**Problem 1:** (8 pts)
(a) (2 pts) Define what it means for the sequence $\{a_n\}_{n=1}^{\infty}$ to converge to a real number $A$.
(b) (2 pts) Define what it means for the sequence $\{a_n\}_{n=1}^{\infty}$ to be Cauchy.
(c) (2 pts) Let $S$ be a set of real numbers. Define what it means for $A$ to be an accumulation point of $S$.
(d) (2 pts) State the Least Upper Bound Property.

**Problem 2:** (6pts) True or False (justify your answer).
(a) (2 pts) $(A \setminus B) \cup (B \setminus A) \subset (A \cup B) \setminus (A \cap B)$.
(b) (2 pts) There exists an infinite subset of $\mathbb{R}$ that has no accumulation points.
(c) (2 pts) Every increasing sequence converges.

**Problem 3:** (8pts)
(a) (4 pts) Define the sequence $\{a_n\}_{n=1}^{\infty}$ by $a_1 = 15$ and $a_n = \sqrt{12 + a_{n-1}}$ for $n \geq 2$. Show that the sequence $\{a_n\}_{n=1}^{\infty}$ is decreasing and bounded.
(b) (4 pts) Is this sequence convergent? Why or why not? If convergent, find its limit.

**Problem 4:** (6 pts) Assume that $y$ is an upper bound for a nonempty bounded from above set $A$. Prove that $y = \sup A$ if and only if for each $\epsilon > 0$, there is $a \in A$ such that $y \geq a > y - \epsilon$.

**Problem 5:** (4 pts)
(a) (2 pts) Give an example of a sequence $\{a_n\}_{n=1}^{\infty}$ of real numbers that is bounded but not convergetn.
(b) (2 pts) Consider your example from (a). Does the set $\{a_n \mid n = 1, 2, ...\}$ have any accumulation points? Explain your answer.

**Problem 6:** (8 pts)
(a) (6 pts) If $\{a_n\}_{n=1}^{\infty}$ converges to $A$ and $\{b_n\}_{n=1}^{\infty}$ converges to $B$, prove that $\{a_n b_n\}_{n=1}^{\infty}$ converges to $AB$. (This is Theorem 1.9 from the textbook. To get credit you must prove it, not just quote the theorem.)
(b) (2 pts) Can $\{a_n b_n\}_{n=1}^{\infty}$ converge without having that both $\{a_n\}_{n=1}^{\infty}$ converges and $\{b_n\}_{n=1}^{\infty}$ converges? Justify your answer.