Project 3: “Hopfield Network”

Sai Thatigotla

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Introduction

In this project we built a Hopfield Network and analyzed the associative memory capacity. We quantified the capacity of the network by looking at the fraction of unstable imprints as a function of the number of imprints and the number of stable imprints as a function of the number of imprints.

Theory

The main idea of this project was to analyze the associative memory capacity of a hopfield network. By imprinting the network with certain input patterns, these patterns can be memorized in the attractors of the network. If a pattern is presented and the network converges/becomes stable then the network recognizes the pattern or has found the associated memory.

Methods

The code used to run the experiments is in hopnet.py and it takes 3 arguments - the number of runs, the number of patterns, and the number of neurons. graph.ipynb was used to create the plots.

The default parameters are 10 runs, P = 50 patterns, and N= 100 neurons. P = 50 & N = 200, P = 100 & N = 200, and P = 200 & N = 400 were also run.

Results

5, 50, 100

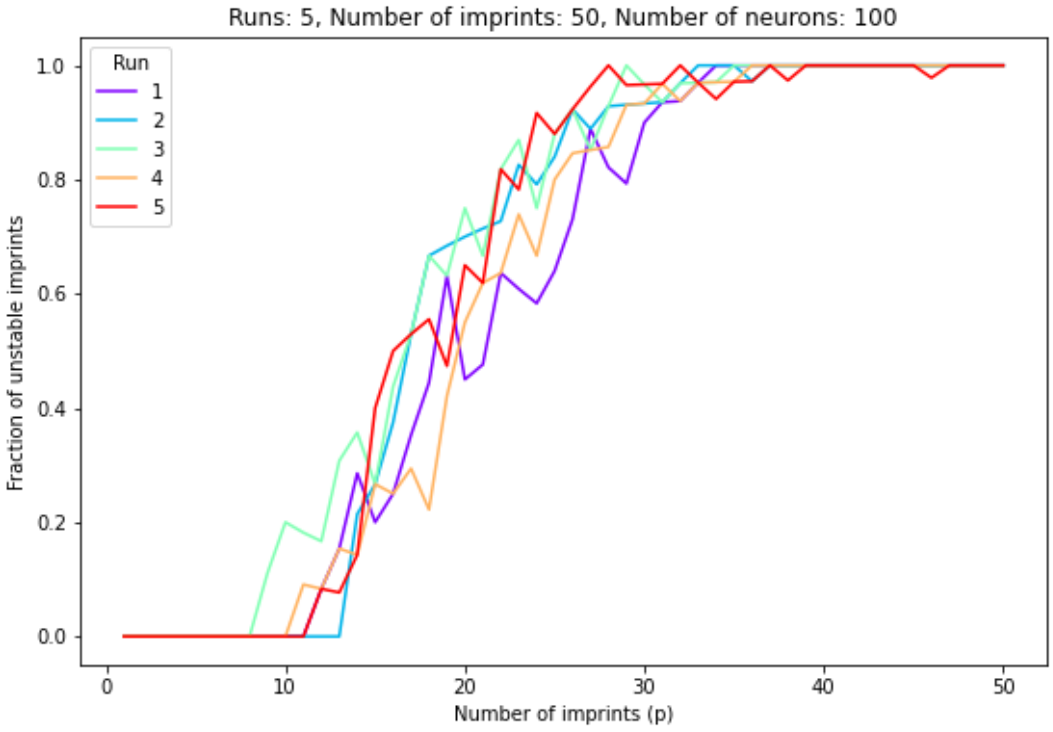
Runs: 5, Number of imprints: 50, Number of neurons: 100
Run
1
2
3
4
5
Fraction of unstable imprints
Number of imprints (p)
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50

Figure 1.

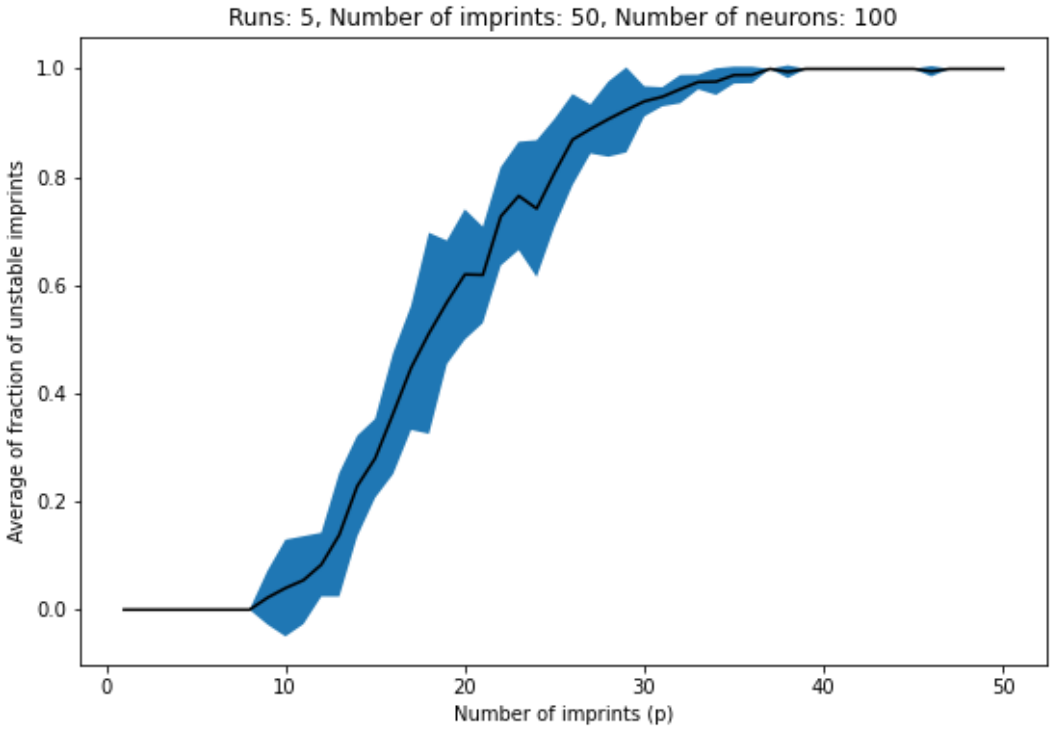
Runs: 5, Number of imprints: 50, Number of neurons: 100
Average of fraction of unstable imprints
Number of imprints (p)
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50

Figure 2.

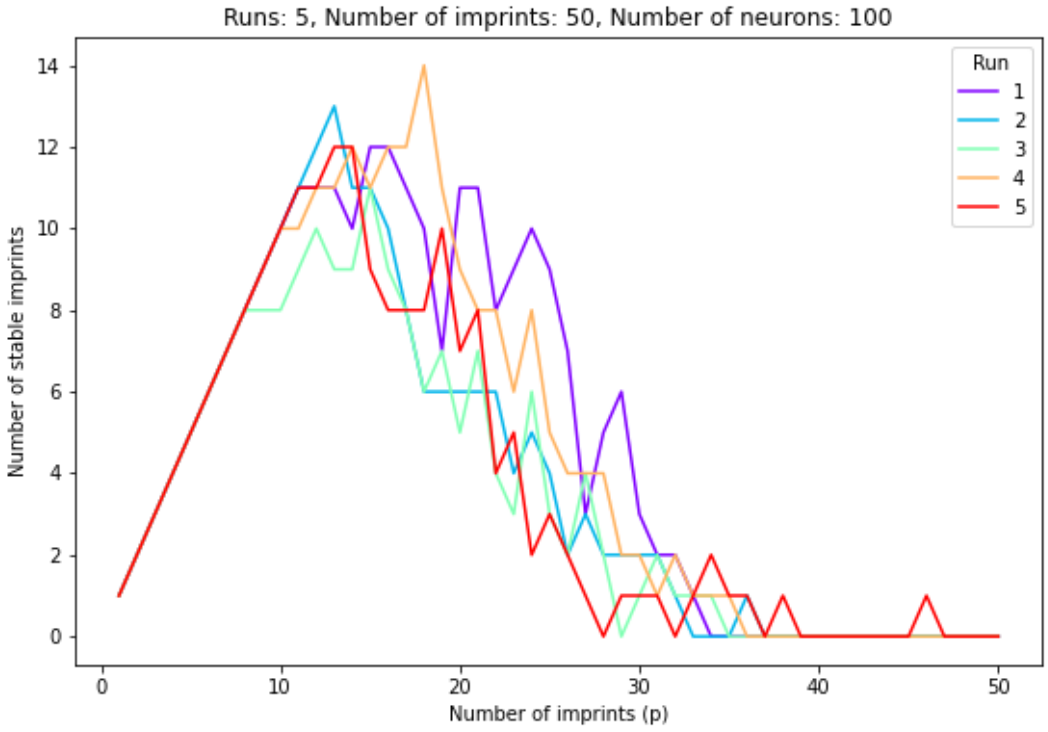
Runs: 5, Number of imprints: 50, Number of neurons: 100
Number of stable imprints
Number of imprints (p)
Run
1
2
3
4
5
0
2
4
6
8
10
12
14
0
10
20
30
40
50

Figure 3.

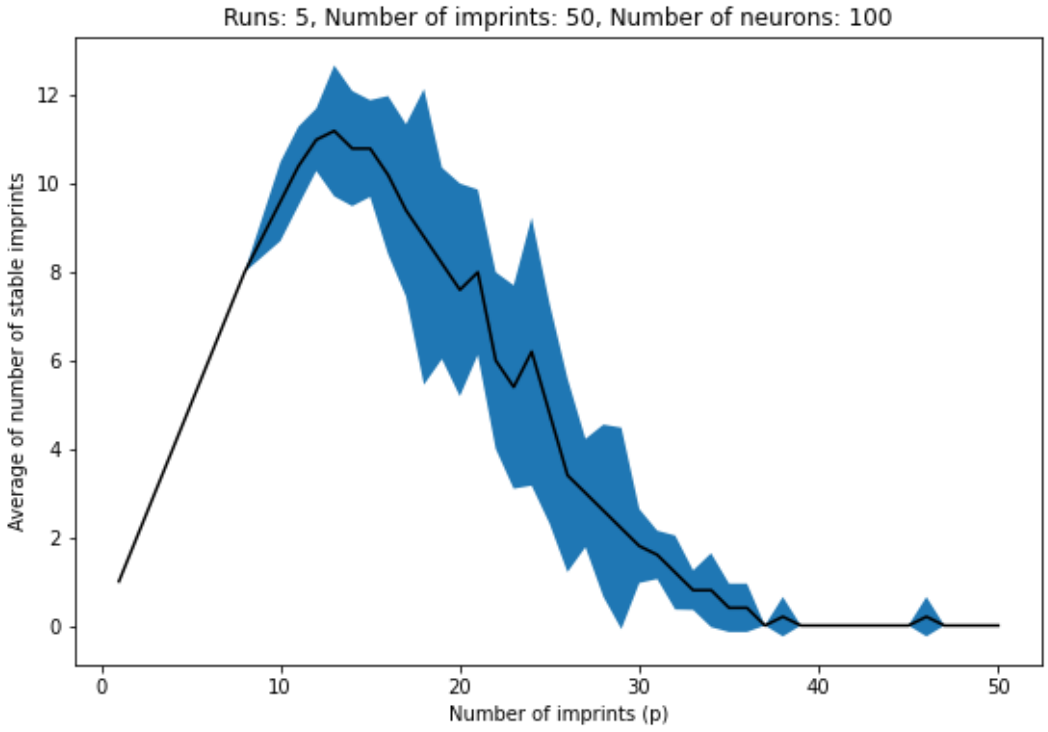

Figure 4.

5, 50, 200

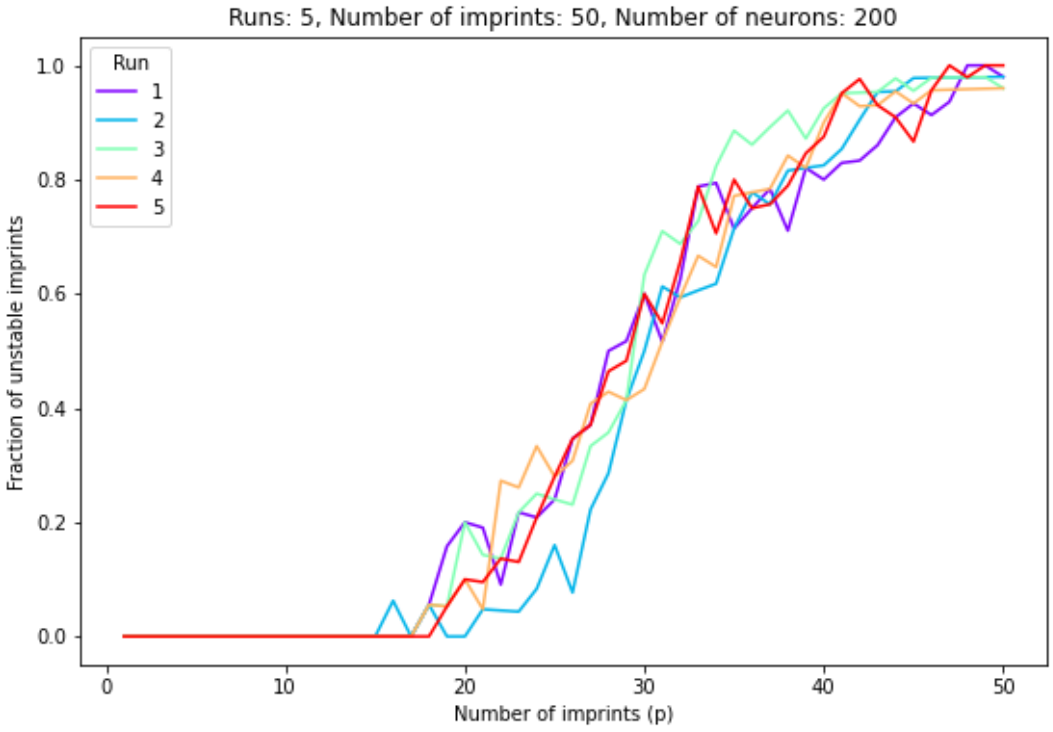
Runs: 5, Number of imprints: 50, Number of neurons: 200
Run
1
2
3
4
5
Fraction of unstable imprints
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
Number of imprints (p)

Figure 5.

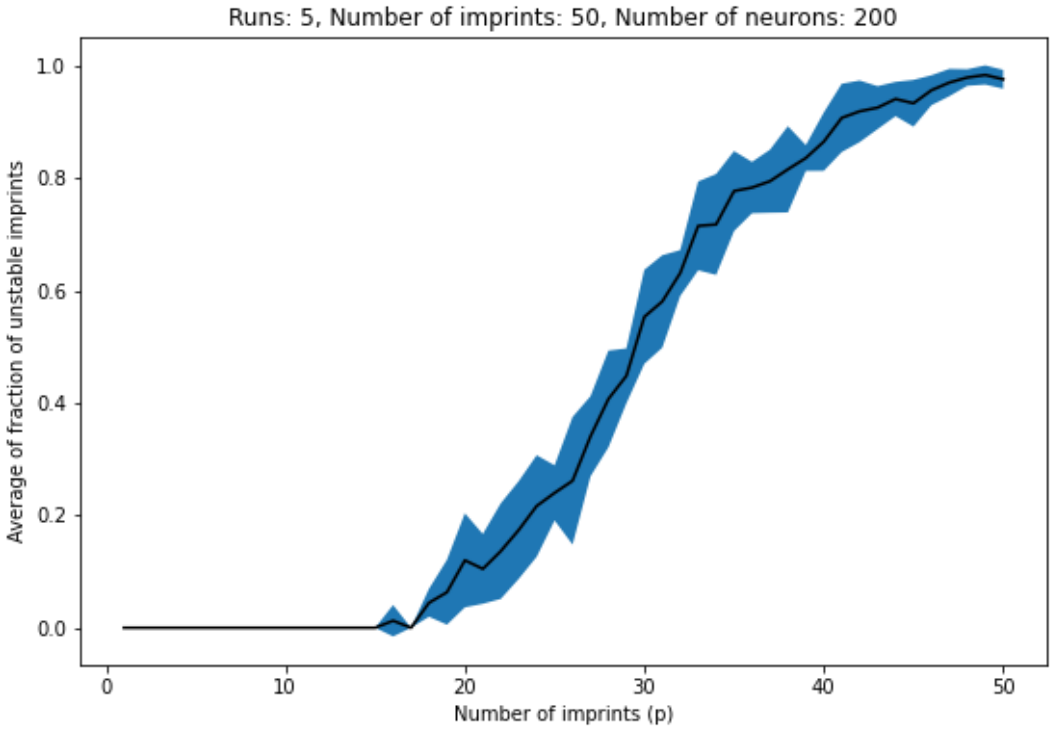
Runs: 5, Number of imprints: 50, Number of neurons: 200
Average of fraction of unstable imprints
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
Number of imprints (p)

Figure 6.

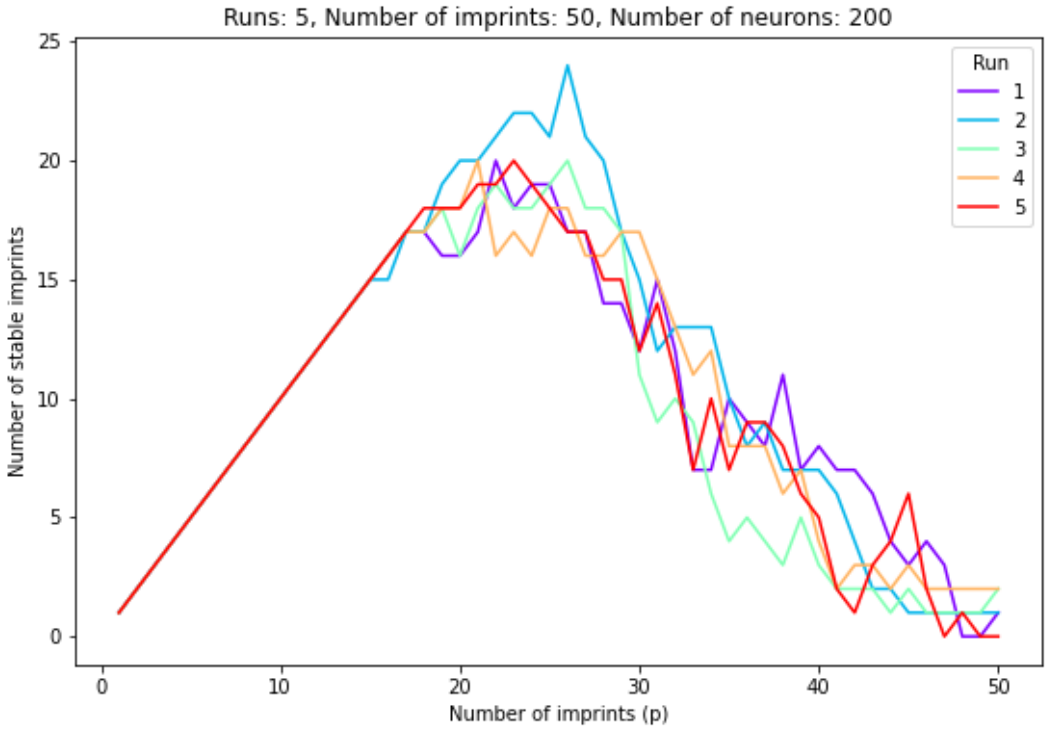
Runs: 5, Number of imprints: 50, Number of neurons: 200
Number of stable imprints
Number of imprints (p)
Run
1
2
3
4
5
0
5
10
15
20
25
0
10
20
30
40
50

Figure 7.

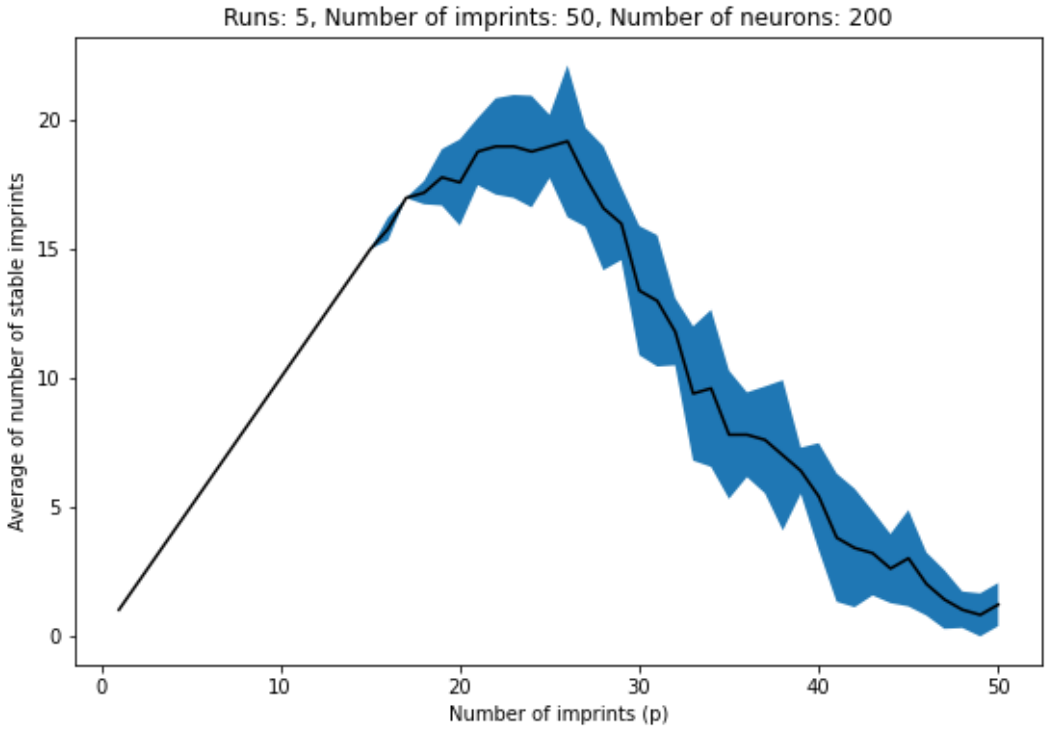

Figure 8.

5, 100, 200

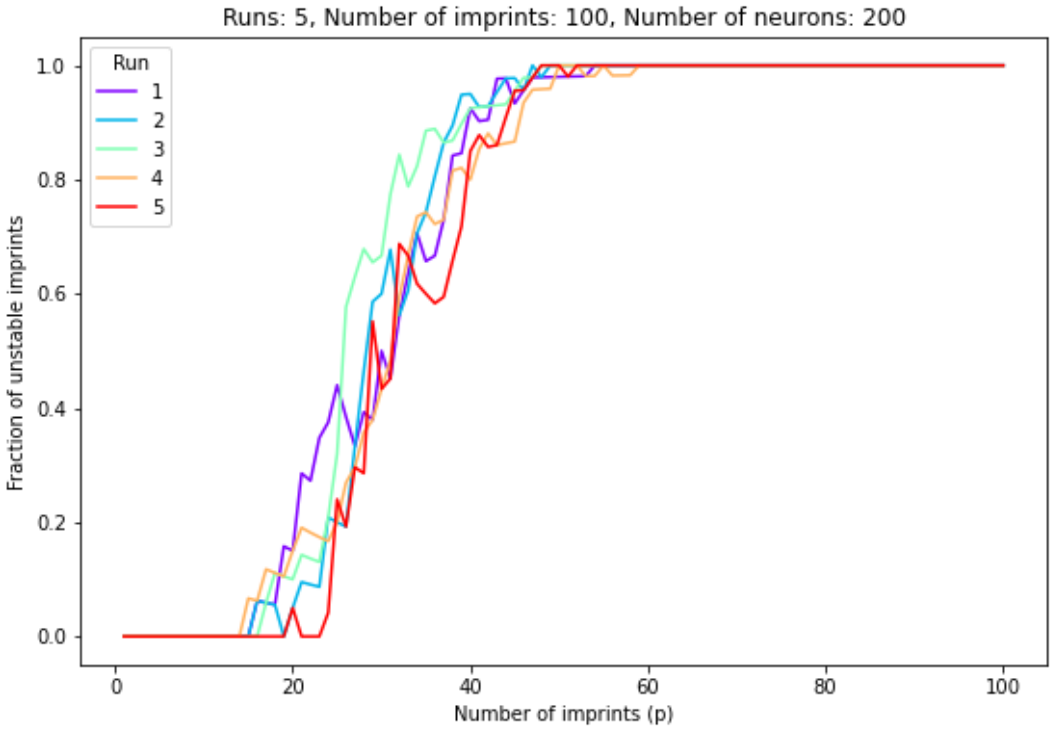
Runs: 5, Number of imprints: 100, Number of neurons: 200
Run
1
2
3
4
5
Fraction of unstable imprints
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
Number of imprints (p)

Figure 9.

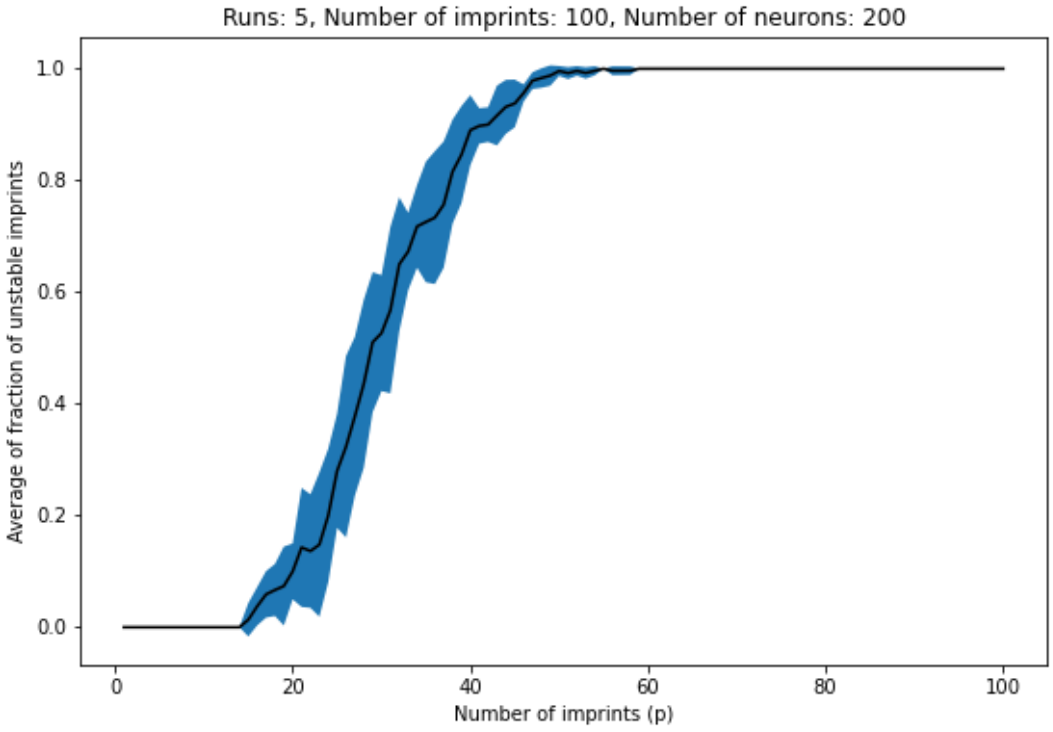
Runs: 5, Number of imprints: 100, Number of neurons: 200
Average of fraction of unstable imprints
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
Number of imprints (p)

Figure 10.

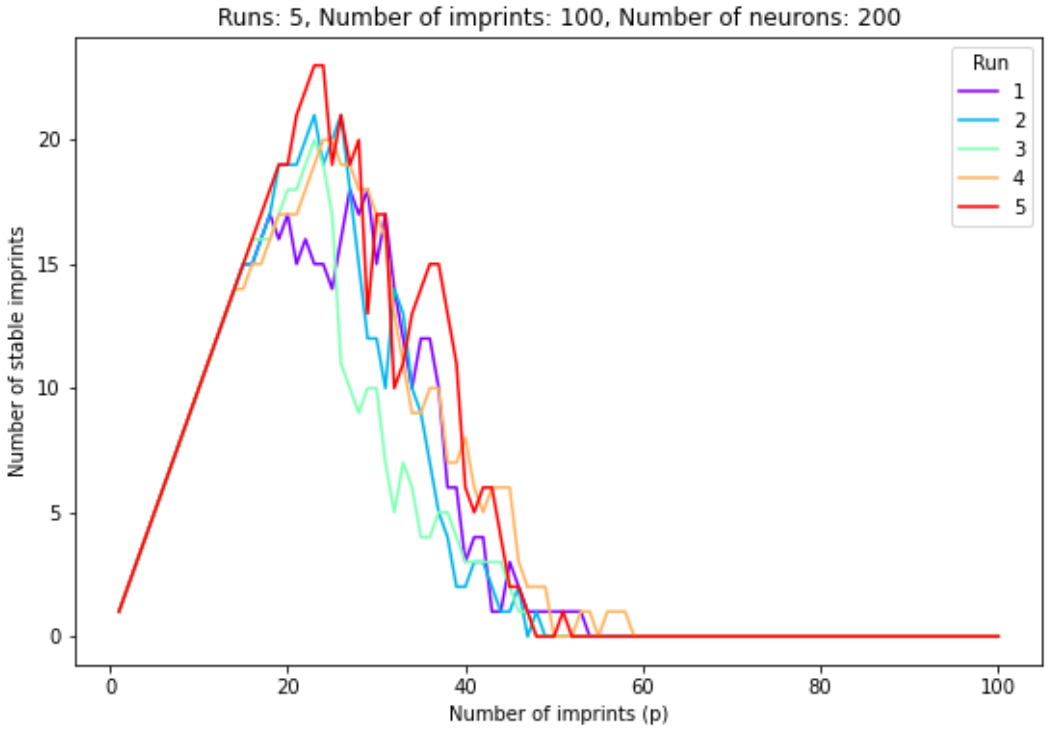
Runs: 5, Number of imprints: 100, Number of neurons: 200
Run
1
2
3
4
5
Number of stable imprints
0
5
10
15
20
0
20
40
60
80
100
Number of imprints (p)

Figure 11.

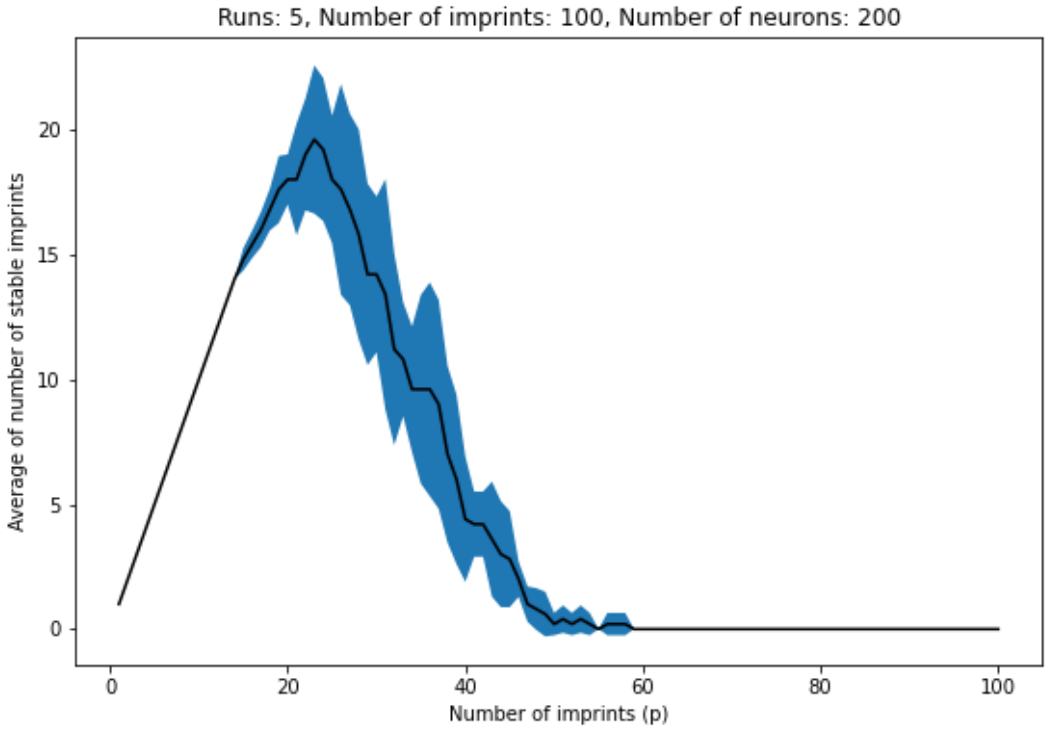

Figure 12.

5, 200, 400

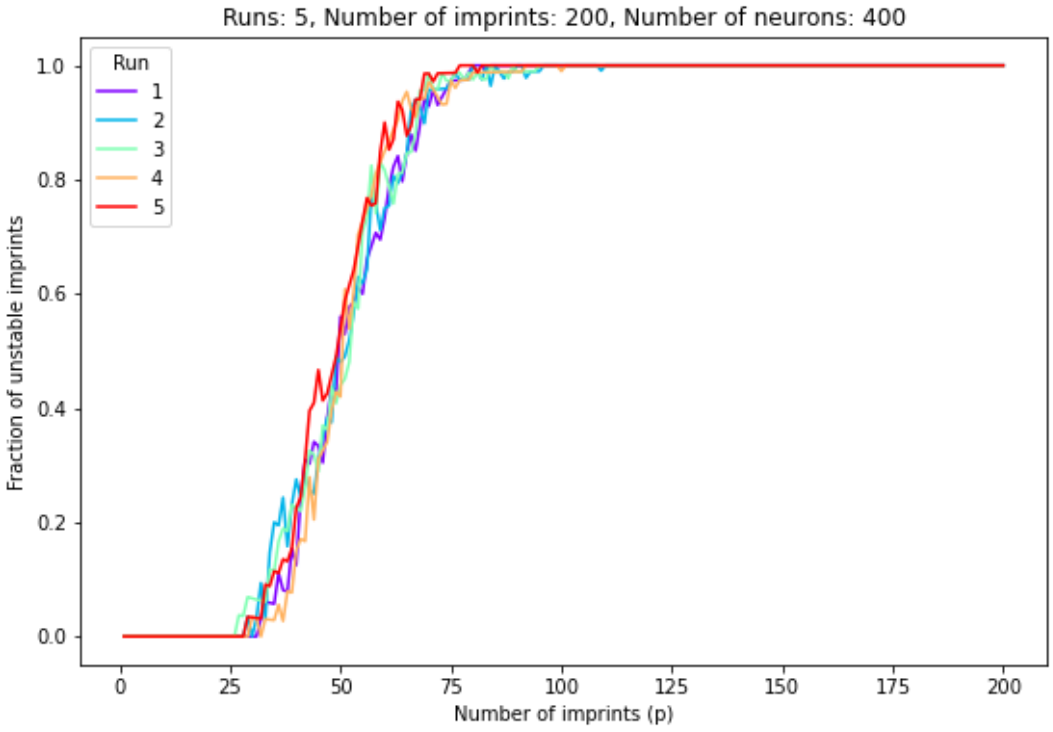
Runs: 5, Number of imprints: 200, Number of neurons: 400
Run
1
2
3
4
5
Fraction of unstable imprints
0.0
0.2
0.4
0.6
0.8
1.0
0
25
50
75
100
125
150
175
200
Number of imprints (p)

Figure 13.

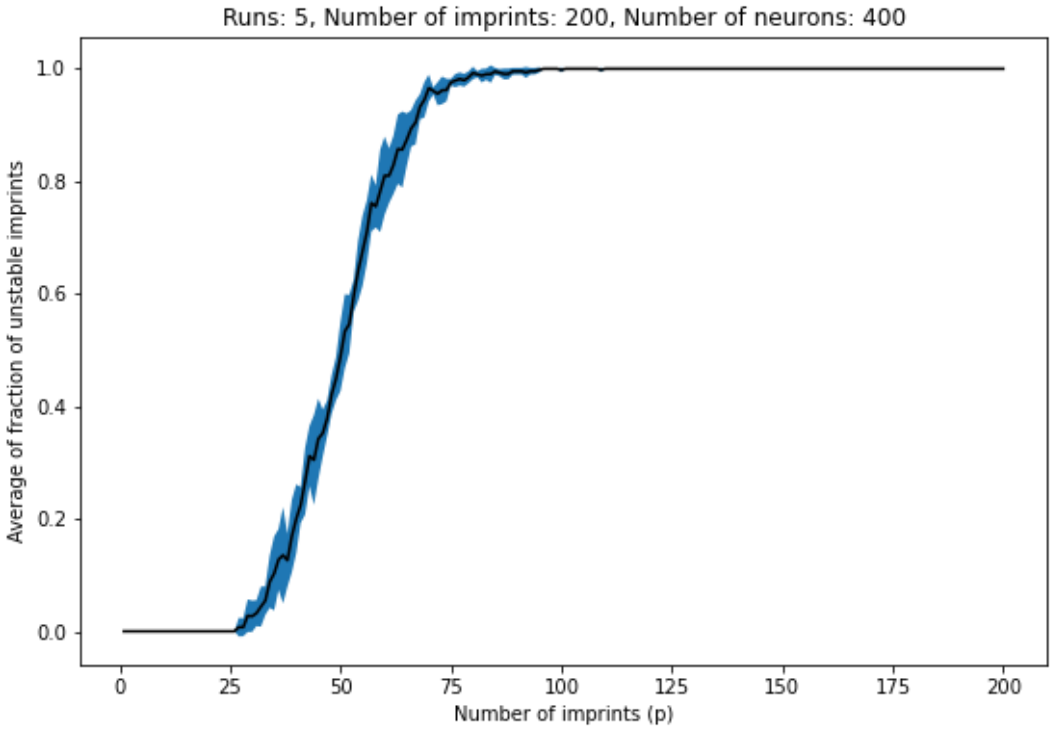
Runs: 5, Number of imprints: 200, Number of neurons: 400
Average of fraction of unstable imprints
Number of imprints (p)
0.0
0.2
0.4
0.6
0.8
1.0
0
25
50
75
100
125
150
175
200

Figure 14.

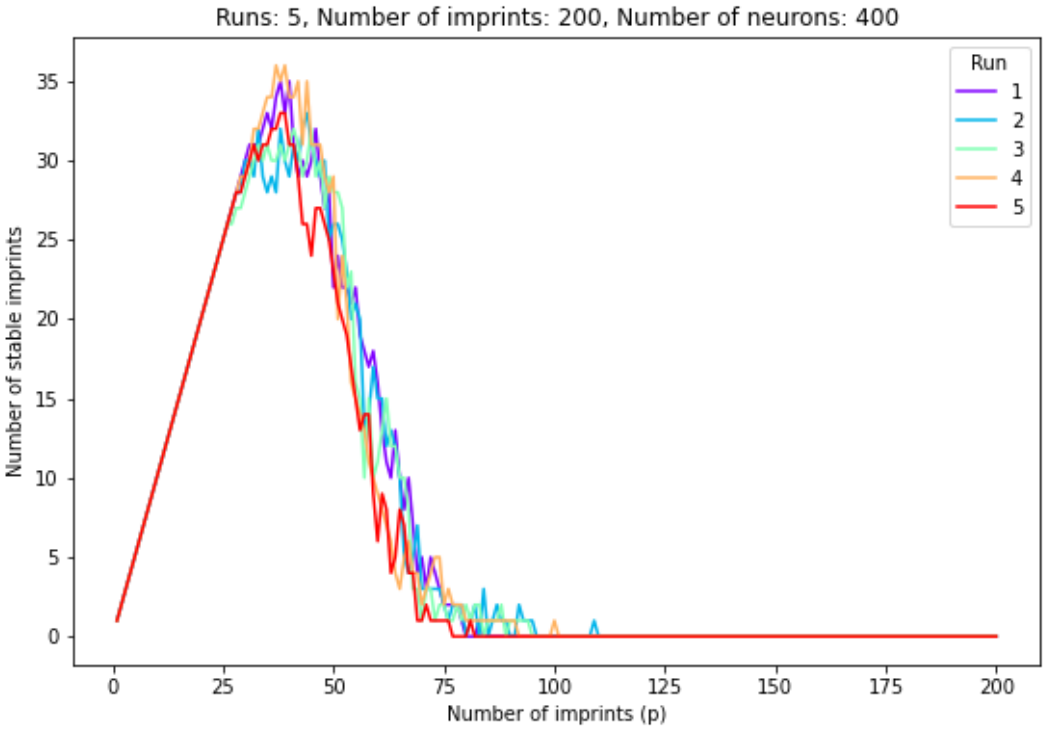
Runs: 5, Number of imprints: 200, Number of neurons: 400
Number of stable imprints
Number of imprints (p)
Run
1
2
3
4
5
0
5
10
15
20
25
30
35
0
25
50
75
100
125
150
175
200

Figure 15.

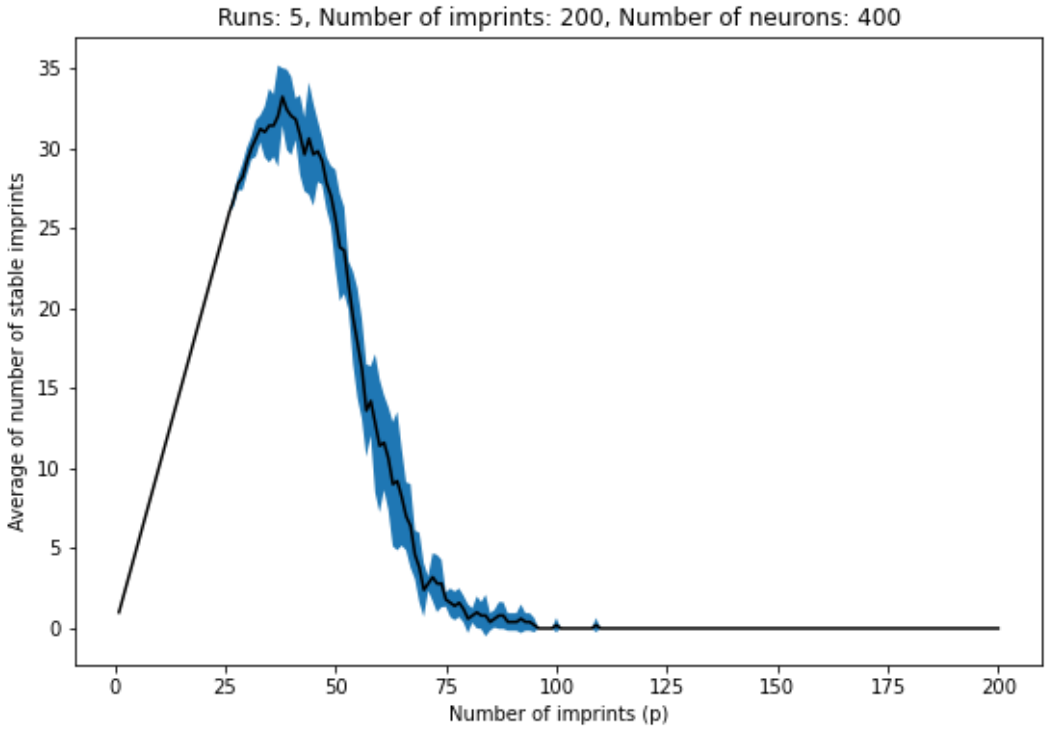

Figure 16.

Discussion

When looking at the default parameters (figures 1-4), there wasn't too much variance between the different runs/experiments. There was the most variation in runs when imprinting between 10 and 40 patterns. The variation was definitely not consistent throughout the number of imprints since they would all converge at a certain point and there wouldn't be much variance. It also didn't seem like there were outliers.

The number of stable patterns does seem to peak indicating a maximum capacity. It peaked when imprinting around 10 patterns and decreased afterwards suggesting the network didn't have enough capacity to learn much beyond that point. The fact that the fraction of unstable imprints also starts to increase around 10 imprinted patterns corroborates this. So, it seems that the maximum capacity of a network with 50 patterns and 100 neurons is when imprinting around 10 patterns.

Changing the number of neurons seems to change the capacity of the network. For example, figure 4 and figure 8 show that increasing the number of neurons from 100 to 200 neurons

changes the peak of stable patterns from around 10 imprinted patterns to 20 imprinted patterns. So, by adding more neurons, the network can retain more information/memory. However, if you compare figure 8 to figure 12, increasing the number of patterns from 50 to 100, the peak capacity (around 20 ish imprinted patterns) didn't change significantly. And while there seems to be a sharper decline afterwards in how many patterns it could imprint, it's misleading as the x-axis extends longer.