VIT-AP AMARAVATI
Assignment, Winter 2020
MAT2005 - Linear Algebra
Due:06.06.2020

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Points: 20

Answer all the questions

1. Find a Jordan canonical form of the following matrix.

$$A = \begin{bmatrix} 5 & 9 & -2 \\ -1 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

2. (a) Find the standard matrix for the stated composition in R^3 given by a rotation of $30°$ about the x-axis, followed by a rotation of $60°$ about the z-axis, followed by a contraction with factor $k = 1/2$. followed by an expansion in the y-direction with factor $k = 4$.

   (b) One obtained the vector $v = (1, 2, 3)$ is the image under the above composite transformation, what is its preimage?

   (c) Let T be the linear operator mapping R^3 into R^3 defined by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}.$$

   If $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$, then what is the transformation matrix B relative to the basis $\{v_1, v_2, v_3\}$ of R^3. Find $Ker(T)$ and $Rang\ T$. Is T an isomorphism?

3. Find the singular value decomposition of the matrix A given below.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 0 & 2 \end{bmatrix}$$

   And hence find a Moore-Penrose Inverse (pseudo-inverse) A^+ and use it to solve $Ax = b$ where $b^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$.

4. Decompose the matrix A into $A = QR$ where Q is an orthonormal basis for $C(A)$, given that

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}.$$

5. Find the canonical form of the quadratic form $3x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 + 4x_2x_3$ and discuss its definiteness.

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