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VIT-AP AMARAVATI
Assignment, Winter 2020
MAT2005 - Linear Algebra
Due:06.06.2020

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Date: June 3, 2020
Points: 20

Answer all the questions

1. Convert the following differential equation into a system, solve the system and use this solution to get the solution to the original differential equation.

$$2y'' + 5y' - 3y = 0, \qquad y\,(0) = -4 \;\; y'\,(0) = 9$$

2.

   (a) For which s, the matrix S is positive definite?

$$S = \begin{bmatrix} s & 3 & 0 \\ 3 & s & 4 \\ 2 & 4 & 6 \\ 0 & 4 & s \end{bmatrix}$$

   (b) Express the matrix S in the form $S = Q\Lambda Q^T$

   (c) Which 3 by 3 symmetric matrix S produce the quadratic $X^TSx = 2(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3)$?

3. Let $A = \begin{bmatrix} 3 & -2 & 1 & 0 \\ 1 & 6 & 2 & 1 \\ -3 & 0 & 7 & 1 \end{bmatrix}$ be the matrix for $T : R^4 \to R^3$ relative to the bases $B = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ and $B = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$, where

   where

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 4 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 6 \\ 9 \\ 4 \\ 2 \end{bmatrix}, \mathbf{w}_1 = \begin{bmatrix} 0 \\ 8 \\ 8 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} -7 \\ 8 \\ 1 \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} -6 \\ 9 \\ 1 \end{bmatrix}$$

   (a) Find $[T(\mathbf{v}_1)]_{B'}, [T(\mathbf{v}_2)]_{B'}, [T(\mathbf{v}_3)]_{B'},$ and $[T(\mathbf{v}_4)]_{B'}$.

   (b) Find $T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3),$ and $T(\mathbf{v}_4)$.

   (c) Find a formula for $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right)$ and use this formula to compute $T\left(\begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}\right)$.

   (d) Find the standard matrix of T.

   (e) Verify the Rank-Nullity theorem.

4. Find orthonormal vectors $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$ such that $\mathbf{q}_1, \mathbf{q}_2$ span the column space of

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}$$

(a) Which of the four fundamental subspaces contains $\mathbf{q}_3$?

(b) Solve $A\mathbf{x} = (1, 2, 7)$ by least squares.

(c) Find the pseudo inverse of A.

5. Given $A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

(a) Find the eigenvalues and eigenvectors of A.

(b) Find the matrix expression for e^A. What are its eigenvalues and eigenvectors?

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