

Today's Content

- Maximum sum without adjacent elements
- No. of ways to go from $(0,0) \rightarrow$ another cells
- No. of ways to go from $(0,0) \rightarrow$ another cells blocked cells
- Min path sum in a 2d Matrix

Quick Revision:

Step: 1 Try it with Recursion

Step: 2 Overlapping Sub Problems

Step: Dp

- dp table: Initialize with invalid val, // not possible to be output.
- If subproblem solve 1st time: Solve it, store it, return it
- If subproblem already solved: retrieve & return it
- TC: $(\# \text{No. of Sub Problems}) * (\text{TC for each subproblem})$

18) Given $arr[N]$ calculate max subsequence sum. {any ele, order index}

Note1: In a subseq 2 adj elements cannot be picked

Note2: Empty SubSequence is also valid = {0}

$$arr[3] = \{9, 14, 3\} = ans: 14$$

Idea:

1. Sum of all odd indices } * fail
Sum of all even indices

$$arr[4] = \{9, 4, 13, 24\} = ans: 33$$

$$arr[3] = \{13, 14, 2\} = ans: 15$$

2. Generate all subsequences:

Consider only non-adjacent subseq sum
get overall max.

$$arr[4] = \{-4, -3, -2, -3\} = ans: 0$$

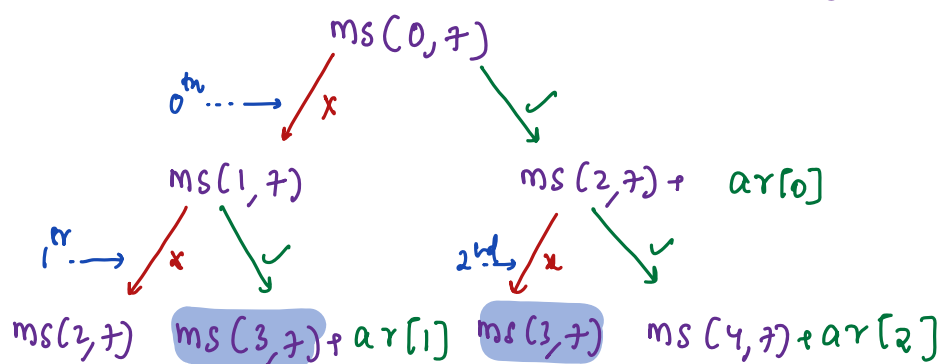
$$Tc: O(2^N * N)$$

Idea:

$$arr[8] = \{2, -1, -4, 5, 3, -1, 4, 2\}$$

1. Recursion

2. Overlapping Sub Problems



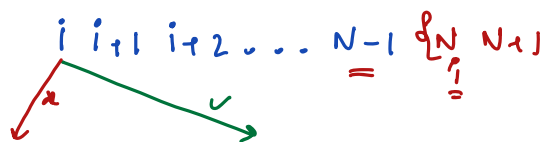
int manSub(int arr[], int i) { // max non-adjacent subseq sum from $[i \dots N-1]$

if ($i \geq N$) { return 0 }

return max { manSub(arr, i+1)

$arr[i] + manSub(arr, i+2)$ }

}



$1+1 \quad i+2 \dots N-1 \quad arr[i] + i+2 \dots N-1$

$manSub(arr, i+1) \quad arr[i] + manSub(arr, i+2)$

Note: In Recursive code, overlapping
apply dp.

int dp[N] = -1; // Invalid

int manSub(int ar[], i){

if(i >= ar.length) {return 0}

if(dp[i] == -1){

dp[i] = max {manSub(ar, i+1) ar[i] + manSub(ar, i+2)}

return dp[i];

}

dp[i] = max non subsequen a sum from {i..N-1}

dp[N] = max sum {N..(N-1)} ✗

dp[N-1] = max sum {N-1... (N-1)} ✓

10:15 break

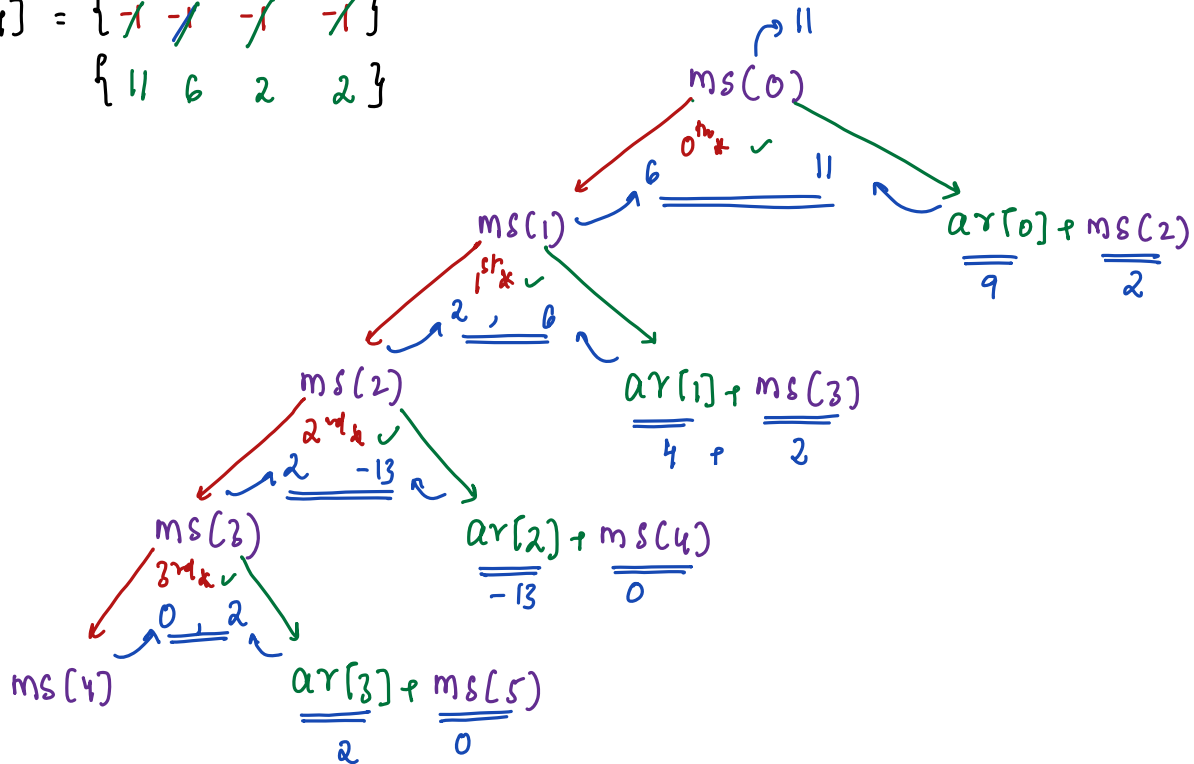
TC: $O(N) * 2 = O(N)$ SC: $O(N)$

Dry Run:

ar[4] = { 9 4 -13 2 }

dp[4] = { ~~-1~~ ~~-1~~ ~~-1~~ ~~-1~~ }

{ 11 6 2 2 }

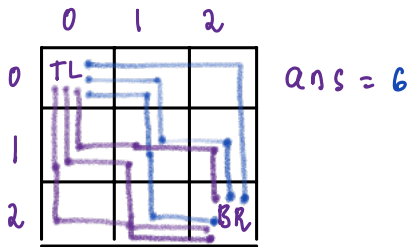


Q8 Number of ways to go from (0,0) → BR cells, in mat[N][M]

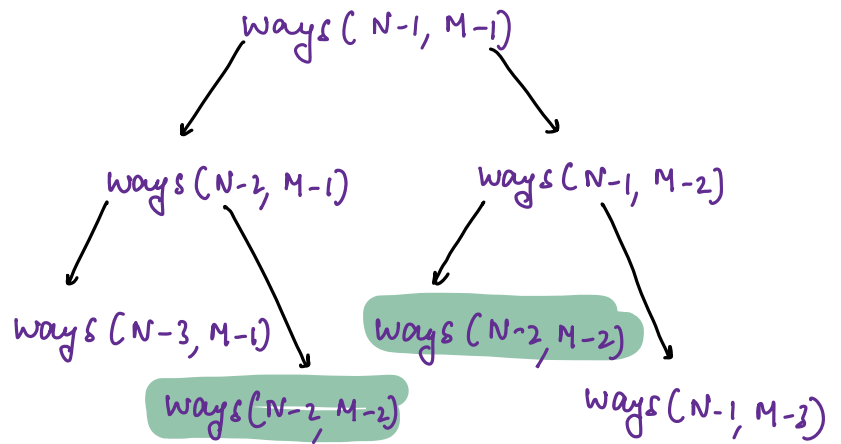
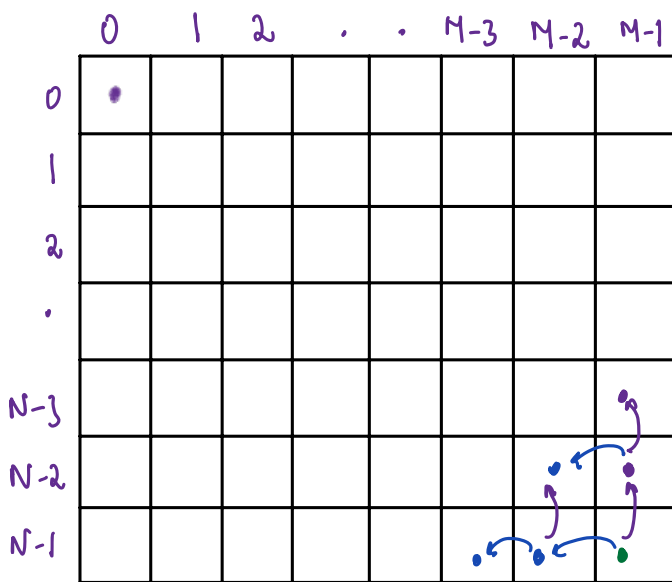
Note: From cell we can go to right or down



Ex:

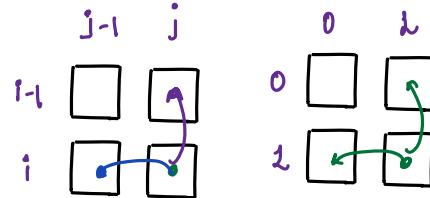


Ex: mat[N][M]



1. Recursion
2. Overlapping Sub Problems.

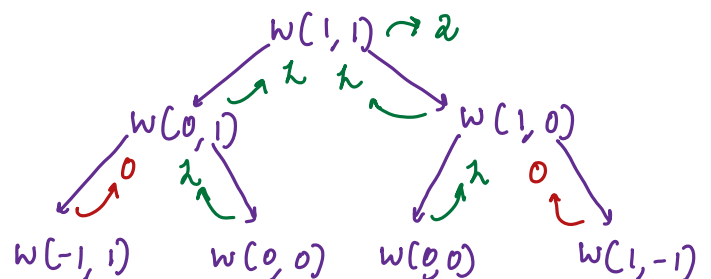
```
int ways(int i, int j) { // Number of ways to reach i, j from 0,0
    if(i < 0 || j < 0) {return 0}
    if(i == 0 && j == 0) {return 1}
    return { ways(i-1, j) + ways(i, j-1) }
}
```



Note: ways(0,0): No. of ways to reach (0,0) from (0,0): 1 way: Stay there

Similar to stair case problem.

Note2: Optimise with dp.



$dp[i][j]$ = no. of ways to reach i, j from $0, 0$

`int dp[N][M] = -1; // Invalid.`

$dp[N][M]$ = ways to reach N, M from $0, 0$ *

$dp[N-1][M-1]$ = ways to reach $N-1, M-1$ from $0, 0$; ✓

`int ways(int i, int j) {`

`if (i < 0 || j < 0) { return 0; }`

`if (i == 0 && j == 0) { return 1; }`

`if (dp[i][j] == -1) {`

`dp[i][j] = ways(i-1, j) + ways(i, j-1)`

`}` `return dp[i][j]`

TC: $(N * M) * (1) = O(N * M)$

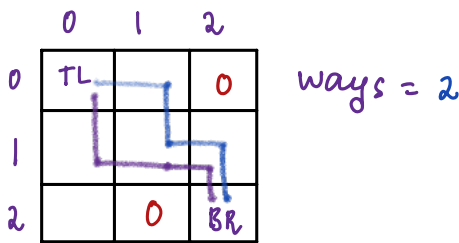
SC: $O(N * M)$

2Q Number of ways to go from (0,0) → BR cells, in mat[N][M]

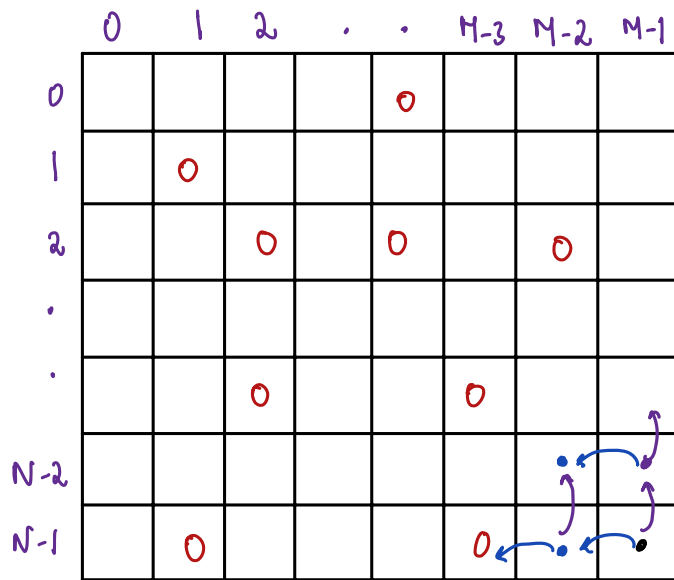
Note: From cell we can go to right or down

Note: Certain cell contain 0, indicates blocked cells, we cannot travel via blocked cells

Ex:



Ex: mat[N][M]



int dp[N][M] = -1; // Invalid.

int ways(int i, int j, int mat[][M]) {

if (i < 0 || j < 0) { return 0; }

if (mat[i][j] == 0) { return 0; } // condition should come above below condition?

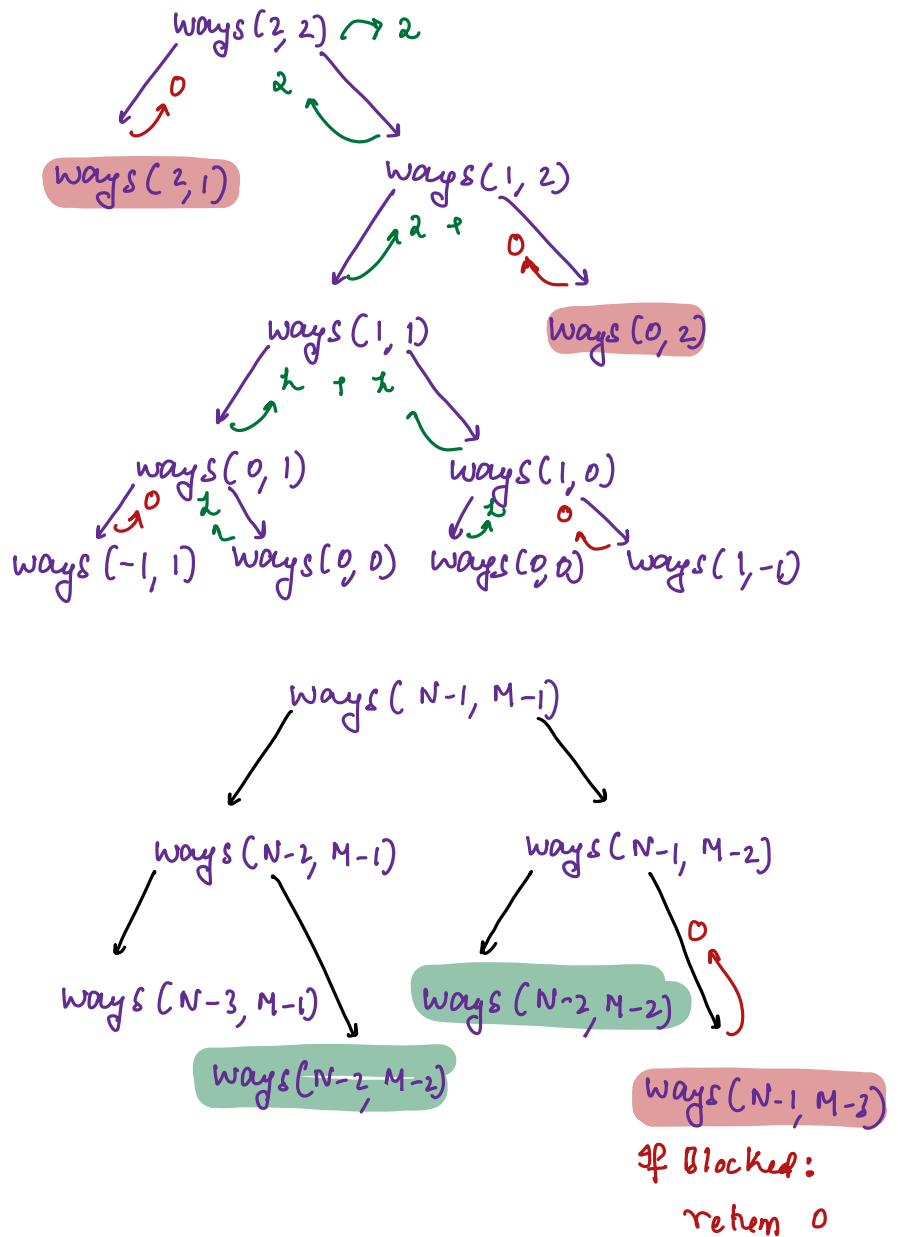
if (i == 0 && j == 0) { return 1; }

if (dp[i][j] == -1) {

dp[i][j] = ways(i-1, j) + ways(i, j-1)

return dp[i][j]

Tc: (N*M)*(1) = O(N*M) Sc: O(N*M)



3Q) Min cost path from $(0,0) \rightarrow (BR)$

Note: from cell we can go to right or down

Note: When we cross/land on a cell, cost associated it

Calculated min cost required to from $(0,0) \rightarrow BR$

Ex: : TODO

	0	1	2
0	2	4	5
1	5	3	2
2	1	4	4

Ex: $mat[N][M]$

	0	1	2	.	.	.	M-2	M-1
0								
1								
2								
.					.	.	.	2
.					.	3	9	3
N-2					.	7	8	6
N-1					.	4	3	10