Review Question

1. Which of the following **cannot** be present in a graph if we wish to find the shortest path?
   (a) negative weight edges
   (b) cycles
   (c) cycles with negative weight edge
   (d) negative weight cycles
2. Which of the shortest path algorithm is greedy?
3. Which algorithm would you use on a general graph that has negative weight on edges?
4. Prove that $\forall n \in \mathbb{Z}^+$ then
$$\sum_{i=1}^{n}(-1)^i i^2 = \frac{(-1)^n(n)(n+1)}{2}$$
5. Show that if f(n) and g(n) are monotonically decreasing functions then so are the sum, f(g(n)).
6. A graph with n vertices and n-1 eges is either disconnected or a tree.

   (TRUE/FALSE)
7. For every n there exists a directed graph on n vertices with $\Omega(n^2)$ edge that has a topological ordering.

   (TRUE / FALSE)
8. Solve the recurrence T(n) = $4T(\frac{n}{2}) + O(n)$ by finding its running time.
9. Design the most efficient algorithm you can to combine M sorted list, each of size N into one sorted list. give the time complexity of your solution in terms of N and M.
10. Show that the function f(n) = $n^2 + n \rightarrow \Theta(n^2)$
11. Given an array of N non unique values, design the most efficient algorithm you can to find the most common value. Give the time complexity of your solution in terms of N.
12. A minimum weight edge in a connected graph G must belong to <u>every</u> minimum spanning tree for G.

    TRUE/FALSE

13. A minimum weight edge in a connected graph G must belong to <u>some</u> minimum spanning tree for G.

TRUE/FALSE

14. If an edge (u,v) is contained in some minimum spanning tree for graph G then it is a light edge crossing some cut of G.

TRUE/FALSE

15. What problem solving strategy do both Kruskal's and Prim's MST algorithms use?

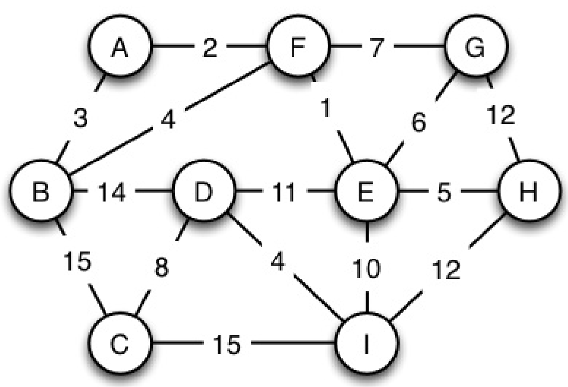
16. Claim: Prim's algorithm correctly finds minimum spanning trees in connected graphs with negative weights.

TRUE/FALSE

17. Claim: Kruskal's algorithm correctly finds minimum spanning trees in connected graphs with negative weights.

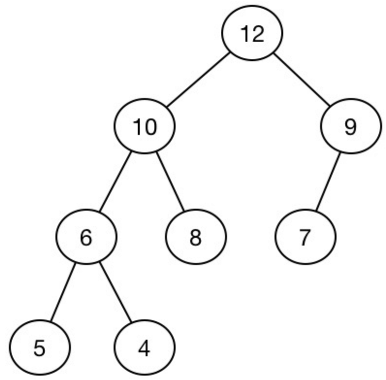
TRUE / FALSE

18. Check off the edges that are included in a Minimum Spanning Tree for this graph.

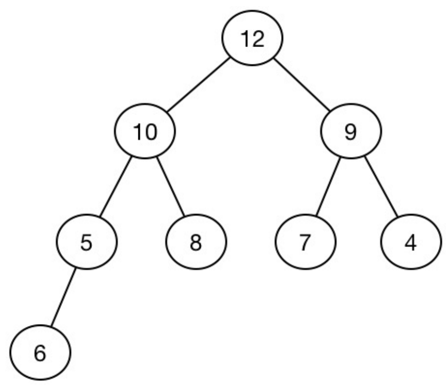


19. In terms of n = $|V|$, Floyd-Warshall's runtime is

20. Is this a Max-Heap? If not, why not?



21. Is this a Max-Heap? If not, why not?



22. Suppose you have a binary heap represented as an array using 1-based indexing (root is at index 1). An item is at index 31 in the array: what is the index of its parent? Write an integer:

23. Suppose you have a binary heap represented as an array using 1-based indexing (root is at index 1). An item is at index 31 in the array: what is the index of its right child? Write an integer:

24. Show the array after Build-Max-Heap has run by entering integers for the keys:

1 2 3 4 5

25. What is the worst-case runtime of randomized quicksort?

26. What is the expected runtime of randomized quicksort?

27. What is the best-case runtime of randomized quicksort?
28. Suppose that you knew that your array was sorted except for the possible misplacement of one or two elements: Which of the sorts that we have studied would be the fastest? What is its expected big-O runtime given your choice in the above question?
29. What is the smallest possible depth of a leaf in a decision tree for a comparison sort of n items?
30. Suppose the deterministic Partition (A, 1, 5) is called on the array A = [7, 3, 9, 4, 5] shown in the table below with 1-based indexing. You will show the state of the array after each swap of elements in the array
31. What is the smallest possible height of a decision tree for a comparison sort of n items?
32. The runtime of counting sort is Θ(n + k). What is k?
33. The runtime of radix sort using counting sort is (d(n + k)). What is d?
34. Which of the sorting algorithms are stable sorts?
35. Use Radix Sort on the following words:

    BOW, DOG, FAX, DIG, BIG, COW

36. Which of the sorting algorithms are not stable sorts?
37. We use Red-Black trees to simultaneously represent what?
38. What of the following properties is violated in a red-black tree when we have underflow of a node in the corresponding 2-4 tree?
39. What of the following properties is necessary to guarantee that the 2-4 tree it represents is balanced, assuming that the other properties hold?
40. What of the following properties is violated in a red-black tree when we have either incorrect representation or overflow of a node in the corresponding 2-4 tree?
41. Match the following with the appropraite sort algorithm
    (a) radix sort
    (b) insertion sort
    (c) merge sort
    (d) Randomized Quick Sort

    (i) application demands a gurantee that every sort finishes within O(n lg n) time even if average cost per higher

(ii) application requires a fast sort on a variety of input sizes and types and distribution of keys and the sort must be done in place to conserve memory

(iii) application will only be very short lists and you want simple code

(iv) you are sorting a set of d digit integers and done mind using a specialized sort to gain speed

42. "Quick Sort can be used as an auxillary sorting routine in Radix Sort, because it operates in places i.e. does not use any extra memory.

TRUE / FALSE

43. Suppose we run DFS on a directed graph using the CLRS version of the algorithm, and while searching vertex v encounter a black node on recursive call. What type of edge is this?

44. Illustrate the operation of PARTITION on the array A $= < 13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11 >$

45. After a given iteration of the loop line 3 of Floyd Warshall, we have computed:

46. Suppose we run DFS on a directed graph using the CLRS version of the algorithm, and while searching vertex v encounter a grey node on recursive call. What type of edge is this?

47. Illustrate a BUILD-MAX-HEAP on the array A $= [5, 3, 17, 10, 84, 19, 6, 22, 9]$

48. What is the **worst case runtime** of Insertion Sort in terms of input size n?

49. If you want a guarantee that an algorithm will never take more than a certain amount of time for a given input size, which analysis do you need?

50. Match the three criteria that a loop invariant must satisfy in order to prove correctness of an algorithm:

(a) Maintenance

(b) Termination

(c) Initialization

(i) The loop invariant must be true before the first iteration of the loop being analyzed

(ii) If the loop invariant is true prior to a given iteration of the loop being analyzed, then it remains true before the next iteration

(iii) If true after the final iteration of the loop being analyzed, the loop invariant states a property that will help show that an algorithm is correct

51. Give the Θ notation for the following expression, simplified as much as possible for $7n^3 + 4n + 99$

52. Consider how Merge Sort implements the Divide and Conquer strategy. At a given call to Merge Sort:
    (a) What is the cost to divide a problem of size n into subproblems?
    (b) What is the cost to combine the solutions to subproblems?
    (c) How many subproblems does Merge Sort divide a problem of size n into?
    (d) What is the size of each subproblem as a function of n?
53. (Ordered DLL) What is the tightest bound on the worst case asymptotic cost of the fastest algorithm for searching an ordered doubly linked list for a given key?
54. What is the worst case and the best case run time for Merge Sort?
55. What is the tightest bound on the worst case asymptotic cost of stack push and pop operations under the array implementations discussed in the text and lecture?
56. What is the tightest bound on the worst case asymptotic cost of queue enqueue and dequeue operations under the array implementations discussed in the text and lecture?
57. (Unordered DLL) What is the tightest bound on the worst case asymptotic cost of the fastest algorithm for searching an unordered doubly linked list for a given key?
58. (SLL) Given a pointer to a list cell in a singly-linked list, what is the expected asymptotic run time of the fastest algorithm to insert a new cell after that cell?
59. (Ordered DLL) What is the tightest bound on the worst case asymptotic cost of the fastest algorithm for insertion of a new key into an ordered doubly linked list? (You have only the key, not the position.)
60. (SLL) Given a pointer to a list cell in a singly-linked list, what is the expected asymptotic run time of the fastest algorithm to delete that cell?
61. (Unordered DLL) What is the tightest bound on the worst case asymptotic cost of the fastest algorithm for insertion of a new key into an unordered doubly linked list?
62. (Binary Tree) Suppose we have a binary tree of n nodes, each node having a key (their ordering is not important for this question), and using a linked node representation (e.g., nodes of class TreeNode as described in the lectures). What is the asymptotic run time of the fastest algorithm to print out the keys of all n nodes of the binary tree?
63. This is a circular doubly linked list with a sentinel referenced by L.nil. What data object does L.nil.prev.prev.key return?

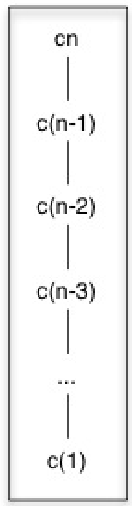
64. Insert the keys 9, 18, and 13 into a hash table using chaining, with m = 5 and h(k) = k mod m. Values to insert (in this order): 9, 18, 13.

65. Solve the following recurrence relation

$$T(n) = 2T(\frac{n}{2}) + n^2$$

66. Insert the keys 9, 18, and 13 into a hash table using open addressing with linear probing, with m = 5 and h(k,i) = (h'(k) + i) mod m. Values to insert (in this order): 9, 18, 13.

67. Write the recurrence relation for the runtime cost of the recursive procedure for which this recursion tree represents the recursion



68. Solve the recurrence relation below

$$T(n) = 4T(\frac{n}{2}) + n^2$$

69. Is the proof correct

Suppose we want to prove that T(n) = 6T(n/3) + n = Θ(n), using proof by substitution. Using c for the constant that is implied in the definition of Θ(n), we guess T(n) ≤ cn, and then prove by substitution of the guess into the formula as follows:

T(n) = 6T(n/3) + n

≤ 6(c(n/3)) + n *// substitution of guess*

≤ 2cn + n *// simplifying*

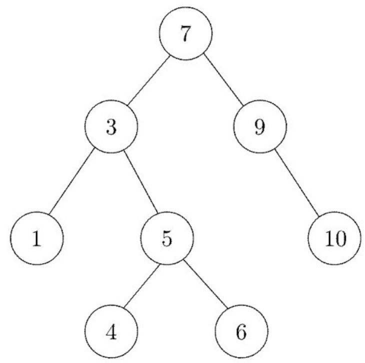
= Θ(n) *// constants don't affect growth rate*

70. How may leaves are there in a complete binary tree of height 10?

71. If G = (V, E) is a connected graph, which of these additional conditions will guarantee that G is a tree?

72. How may internal vertices are there in a complete binary tree of height 10?

73. Is this a valid BST?



74. Which traversal would you use to get the keys of a binary search tree printed out in sorted order

75. If you have an optimal solution to a sub problem, this solution must be part of the optimal solution to the overall problem.

(TRUE / FALSE)

76. Solve the following recurrence relation

(a)

$$T(n) = 2T(\frac{n}{2}) + n^4$$

(b)

$$T(n) = T(n-2) + n^2$$

(c)

$$T(n) = 4T(\frac{n}{2}) + n^2\sqrt{n}$$

77. Suppose that we have an array of n data records to sort and that the key of each record has the value 0 or 1.
An algorithm for sorting such a set of records might possess some subset of the following three desirable characteristics:

   (i) runs in O(n) times
   (ii) stable
   (iii) sorts in place, using no more than a constant amount of storage space in addition to the original array

   Given an algorithm such that it fits

   (a) criteria i and ii
   (b) criteria i and iii
   (c) criteria ii and iii

78. Characterize the running time of the following algorithm with justification

```
TripleLoop(A, n)
  for k = 1 to 10
    for i = 1 to n
      for j = n downto i
        A[i] = A[j] + 1
```

79. How does the growth rate of these pairs of functions compare (O, o, Ω, ω, Θ, or incomparable) and why?

   (a) $n^2 \& 4^{\log(n)}$
   (b) 2^{999} and $lg(lg(n))$
   (c) $lg(n)$ and $\log_{10}(n)$

80. Use induction to prove $d! > 2d$ for all large integers d

81. Given an adjacency list representation of a graph G=(V,E), and assuming randomly distributed graphs, what is the time complexity in terms of V and E to determine whether an edge exists from vertex u to vertex v, and why?

82. Given an array representation A of a graph G=(V,E), and assuming randomly distributed graphs, what is the time complexity in terms of V and E to determine whether an edge exists from vertex u to vertex v, and why?

83. Given a hash table with chaining, having m cells and n items have been inserted into the hash table, what is the expected time in terms of n and m to find an element x in the hash table, and why?

84. The value $|f|$ of a flow f : VxV$\rightarrow \mathbb{R}$ with source s and sink t is:

85. Given an n x n array A, what is the Θ time complexity in terms of n to access a cell A[i,j] of the array, and why?

86. Explain the type of the edges:

    (a) Forward Edges

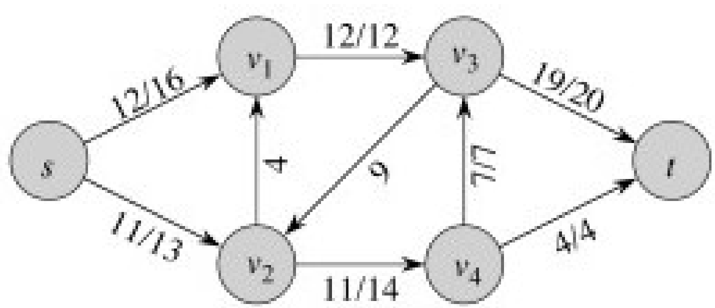
    (b) Backward Edges

    (c) Cross Edges

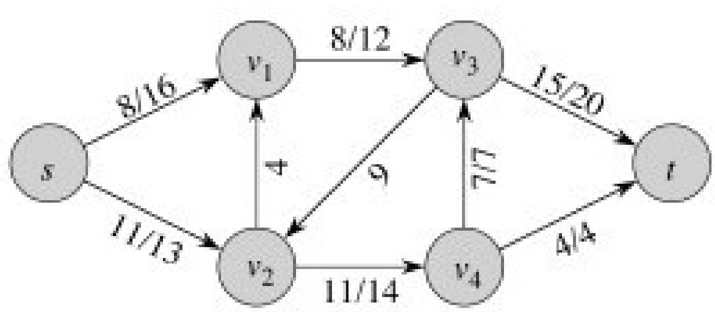
    (d) Tree

87. In a flow network, the flow f(u, v) assigned to an edge has what kind of relationship to the capacity c(u,v).

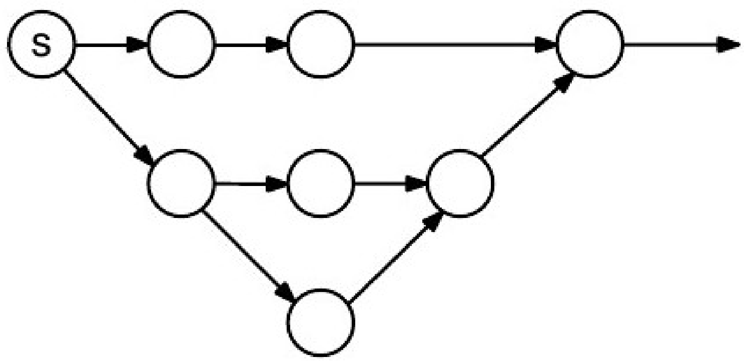
88. What is the min-cut corresponding to the max-flow in this graph?



89. Write the correct residual network for the flow network shown below?



90. Write the definitions for the following terms
    (a) dynamic threading
    (b) static thereading
    (c) race condition
    (d) strand
    (e) dependency
91. Which keywords can a compiler **always** safely ignore?
92. Consider the following multithreaded computation DAG where the computation starts at s and each vertex represents a task that takes 1 unit of time to execute.



Write the span and work of this computation.

93. The work of an algorithm is T_1 = n log n. The span is T_∞ = log n. What is the maximum number of processors for which linear speedup is possible?
94.