Problem 1: (20 pts)
(a) (5 pts) Complete the definition. Suppose $f : D \to \mathbb{R}$ and x_0 is ... Then f has a limit L at x_0 iff ...
(b) (5 pts) State the Bolzano-Weierstrass Theorem.
(c) (5 pts) Define what it means for the sequence $\{a_n\}_{n=1}^{\infty}$ to be Cauchy.
(d) (5 pts) State the Sequential Limit Theorem.

Problem 2: (14 pts) Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 - 5$. Use $\epsilon - \delta$ definition to prove that $\lim_{x\to 1} f(x) = -4$.

Problem 3: (15 pts) Suppose x is an accumulation point of the set $\{a_n \mid n \in \mathbb{N}\}$. Prove that there is a subsequence of $\{a_n\}_{n=1}^{\infty}$ that converges to x.

Problem 4: (15 pts) Suppose that $f : \mathbb{R} \to \mathbb{R}$, $f(x) < 0$ if $x < 0$, $f(x) > 0$ if $x > 0$ and $\lim_{x\to 0} f(x)$ exists. Call that limit L. Prove that $L = 0$. (*Hint:* Use the Sequential Limit Theorem.)

Problem 5: (12 pts) Suppose $E \subset R$ is non-empty and that $E \cup [0, 1] = \varnothing$, where $[0, 1]$ denotes a closed interval.
(a) (6 pts) Is it possible that $\sup E = 1$? If yes, give an example of such a set. If no, explain why not.
(b) (6 pts) Is it possible that $\sup E = 0$? If yes, give an example of such a set. If no, explain why not.

Problem 6: (24 pts) Indicate by writing **T** or **F** whether each statement is true or false. **Give no proofs.**
(1) The function $f : (0, 1) \to \mathbb{R}$, defined by $f(x) = x \cos\left(\frac{1}{x}\right)$ does not have a limit at 0.
(2) The function $f : (0, 1) \to \mathbb{R}$, defined by $f(x) = \cos\left(\frac{1}{x}\right)$ does not have a limit at 0.
(3) If x_0 is an accumulation point of a set $S \subset \mathbb{R}$, then $x_0 \in S$.
(4) Every subsequence of a Cauchy sequence is Cauchy.
(5) If $\varnothing \neq A \subset B \subset \mathbb{R}$ then $\inf A \leq \inf B$?
(6) Let A be the limit of the sequence $\{a_n\}_{n=1}^{\infty}$. Then every neighborhood of A contains all but finitely many members of the sequence $\{a_n\}_{n=1}^{\infty}$.
(7) Let A be an accumulation point of the set $\{a_n \mid n \in \mathbb{N}\}$. Then every neighborhood of A contains all but finitely many elements of the set $\{a_n \mid n \in \mathbb{N}\}$.
(8) If x and y are real numbers with $x \neq y$, then there is a neighborhood P of x and a neighborhood Q of y such that $P \cap Q = \varnothing$.