Topic 5: Probabilities, Expected Values, IRV, Group __ (Section 1)

Group Members Present: Firstname Lastname <uhemail@hawaii.edu>

- Name
- Name
- Name
- Name

These problems require equations: use View/Show Equation Toolbar if needed to get the toolbar.

1. Probabilities

a. Let X be a random variable that takes values {5, 10, 15} with the following probabilities $\Pr[X=5] = \frac{3}{5}$, $\Pr[X=10] = \frac{1}{5}$, $\Pr[X=15] = \frac{1}{5}$. Compute E[X], the expected value of X. Show your work, including the use of the definition of expected value (linearity of expectations and Lemma 1). *Hint: you don't need* $\boldsymbol{\Sigma}$ *here: write out each term.*

$E[X] =$

b. Let Y be a random variable that takes values {1, 2, 3, 4, 5, 6} with equal probabilities. State this probability, write the formula for expected value of Y, and solve the formula to get the value. Show your work. (This is the expected value of a 6 sided dice.)

$Pr[Y = 1] = ?$ (and the same for 2 ... 6)

$E[Y] = \sum$

2. Permutation search

Let A be an array of size n that contains integers 1 through n, which are ***randomly*** permuted.

Here is an algorithm that takes as input the array A and an integer k, where $1 \le k \le n$, and returns the index i such that A[i] = k.

```
int linear_search (A, k)
    for i = 1 to n
       if A[i] == k
          return i
```

Analyze the expected running time of this algorithm as follows.

a. Define an indicator random variable X_i = I{ $A[i] = k$ }. What is $E[X_i]$, and why? *Hint: don't use* $\boldsymbol{\Sigma}$*: this is the expected value of a single event.*

$E[X_i] =$

b. Let *Y* be a random variable that denotes the number of elements checked by linear search when searching for key *k*. Determine the expression for *Y* in terms of X_i's.

$$Y = \sum$$

c. Use parts (i) and (ii) to compute the expected runtime of your algorithm E[Y]. *(Remember that E[a*X] = a*E[X] for any constant a and random variable X)*

$$E[Y] = E\left[\sum\right]$$

Challenge Problems. *Finished early? Work on the following problem*

3. Let Y be a random variable that takes values {1, 2, 3, … n} with equal probabilities. Compute E[*Y*], the expected value of *Y*. Show your work. (This is the expected value of an n-sided dice.)

$$E[Y] = \sum$$

If you have *m* identical *n*-sided die, what would be the expected value of rolling all *m* of them and summing the result?

4. How would your analysis in 1b change if the dice were weighted such that the "6" was twice as likely to come up as the other numbers {1, 2, 3, 4, 5}, which were otherwise equally likely? Write a modified expectation formula and solve.

$$E[Y] = \sum$$

5. In expectation, how many times must you roll a die to get two heads in a row?