Section 4.4 Exercise 32

Sol.

For (a), we get
$$\lim_{x \to 1} \frac{\ln x}{x - 1} = \lim_{x \to 1} \frac{1/x}{1} = 1.$$
For (b), we get
$$\lim_{x \to 0} \frac{x}{e^x - 1} = \lim_{x \to 0} \frac{1}{e^x} = 1.$$
For (c), we get
$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\cos x}{1} = 1.$$

Section 4.4 Exercise 35

Sol.

The Inverse-Function Theorem gives
$$(f^{-1})'(f(1)) = \frac{1}{f'(1)} = \frac{1}{6}.$$
This is the slope of the tangent line at (3,1). It follows that the equation of the tangent line at the point (3,1) is
$$y = \frac{1}{6}x + \frac{1}{2}.$$

Section 4.4 Exercise 36

Sol.

If $f(x) = \sin x$, then the Inverse-Function Theorem gives
$$(f^{-1})'(\sin x) = \frac{1}{f'(x)} = \frac{1}{\cos(x)} = \frac{1}{\sqrt{1 - \sin x^2}}.$$
Replacing $\sin x$ by x then gives
$$(f^{-1})'(x) = \frac{1}{\sqrt{1 - x^2}}.$$