Writing Problem

Sol.

Let $x = \sup S$, and let $\epsilon > 0$. Suppose for a contradiction that there is no $a \in S$ such that $a > x - \epsilon$. Then $a \leq x - \epsilon$ for all $a \in S$, i.e., $x - \epsilon$ is an upper bound for the set S. But $x - \epsilon < x$, contradicting the fact that x is the least upper bound for S.
This contradiction shows that there exists $a \in S$ such that $x - \epsilon < a$. Lastly, since x is an upper bound for S, we must have $a \leq x$, so that

$$x - \epsilon < a \leq x$$

as required.