MidTerm Exam #2 Solutions

Monday, November 18, 2019

Name_____

Solve all six problems. Show your work to receive full credit.

1. *Pilates* is a popular set of exercises for the treatment of individuals with lower back pain. The method has six basic principles: centering, concentration, control, precision, flow, and breathing. The article *"Efficacy of the Addition of Modified Pilates Exercises to a Minimal Intervention in Patients with Chronic Low Back Pain: A Randomized Controlled Trial"* (*Physical Therapy*, 2013: 309-321) reported on an experiment involving 86 subjects with nonspecific low back pain. The participants were randomly divided into two groups of equal size. The first group received just educational materials, whereas the second group participated in 6 weeks of *Pilates* exercises. The sample mean level of pain (on a scale from 0 to 10) for the control group at a 6-week follow-up was 5.2 and the sample mean for the treatment group was 3.1; both sample standard deviations were 2.3. Carry out a hypothesis test using a significance level of $\alpha = .01$. Does it appear that true average pain level for the control condition exceeds that for the treatment condition by more than 1?

   $H_0 : \mu_{\text{control}} - \mu_{\text{treatment}} =$ _____

   $H_a : \mu_{\text{control}} - \mu_{\text{treatment}}$ _____ 1

   test statistic value = _____

   P-value = _____

   Accept or Reject H_0 at $\alpha = .01$: _____

Let μ_1 = the population mean pain level under the control condition and μ_2 = the population mean pain level under the treatment condition.

a. The hypotheses of interest are H_0: $\mu_1 - \mu_2 = 0$ versus H_a: $\mu_1 - \mu_2 > 0$. With the data provided, the test statistic value is $z = \dfrac{(5.2 - 3.1) - 0}{\sqrt{\dfrac{2.3^2}{43} + \dfrac{2.3^2}{43}}} = 4.23$. The corresponding P-value is $P(Z \geq 4.23) = 1 - \Phi(4.23) \approx 0$.

Hence, we reject H_0 at the $\alpha = .01$ level (in fact, at any reasonable level) and conclude that the average pain experienced under treatment is less than the average pain experienced under control.

b. Now the hypotheses are H_0: $\mu_1 - \mu_2 = 1$ versus H_a: $\mu_1 - \mu_2 > 1$. The test statistic value is $z = \dfrac{(5.2 - 3.1) - 1}{\sqrt{\dfrac{2.3^2}{43} + \dfrac{2.3^2}{43}}} = 2.22$, and the P-value is $P(Z \geq 2.22) = 1 - \Phi(2.22) = .0132$. Thus we would reject H_0 at the $\alpha = .05$ level and conclude that mean pain under control condition exceeds that of treatment condition by more than 1 point. However, we would not reach the same decision at the $\alpha = .01$ level (because $.0132 \leq .05$ but $.0132 > .01$).

2. Anorexia Nervosa (AN) is a psychiatric condition leading to substantial weight loss among women who are fearful of becoming fat. The article *"Adipose Tissue Distribution After Weight Restoration and Weight Maintenance in Women with Anorexia Nervosa"* (*Amer. J. of Clinical Nutr.*, 2009: 1132–1137) used whole-body magnetic resonance imagery to determine various tissue characteristics for both an AN sample of individuals who had undergone acute weight restoration and maintained their weight for a year and a comparable (at the outset of the study) control sample. Here is summary data on intermuscular adipose tissue (IAT; kg).

Condition	Sample Size	Sample Mean	Sample SD
AN	16	.52	.26
Control	8	.35	.15

Assuming that both the AN and Control populations are normal, give a 99% confidence interval for $\mu_{AN} - \mu_C$, the difference between the true mean AN IAT and the true mean Control IAT

under the described AN protocol: $\alpha =$ _____ $\frac{\alpha}{2} =$ _____ $\nu \approx$ _____

99% CI: (_____, _____)

Let's construct a 99% CI for $\mu_{AN} - \mu_C$, the difference between true mean AN IAT and true mean control IAT. Assuming the data come from normal populations, the CI is given by

$$(\bar{x} - \bar{y}) \pm t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}} = (.52 - .35) \pm t_{.005,21}\sqrt{\frac{(.26)^2}{16} + \frac{(.15)^2}{8}} = .17 \pm 2.831\sqrt{\frac{(.26)^2}{16} + \frac{(.15)^2}{8}} = (-.07, .41).$$

Since this CI includes zero, it's plausible that the difference between the two true means is zero (i.e., $\mu_{AN} - \mu_C = 0$). [Note: the df calculation $\nu = 21$ comes from applying the formula in the textbook.]

3. Antipsychotic drugs are widely prescribed for conditions such as schizophrenia and bipolar disease. The article *"Cardiometabolic Risk of Second-Generation Antipsychotic Medications During First-Time Use in Children and Adolescents"* (*J. of the Amer. Med. Assoc.*, 2009) reported on body composition and metabolic changes for individuals who had taken various antipsychotic drugs for short periods of time.

   a. A (large) sample of 41 individuals who had taken *aripiprazole* had a mean change in total cholesterol (mg/dL) of 3.75, and the estimated standard error $s_D/\sqrt{n}$ was 3.878. Calculate a 95% confidence interval for the true mean change in total cholesterol μ_D under the *aripiprazole* regimen.

   95% CI: (_____, _____)

   b. The article reported that for a sample of 36 individuals who had taken *quetiapine*, the sample mean cholesterol level change and estimated standard error were 9.05 and 4.256, respectively, and the *P*-value is .02. Making necessary assumptions about the distribution of change in cholesterol level, does the choice of significance level impact your conclusion as to whether true average cholesterol level increases? Explain.

a. Let μ_D denote the true mean change in total cholesterol under the aripiprazole regimen. A 95% CI for μ_D, using the "large-sample" method, is $\bar{d} \pm z_{\alpha/2}\frac{s_D}{\sqrt{n}} = 3.75 \pm 1.96(3.878) = (-3.85, 11.35)$.

b. Now let μ_D denote the true mean change in total cholesterol under the quetiapine regimen. The hypotheses are H_0: $\mu_D = 0$ versus H_a: $\mu_D > 0$. Assuming the distribution of cholesterol changes under this regimen is normal, we may apply a paired *t* test:

$$t = \frac{\bar{d} - \Delta_0}{s_D/\sqrt{n}} = \frac{9.05 - 0}{4.256} = 2.126 \Rightarrow P\text{-value} = P(T_{35} \geq 2.126) \approx P(T_{35} \geq 2.1) = .02.$$

Our conclusion depends on our significance level. At the $\alpha = .05$ level, there is evidence that the true mean change in total cholesterol under the quetiapine regimen is positive (i.e., there's been an increase); however, we do not have sufficient evidence to draw that conclusion at the $\alpha = .01$ level.

4. It is well-known that a placebo, a fake medication or treatment, can sometimes have a positive effect just because patients often expect the medication or treatment to be helpful. The article *"Beware the Nocebo Effect"* (*New York Times*, Aug. 12, 2012) gave examples of a less familiar phenonmenon, the tendency for patients informed of possible side effects to actually experience those side effects. The article cited a study reported in *The Journal of Sexual Medicine* in which a group of patients diagnosed with benign prostatic hyperplasia (BPH) was randomly divided into two subgroups. One subgroup of size 55 received a compound of proven efficacy along with counseling that a potential side effect of the treatment was erectile dysfunction. The other subgroup of size 52 was given the same tretment without counseling. The percentage of the no-counseling subgroup that reported one or more sexual side effects was 15.3%, whereas 43.6% of the counseling subgroup reported at least one sexual side effect. State and test the appropriate hypotheses at significance level .05 to decide whether the nocebo effect is operating here.

Let p_1 = the true proportion of patients that will experience erectile dysfunction when given no counseling, and define p_2 similarly for patients receiving counseling about this possible side effect. The hypotheses of interest are H_0: $p_1 - p_2 = 0$ versus H_a: $p_1 - p_2 < 0$.

The actual data are 8 out of 52 for the first group and 24 out of 55 for the second group, for a pooled proportion of $\hat{p} = \dfrac{8+24}{52+55} = .299$. The two-proportion z test statistic is $\dfrac{(.153-.436)-0}{\sqrt{(.299)(.701)\left[\frac{1}{52}+\frac{1}{55}\right]}} = -3.20$, and the P-value is $P(Z \le -3.20) = .0007$. Since $.0007 < .05$, we reject H_0 and conclude that a higher proportion of men will experience erectile dysfunction if told that it's a possible side effect of the BPH treatment, than if they weren't told of this potential side effect.

5. Taxophene is an insecticide that has been identified as a pollutant in the Great Lakes ecosystem. To investigate the effect of toxaphene exposure on animals, groups of rats were given toxaphene in their diet. The article *"Reproduction Study of Toxaphene in the Rat"* (*J. of Environ. Sci. Health*, 1988: 101-126) reports weight gains (in grams) for rats given a low dose (4 ppm) and for control rats whose diet did not include the insecticide. The sample standard deviation for 23 female control rats was 32 g and for 20 female low-dose rats was 54 g. Does this data suggest that there is mor variability in low-dose weight gains than in control weight gains? Assuming normalilty, carry out a test of hypotheses at significance level .05.

Let $\sigma_1^2 =$ variance in weight gain for low-dose treatment, and $\sigma_2^2 =$ variance in weight gain for control condition. We wish to test $H_0 : \sigma_1^2 = \sigma_2^2$ v. $H_a : \sigma_1^2 > \sigma_2^2$. The test statistic is $f = \frac{s_1^2}{s_2^2} = \frac{54^2}{32^2} = 2.85$. From Table A.9 with df = (19, 22) ≈ (20, 22), the *P*-value is approximately .01, and we reject H_0 at level .05. The data do suggest that there is more variability in the low-dose weight gains.

6. The lumen output was determined for each of $I = 3$ different brands of lightbulbs having the same wattage, with $J = 8$ bulbs of each brand tested. The sums of squares were computed as $\text{SSE} = 4773.3$ and $\text{SSTr} = 591.2$. State the hypotheses of interest (including word definitions of parameters), and use the *F* test of ANOVA with $\alpha = .05$ to decide whether there are differences in true average lumen outputs among the three brands for this type of bulb by obtaining as much information as possible about the *P*-value.

With μ_i = true average lumen output for brand *i* bulbs, we wish to test $H_0 : \mu_1 = \mu_2 = \mu_3$ v. H_a: at least two μ_i's are different. $\text{MSTr} = \hat{\sigma}_B^2 = \dfrac{591.2}{2} = 295.60$, $\text{MSE} = \hat{\sigma}_W^2 = \dfrac{4773.3}{21} = 227.30$, so

$$f = \frac{\text{MSTr}}{\text{MSE}} = \frac{295.60}{227.30} = 1.30.$$

For finding the *P*-value, we need degrees of freedom $I - 1 = 2$ and $I(J - 1) = 21$. In the 2nd row and 21st column of Table A.9, we see that $1.30 < F_{.10,2,21} = 2.57$, so the *P*-value > .10. Since .10 is not < .05, we cannot reject H_0. There are no statistically significant differences in the average lumen outputs among the three brands of bulbs.