

Phases of Compiler

1. Lexical phase → scanner → tokens → file
2. Syntax phase → parser → tree → grammar
3. Semantic phase
4. Intermediate code generation
5. Code optimization
6. Final code generation

7. Error handling

Output → asm file
 $a = b + c * d$ → actual $a = (int)(b + float)c * d$



Syntax tree



Semantic tree

Automata-machines (which says whether input is recognised or not i.e Yes/No as output)

→ Finite automata → no memory

→ Pushdown automata → some stack (memory)

→ Turing Machine → magnetic tape

Deterministic
Non-deterministic

1. Sub
2. Infix
3. Prefix

$$f = \{x^k : k \in \mathbb{N}\}$$

Derivation

Infix

Prefix

Suffix

Postfix

$$\text{SUS} = \{x : x \in S_1 \text{ and } x \in S_2\}$$

$$\text{SUS}_1 = \{x : x \in S_1 \text{ and } x \notin S_2\}$$

$$S_1 - S_2 = \{x : x \in S_1 \text{ and } x \notin S_2\}$$

$$S_1 \cup S_2 = \{x : x \in S_1 \text{ and } x \in S_2\}$$

Show that $S_1 = S_2 = n(\frac{n+1}{2})$
for n be the no. of elements
for $n=10$

$$\text{LHS} = S_1 = \binom{10}{2} = \frac{10!}{2!} = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 10!$$

$$\text{RHS} = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 10!$$

Let $P(k)$ be true

$$\sum_{i=0}^{k-1} i = \frac{k(k+1)}{2}$$

Add $k+1$ on both sides

$$\sum_{i=0}^k i = \frac{k(k+1)(k+2)}{2}$$

$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)+1}{2}$$

$\therefore P(k+1)$ is true

$\therefore P(n)$ is true $\forall n \in \mathbb{N}$

Three Basic Concepts

Language \rightarrow collection of strings/sequences
of characters

Grammar

Automata

Operation on Strings

1. Reverse

2. Substring

3. Prefix

4. Suffix

5. Proper prefix

6. Proper suffix

7. Proper substring

8. Length

Let $S = \{\text{computer}\}$

now $\Rightarrow \text{return proc}$

len $\Rightarrow S$

prefix $\Rightarrow \{\text{com}, \text{comp}, \text{comput}, \text{computer}, \text{computer}\}$

proper prefix $\Rightarrow \{\text{co}, \text{com}, \text{comp}, \text{comput}, \text{computer}\}$
(except & & computer(s))

suffix $\Rightarrow \{\text{er}, \text{riter}, \text{iter}, \text{puter}, \text{mputer}, \text{computer}\}$
& & computer(s)

Operations on languages

(1)

UNION

(2)

Intersection

(3)

CONCERNATION
(Kleene closure)

→ star closure {only one}

→ positive closure {one by one}

→ release

→ complement

$L_1 = \{a, b, c\}$

$L_2 = \{1, 2, 3\}$

find all operations on languages

$$L_1 \cup L_2 = \{ab, c, 1, 2, 3\}$$

$$L_1 \cap L_2 = \{\} \phi$$

$$L_1 \cdot L_2 = \{abl, ab23, cl, c23\}$$

$$L_1^R = \{ba, c\}$$

$$\bar{L}_1 = \epsilon - L_1$$

Star closure = one more occurrence of L

$$L_1^* = L_1^0 \cup L_1^1 \cup L_1^2 \cup L_1^3 \quad \{ \phi, ab, c, \dots \}$$

$$L_1^0 = \phi$$

$$L_1^1 = \{ab\}$$

$$L_1^2 = L_1 \cdot L_1 = \{abb, abc, cab, ca\}$$

positive closure

$$L_1^{(+)^+} = L_1^+ - \phi$$

Finite state generated by

$L = \{a^n b^m : n+m \leq 3, n \geq 0\}$

States: $S \rightarrow$ Rainy days
Cloudy days

$L = \{a^n b^m : n \geq 0\}$

From the proof of closure from the start of
any string of the language is called
one is present in the language

$L = \{a^n b^m : n \geq 0\}$

$S \rightarrow aSb$

$\{S, S \rightarrow aSb, S \rightarrow aSb|b\}$

$G: \{S, S \rightarrow aSb, S \rightarrow a^n b\}$

$S \rightarrow aSb$

$S \rightarrow$

$G: \{S, S \rightarrow aSb, S, \{S \rightarrow aSbb, S \rightarrow \lambda\}\}$

$G: \{S, S \rightarrow aSb, S, \{S \rightarrow aSbb, S \rightarrow \lambda\}\}$

$L = \{a^n b^m : n+m \leq 3, n \geq 0\}$

$S \rightarrow aSb|b$

$b \rightarrow ab|a$

$a \rightarrow S^n b$
 $b \rightarrow a$
 $a \rightarrow b$

$S \rightarrow aab|aab$

$S \rightarrow aS|A$

$A \rightarrow aa$

Generate the grammar on

$\Sigma = \{a, b\}$

- ① starts with exactly one a
- ② starts with at least one a
- ③ no more than three a 's.
- ④ at least three a 's

① $S \rightarrow aBaB\lambda$

$B \rightarrow bB|\lambda$

② $A \rightarrow aA|\lambda$

$S \rightarrow ABaBA$

$B \rightarrow bB|\lambda$

$S \rightarrow BaB aB aB | BaB$
 $B \rightarrow bB|\lambda$

$S \rightarrow$ $\{w : |w| \bmod 3 = 0\}$ on $\Sigma = \{a\}$

$S \rightarrow aaaS / \lambda$

$|w| \bmod 3 = 0$ on $\Sigma = \{a\}$

$S \rightarrow a^A / aaA$
 $A \rightarrow aaA / \lambda$

$S \rightarrow aaaS / aala$

$L = \{uw^R : w \in \{a, b\}^*$

$S \rightarrow asalbsb / \lambda$

$L = \{w : n_a(w) = n_b(w)$
no of a's no of b's
on w on w

$S \rightarrow asb / bsaw / \cancel{abs} / \cancel{bas}$

$S \rightarrow asb / bsaw / ss / \lambda$

~~abs~~
~~bas~~

Check Derive aab

$\Rightarrow as \downarrow b \quad aabb$
 $\Rightarrow as \downarrow b \quad \swarrow ab$

* Find the language generated by grammar

$$\begin{array}{l} \text{(i) } S \rightarrow a^A \\ A \rightarrow bS \\ S \rightarrow \lambda \end{array}$$

$$\begin{array}{l} \text{(ii) } S \rightarrow A_a \\ A \rightarrow B \\ B \rightarrow A_a \end{array}$$

Undeterm.

$$L = \{(ab)^n : n \geq 0\}$$

$$\text{(i) } S \rightarrow aSb|ab$$

$$\text{(ii) } S \rightarrow aAb|ab$$
$$A \rightarrow aAb|\lambda$$

$$\Sigma = \{a, b\} \text{ and}$$
$$L = \{a^n b^n : n \geq 0\}$$

$$\text{Find } L^R \quad L^2$$

$$L^R = \{b^n a^n : n \geq 0\}$$

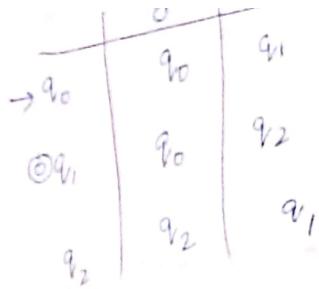
$$L^2 = \{a^n b^n a^m b^m : m, n \geq 0\}$$

Automata:-

Finite Automata

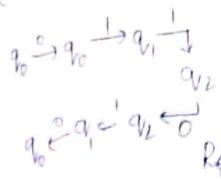
Transducer

Accepts
(language)



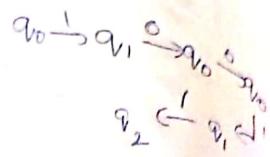
Check if the string

(i) 011010



Reject

(ii) 10011



Reject

Default transition: The default transition is $\delta^*(q_i, \lambda) = q_i$

$$\delta^*(q_0, 011010) \rightarrow \delta(\delta^*(q_0, 0110), 0)$$

$$= \delta(\delta(\delta^*(q_0, 0110), 1), 0)$$

$$= \delta(\delta(\delta(\delta^*(q_0, 0110), 1), 0), 1)$$

$$= \delta(\delta(\delta(\delta(\delta^*(q_0, 0110), 1), 0), 1), 0)$$

$$= \delta(\delta(\delta(\delta(\delta(\delta^*(q_0, 0110), 1), 0), 1), 0), 1)$$

$$= \delta(\delta(\delta(\delta(\delta(\delta(\delta^*(q_0, 0110), 1), 0), 1), 0), 1), 0), 1)$$

1001

$$q_0 \xrightarrow{\text{(Accept)}} q_1 \xrightarrow{\delta} q_0 \xrightarrow{1} q_1$$

1001

$$\delta^*(q_0, 1001)$$

$$\delta(\delta^*(q_0, 100), 1)$$

$$\delta(\delta(\delta^*(q_0, 10), 0), 1)$$

$$\delta(\delta(\delta(\delta^*(q_0, 1), 0), 0), 1)$$

$$\delta(\delta(\delta(\delta(\delta^*(q_0, 1), 0), 0), 0), 1)$$

$$\delta(\delta(\delta(\delta(\delta^*(q_0, 1), 0), 0), 0), 1)$$

$$\delta(\delta(\delta(\delta(q_0, 1), 0), 0), 1)$$

$$\delta(q_0, 1)$$

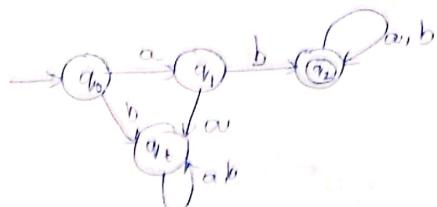
$$\begin{aligned} & \Rightarrow q_1 \in F \\ & \text{(Accepted)} \end{aligned}$$

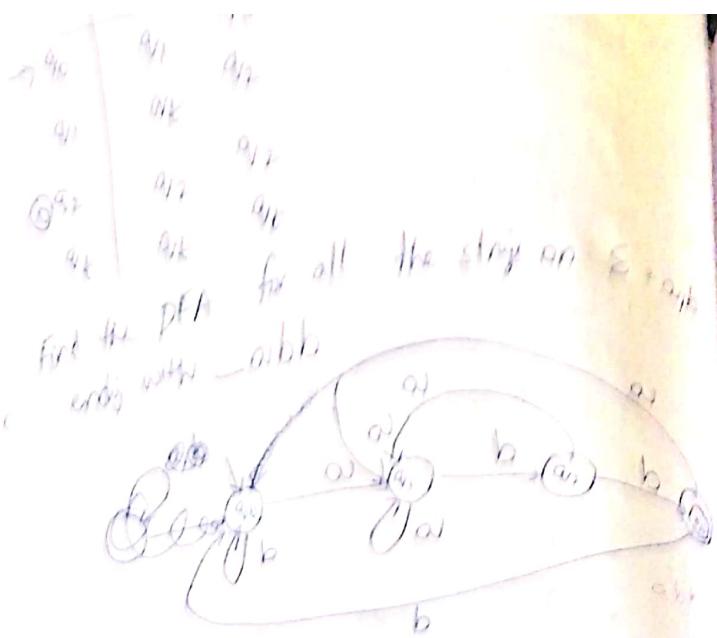
The language accepted by DFA M is defined as

$$L(M) = \{ w \in \Sigma^* : \delta^*(q_0, w) \in F \}$$

$$\overline{L(M)} = \{ w \in \Sigma^* : \delta^*(q_0, w) \notin F \}$$

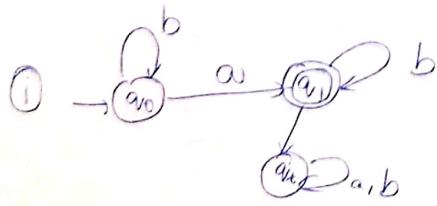
Find the DFA for set of all strings of $\Sigma = \{a, b\}$ that starts with prefix ab .





DFA on

- $E = \{a, b\}$ find Density one a,
 (i) at least one a,
 (ii) no more than three a,
 (iii) at least three a,



fix all the steps on E

fix the orbits

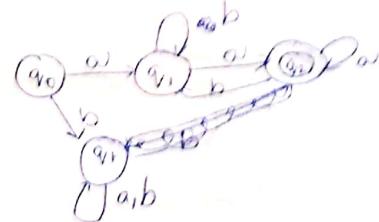
(x) $\left(\frac{d}{dx} - B(x) \right) \Phi(x) = B(\Phi(x))$



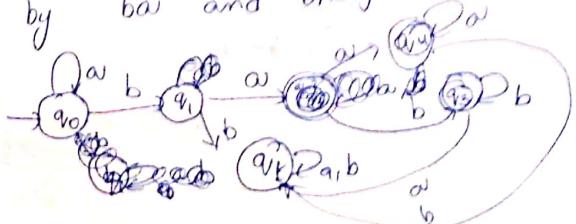
A language L is said to be regular if and only if there exist a NFA D to

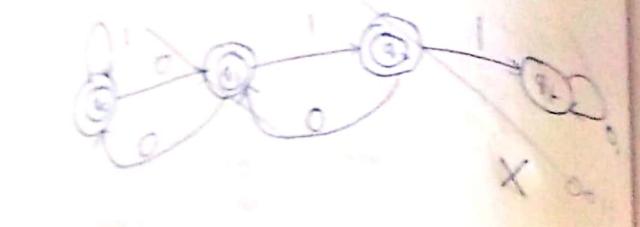
Show that

$$L = \{a^m w b^n \mid m, n \in \mathbb{N}, w \in \{a, b\}^*\}$$



Find the DFA which accept any number of 0's followed by 0's and ending with 0's





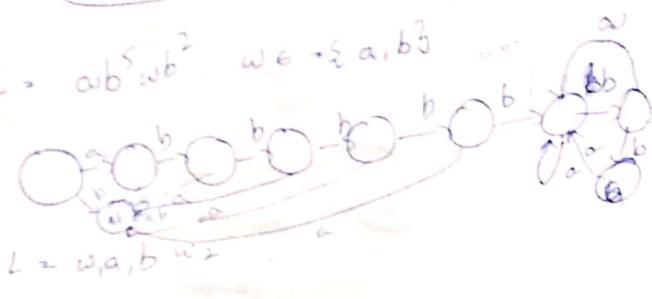
$L = \{ w \mid \text{mod } 3 = 0 \}$



$L = \{ w \mid \text{mod } 3 = 0 \text{ on start} \}$

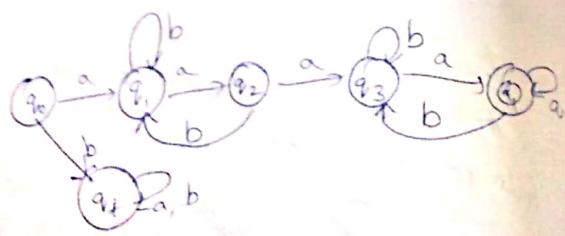
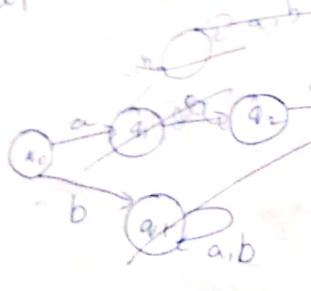


$L = ab^5 b^2$ where $a, b \in \{a, b\}$



$L = \{a, b\}^{12}$

$a \cdot w_1 a a w_2 a \quad w_1, w_2$



aba
ababaa

DFA for $\{a, b\}^*$ with

i) atleast one b

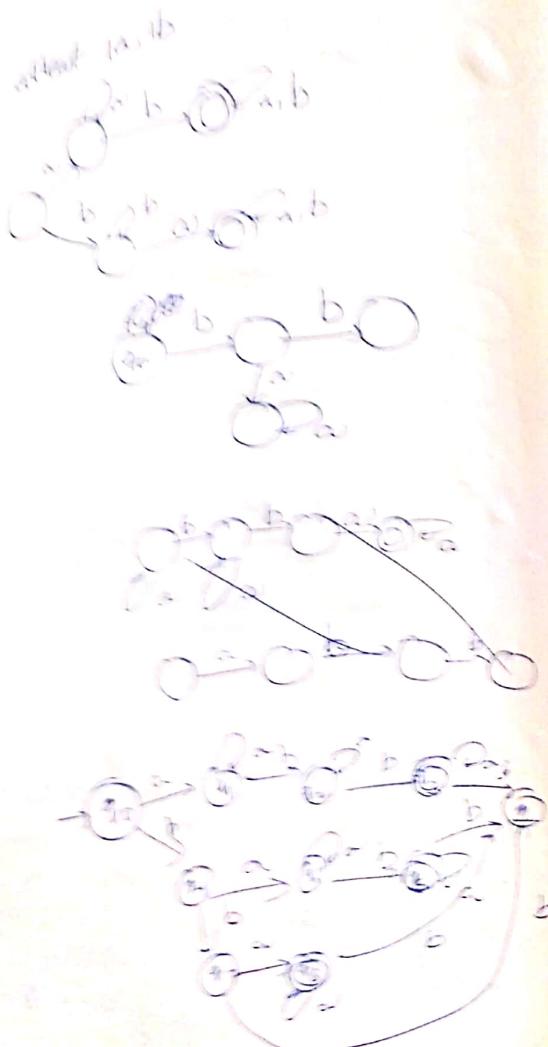
ii) with atleast one a

iii) atleast one a, b

iv) at least one a, exactly two b's

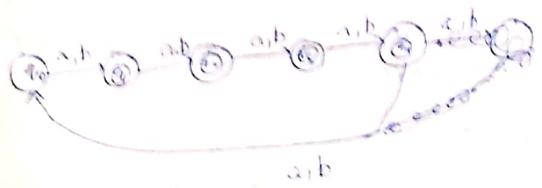


Q



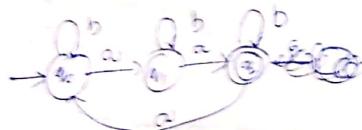
DFA for all the strings with exactly two 'a's and more than three 'b's

L_{ab} where L_a maps to {a, b}



	a	b
→ q ₀	q ₁	q ₅
q ₀	q ₂	q ₅
q ₁	q ₃	q ₅
q ₂	q ₄	q ₅
q ₃	q ₅	q ₅
q ₄	q ₅	q ₅

L = L_{ab} $\cap \{a\}^m \text{ mod } 3 > 1 \text{ in } \mathbb{Z}_{(3)}$

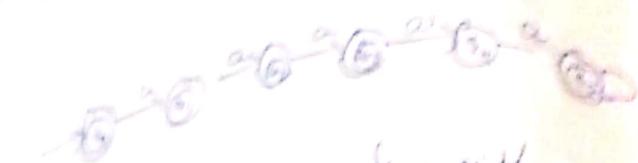


Show that

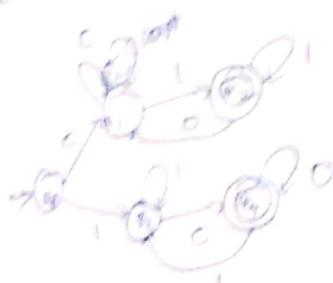
$$L = \{a^n : n \geq 4\} \text{ in } \mathbb{Z}_{(3)}$$



20.10



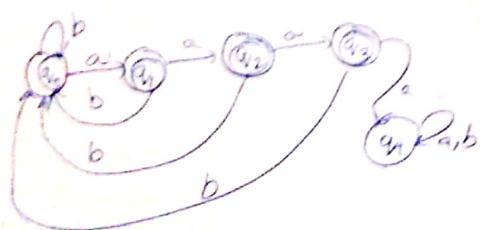
left most symbol differs from rightmost
on 2-10-17



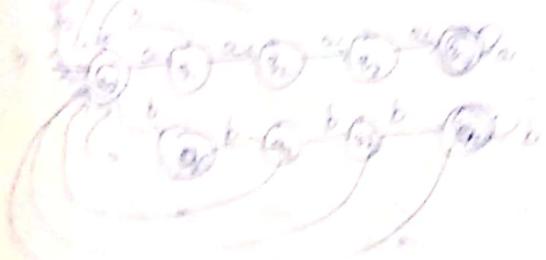
run - A run in a string is a substring of length at least two as long as possible consisting of entirely of some symbol.

Find DFA for

$L = \{w : \text{every run or } a's \text{ has length either } 2 \text{ or } 3 \text{ on } \Sigma = \{a, b\}\}$



$L = \{w : \text{no run or length less than 4 occurs}\}$



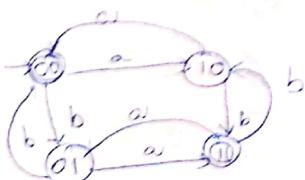
All my states do not do 000
on 2-10-17

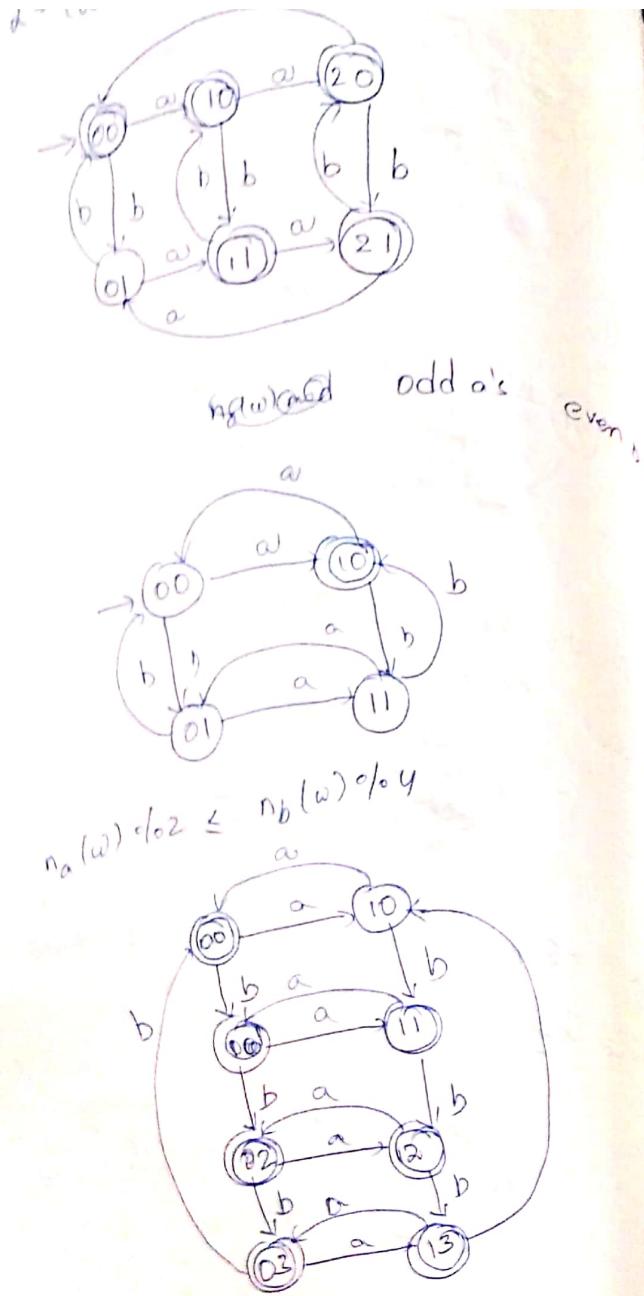
abba



$\Rightarrow \{p \in \Sigma^* : n_0(p) \bmod 2 = n_1(p) \bmod 2 \equiv 1 \pmod 3\}$

modulo logic -





Draw DFA to accept the binary strings divisible by 5.

Division by k

$$j = \frac{(r+i)+d}{k}$$

↓ states digits
index

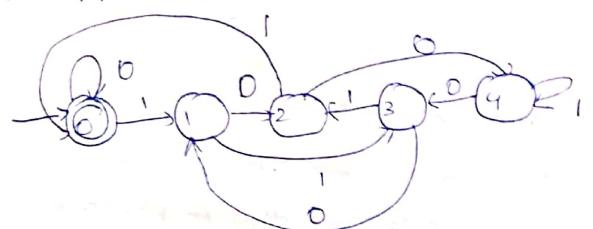
$$k = 5$$

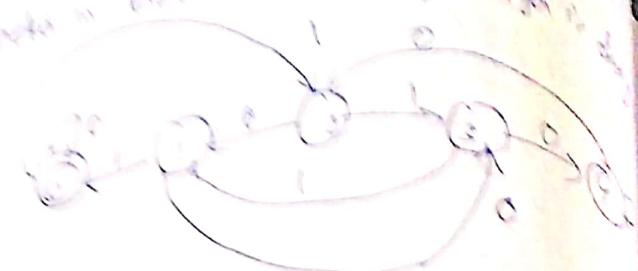
$$i = 0, 1, 2, 3, 4$$

d

i	d	j
0	0	0
0	1	1
1	0	2
1	1	3
2	0	4
2	1	0
3	0	1
3	1	2
4	0	3
4	1	4

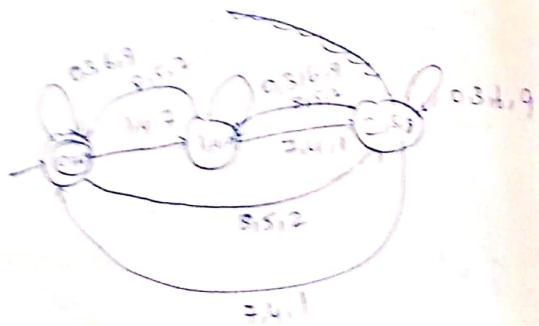
$$s = (q_i, d) = a_j$$





Ques to find the decimal digit demand in $(\{0,1,2,3\})^*$

α	0	1	2	3
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1



Regular expression (RE) :-

- * \rightarrow zero or more occurrences a^*
- + \rightarrow one concatenation $(a.b) \rightarrow b$ followed by a
- \cup \rightarrow or $(a+b) \rightarrow a \cup b$

1. ϕ, a are primitive regular expressions.
2. $x_1, x_2, x_3, \dots, x_n$ are also regular expressions if x_i is a regular expression.

Find the language generated by regular expression

$$L(a+bc) \rightarrow L(a) + L(b.c)$$

$$\{a\} \cup \{ab\}$$

$$L(a(c+b)c^*)$$

$$L(a^*) \cdot L(c+b) \cdot L(c^*)$$

$$\{L(a)^*\}^* \cdot L(c) + L(b) \cdot \{L(c)\}^*$$

$$a^* \cdot c + b \cdot c^*$$

$$\{a, ab\} \quad \{c, bc\}$$

$$(ab)^* \neq a^*b^*$$

$$L((ab)^*)$$

$$\{L(ab)\}^*$$

$$\{L(a) \cdot L(b)\}^*$$

$$(ab)^*$$

$$\{\lambda, ababab, \dots\}$$

$$L(a^*) \cdot L(b^*)$$

$$\{L(a)\}^* \cdot \{L(b)\}^*$$

$$a^* \cdot b^*$$

$$\{a, ab, ab^2, \dots, babb, \dots\}$$

$(a+b)^m$
 $1 + (a+b)^1$
 $(a+b)^2 = (a+b)^1 + (a+b)^1$
 (at least 1) a and b only
 $(a+b)^3 = (a+b)^1 + (a+b)^2$
 (at least 1) a and b only
 $(a+b)^4 = (a+b)^1 + (a+b)^2 + (a+b)^3$
 (at least 1) a and b only
 $(a+b)^5 = (a+b)^1 + (a+b)^2 + (a+b)^3 + (a+b)^4$
 (at least 1) a and b only
 \vdots

(at least 1) a and b only
 $(a+b)^n = \text{any number of } a \text{ followed by } b$
 $(a+b)^n = \text{any number of } b \text{ followed by } a$
 $(a+b)^n = \text{any number of } ab$
 $(a+b)^n = \text{any number of } ba$
 \vdots

$a^k b^l c^m$
 at least one a followed by odd no of b's
 $(a+b)^n = \text{any number of } a \text{ followed by } b$
 $(a+b)^n = \text{any number of } b \text{ followed by } a$
 $(a+b)^n = \text{any number of } ab$
 $(a+b)^n = \text{any number of } ba$
 \vdots

stay a, b either a or bb
 $(a+b)^n \cdot (a+bb)$
 even no of a's followed by odd no of b's

a, b, c
 $(a+b)^n ((a+b)^1 + b)$ or $(a+b)^n b ((a+b)^1)$
 1 or 0 followed by any number of 1's
 $(1+0)^1$
 $(1+0)^2$ without any consecutive 1's
 \vdots

a, b, c
 $(1+0)^n (1+c)$
 all digits even except on 1(c, 1)
 $((1+0)^n)^* + 1^*$
 \vdots

$a = \{a, b\}$ of length 2
 $aa + bb + ab + ba$
 $\frac{(ab)}{(a+b)(a+b)}$

even length
 $((a+b)(a+b))^*$
 $((a+b)(a+b)(a+b))^* + (a+b)$
 odd length
 $((a+b)(a+b))^* + (a+b)$

stay a's and b's with other
 $(ab)^* + (ba)^*$

$$L = \{(a^n)(a-q)^*\}$$

Set of starting point

$$(1+1^*) \cdot (a-q)^* \cdot (1^*) (a-q)^*$$

? ≥ 0 no. of occurrence of previous symbol preceding

All strings with exactly one exactly a or exactly b on $\Sigma_{a,b}$

$$(b+c)^* a (b+c)^*$$

All strings with no more than 3 a's on $\Sigma_{a,b,c}$

$$(b+c)^* a? (b+c)^* a? (b+c)^* a? (b+c)^*$$

$$L = \{w : n_1 \text{ mod } 3 = 0\}$$

$$((a+b)(a+b)(a+b))^*$$

$$L = \{w : n_1(w) \% 3 = 0\}$$

$$(bab^*ab^*ab^*)^* + b^*$$

$$L = \{a^n b^m : n, m \geq 1, nm \geq 3\}$$

$$abb^* + ab^*b^* + aabb^*$$

$$L = \{ab^n w : \text{where } n \geq 3, w \in \Sigma^*\}$$

$$abb^*(a+b)^*$$

$$L = \{a^n b^m : n \leq 4, m \leq 3\}$$

$$a? a? a? b? b? b?$$

$$L = \{a^n b^m : n \leq 4, m \leq 3\}$$

$$aada + b?b?b?$$

$$L = \{a^n b^m : m+n \text{ is even}\}$$

$$(aa)^* (bb)^* + a(ba)^* \cdot b(bb)^*$$

$$L = \{w : n_1(w) \% 3 = 0\}$$

$$\Sigma = \{a, b\}$$

$$b^* ab^* a? b^*)$$

$$\begin{aligned} & b^* ab^* a \\ & b^* ab^* a b^* a b^* a \\ & (a+b) ((a+b)(a+b)(a+b)) \\ & + (a+b)(a+b) ((a+b)(a+b)(a+b)) \end{aligned}$$

$$(b^* a-b^*)^* (b^* a-b^*)^*$$

$$b^* a b^* a? b^*$$

strings of zeros & ones with at most one pair of consecutive zeros.

$$\begin{aligned} & 1^* (01^*)^* (00+1^*)^* (10)^* 1^* \\ & (1+01)^* (1+0+00) (1+10)^* \end{aligned}$$

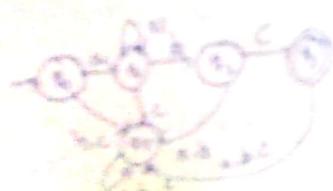
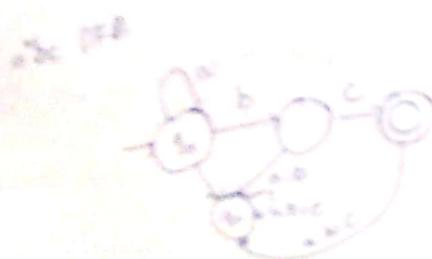
Now with the solution
to get direct $(ab)(ba)$

$a(b)(b(ab)) \cdot (abc)^*$

$b^* = (abc)^* b$

but you can't do it with
just weighted ones atleast

$(abc)^* \text{ code } ((ab)^*)^*$



① $(ab)^* b$



② $(abc)^*$



③



$(ab)^*$



$b(a)^*$



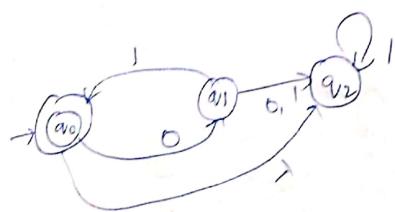
NFA
Nondeterministic finite acceptor

$M = (Q, \Sigma, \delta, q_0, F)$ is defined as

Q = represent of finite set of internal states
 Σ = finite set of input symbols
 δ = It is a transition function denoted by
 $Q \times (\Sigma \cup \{\lambda\}) \rightarrow Q^Q$

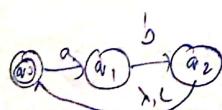
$q_0 \in Q$ denoting starting state

$F \rightarrow$ it is a subset of Q denoting final states



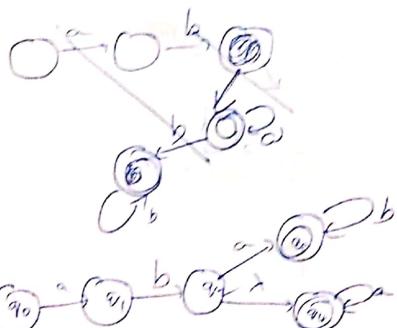
	0	1
q_0	q_1	\emptyset
q_1	q_2	q_0, q_2
q_2	\emptyset	q_2

Write NFA with three states that accepts
 $\{ab, abc\}^*$



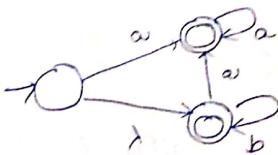
NFA with no more than five states to accept

$\{babab^n : n \geq 0\} \cup \{bab^n : n \geq 0\}$



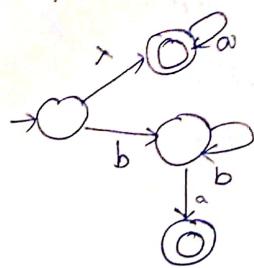
NFA with three states with three states

$L = \{a^n : n \geq 1\} \cup \{b^m : m \geq 0, k \geq 0\}$

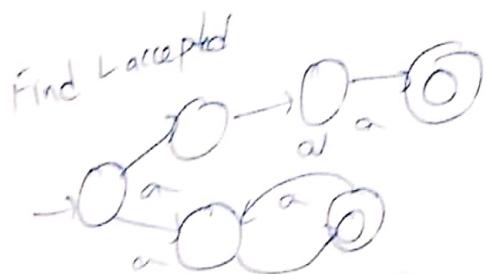
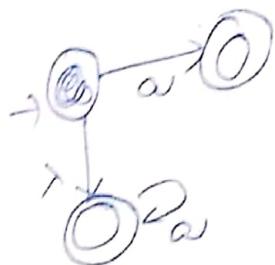
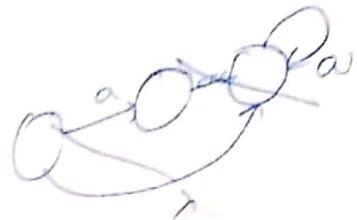


NFA with four states for the language

$L = \{a^n : n \geq 0\} \cup \{b^n : n \geq 1\}$



Draw NFA that accept a language that resulting only a removed



$$L = \{ a^3 \cup a^{2n} : n \geq 1 \}$$

$$\begin{array}{l} a^3 \\ a^2 a \\ a^3 \cup a^{2n} \end{array}$$

The language accepted by NFA 'M' is defined as
 $L(M) = \{ w \in \Sigma^* : \delta^*(q_0, w) \cap F \neq \emptyset \}$

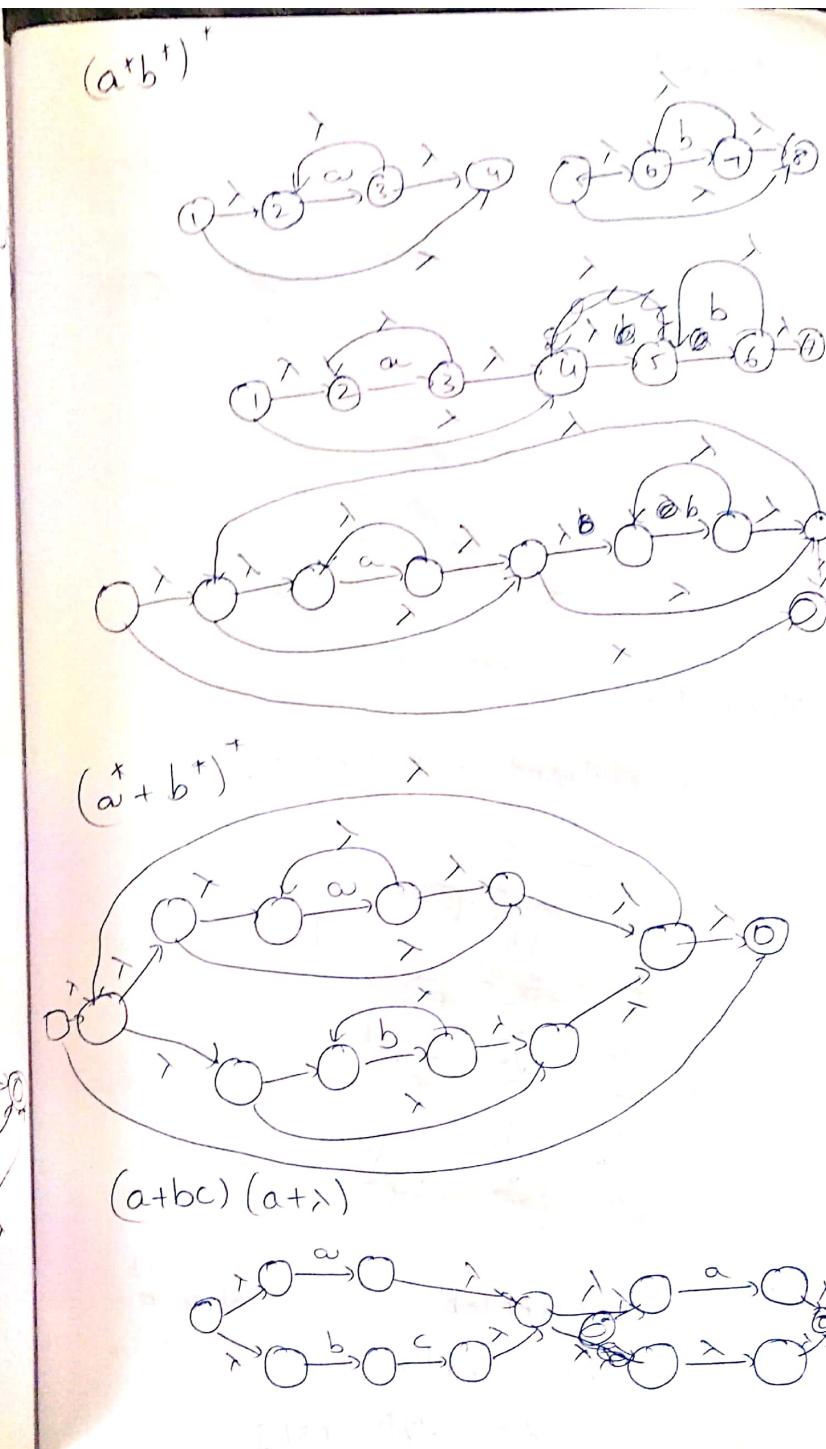
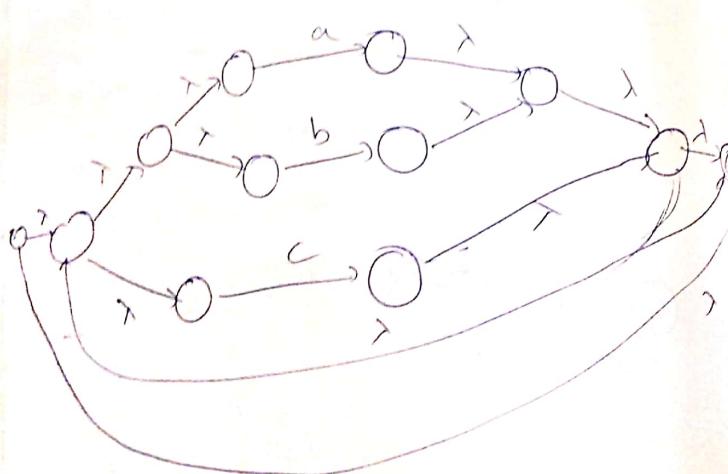
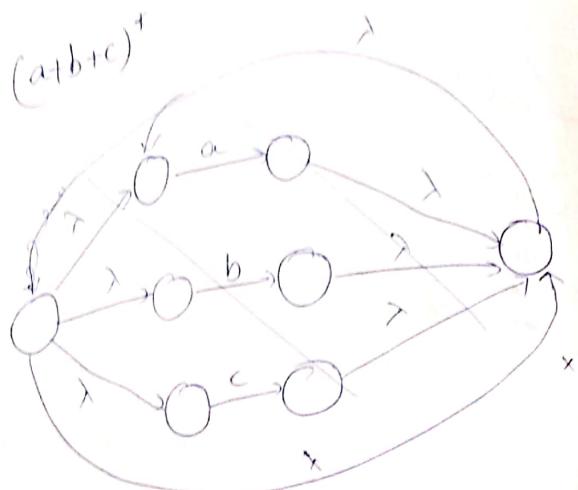
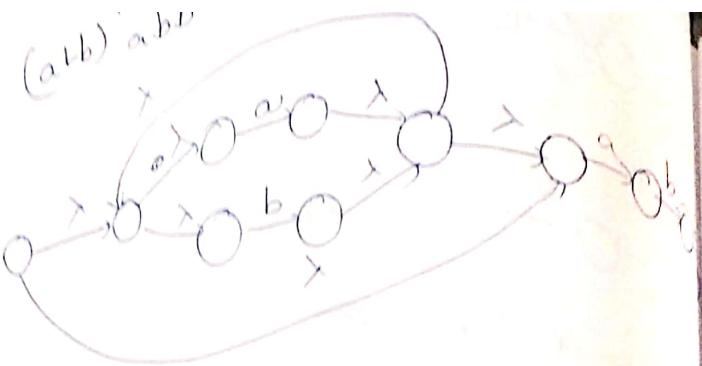
Regular expression to NFA :-

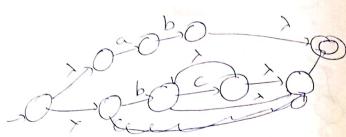
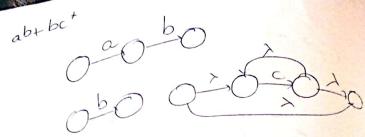
RE → Thompson's Algorithm

↓
NFA → subset construction

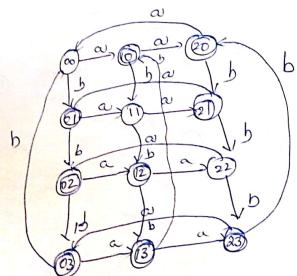
↓
DFA → Tabulation

↓
minimized DFA





Q) Given a DFA
 $L = \{w \mid n_a(w) \equiv n_b(w) \pmod{4}\}$



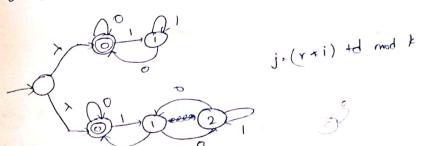
Q)

$S \rightarrow aAb \mid ab$
 $A \rightarrow a^n A b \mid \lambda$

$$L = \{a^n b^n : n \geq 1\}$$

aabb
 $\overline{ab}, \overline{aabb}$

Write DFA that only binary which are divisible by
 $(INEA)$
 $2 \oplus 3$

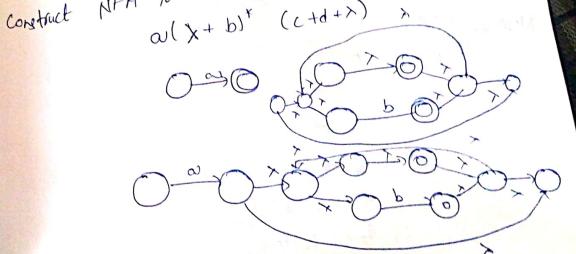


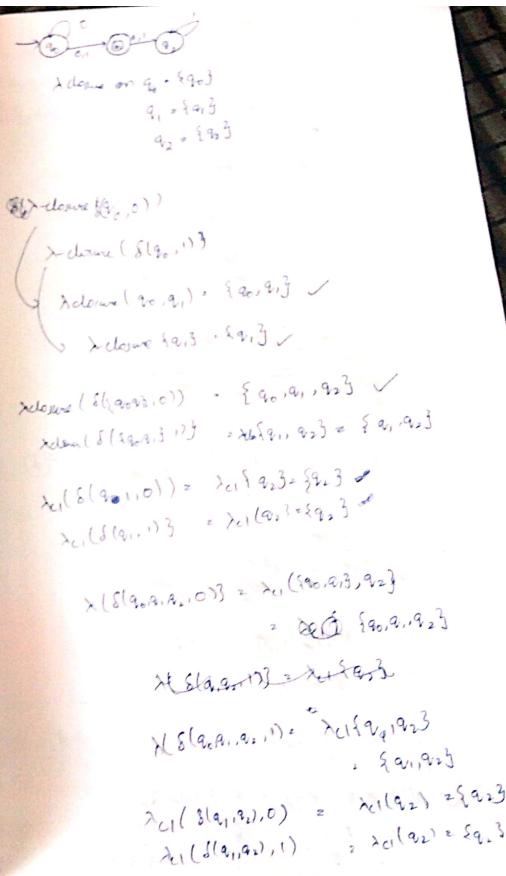
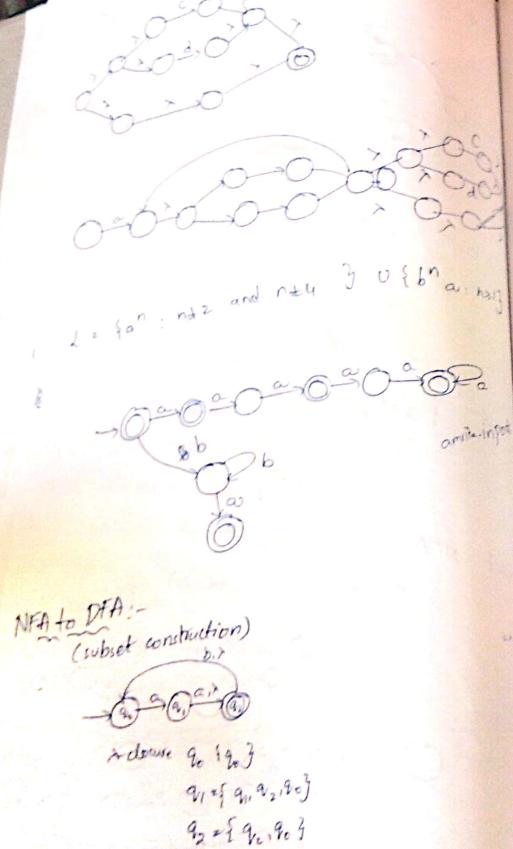
1	0	0
1	1	2
2	1	1

$$L = \{a^{2n} b^{2m+1} : n, m \geq 0\}$$

$$(aa)^* (bb)^* b$$

Construct NFA for

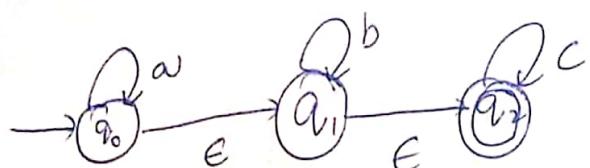
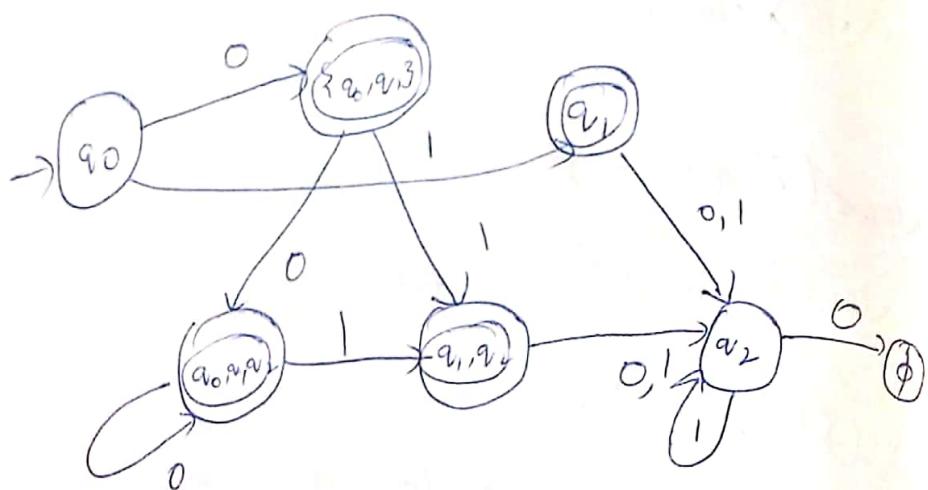




$$\lambda_1 S(q_2^{(t)}) = \{q_2\}$$

$$\lambda_1(S_{\text{last}}) = \{q_2\}$$

	0	1
0	$\{q_0, q_3\}$	$\{q_2\}$
q_0	$\{q_0, q_1\}$	$\{q_1, q_2\}$
q_1	$\{q_2\}$	$\{q_1, q_2\}$
q_0, q_1, q_2	$\{q_0, q_1, q_2\}$	q_2
q_1, q_2	\emptyset	$\{q_2\}$
q_2	\emptyset	$\{q_2\}$



$$\lambda_{C1}\{q_0\} = \{q_0, q_1, q_2\} \rightarrow A$$

$$\lambda_{C1}\{q_1\} = \{q_1, q_2\}$$

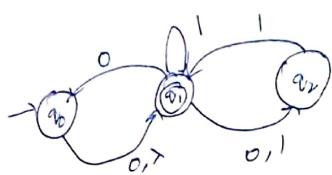
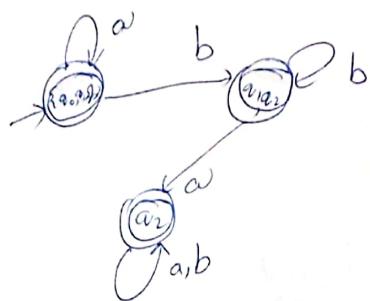
$$\lambda_{C1}\{q_2\} = \{q_2\}$$

$$\lambda_{C1}(S(q_0, q_1, q_2), \emptyset) \in \lambda_{C1}\{q_0, q_1, q_2\}$$

$$\lambda_{C1}(A, a) \rightarrow \lambda_{C1}\{q_0, q_1, q_2\}$$

$$\lambda_{C1}(A, b) \rightarrow \lambda_{C1}\{q_1\} = \{q_1, q_2\}$$

$$\begin{array}{c} \lambda_1(a_0, a_1, a_2) \\ \lambda_1(a_0, a_1, a_2), b = \lambda_1(a_1, a_2) \\ \lambda_1(a_0, a_1, a_2), b = \{a_1, a_2\} \end{array}$$



$$\lambda_1(q_0) = \{q_0, q_1\}$$

$$\lambda_1(q_1) = \{q_1\}$$

$$\lambda_1(q_2) = \{q_2\}$$

$$\begin{aligned} \lambda_1((q_0, q_1), 0) &= \lambda_1\{q_1, q_2\} \\ &= \{q_1, q_2\} \end{aligned}$$

$$\begin{aligned} \lambda_1((q_0, q_1), 1) &= \lambda_1\{q_0, q_2\} \\ &= \{q_0, q_2\} \end{aligned}$$

$$\begin{aligned} \lambda_1\{q_1, q_2\}, 0 &= \{q_2\} \\ \lambda_1\{q_1, q_2\}, 1 &= \lambda_1\{q_1, q_2\} \end{aligned}$$

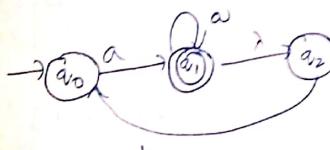
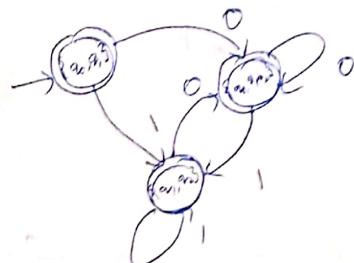
	0	1
0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
1	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$

$$\lambda_1(q_0, q_1, q_2), 0$$

$$\lambda_1\{(q_0, q_1, q_2)\}$$

$$\lambda_1\{q_0, q_1, q_2, 0\}, 0$$

$$\lambda_1\{(q_0, q_1, q_2), 0\} = \{q_1\}$$



$$\lambda_1\{q_0\} = \{q_0\}$$

$$\lambda_1\{q_1\} = \{q_1\}$$

$$\lambda_1\{q_2\} = \{q_2\}$$

$$\begin{array}{c|cc|c} & a & b & \\ \hline q_0 & \{q_0\} & \{q_0, q_2\} & \emptyset \\ q_1 & \{q_1\} & \{q_1, q_2\} & \emptyset \\ q_2 & \{q_2\} & \{q_2\} & \emptyset \end{array}$$

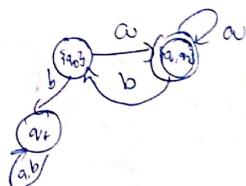
$$\lambda_1\{q_0, a\} = \lambda_1\{q_1\}$$

$$+ \{q_0, q_2\}$$

$$\lambda_1\{q_0, b\} = \emptyset$$

$$\lambda_1\{q_1, q_2, a\} = \{q_1\} = \{q_1, q_2\}$$

$$\lambda_1\{q_1, q_2, b\} = \lambda_1\{q_0\} = \emptyset$$





$$\lambda_1 \{ a \}^* \{ a \}$$

$$\lambda_1 \{ a \} = \{ a \}$$

$$\lambda_1 \{ a \} = \{ a \}$$

$$\lambda_1 \{ q_0 a \} = \{ q_1, q_2 \}$$

$$\lambda_1 \{ q_0 a \} = \{ q_1 \}$$

$$\lambda_1 \{ q_1, a \} = \{ q_1, q_2 \}$$

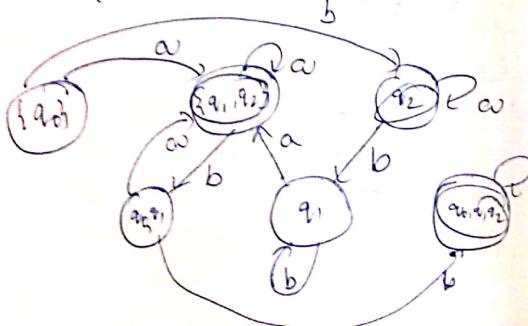
$$\lambda_1 \{ q_1, a \}, b = \{ q_0, q_1 \}$$

$$\{ q_2, a \} = \{ q_2 \}$$

$$\{ q_2, b \} = \{ q_2 \}$$

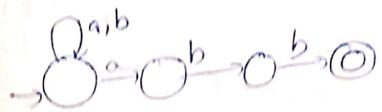
$$\{ q_0, q_1 \} = \{ q_1, q_2 \}$$

$$\{ q_0, q_1 \}, b = \{ q_2, q_0, q_1 \}$$

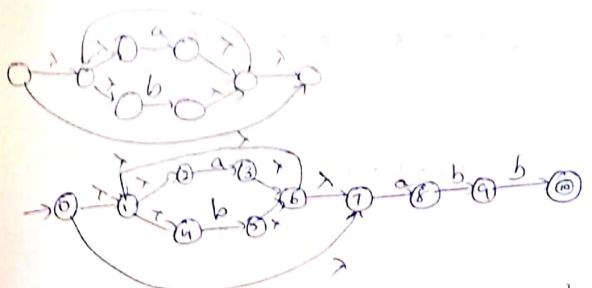


convert the regular expression

$$(a+b)^* a b b$$



$$a \rightarrow a \rightarrow b \rightarrow b \rightarrow \text{final}$$



$$\lambda_1(0) = \{ 0, 1, 2, 4, 7 \} \rightarrow \{ 0, 1, 2, 4, 7 \} \rightarrow A$$

$$\lambda_1(1) = \{ 1, 2, 4, 7 \}$$

$$\lambda_1(2) = \{ 2 \}$$

$$\lambda_1(3) = \{ 3, 6, 1, 2, 4, 7 \} \rightarrow \{ 1, 2, 3, 4, 6, 7 \}$$

$$\lambda_1(4) = \{ 4 \}$$

$$\lambda_1(5) = \{ 5, 6, 1, 2, 4, 7 \} \rightarrow \{ 1, 2, 4, 5, 6, 7 \}$$

$$\lambda_1(6) = \{ 6, 7, 1, 2, 4 \} = \{ 1, 2, 4, 6, 7 \}$$

$$\lambda_1(7) = \{ 7 \}$$

$$\lambda_1(8) = \{ 8 \}$$

$$\lambda_1(9) = \{ 9 \}$$

$$\lambda_1(10) = \{ 10 \}$$

$$\lambda_1(0, 1, 2, 4, 7, a) = \lambda_1 \{ 3, 8 \} \rightarrow \{ 1, 2, 3, 4, 6, 7, 8 \} \rightarrow B$$

$$\lambda_1(0, 1, 2, 4, 7, b) = \lambda_1 \{ 5 \} \rightarrow \{ 1, 2, 4, 5, 6, 7 \} \rightarrow C$$

$$\lambda_{c1} \{1, 2, 3, 4, 6, 7, 8, b\} = \lambda_3 \{5, 9\}$$

$\rightarrow D$

$$\lambda_{c1} \{1, 2, 4, 5, 6, 7, a\} = \lambda_3 \{3, 8\}$$

$\rightarrow B$

$$\lambda_{c1} \{1, 2, 4, 5, 6, 7, b\} = \lambda_{c1} \{5\}$$

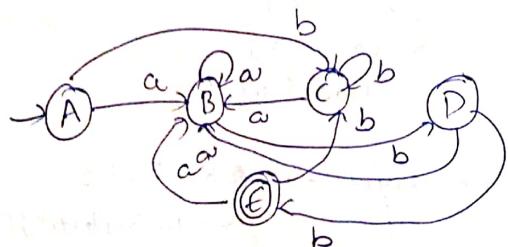
$$\lambda_{c1} \{1, 2, 4, 5, 6, 7, 9, a\} = \lambda_3 \{3, 8\}$$

$$\lambda_{c1} \{1, 2, 4, 5, 6, 7, 9, b\} = \lambda_3 \{5, 10\}$$

$\rightarrow \{1, 2, 4, 5, 6, 7, 10\}$

$$\lambda_{c1} \{1, 2, 4, 5, 6, 7, 10, a\} = B$$

$$\lambda_{c1} \{1, 2, 4, 5, 6, 7, 10, b\} = \{5\}$$



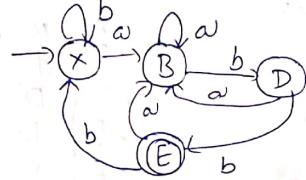
Mark & Reduce or Tabulation method :-

B			
C			
D			
E			
A	B	C	D

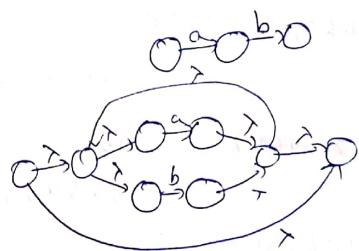
B	X			
C		X		
D	X	X	X	
E	X	X	X	X
A	B	C	D	

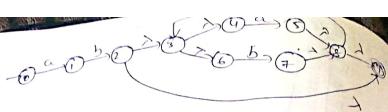
$$X = \{A, C\}$$

x	a	b
x	B	x
B	B	D
D	B	E
E	B	x



$$ab(a+b)^*$$





$$\begin{aligned}
 \lambda_C(\emptyset) &= \{1\} \rightarrow A \\
 \lambda_C(1) &= \{1\} \\
 \lambda_C(2) &= \{2, 3, 4, 6, 9\} \\
 \lambda_C(3) &= \{3, 4, 6\} \\
 \lambda_C(4) &= \{4\} \\
 \lambda_C(5) &= \{5, 8, 3, 4, 6\} = \{3, 4, 5, 6, 8, 9\} \\
 \lambda_C(6) &= \{6\} \\
 \lambda_C(7) &= \{7, 8, 3, 4, 6\} \rightarrow \{3, 4, 6, 7, 8, 9\} \\
 \lambda_C(8) &= \{8, 9, 3, 4, 6\} \rightarrow \{3, 4, 6, 7, 8, 9\} \\
 \lambda_C(9) &= \{9\}
 \end{aligned}$$

$$\begin{aligned}
 \lambda_U(0, a) &= \{1\} \rightarrow B \\
 \lambda_U(0, b) &= \emptyset
 \end{aligned}$$

$$\lambda_U(1, a) = \emptyset$$

$$\begin{aligned}
 \lambda_U(1, b) &= \lambda_U\{\emptyset\} \\
 &= \{2, 3, 4, 6, 9\} \rightarrow C
 \end{aligned}$$

$$\begin{aligned}
 \lambda_U\{\emptyset, 3, 4, 6, 9, a\} &= \lambda_U\{\emptyset\} \\
 &= \{3, 4, 5, 6, 8, 9\} \rightarrow D
 \end{aligned}$$

$$\lambda_U\{\emptyset, 3, 4, 6, 9, b\} = \lambda_U\{\emptyset\}$$

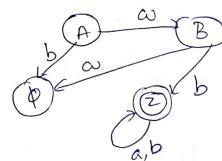
$$= \{3, 4, 5, 6, 8, 9\}$$

$$\begin{aligned}
 \lambda_U\{3, 4, 5, 6, 8, 9\} &= \lambda_U\{\emptyset\} \\
 \lambda_U\{3, 4, 5, 6, 8, 9\} &\rightarrow \lambda_U\{\emptyset\}
 \end{aligned}$$

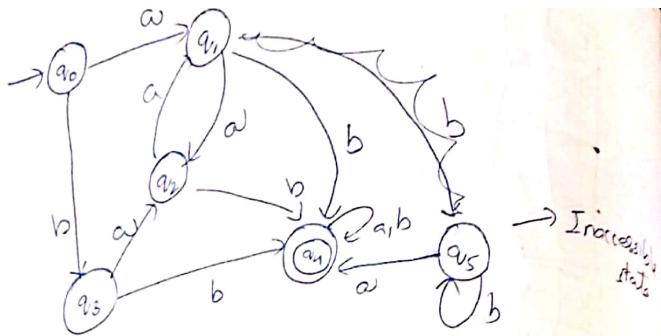
	A	a	b
B	B	b	
C	φ	φ	
D	D'	c	
E	D	E	
	D	E	
Z	Z	Z	

B	X		
C	X	X	
D	X	X	
E	X	X	
	A	B	C
			D

A	a	b
B	B	φ
C	φ	z
Z	z	z



$\alpha = \text{let } \beta = \text{in } \dots$

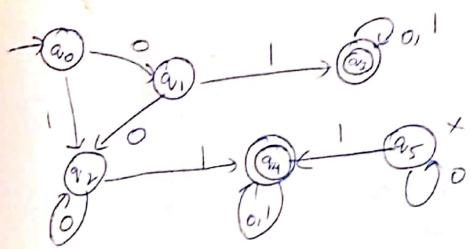
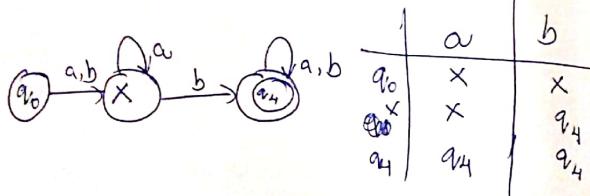


	a	b	
q_0	q_1	q_3	
q_1	q_2	q_4	
q_2	q_1	q_4	
q_3	q_2	q_4	
q_4	q_3	q_1	
	q_5	q_5	

$q_{0,1}$	X				
$q_{1,2}$	X				
$q_{2,3}$	X				
$q_{3,4}$	X	X	X	X	
$q_{4,5}$					
	q_0	q_1	q_2	q_3	q_4

$X \rightarrow q_1, q_2, q_3$

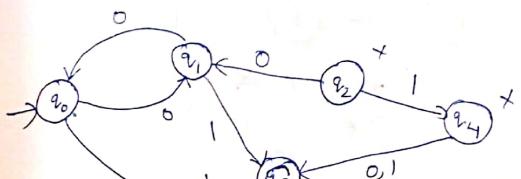
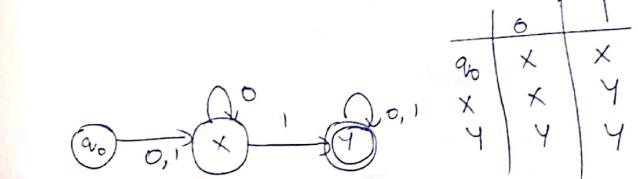
$q_4 \rightarrow$



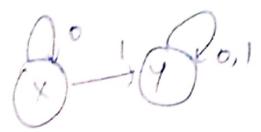
	0	1
q_0	q_1	q_2
q_1	q_2	q_3
q_2	q_2	q_4
q_3	q_3	q_4
q_4	q_4	q_4

$X \rightarrow q_1, q_2$

$Y \rightarrow q_3, q_4$

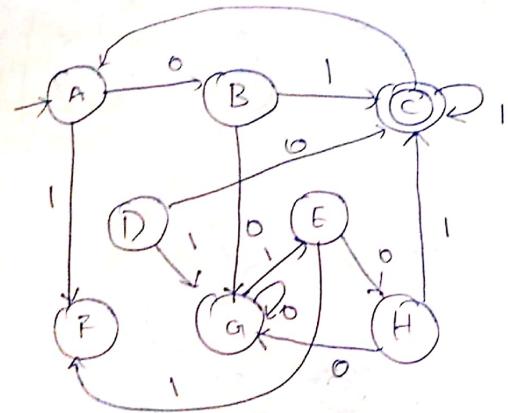


	0	1	
q_0	q_1	q_3	
q_1	q_0	q_2	
q_2	q_0	q_3	
q_3	q_2	q_5	
q_4	q_5	q_5	
	q_0	q_1	q_2



$$\begin{array}{c|cc|c} x & x & & \\ \hline y & y & y & y \end{array}$$

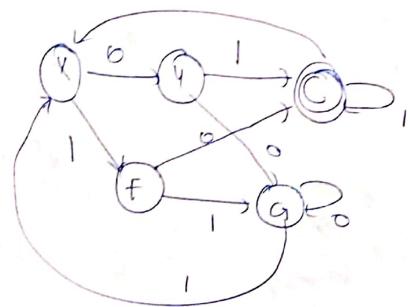
	0	1
A	B	F
B	G	C
C	A	C
D	C	G
E	H	F
F	C	G
G	G	E
H	G	C



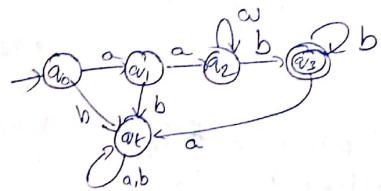
$x \rightarrow A, E$
 $y \rightarrow B, H$

B	X					
C	X	X				
E	.	X	X			
F	X	X	X	X		
G	X	X	X	X	X	
H	X	.	X	X	X	X
A		B	C	E	F	G

	0	1
x	y	F
y	G	C
c	x	C
f	c	G
g	g	x
o		



$$L = \{a^n b^m : n \geq 2, m \geq 1\}$$

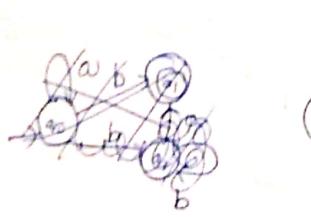


	a	b
q_0	ϕ	ϕ
q_1	ϕ	ϕ
q_2	q_3	q_3
q_3	q_3	q_3

q_1	X	
q_2	X	X
q_3	X	X
q_0	q_1	q_2

Find minimal DFA

$$L = \{a^n b^n \mid n \geq 0\} \cup \{b^m a^m \mid m \geq 1\}$$



$$\lambda_1 \{q_0\} = \{q_0, q_1\}$$

$$\lambda_1 \{q_1\} = \{q_1\}$$

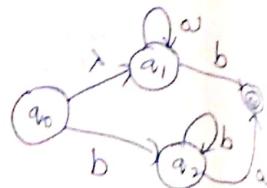
$$\lambda_1 \{q_2\} = \{q_2\}$$

$$\lambda_1 \{q_3\} = \{q_3\}$$

$$\lambda_2 (\{q_0, q_1\}, a) = \{q_1\}$$

$$\lambda_2 (\{q_0, q_1\}, b) = \{q_2, q_3\}$$

	a	b
$\{q_0, q_1\}$	q_1	q_2, q_3
q_1	q_1	q_3
q_2	q_3	q_2
q_3	ϕ	q_1

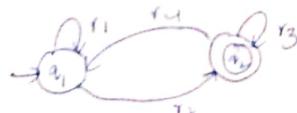


Indistinguishable -

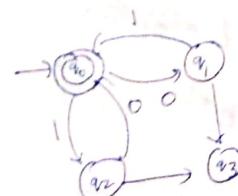
Two states p and q of a DFA are indistinguishable if $\delta^*(p, w) \in F$ and $\delta^*(q, w) \in F$ or
 $\delta^*(p, w) \notin F$ and $\delta^*(q, w) \notin F$. Note L^*

Two states p and q of a DFA are distinguishable if
 i) $\delta^*(p, w) \in F$ and $\delta^*(q, w) \notin F$ or
 ii) $\delta^*(p, w) \notin F$ and $\delta^*(q, w) \in F$ for $w \in L^*$

Finite Automata to Regular Expression



$$r_1^* r_2 [r_3 + r_4 r_2^*]^*$$



$$(r_1^* r_2)^* r_3^*$$

$$((01)^k + (10)^k)^*$$

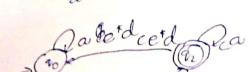


$$0^* + (0^* 1 1^*)$$

$$0^* 1 ? 1^*$$

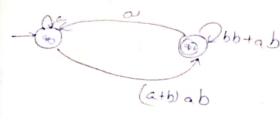
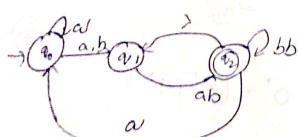


$$(1 + 01^*0)^* 01^* 1 (0^*1)^*$$

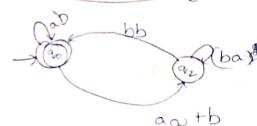
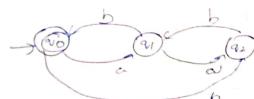


$$(ae^*d)(ae^*b + (ce^*b)^*)$$

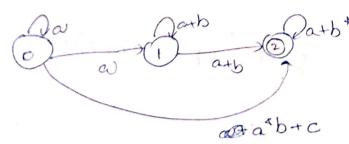
$$(ae^*d)^* ae^* b [ce^*b + ce^*d + ee^*d]$$



$$a^* (ab)ab [(bb+ab)a^*] a^* (aa^*bb)$$

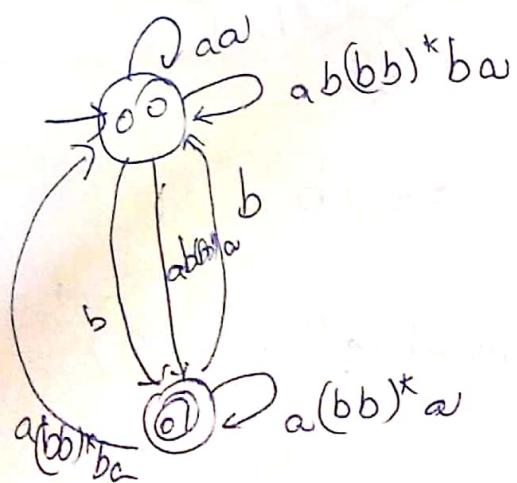
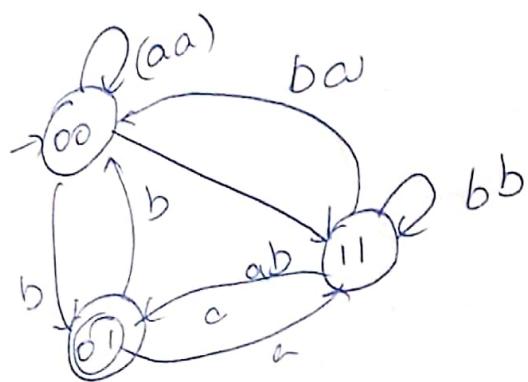
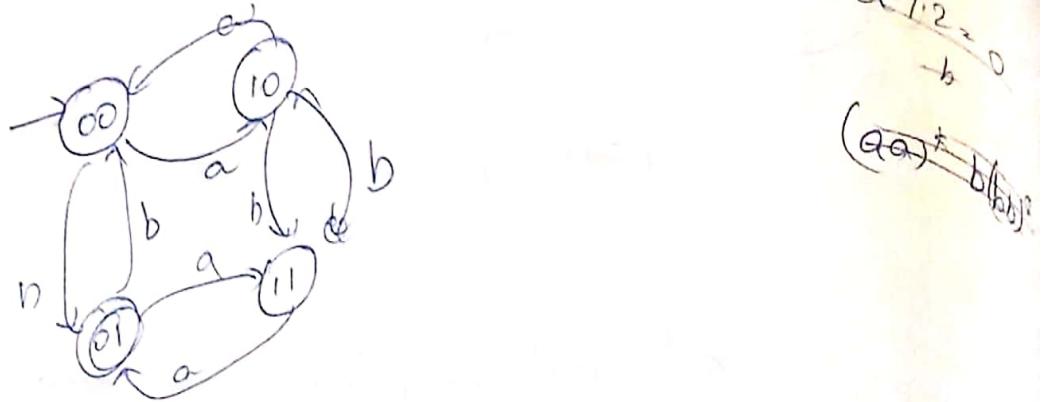


$$(ab^* (aa+b) + (ba)^* . bb)^*$$



$$a^* (a^*b+c) + (a.(a+b)^*)^* (a+b)^*$$

$$a^* ((a^*b+c) + (a.(a+b)^*(a+b)))$$



$$(aa + ab(bb)^*ba)^* (b + ab(bb)^*a) \left[\begin{array}{l} a(bb)^*a \\ b + a(bb)^*ba \\ ((aa + ab(bb)^*ba)^*)^* \end{array} \right]$$

Regular Grammar :-

Right linear Grammar

Left linear Grammar

$S \rightarrow aA \mid b$
 $A \rightarrow aA \mid bB \mid \lambda$
 context free grammar

Right & left linear grammar for
 $L = \{(ab)^n a\}$

$S \rightarrow abS \mid a \rightarrow$ Right linear

$S \rightarrow S \overset{ba}{\underset{aa}{\mid}} a \rightarrow$ Left linear
 $\underline{ab} \underline{ba} \quad \underline{aa} \underline{ba}$

② $aa^k(ab+a)^*$

$S \rightarrow ax_1 \mid a$
 $x_1 \rightarrow ax_1 \mid \lambda \mid x_2$
 $x_2 \rightarrow abx_2 \mid a^k x_2 \mid \lambda$

$S \rightarrow x_1 \mid x_2 \mid a$
 $x_1 \rightarrow x_1 ab \mid x_1 a \mid x_2 \mid \lambda$
 $x_2 \rightarrow x_2 a \mid \lambda$

$L = \{a^n b^m \mid n \geq 2, m \geq 3\}$ $aaa^k bbb^k$

$S \rightarrow aa \mid a a A$
 $A \rightarrow aA \mid B \mid \lambda$
 $B \rightarrow bB \mid bbb$

$S \rightarrow Sb \mid Abbb$
 $A \rightarrow Aa \mid aa$

$L = (aab^*ab)^*$

$S \rightarrow \lambda \mid A$
 $A \rightarrow aa \mid B$
 $B \rightarrow bB \mid \lambda \mid abS \mid \lambda$

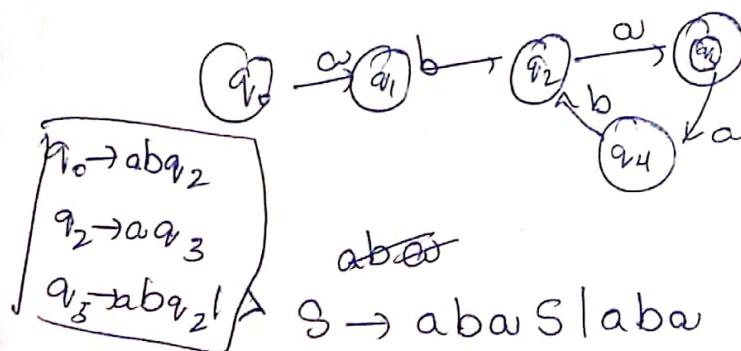
$S \rightarrow \lambda \mid Aab$
 $A \rightarrow bA \mid aa$

$S \rightarrow Sab \mid Sb \mid B$ $B \rightarrow Bb \mid A$ $A \rightarrow \alpha$

Find the language generated by given grammar

 $S \rightarrow Sab$ $S_1 \rightarrow S_1 ab \mid S_2$ $S_2 \rightarrow \alpha$ $L(\underline{a(ab)^*ab})$ $L = \{a(ab)^n : n \geq 1\}$

Find the language

 $S \rightarrow abA$ $A \rightarrow baB$ $B \rightarrow aAb \mid bb$ $abbaabb$ $abbaaababb$ $abba(a\bar{a})^*bb$ $L(abba\underline{a(ba)^*bb})$ $S \rightarrow S_1 bb$ $S_1 \rightarrow S_1 aba \mid abba$ 

$$\overbrace{aba(ba)^*}^+ + \overbrace{aba(aba)^*}^+$$

Properties of regular grammar

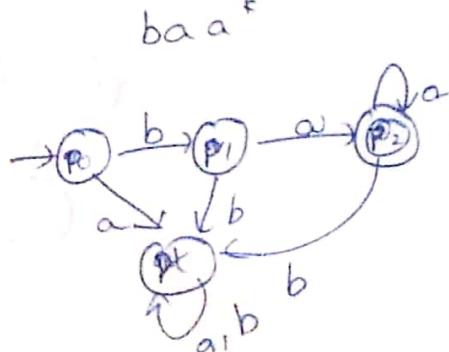
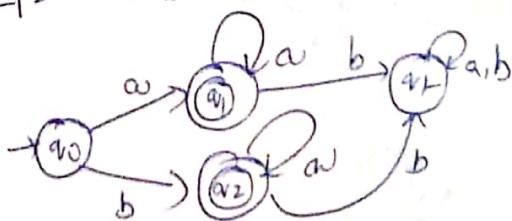
1. If $L_1 \& L_2$ are regular languages then $L_1 \cup L_2$, $L_1 \cap L_2$, $L_1 \cdot L_2$, \bar{L}_1 , L_1^* are also regular i.e. family of regular languages is closed under union, intersection, concatenation, complement, star closure.
2. Family of regular languages is closed under difference ($L_1 - L_2 = L_1 \cap \bar{L}_2$)
3. Family of regular languages is closed under reversal L_1^R
4. If $L_1 \& L_2$ are regular languages then L_1 / L_2 called as right quotient is also regular.
 $L_1 / L_2 = \{ x : xy \in L_1 \text{ for some } y \in L_2 \}$
5. Family of regular languages is closed under homomorphism.

Every regular grammar is linear but vice-versa is not true.

Show that ^{union of} two languages $L_1 \& L_2$ is regular

$$L_1 = L((a+b)a^*)$$

$$L_2 = L(ba^*)$$



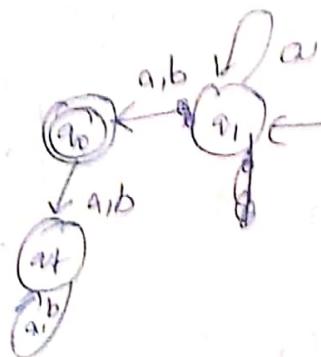
ii) $L_1 \cdot L_2$



v) L_1



vi) L_1^F



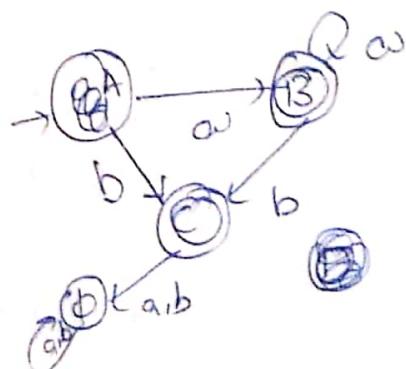
$$\lambda(q_1) = q_1$$

$$(q_1, a)$$

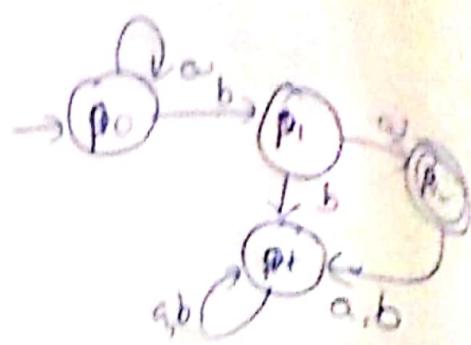
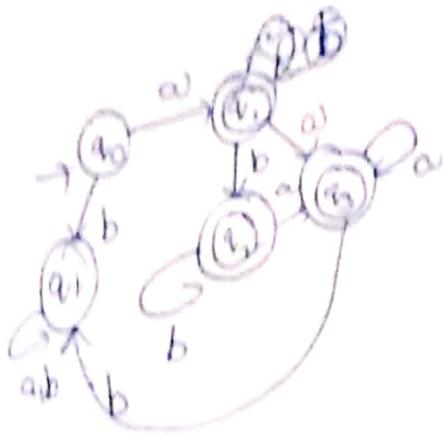
	a	b
A q_1	q_1, q_0	q_0
B q_1, q_0	q_1, q_0	q_0
C q_0	\emptyset	\emptyset

$$(q_1, q_0), a =$$

$$(q_1, q_0), b = q_0$$



$$L_1 = L(a^b b^a)$$



$$((q_0, p_0), a) = (q_1, p_0)$$

$$((q_0, p_0), b) = (q_3, p_1)$$

$$((q_1, p_0), a) = (q_2, p_0)$$

$$((q_1, p_0), b) = (q_2, p_1)$$

$$((q_2, p_0), a) = (q_3, p_1)$$

$$((q_2, p_0), b) = (q_3, p_t)$$

$$((q_3, p_0), a) = (q_3, p_0)$$

$$((q_3, p_0), b) = (q_t, p_1)$$

$$((q_2, p_1), a) = (q_3, p_2)$$

$$((q_2, p_1), b) = (q_t, p_t)$$

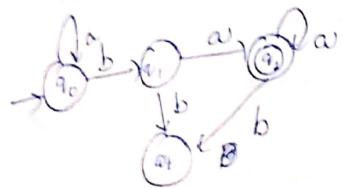
$$((q_t, p_2), a) = (q_t, p_t)$$

$$((q_t, p_2), b) = (q_t, p_t)$$

$$((q_3, p_2), a) = (q_3, p_t)$$

$$((q_3, p_2), b) = (q_t, p_t)$$

pr 4.16.2 "regular"

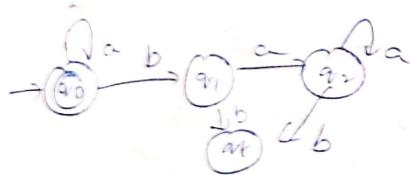


$$L(q_0) \cap L_2 = \{abaa^*\} \neq \emptyset$$

$$L(q_1) \cap L_2 = \emptyset$$

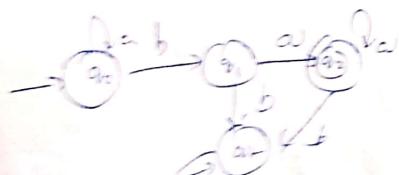
$$L(q_2) \cap L_2 = \emptyset$$

$$L(q_3) \cap L_2 = \emptyset$$



$$L_1 = L(a^*baa^*)$$

$$L_2 = L(ab^*)$$

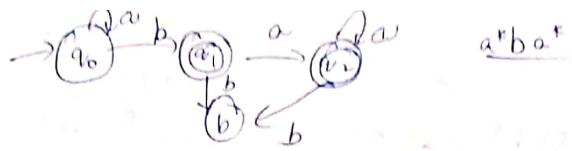


$$L(q_0) \cap L_2 = \emptyset$$

$$L(q_1) \cap L_2 = \{a\}$$

$$L(q_2) \cap L_2 = \{a^2\}$$

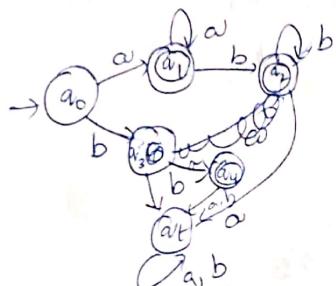
$$L(q_3) \cap L_2 = \emptyset$$



$$L_1 = \{a^n b^m : n \geq 1, m \geq 0\} \cup \{b^m\}$$

$$L_2 = \{b^m : m \geq 1\}$$

L_1 / L_2



$$L(q_0) \cap L_2 = \emptyset$$

$$L(q_1) \cap L_2 = \{b\}$$

$$L(q_2) \cap L_2 = \{b\}$$

$$L(q_3) \cap L_2 = \emptyset$$

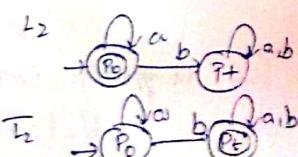
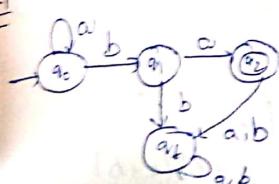
$$L(q_4) \cap L_2 = \emptyset$$

$$L = L(a^*ba)$$

$$L_2 = L(a^*)$$

$$L_1 - L_2$$

$$L_1 \cap \overline{L_2}$$



$$\begin{aligned} ((q_0, p_0), a) &\rightarrow (q_0, p_0) \\ ((q_0, p_0), b) &\rightarrow (q_1, p_1) \end{aligned}$$

$$\begin{aligned} ((q_1, p_1), a) &\rightarrow (q_2, p_1) \\ ((q_1, p_1), b) &\rightarrow (q_2, p_1) \end{aligned}$$

$$\begin{aligned} ((q_2, p_1), a) &\rightarrow (q_3, p_1) \\ ((q_2, p_1), b) &\rightarrow (q_4, p_1) \end{aligned}$$

$$\begin{aligned} ((q_0, p_0), a) &\rightarrow (q_0, p_0) \\ ((q_1, p_1), a) &\rightarrow (q_1, p_1) \\ ((q_1, p_1), b) &\rightarrow (q_2, p_1) \\ ((q_2, p_1), a) &\rightarrow (q_3, p_1) \\ ((q_2, p_1), b) &\rightarrow (q_4, p_1) \end{aligned}$$

$$h \in 12 + (23)^* \quad (12+23)^*$$

does $h(\tau)$ accept 1223

Show that family of regular languages is closed under
nⁿ operation

$$nor(L_1, L_2) = \{w : w \notin L_1 \text{ and } w \notin L_2\}$$

$$nor(L_1, L_2) = \overline{L_1 \cap L_2}$$

$$\overline{x \cdot y} = \overline{x} \cup \overline{y}$$

$$nor(L_1, L_2) = \{w \in \overline{L_1} \text{ or } w \in \overline{L_2}\}$$

$$\overline{L_1 \cup L_2}$$

$$s, \exists s_2 - \{x : x \in s_1 \text{ or } x \in s_2 \text{ but } x \notin s_1 \text{ and } s_2 \text{ both}\}$$

$$L_1 \cup L_2 - (L_1 \cap L_2)$$

$$L = \{a^n b^n : n \geq 1\}$$

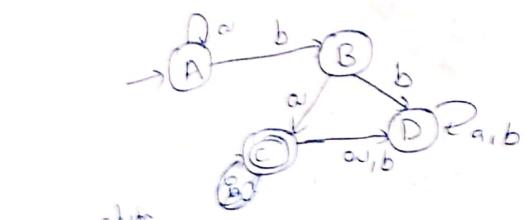
not regular language

~~aaabb~~

Pumping Lemma:-

Let L be an infinite regular language then there exists some positive integer n such that any $w \in L$ with $|w| \geq n$ can be decomposed as $w = xyz$ with $y \neq \emptyset$ and $xy \leq n$ and by taking $y \neq \emptyset$ such that $|xyz| \leq n$ and $|y| \geq 1$.

$$xyz \in L \quad \forall k \geq 0, xy^k z \in L$$



$$h: S \rightarrow T^*$$

Suppose S and T are alphabets from which $h: S \rightarrow T^*$ is called homomorphism

$$h: S = \{a, b\} \rightarrow T = \{1, 2, 3\}$$

$$\text{and } h(a) = \{1, 2\}$$

$$h(b) = \{2, 3\}$$

Final homomorphic lang

$$h(L_{(a+b)}) = L(12+23)$$

$$h(x) = a+b^*(a+b)$$

$L = \{a^n b^n : n \geq 0\}$
Assume L is regular

$$n=5$$

$w = \underline{\underline{aa}} \underline{\underline{abb}} \in L$

$$|w|=6 \geq 5$$

$$|xy|=3 \leq 5 \quad |y|=1$$

$aa(a^i)^*bbb$

if $i > 0$ $aabb \notin L$

Hence our assumption is wrong.

Hence L is not regular.

$L = \{ww^T : w \in \{a,b\}^*\}$

At L be regular

$$n=5$$

$\underline{\underline{abb}} \underline{\underline{bba}}$

$$|xy|=3 \geq 5 \quad |y|=1$$

$x^i y^i z \quad i=0$

$abbba \notin L$
Our assumption is wrong.
So L is not regular

$L = \{a^n b^l : n+l\}$

Let b be regular

$$\text{let } n=3$$

$$l=4$$

$w \in \underline{\underline{aaa}} \underline{\underline{bbb}} \underline{\underline{b}}$

$$|xy|=4 \geq 5 \quad |y|=1$$

$x(y^i)^*z$

for $i=0$
 $aaa bbb \notin L$

so L is not regular

$L = \{a^n : n \geq 3\}$

$$n \rightarrow \text{BS}$$

$w \in \underline{\underline{aaaaa}}$

$$|xy|=5 \quad |y|=1$$

$x(y^i)^*z \quad \text{for } i=0$

$aaaaa \notin L$

so L is not regular.

$L = \{a^n b^m c^{m+n} : n, m \geq 0\}$

Assume L is regular

homomorphism

$$h(a)=a \quad h(b)=a \quad h(c)=c$$

$L = \{a^n a^m a^{m+n}\}$

$$= \{a^{m+n} a^{m+n}\} \quad a^x c^x$$

$L = \{ a^n : n \geq 2 \text{ & } n \text{ is prime} \}$

Identify the following languages are regular.

① $L = \{ a^m b^l c^k : m+l+k > 5 \} \quad R$

② $L = \{ a^n b^k a^k : n > 5, k > 3, k \geq 0 \} \quad R$

③ $L = \{ a^n b^n c^n : n \geq 5, (n \geq 3, k \leq n) \} \quad NR$

④ $L = \{ uuvv^T : u, v \in \{a, b\}^+ \} \quad R$

⑤ $L = \{ uuvv^T : u, v \in \{a, b\}^+ \} \quad NR$

Context Free Grammars (CFG)

A grammar G is (V,T,P) is said to be context free grammar if all the products are of the form

$A \rightarrow d$

where $A \in V$

$d \in (VUT)^*$

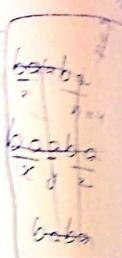
$s \rightarrow asbl \lambda$

Generate a context free grammar

$L = \{ a^n b^m c^{n+m} : n, m \geq 0 \}$

$S \rightarrow aScB$

$B \rightarrow bBc \lambda$



Generate a context free grammar for the language

$L = \{ a^n b^m c^k : n = m \text{ or } m \leq k \}$
 $n, m, k \geq 0$

~~to do~~ ~~start~~

$S \rightarrow S_1 | S_2$

$S_1 \rightarrow \cancel{aS_1b} | A B$

~~cancel~~ ~~from~~

$A \rightarrow aAb | \lambda$

$B \rightarrow cB | \lambda$

$S_2 \rightarrow aS_2 | E$

$E \rightarrow bEc | \cancel{aEa} | z$

$z \rightarrow cz | \lambda$

$L = \{ a^n b^m c^k \text{ when } k = 1(n-m) \}$
 $k = n - m \text{ or } k = n + m$

$S \rightarrow S_1 | S_2$

$S_1 \rightarrow aS_1c | A$

$A \rightarrow aAb | \lambda$

$S_2 \rightarrow aS_2b | B \times Y$

~~B~~ ~~aS2b~~

$X \rightarrow axb | \lambda$

$Y \rightarrow bYc | \lambda$

$$L = \{a^{n_0} \sim$$

$$\begin{aligned} S &\rightarrow aSc \mid B \\ B &\rightarrow bBc \mid \lambda \end{aligned}$$

$$L = \{a^n w w^R b^n : n \geq 0 \text{ and } w \in \{a, b\}^*\}$$

$$\begin{aligned} S &\rightarrow aSb \mid aSb \mid w \\ w &\rightarrow aWa \mid bWb \mid \lambda \end{aligned}$$

Show L is context free

$$L = a^n b^n : n \geq 0$$

$$L \subseteq a^n b^n a^m b^m$$

$$\begin{aligned} S &\rightarrow AaA \\ A &\rightarrow aAb \mid \lambda \end{aligned}$$

Show L is context free for $k \geq 1$

$$S \rightarrow \lambda$$

$$\begin{aligned} S &\rightarrow AS \mid A \\ A &\rightarrow aAa \mid \lambda \end{aligned}$$

$$L = \{w : w \in \{a, b\}^*, n_a(w) = n_b(w) + 1\}$$

$$\begin{aligned} S &\rightarrow aSb \mid bSa \mid Sab \mid labS \mid bas \\ S &\rightarrow lab \end{aligned}$$

$$L = \{uvwv^R : u, v, w \in \{a, b\}^*, |uv| = k+2\}$$

$$\boxed{\begin{aligned} S &\rightarrow aaVlabv \mid baV \mid bbV \\ V &\rightarrow aVa \mid bVb \mid aa \mid bb \mid ba \mid ab \end{aligned}}$$

$$S \rightarrow TT \mid V$$

$$T \rightarrow a \mid b$$

$$V \rightarrow aVa \mid bVb \mid TT$$

$$L = \{a^n b^m c^k : n+m+k = k, n, m, k \geq 0\}$$

$$\begin{aligned} S &\rightarrow aSc \mid B \\ B &\rightarrow bBcc \mid \lambda \end{aligned}$$

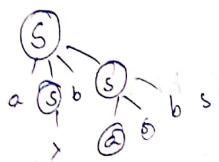
Context free grammar to generate valid

$$S \rightarrow (S) \mid [S] \mid \{S\} \mid \lambda$$

Check whether grammar has two or more parse trees
Ambiguous leftmost derivation:
 $S \rightarrow aSbS \mid bSaS \mid \lambda$

left most derivation
right most derivation

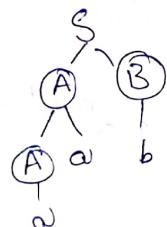
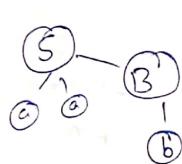
abab



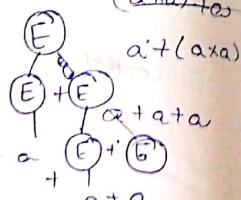
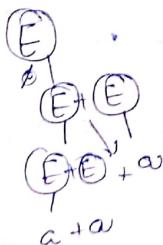
$S \rightarrow AB \mid aaB$

$A \rightarrow a \mid Aa$

$B \rightarrow b$



$E \rightarrow E+E \mid E\times E \mid \alpha \mid \omega$



$(id+id) \times id$
 $(\alpha+\alpha) \times \alpha$

$$\begin{aligned} E &\Rightarrow E+E \\ &\Rightarrow E+E+\alpha \\ &\Rightarrow \alpha+\alpha+\alpha \end{aligned}$$

$$\begin{aligned} E &\Rightarrow E+E \\ &\Rightarrow \alpha+E+E \\ &\Rightarrow \alpha+\alpha+\alpha \end{aligned}$$

Unambiguous grammar

$$\begin{aligned} E &\rightarrow E+E \mid T \mid T \\ T &\rightarrow T+F \mid F \\ F &\rightarrow id \end{aligned}$$

$$E \rightarrow E+E \mid E \rightarrow T \mid T$$

$$T \rightarrow T+F \mid T/F \mid F$$

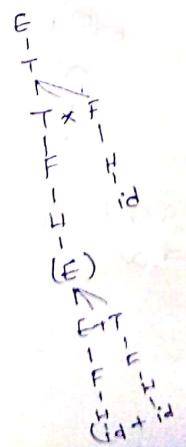
$$F \rightarrow H^*F \mid H$$

$$H \rightarrow id \mid (E) \mid -E$$

$$(id+id) \times id$$

$$\begin{array}{c} + \\ \diagdown \quad \diagup \\ T \quad F \\ \diagup \quad \diagdown \\ id \end{array}$$

$$\begin{array}{c} E \\ | \\ E+T \\ | \\ F \\ | \\ U \\ | \\ P \\ | \\ id \\ \downarrow \\ id+id \times id \end{array}$$



$(id+id) \times id$

Find the grammar and derive the string from S₀

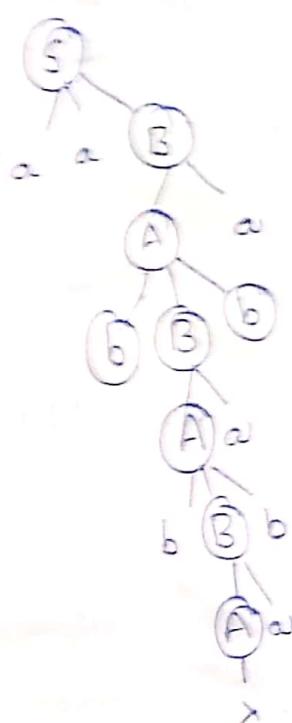
S → aSbA1λ

aab
aabbb

S → aaB
A → bBb1λ
B → Ba

Derive

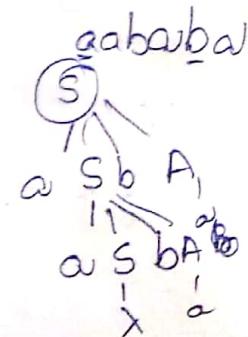
aabbababa



S ⇒ aaB

⇒ aaAa [B → a]
⇒ aabBbω [A → b]
⇒ aabAabe [B → e]
⇒ aaabbBbbe [A → e]
⇒ aabbAabbω [B → e]
⇒ aabbabbω [A → e]

S → aSbA1λ
A → ω



(Push Down Automata)
PDA Automata

Simplification of PDA's -

- Substitution rule through
- elimination of left recursion
- elimination of 2 production
- elimination of unit production
- elimination of useless production.

Simplify the grammar

$$S \rightarrow aS/A$$

$$A \rightarrow ab\alpha$$

$$S \rightarrow aS/bA/Ab$$

$$A \rightarrow a\beta A$$

Elimination

$$S \rightarrow aS/a\beta ab\alpha$$

$$A \rightarrow a\beta bA$$

no minimization
(S' → aS')

Elimination of left recursion:

$$A \rightarrow Aa/B$$

$$S \rightarrow Sa/b/b/Sba$$

$$S \rightarrow aS'/bs'$$

$$S' \rightarrow obS'/baS'/\lambda$$

$$S \rightarrow aSa/bSb/aA$$

$$A \rightarrow Aa/bAb/ba$$

$$S \rightarrow aS/b$$

$$S \rightarrow aSa/bSb/aA$$

$$A \rightarrow BaA'/aA'$$

$$A' \rightarrow aA'/ba/\lambda$$

$$S \rightarrow aB/b$$

$$S \rightarrow Aa/b$$

$$A \rightarrow Sa/b$$

$$S \rightarrow Sa/ba/b$$

$$S \rightarrow baS'/bs'$$

$$S' \rightarrow aas'/\lambda$$

$$S \rightarrow Ab/\alpha$$

$$A \rightarrow Ab/Sa/b$$

$$S \rightarrow Ab/\alpha$$

$$A \rightarrow SaA'/bA'$$

$$A' \rightarrow bA'/\lambda$$

$$\downarrow$$

$$S \rightarrow SaA'b/bA'b/\alpha$$

$$A \rightarrow SaA'/ba$$

$$A' \rightarrow bA'/\lambda$$

$$\begin{cases} S \rightarrow bA'b/\alpha \\ A' \rightarrow aA'/a \\ A' \rightarrow bA'/ba \\ A' \rightarrow \lambda/\lambda \end{cases}$$

$A \rightarrow A\beta_1\beta_2\ldots$

Sub Ø in Ø

$S \rightarrow Ab\beta_1S\alpha_1b\beta_2\alpha_2\ldots$ Both are left recursive
 $A \rightarrow Ab\beta_1\alpha_1b$

Sub Ø in Ø

$S \rightarrow Ab\beta_1\alpha_1$
 $A \rightarrow Ab\beta_1\alpha_1b$

$S \rightarrow Ab\beta_1\alpha_1$
 $A \rightarrow aaA'\beta_1/bA'$
 $A' \rightarrow bA'/baA''\beta_2$

$E \rightarrow E + T \mid T$

$T \rightarrow id$

$F \rightarrow id$

$E \rightarrow E + T \mid id$

$T \rightarrow id$

$F \rightarrow id$

$E \rightarrow id \quad E'$

$E' \rightarrow +TE' \mid \lambda$

$T \rightarrow id$

$F \rightarrow id$

Nullable variables
 E'

$S \rightarrow ABc$

$A \rightarrow BC$

$B \rightarrow b\beta$

$C \rightarrow D\lambda$

$D \rightarrow d$

$E \rightarrow d$

A,B,C are nullable

eliminate Ø

$S \rightarrow ABc \mid Bac \mid Aac \mid ABC \mid$

$\quad ac \mid Aa \mid Ba \mid a$

$A \rightarrow BC \mid Bc \mid C \beta$

$B \rightarrow b$

$C \rightarrow D \beta$

$D \rightarrow d$

$E \rightarrow d$

eliminate unit productions

$S \rightarrow ABc \mid Bac \mid Aac \mid ABC \mid$

$\quad ac \mid Aa \mid Ba \mid a$

Production Graph

$A \rightarrow B$

$C \rightarrow D$

$A \rightarrow BC \mid b \mid d$

$B \rightarrow b$

$C \rightarrow d$

$D \rightarrow d$

$E \rightarrow d$

Elimination of useless production

$S \rightarrow ABc \mid Bac \mid Aac \mid ABC \mid ac \mid Aa \mid Ba \mid a$

$B \rightarrow b$

$C \rightarrow d$

$S \rightarrow aA/a/Bb/c$
 $a \rightarrow aS$
 $S \rightarrow a/Aaa$
 $C \rightarrow e/CD$
 $D \rightarrow dd$

Q) $S \rightarrow a/Bb/aA$
 $S \rightarrow a/Aa$
 $D \rightarrow dd$
 $A \rightarrow aB$

II) $S \rightarrow a/b/aA$
 $S \rightarrow bSb$
 $S \rightarrow aA$
 $B \rightarrow a/Aa$
 $A \rightarrow aB$
 ↵
 $S \rightarrow a/Bb/aA$
 $A \rightarrow aB$
 $B \rightarrow a/Aa$

$S \rightarrow BAIB$
 $a \rightarrow oA2/12A0/1$
 $S \rightarrow AB1/1B/1A$
 When start deriving X
 you cannot eliminate it
 No left recursion

like * λ-production
 $S \rightarrow *BAAB1/AB1BABA/A1G1BB1AA/AB/BA$
 $A \rightarrow oA2/12A0/1 o2/20$
 $B \rightarrow AB1/1B1A/B/1$

like unit production
 $BAB1A1B1BABA/AB1BABA/o2/20$
 $S \rightarrow oS1B1oA2/2A00/1X$
 $B \rightarrow AB1/1B/oA2/2A00/20/1B/1$
 $A \rightarrow oA2/12A0/1o2/20$
 No useless productions

$S \rightarrow aA/aBB$
 $A \rightarrow aaA/1X$
 $B \rightarrow bB/bbc$
 $C \rightarrow B$

$E \rightarrow E + T / T$
 $T \rightarrow id$
 $F \rightarrow num$

$S \rightarrow aAb$
 $a \rightarrow ab$
 $b \rightarrow b$

No left recursion

Markable variable

A

$S \rightarrow aA|aB|a$

$A \rightarrow aaA|aa$

$B \rightarrow bbB|bbC$

$C \rightarrow B$

precedence graph

$C \rightarrow B$

$S \rightarrow aA|aB|a$

$A \rightarrow aaA|aa$

$B \rightarrow bbB|bbC$

$C \rightarrow B|bbB|bbC$

$S \rightarrow aA$

$A \rightarrow aa$

$S \rightarrow abSbTa$

↓

$S \rightarrow abSb$

$S \rightarrow aa$

$S \rightarrow A_1 B$

$A_1 \rightarrow a$

$S \rightarrow A_1 A_1$

$B \rightarrow B_1 B_2$

$B_1 \rightarrow b$

$B_2 \rightarrow SB_1$

CNF

$S \rightarrow AB | aB$

$A \rightarrow aab | \lambda \Rightarrow$

$B \rightarrow bb | BA$

~~$S \rightarrow AB | aB$~~

$S \rightarrow AB | aB | B$

$A \rightarrow aab$

$B \rightarrow bb | B | bb | bbaab$

$S \rightarrow AB | aB | bbA | bb | bbaab$

$A \rightarrow aab$

$B \rightarrow bbA | bb | bbaab$

$S \rightarrow AB | A_1 B_1 | B_1 B_2 | B_1 B_1 | B_1 C_1$

$A_1 \rightarrow a$

$B_1 \rightarrow b$

$A \rightarrow A_1 C_3$

$B \rightarrow B_1 D_1 | B_1 B_1$

$D_1 \rightarrow B_1 A$

$B_2 \rightarrow B_1 A$

$C_1 \rightarrow B_1 C_2$

$C_2 \rightarrow A_1 C_3$
 $C_3 \rightarrow \emptyset, B_1$

④

$$S \rightarrow \omega^* SB / b S A / \alpha^* b$$

$$D \rightarrow \alpha^*$$

$$B \rightarrow b$$

⑤ $s \rightarrow abla \beta a \alpha s$

$$S \rightarrow \alpha B / \alpha S / \alpha A S$$

$$B \rightarrow b$$

$$A \rightarrow \alpha$$

⑥

$$E \rightarrow E + T / T$$

$$T \rightarrow T * F / F$$

$$F \rightarrow \omega$$

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' / \lambda$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' / \lambda$$

$$F \rightarrow \omega$$

$$E \rightarrow TE / T$$

$$E' \rightarrow +TE' / +T$$

$$T \rightarrow FT' / F$$

$$T' \rightarrow *FT' / *F$$

$$F \rightarrow \omega$$

$$E \rightarrow T \rightarrow_f$$
 ~~$E \rightarrow FE' / F$~~
 ~~$E' \rightarrow +FE' / F$~~
 ~~$T \rightarrow FT / F$~~
 ~~$T' \rightarrow *FT' / *F$~~
 ~~$F \rightarrow \omega$~~

$$E \rightarrow TE' / FT' / \omega$$

$$E' \rightarrow +TE' / +T$$

$$T \rightarrow FT' / \omega$$

$$T' \rightarrow *FT' / *F$$

$$F \rightarrow \omega$$
 \Leftarrow

$$E \rightarrow TE' / FT' / \omega$$

$$E' \rightarrow X / XT$$

$$X \rightarrow +$$

$$Y \rightarrow TE'$$

$$T \rightarrow FT' / \omega$$

$$T' \rightarrow Z / ZF$$

$$Z \rightarrow *$$

$$A \rightarrow FT'$$

$$F \rightarrow \omega$$
 \Rightarrow

$$E \rightarrow \alpha T' / \alpha E'$$

$$\alpha T' / \omega$$

$$E' \rightarrow +TE' / +T$$

$$T \rightarrow \alpha T' / \omega$$

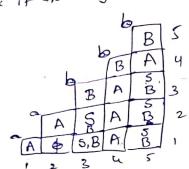
$$T' \rightarrow *FT' / *$$

$$F \rightarrow \omega$$

L = {a^n b^n}

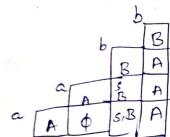
$$\begin{array}{l} S \rightarrow AB \\ A \rightarrow BB/a \\ B \rightarrow AB/b \end{array}$$

check if the string aabbabb



aabbabb

aabb



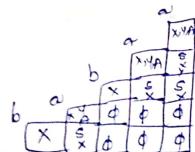
~~S → aB~~ XY

~~0X → XAa/b~~

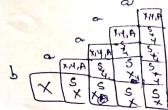
~~Y → AY/a~~

~~a → a~~

baba



baba



Push Down Automata (PDA)

A PDA is defined as

$$P = (Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

where

Q = finite set of internal states

Σ = finite set of input symbols

T = finite set of stack symbols

δ = It is a transition function

$$Q \times (\Sigma \cup \lambda) \times T \rightarrow Q \times T^*$$

$q_0 \in Q$ is an initial state

ZGT = stack initial top symbol

FQ = set of final states

$$L = \{a^n b^n : n \geq 1\}$$

$$\delta(q_0, a, z) = (q_1, az)$$

$$\delta(q_1, a, a) = (q_2, aa)$$

$$\delta(q_2, b, a) = (q_2, \lambda)$$

$$\delta(q_2, \lambda, z) = (q_2, z) \rightarrow \text{Accept}$$

Set initial
Intermediate memory (ZD)
 $\{ \text{q}_0, w_0 \}$

$\{ \text{q}_0, \text{abb}, z \} \vdash \{ \text{q}_0, \text{bb}, z \}$
 $\{ \text{q}_0, \text{bb}, z \} \vdash \{ \text{q}_0, b, az \}$
 $\{ \text{q}_0, b, az \} \vdash \{ \text{q}_0, a, az \}$

aaba

$\{ \text{q}_0, \text{aaba}, z \} \vdash \{ \text{q}_0, \text{ab}, az \}$
 $\{ \text{q}_0, \text{ab}, az \} \vdash \{ \text{q}_0, a, az \}$

No such derivation
so string is rejected

PDA acceptance

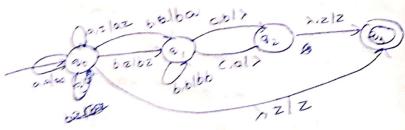
accept by final state
accept by empty state

~~(q₁, ababb, z) + (q₂, bbabb, az)~~
~~+ (q₃, babbba, az) + (q₄, cabb, bba₂)~~
~~+ (q₅, abab)~~
~~ab:ba~~
~~(q₆, abiba₂) + (q₇, biba, a₂)~~
~~+ (q₈, cba, ba₂) + (q₉, ba, baa₂)~~
~~+ (q₁₀, a₂a) + (q₁₁, A, z) + (q₁₂, z)~~

for $\frac{ab^2}{ab} \rightarrow$ in DFA
 $\text{and } ab \rightarrow \text{NFA}$
 $\text{in } A, a \rightarrow (q_1, a)$
 $(q_1, b) \rightarrow (q_2, b)$

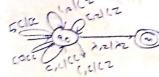
$\delta = \text{amb}^m c^{n+m}$: n mzo g

$$\begin{aligned}
 \delta(q_0, a, z) &= \delta(q_0, az) \\
 \delta(q_0, aa, z) &= \delta(q_0, aa) \\
 \delta(q_0, a, a) &= \delta(q_0, a) \\
 \delta(q_0, b, z) &= \delta(q_1, bz) \\
 \delta(q_0, b, a) &= (q_1, ba) \\
 \delta(q_0, b, b) &= (q_1, bb) \\
 \delta(q_0, b, b) &= (q_2, \lambda) \\
 \delta(q_0, c, b) &= (q_2, \lambda) \\
 \delta(q_0, c, a) &= (q_2, \lambda) \\
 \delta(q_0, c, b) &= (q_2, \lambda) \\
 \delta(q_0, c, a) &= (q_2, \lambda) \\
 \delta(q_0, c, b) &= (q_2, \lambda) \\
 \delta(q_0, \lambda, z) &= (q_3, z) \text{ Intercept} \\
 \delta(q_0, \lambda, z) &= (q_3, z) \text{ Intercept}
 \end{aligned}$$



$$\begin{aligned}
 \delta(q_0, (, z) &= (q_0, (z)) \\
 \delta(q_0, E, z) &= (q_0, [z]) \\
 \delta(q_0, [, C) &= (q_0, (C]) \\
 \delta(q_0, L, C) &= (q_0, (L)) \\
 \delta(q_0, L, () &= (q_0, (L)) \\
 \delta(q_0, E, () &= (q_0, ((L))) \\
 \delta(q_0, E, C) &= (q_0, ((C))) \\
 \delta(q_0, (, B, C) &= (q_0, (B, C)) \\
 \delta(q_0, (, B, B) &= (q_0, (B, B)) \\
 \delta(q_0, (, B,)) &= (q_0, (B))
 \end{aligned}$$

$(q_0, \lambda, z) \rightarrow (q_3, z)$ Intercept



$q_2: ab^+aba^*$

$$\begin{aligned}
 \delta(q_0, a, z) &= (q_0, z) \\
 \delta(q_1, a, z) &= (q_2, z) \\
 \delta(q_2, b, z) &= (q_2, bz) \\
 \delta(q_2, a, z) &= (q_3, z) \\
 \delta(q_3, b, z) &= (q_4, z) \text{ Intercept} \\
 \delta(q_4, z, z) &= (q_4, z) \text{ Intercept}
 \end{aligned}$$

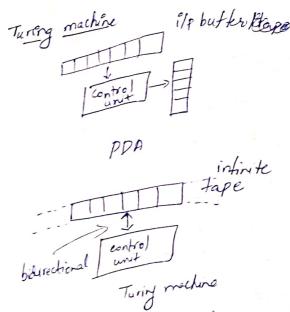
languages accepted by PPA

$$L(P) = \{w : w \in \Sigma^* \text{ and } \delta(q_0, w, z) \stackrel{*}{\in} \{q_3\}\}$$

$$n_a(w) = n_b(w)$$

$$\begin{aligned}
 \delta(q_0, a, z) &= (q_1, az) & \delta(q_2, a, z) &= (q_0, az) \\
 \delta(q_0, b, z) &= (q_1, bz) & \delta(q_2, b, z) &= (q_0, bz) \\
 \delta(q_1, a, a) &= (q_1, aa) & \delta(q_2, AA) &= (q_0, AA) \\
 \delta(q_1, b, B) &= (q_1, BB) & \delta(q_2, BB) &= (q_0, BB) \\
 \delta(q_1, a, b) &= (q_1, bb) & \delta(q_2, a, b) &= (q_0, ab) \\
 \delta(q_1, b, A) &= (q_1, BA) & \delta(q_2, BA) &= (q_0, BA)
 \end{aligned}$$

accept / $\delta(q_0, z, z) = (q_3, z)$

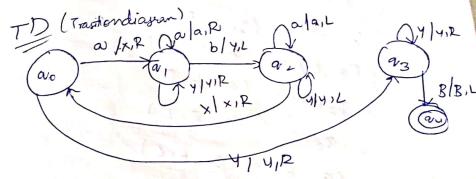


A turing machine is represented as
 $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where
 - Q = finite set of internal states
 - Σ = set of input symbols
 - Γ = finite set of tape symbols
 - $\delta: Q \times (\Sigma \cup \{\#\}) \rightarrow Q \times \Gamma \times \{L, R\}$
 - $q_0 \in Q$ initial state
 - $\#\in \Gamma$ represents symbol on tape
 $F \subseteq Q$ which denote set of final states.

It is also called as nondeterministic turing machine

$L \vdash \text{Sarban} : n \geq 3$

$$\begin{aligned}
 (q_0, a) &= (q_1, x, R) \\
 (q_1, a) &= (q_1, a, z) \\
 (q_1, b) &= (q_2, y, B) \\
 (q_2, a) &= (q_2, a, \bar{B}) \\
 (q_2, x) &= (q_3, x, R) \\
 (q_3, y) &= (q_1, y, R) \\
 (q_2, y) &= (q_2, y, L) \\
 (q_0, y) &= (q_2, y, R) \\
 (q_3, y) &= (q_3, y, R) \\
 (q_3, B) &= (q_4, B, L) \quad \text{Accept}
 \end{aligned}$$



Transition table -

	a	b	x	y	B
q0	a1, x/R				q3, y/R
q1	q1, a/B	q2, y/L			q1, y/R
q2	q2, a/L		q0, x/R	q3, y/L	
q3	(q3, a, B)			q3, y/R	q4, B, L
q4	(q4, y, B)				

$x_{aabb} + x_{aabb} + x_{aabb}$
 $+ x_{aabb} \rightarrow$
 Accept

$db^n \in L$

$$\delta(a_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, b) = (q_2, y, R)$$

$$\delta(q_2, b) = (q_2, b, R)$$

$$\delta(q_2, c) = (q_3, z, L)$$

$$\delta(q_3, b) = (q_3, b, L)$$

$$\delta(q_3, y) = (q_3, y, L)$$

$$\delta(q_3, a) = (q_3, a, L)$$

$$\delta(q_3, x) = (q_0, x, R)$$

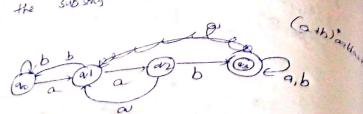
$$\delta(q_0, y) = (q_1, y, R)$$

$$\delta(q_1, z) = (q_2, z, R)$$

$$\delta(q_2, z) = (q_3, z, L)$$

$$\delta(q_3, y) = (q_4, y, R)$$

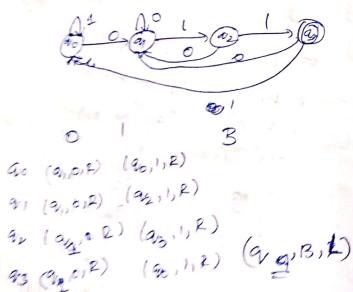
Obtain string minimum
containing the substring 'cab'



	a	b	B
q_0	(q_1, a, R)	(q_0, b, R)	(q_0, c, Q, R)
q_1	(q_2, a, R)	(q_0, b, R)	
q_2	(q_3, a, R)	(q_2, b, R)	
q_3	(q_4, a, R)	(q_3, b, R)	(q_4, B, L)
q_4			
q_5			

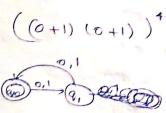
Constructs string machine all the strings on $0 > 1$
ending with '011'

$$(0+1)^* 011$$



	a	b	B
q_0	(q_1, a, R)		(q_0, l, R)
q_1			(q_2, l, R)
q_2			(q_3, l, R)
q_3			(q_4, B, L)
q_4			
q_5			

$$L = \{w\omega^k : w \in ab^* \}$$



	0	1	B
q_0	$(q_1, 0, R)$		
q_1			(q_2, B, L)

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a,b} q_2$$

	a	b	B
q_0	(q_1, a, R)		(q_1, B, R)
q_1	(q_1, a, R)	(q_1, b, R)	(q_2, B, L)

$$L = \{w\omega^k : w \in ab^*\}$$

abbbba

$$\begin{aligned}\delta(q_0, a) &= (q_1, B, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, b) &= (q_1, b, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, a) &= (q_3, B, L) \\ \delta(q_3, a) &= (q_3, a, L) \\ \delta(q_3, b) &= (q_3, b, L)\end{aligned}$$

$$\begin{aligned}
 \delta(q_3, B) &= (q_0, B, R) \\
 \delta(q_0, b) &= (q_4, B, R) \\
 \delta(q_4, a) &= (q_4, a, R) \\
 \delta(q_4, b) &= (q_4, b, R) \\
 \delta(q_4, B) &= (q_5, B, L) \\
 \delta(q_5, b) &= (q_6, B, L) \\
 \delta(q_6, a) &= (q_6, a, L) \\
 \delta(q_6, b) &= (q_6, L) \\
 \delta(q_6, B) &= (q_0, B, R) \\
 \delta(q_0, B) &= (q_7, B, L) \quad // \text{Accept}
 \end{aligned}$$

abba

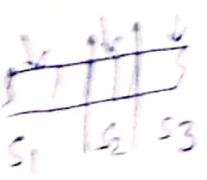
$(q_0, abba)$

$$\begin{aligned}
 q_0 abba &\vdash B q_1 bba \vdash B b q_1 ba \vdash B b b q_1 a \\
 &\vdash B b b q_1 \vdash B b b q_2 a \vdash B b b q_3 B B \vdash B q_3 b b B \\
 &\vdash q_3 B b b B \vdash B q_0 b b B \vdash B B q_4 b B \\
 &\vdash B B b q_4 B \vdash B B q_5 b B \vdash B B B q_6 B B B \\
 &\vdash q_6 B B B B \vdash B q_0 B B B B \vdash q_7 B B B B
 \end{aligned}$$

$$n_a(\omega) = n_b(\omega)$$

Type of TM

- ① Standard Turing Machine ($\delta: Q \times \{0,1\}^2 \rightarrow Q \times \{0,1\}$)
- ② Non-deterministic TM ($\delta: Q \times \{0,1\}^2 \rightarrow Q^{*}$)
- ③ Turing Machine with stay option
($\delta: Q \times \{0,1\} \rightarrow Q \times \{0,1\}$)
- ④ multi-track Turing machine
($\delta: Q \times \{0,1\}^n \rightarrow Q \times \Gamma^n \times \{LR\}^n$)
- ⑤ Multi-Dimensional TM
($\delta: Q \times \{0,1\}^n \rightarrow Q \times \Gamma \times \{L,R\}^n$)
- ⑥ Multi-Tape TM

($\delta: Q \times \{0,1\}^{3^n} \rightarrow Q \times \Gamma^n \times \{L,R\}^n$)
- ⑦ Multi-stack TM

($\delta: Q \times \{0,1\}^7 \rightarrow Q \times \Gamma^7 \times \{L,R\}^7$)

A multi-stack TM is similar to multi-tape but its tape works like a stack and has a limit in accepting.

Recursively enumerable \subseteq Recursively

Language (decidable) -

There exist a TM that accepts every string in the language and rejects every string not in the language.

Language (undecidable) -

There exist a TM which accepts inputs in many more than a finite step than the language is recursively enumerable.

Many examples of undecidable languages are given in the notes.

Type	Grammar	Syntax	Language	Machine	States unit	Parsing complexity
Type 0	Unrestricted	$\alpha \rightarrow \beta$ $\alpha \in (VUT)^*$ $\beta \in (VUT)^*$	Recursive Enumerable	Turing Machine	Infinite tape	undecidable
Type 1	Context-sensitive	$\alpha \rightarrow \beta$ $\alpha, \beta \in (VUT)^*$ $ \beta \geq \alpha $ α contains a variable	Context sensitive	Linear Bound Automation	Restricted tape	NP complete
Type 2	Context free	$\alpha \rightarrow \beta$ $\alpha \in V$ $\beta \in (VUT)^*$	Context free	Push down Automata	Stack	$O(n^3)$
Type 3	Regular	$\alpha \rightarrow a\beta\alpha$ $\alpha \rightarrow \beta a$ $a \in V$ $\alpha \in T^*$	Regular	Finite Automata	None	$O(n)$