MATH 331 (Introduction to Real Analysis), Spring 2020
Midterm Exam 1
Monday, February 10, 2020

Name: ____________________

Student ID Number: ____________________

I understand it is against the rules to cheat or engage in other academic misconduct during this test.

(SIGN HERE)

Question 1	20	
Question 2	20	
Question 3	20	
Question 4	20	
Question 5	20	
Total	100	

- There are 5 questions. Make sure your exam contains all these questions.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- If you need more room, use the backs of the pages and indicate that you have done so.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam.

Problem 1 (20 pts).

Let $A \subset \mathbb{R}$.

a. Give the definition of $\sup A$ and $\inf A$.

b. Suppose that $A \cap [0,1] = \emptyset$. Is it possible that $\sup A = 0$?
If yes, give an example of such a set. If no, explain why.

c. Suppose that $A \cap [0,1] = \emptyset$. Is it possible that $\sup A = 1$?
If yes, give an example of such a set. If no, explain why.

Problem 2 (20 pts).

Let $A \subset \mathbb{R}$.

a. Give the definition of accumulation point of A.

b. State the Bolzano-Weierstrass theorem.

c. Give an example of a set A that has exactly two accumulation points.

Problem 3 (20 pts).

Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of real numbers.

a. Define what it means for $(a_n)_{n\in\mathbb{N}}$ to converge to a real number a.

b. Suppose that $(a_n)_{n\in\mathbb{N}}$ converges to a, and define a new sequence by

$$b_n := \frac{a_n + a_{n+1}}{2} \qquad (n \in \mathbb{N}).$$

Prove that $(b_n)_{n\in\mathbb{N}}$ also converges to a.

Problem 4 (20 pts).

Using arithmetic on sequences, find the limits of the following sequences:

a. $a_n = \frac{n^3-7n+4}{-2n^3+2n^2+1}$.

b. $a_n = \frac{(-1)^n\, n \sin(n^3)}{n^2}$.

c. $a_n = \sqrt{n+1} - \sqrt{n}$.

d. $a_n = \frac{\cos(\sin n)}{\sqrt{n}}$.

Problem 5 (20 pts).

True or False? Give a **short** justification.

a. Every nonempty set of real numbers that is bounded from above has a greatest lower bound.

b. If $(a_n)_{n\in\mathbb{N}}$ is a sequence, then $a_n \to a$ if and only if $|a_n| \to |a|$.

c. Every Cauchy sequence is convergent.

d. Every infinite set of real numbers has a least one accumulation point.