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Math 331
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QUIZ 1 Solutions

(8) **1.** Give the definitions:

a. A sequence $\{a_n\}_{n=1}^{\infty}$ is *convergent* iff ...

there is a real number A such that for every $\varepsilon > 0$ there exists an integer N such that if $n \geq N$, then $|a_n - A| < \varepsilon$.

b. Let S be a set of real numbers. A real number A is an *accumulation point* of S iff ... (For this part only: full credit for any statement equivalent to the definition.)

every neighborhood of A contains infinitely many points of S.

c. State the Least Upper Bound Property of $\mathbb{R}$.

Every non-empty subset of $\mathbb{R}$ that is bounded from above has a least upper bound.

d. State the Bolzano-Weierstrass Theorem, which concerns sets of real numbers that have accumulation points.

Every bounded and infinite set of real numbers has at least one accumulation point.

(6) **2.** Suppose $E \subset \mathbb{R}$ is non-empty and that $E \cap [0,1] = \emptyset$.

a. Is it possible that $\sup E = 0$? If "yes", give an example of such a set. If "no", explain why not.

Yes. An example is the set $E = (-1, 0)$.

b. Is it possible that $\sup E = 1$? If "yes", give an example of such a set. If "no", explain why not.

No. If $\sup E = 1$, then from hw #0.44 $(1-\varepsilon, 1] \cap E \neq \emptyset$ for all $\varepsilon > 0$. But $(0,1] \cap E = [0,1] \cap E = \emptyset$, from the hypothesis, so taking $\varepsilon = 1$ in hw #0.44 gives a contradiction.

(4) **3.** Give an example of a set S of real numbers that has exactly two accumulation points, 0 and 1.

An example is

$$S = \left\{ \frac{1}{n} \,:\, n \in \mathbb{N} \right\} \cup \left\{ 1 + \frac{1}{n} \,:\, n \in \mathbb{N} \right\}.$$

Compare to hw #1.22, and examples from your class notes.

(7) **4.** Prove that every convergent sequence is a Cauchy sequence. (This is a theorem in the text. Don't just refer to another theorem coming after this in the book; give a proof based

on the definitions, using $\varepsilon > 0$.)

This is Theorem 1.3 in the text. The proof is a "standard $\varepsilon/2$ argument", which you can find in the text.