VIT-AP AMARAVATI
Assignment, Winter 2020
MAT2005 - Linear Algebra

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Due:06.06.2020 Points: 20

Answer all the questions

1. Convert the following differential equation into a system, solve the system and use this solution to get the solution to the original differential equation.

$$2y'' + 5y' - 3y = 0, \qquad y(0) = -4 \;\; y'(0) = 9$$

2. A car rental agency has three rental locations, denoted by 1, 2, and 3. A customer may rent a car from any of the three locations and return the car to any of the three locations after, say, t days. The manager finds that customers return the cars to the various locations according to the probabilities given in the transition matrix:

$$P = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.6 \\ 0.1 & 0.5 & 0.2 \end{bmatrix}$$

   (a) If the car was at rental location 1 initially, what is the probability that it will be at rental location 3 after 3 days? What is the probability that it will be at rental location 2 after 3 days?

   (b) What is the state vector for the system after 4 years?

   (c) Discuss the behaviour of $\mathbf{x}(n) = P^n\mathbf{x}(0)$ as $n \to \infty$.

3. Find the canonical form of the quadratic form $3x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 + 4x_2x_3$ and discuss its definiteness.

4.

   (a) Find an orthogonal basis for the column space of the matrix

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

   (b) Find QR-decomposition of A such that $A = QR$.

   (c) For $u = (0, 2, 1, 0)$, find $u = w_1 + w_2$ such that $w_1 \in W = C(A)$ and $w_2 \in W^{\perp}$.

   (d) Find the Jordan form of A.

5. Given $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & 0 \end{bmatrix}$

   (a) Let $T : R^3 \to R^2$ be the linear transformation whose representation is A with respect to the ordered bases $B = \{(1, 0, -1), (0, 2, 0), (1, 2, 3)\}$ and $B' = \{(1 - 1), (2, 0)\}$. Find the representation of T with respect to the natural bases for R^3 and R^2. Also, find bases for kernel and Range of T.

   (b) Find the Psuedoinverse of the A through SVD.

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