to use for variables