

Todays Content: → 9:05 pm Start

- a) Sorting Intro
- b) Sorting Algorithms
 - a) Bubble Sort
 - b) Insertion Sort
 - c) MergeSort → Recursion 9:07pm

New Students:

- a) Pre-Requests Recursion
- b) Even if you are not aware you will get 80%.

Sorting: Arranging data in inc/dec order based on parameter.

1 2 3 5 7 : inc, parameter = value

9 6 3 2 1 : dec, parameter = value

1 7 2 9 24 : inc, parameter = factor

fact: 1 2 2 3 8 :

Why Sorting: It helps in searching data.

Property: 1

Sort data increasing order based on marks {parameter}

Name	Marks	Name	Marks
→ Mohit	0	mohit	0
Aman	100	kajal	32
Shrush	45 ↪	S Algo1: Aayush 45	S Algo2: Shrush 45
Kajal	32	Shrush 45	Aayush 45
Aayush	45 ↪		
Aditya	92		

Stable Sorting: When 2 data points have same parameter value, their relative order should be same, before & after sorting.

Ex: $ar[5] = \{ 1 \ 5 \ 2 \ 6 \ 2 \}$ Sort inc based on value.

Sort1 = { 1 2 2 5 6 } Sort1 is not stable

Sort2 = { 1 2 2 5 6 } Sort2 is stable

Inplace Sorting:

Sorting algorithm with SC: O(1), it's known Inplace.

Sorting :
Inplace

Bubble Sort:

1

Given $ar[N]$, Sort $ar[]$ in inc order, by swapping only adjacent elements.

0 1 2 3 4 5 6 7 8

$ar[9] = \{ 8 2 4 -1 6 7 5 10 -1 \}$ obs:

ite₁ { 2 4 -1 6 7 5 8 -1 10 } last 2 ele in correct pos.

ite₂ { 2 -1 4 6 5 7 -1 8 10 } last 2 ele in correct pos.

ite₃ { -1 2 4 5 6 -1 7 8 10 } last 3 ele in correct pos.

obs: Repeat above iterations N times, entire $ar[]$ sorted in inc order

void BubbleSort(int ar[]){ TC: O(N²) SC: O(1) }

int N = ar.length;

for(int i=0; i < N; i++) {

int swap = 0

for(int j=0; j < N-1; j++) { // j = N-1, $ar[j] > ar[j+1] \Rightarrow ar[N-1] > ar[N]$: Err

if ($ar[j] > ar[j+1]$) {

Swap $ar[j]$ & $ar[j+1]$

swap++;

Dry Run = 0 1 2 3 4

$ar[5] = \{ 3 1 6 10 8 \}$

i=0 | it₁ = 1 3 6 8 10 : 2 swaps

i=1 | it₂ = 1 3 6 8 10 : 0 swaps

Obs: In a full iteration, if there are no swaps, data is sorted.

}

} if (swap == 0) {

// data sorted;

} break;

Check Stability: Parameters same: Relative order maintained

0 1 2 3

$ar[4] = \{ 2 4 2 1 \}$

BubbleSort

ite₁ = 2 2 1 4

a) StableSort

ite₂ = 2 1 2 4

b) InplaceSort

ite₃ = { 1 2 2 4 }

Insertion Step:

Given an $\text{arr}[N]$, first $N-1$ elements are sorted, sort entire $\text{arr}[]$.

Note: No Extra Space is allowed.

Examples:

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$ $\text{arr}[6] = \{ 2 \quad 6 \quad 10 \quad 14 \quad 20 \quad 4 \}$ After Sorting $= \{ 2 \quad 4 \quad 6 \quad 10 \quad 14 \quad 20 \}$	$N=8$ $\text{arr}[8] = \{ 2 \quad 7 \quad 12 \quad 19 \quad 24 \quad 30 \quad 34 \quad 4 \}$ $\text{arr}[7] = \{ 2 \quad 4 \quad 7 \quad 12 \quad 19 \quad 24 \quad 30 \quad 34 \}$
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$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$ $\text{arr}[7] = \{ 3 \quad 6 \quad 10 \quad 14 \quad 20 \quad 24 \quad 7 \}$ $= \{ 3 \quad 6 \quad 7 \quad 10 \quad 14 \quad 20 \quad 24 \}$

Tracing: $[0 \dots 5]$ sorted:

$j \quad \text{arr}[j] > \text{arr}[j+1] \quad T/F \quad \text{Swap}$ $5 \quad \text{arr}[5] > \text{arr}[6] \quad T \quad \text{Swap}$ $4 \quad \text{arr}[4] > \text{arr}[5] \quad T \quad \text{Swap}$ $3 \quad \text{arr}[3] > \text{arr}[4] \quad T \quad \text{Swap}$ $2 \quad \text{arr}[2] > \text{arr}[3] \quad T \quad \text{Swap}$ $1 \quad \text{arr}[1] > \text{arr}[2] \quad F \quad \text{break}$

TC: $O(N)$ SC: $O(1)$

```
void SortStep(int arr[]){
    int N = arr.length;
    /* First N-1 elements sorted.
       Means from [0...N-2]
       Initialize j at last sorted index */

```

```
for (int j=N-2; j>=0; j--) {
    if (arr[j] > arr[j+1]) {
        swap arr[j] & arr[j+1];
    } else {
        break;
    }
}
```

3

Insertion Step:

Insert 1 single ele in sorted data to make entire data sorted its called

3Q) Given $\text{arr}[N]$ sort it using Insertion Step.

$\text{arr}[] =$	0	1	2	3	4	5
	10	3	6	8	2	5

Step 1: $[0-0]$ sorted, Insert $\text{arr}[1]$, Sort $[0-1]$: Insertion Step

0	1	2	3	4	5
3	10	6	8	2	5

Step 2: $[0-1]$ sorted, Insert $\text{arr}[2]$, Sort $[0-2]$ Insertion Step

0	1	2	3	4	5
3	6	10	8	2	5

Step 3: $[0-2]$ sorted, Insert $\text{arr}[3]$, Sort $[0-3]$ Insertion Step

0	1	2	3	4	5
3	6	8	10	2	5

Step 4: $[0-3]$ sorted, Insert $\text{arr}[4]$, Sort $[0-4]$ Insertion Step

0	1	2	3	4	5
2	3	6	8	10	5

Step 5: $[0-4]$ sorted, Insert $\text{arr}[5]$, Sort $[0-5]$ Insertion Step

0	1	2	3	4	5
2	3	5	6	8	10

void insertionSort (int arr[]) { TC: O(N^2) SC: O(1) } \rightarrow Inplace Sorting

int N = arr.length;

stable \rightarrow TODO / Yes Try Example.

for (int i=1; i < N; i++) {

// Insert $\text{arr}[i]$ in sorted data $\{0 .. i-1\}$ sorted, Insertion Step.

for (int j=i-1; j >= 0; j--) {

$\{0 \ 1 \ 2 \ 3 \dots i-2 \ i-1 \ i\}$

if ($\text{arr}[j] > \text{arr}[j+1]$) {

swap $\text{arr}[j]$ & $\text{arr}[j+1]$

else { break }

}

}

Q8) Given 2 sorted arrays $A[N]$, $B[M]$, create $C[N+M]$ which contains overall sorted data.

$$A[4] = \underline{7 \ 10 \ 11 \ 14}$$

$$A[3] = \underline{3 \ 6 \ 10}$$

$$B[3] = \underline{3 \ 8 \ 9}$$

$$B[3] = \underline{5 \ 8 \ 14}$$

$$C[7] = \underline{3 \ 7 \ 8 \ 9 \ 10 \ 11 \ 14}$$

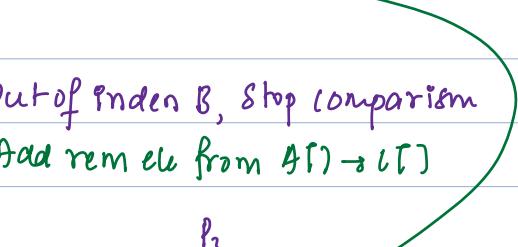
$$C[6] = \underline{3 \ 5 \ 6 \ 8 \ 10 \ 14}$$

E₁: $\frac{P_1}{0 \ 1 \ 2 \ 3 \ 4 \ 5}$

$$A[6] = \cancel{7} \ \cancel{8} \ \cancel{9} \ \underline{14 \ 19 \ 21}$$

while($P_1 < N$) {
 |
 3
 | } 

$B[5] = \cancel{5} \ \cancel{8} \ \cancel{11} \ \cancel{14} \ \cancel{15}$ P_2 : Out of index B, Stop comparison
Add rem ele from $A[7] \rightarrow C[7]$

while($P_2 < M$) {
 |
 3
 | } * 

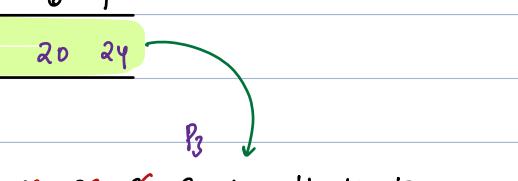
$\frac{P_3}{0 \ X \ 2 \ 3 \ X \ 9 \ 16 \ 7 \ 8 \ 9 \ 10}$
 $C[11] = \underline{2 \ 3 \ 5 \ 6 \ 8 \ 9 \ 11 \ 13 \ 14 \ 19 \ 21}$

E₂: $\frac{P_1}{0 \ 1 \ 2 \ 3 \ 4 \ 5} \ P_1$: Out of index A: Stop comp

$$A[6] = \cancel{7} \ \cancel{8} \ \cancel{11} \ \cancel{13} \ \cancel{17} \ \cancel{20} \ \cancel{24}$$

while($P_1 < N$) {
 |
 3
 | } *

$\frac{P_2}{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7}$
 $B[8] = \cancel{5} \ \cancel{8} \ \cancel{11} \ \underline{13 \ 17 \ 20 \ 24}$

while($P_2 < M$) {
 |
 3
 | } 

$$C[14] = \underline{1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 8 \ 8 \ 10 \ 11 \ 13 \ 17 \ 20 \ 24}$$

int[] merge(int A[], int B[]) { Tc: O(N+M) Sc: O(1)

int N = A.length

int M = B.length

int C[N+M]; → // asked by question.

int p₁ = 0 p₂ = 0 p₃ = 0.

while(p₁ < N && p₂ < M) { // Compare till both are valid

 if(A[p₁] < B[p₂]) { C[p₃] = A[p₁]; p₁++; p₃++; }

} else { C[p₃] = B[p₂]; p₂++; p₃++; }

while(p₁ < N) { // Elements in A[] are remaining

 C[p₃] = A[p₁]; p₁++; p₃++;

while(p₂ < M) { // Elements in B[] are remaining

 C[p₃] = B[p₂]; p₂++; p₃++;

} return C;

508) Given $\text{arr}[N]$ elements & 3 indices s, m, e

→ Subarray [s m] is sorted

→ Subarray $[m+1, e]$ is sorted

Sort Entire Subarray from [s...e]

A diagram illustrating a stack structure. It consists of four colored rectangular boxes arranged horizontally. From left to right: a green box labeled 'S', a blue box labeled 'M', a light blue box labeled 'M+1', and a purple box labeled 'e'. To the right of the 'e' box is a bracket spanning all four boxes, with the label 'n-1' positioned above it. Below the green and blue boxes is a dashed horizontal line, and below the light blue and purple boxes is another dashed horizontal line.

		<i>s</i>		<i>m</i>	<i>m₁₁</i>	<i>e</i>	
	0	1	2	3	4	5	6
<i>ar[12]</i> =	4	8	X	X	X	X	13
							0
<i>s m e</i>				P ₁			P ₂
2 6 10							11

$$\# \text{ele} = l - S + 1 \quad 0 \quad 1 \quad x \quad x \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

tmp[9] -1 2 3 4 7 8 9 11 13

Copy $\text{tmp}[\cdot]$ \Rightarrow $\text{arr}[s \dots e]$

		<i>s</i>		<i>m</i>		<i>e</i>
0	1	2	3	4	5	6
4	8	-1	2	3	4	7

PseudoCode:

$\rightarrow A[]$: Subarray $[s \dots m]$ is sorted
 : Subarray $[m+1 \dots e]$ is sorted
 : Sort entire subarray from $[s \dots e]$

void merge(int A[], int s, int m, int e) { $Tc: O(N)$ $Sc: O(N)$ }

$\text{int temp}[e-s+1]; // [s \underset{P_1}{\cancel{s+1 \dots m}} \underset{P_2}{\cancel{m+1 \dots e}}]$
 $\text{int } P_1 = s, P_2 = m+1, P_3 = 0;$

$\text{while}(P_1 <= m \text{ and } P_2 <= e) \{ // \text{Compare till both are valid}$
 $\quad \text{if } (A[P_1] < A[P_2]) \{ \text{temp}[P_3] = A[P_1]; P_1++; P_3++ \}$
 $\quad \text{else} \{ \text{temp}[P_3] = A[P_2]; P_2++; P_3++ \}$

$\text{while}(P_1 <= m) \{ // \text{Copy rem ele to temp[]}$
 $\quad \text{temp}[P_3] = A[P_1]; P_1++; P_3++$

$\text{while}(P_2 <= e) \{ // \text{Copy rem ele to temp[]}$
 $\quad \text{temp}[P_3] = A[P_2]; P_2++; P_3++$

$\text{// temp[] sorted } \xrightarrow{\text{Copy}} A[s \dots e]$
 $\text{int } k = 0;$
 $\text{for } (\text{int } i=s; i<=e; i++) \{$
 $\quad A[i] = \text{temp}[k];$
 $\quad k++;$

$\text{temp}[e-s+1]:$	0	1	2	$e-s$	TODD
	\downarrow	\downarrow	\downarrow			\downarrow
$A[s \dots e]:$	s	$s+1$	$s+2$	e	
	\downarrow	\downarrow	\downarrow			

Idea of Merge Sort : { Tomorrow 9PM : 9:40PM }

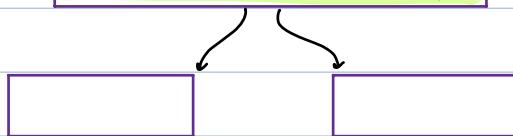
1

Given $arr[100]$ elements sort them.

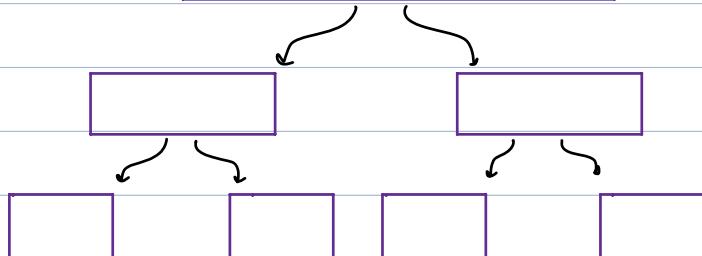
Case1 : $a_1 a_2 a_3 \dots a_{50} a_{51} \dots a_{99} a_{100}$ Iterations : 10^4

1. Apply BS/IS on above ele : $O(N^2) = 100^2 = 10^4$

Case2 : $a_1 a_2 a_3 \dots a_{50} a_{51} \dots a_{99} a_{100}$ Iterations



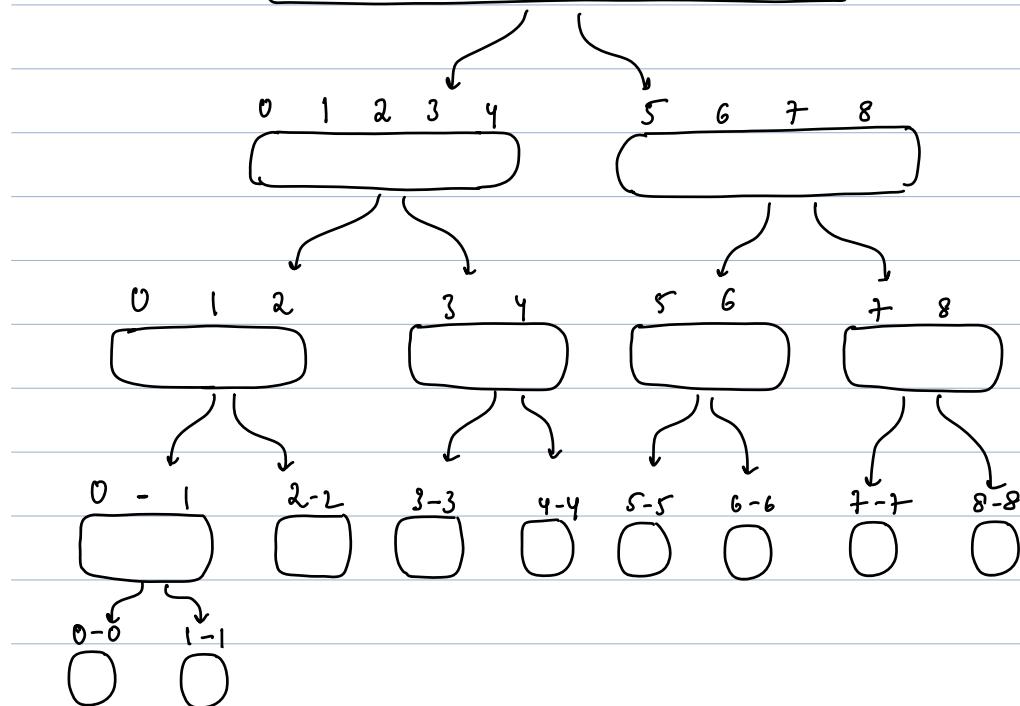
Case3 : $a_1 a_2 a_3 \dots a_{50} a_{51} \dots a_{99} a_{100}$ Iterations



Idea: keep dividing $arr[]$, till it contains 1 element & merge them.

Idea:

ar[9] : 0 1 2 3 4 5 6 7 8
3 10 6 15 8 12 2 18 17



main() {

// Given arr[n] sorted entirely

 mergeSort(arr, 0, n-1)

1. Ass: Given subarray from [s..e] sort it.

void mergeSort(int arr[], int s, int e) {

}

→ A[] : Subarray [s m] is sorted
: Subarray [m+1 e] is sorted
: Sort entire subarray from [s...e]

void merge(int A[], int s, int m, int e) {

}