Problem 1: (15 pts) Give the following definitions

(a) (5 pts) Assume $f : D \to \mathbb{R}$, $x_0 \in D$ and x_0 is an accumulation point of D. Define what it means for f to be differentiable at x_0.
(b) (5 pts) Assume $f : D \to \mathbb{R}$, and $E \subset D$. Define what it means for f to be uniformly continuous on E.
(c) (5 pts) Assume $f : D \to \mathbb{R}$, and $x_0 \in D$. Define what it means for f to be continuous at x_0.

Problem 2: (10 pts) Give an example of an open cover of the set $[1, 5)$ that has no finite subcover.

Problem 3: (15 pts) State the following theorems:

(a) (5pts) The Mean Value Theorem
(b) (5pts) The Extreme Value Theorem
(c) (5pts) The Intermediate Value Theorem

Problem 4: (12 pts) Prove that the equation $x^3 + 3x + 1 = 0$ has exactly one root in the interval $[-2, 2]$.

Problem 5: (10 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Prove that the set $A = \{x \in \mathbb{R} \mid f(x) = 0\}$ is a closed subset of $\mathbb{R}$.

Problem 6: (10 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0 & x = 0 \end{cases}$.
Show that $f'(x)$ exists for all $x \in \mathbb{R}$, but the function $f' : \mathbb{R} \to \mathbb{R}$ is not continuous at 0.

Problem 7: (24 pts) Indicate by writing **T** or **F** whether each statement is true or false. **Give no proofs.**
(1) Every uniformly continuous function is differentiable. (F)
(2) If $f : J \to \mathbb{R}$ is defined by $f(n) = n^2$, then f is uniformly continuous. (T)
(3) If $f : \mathbb{R} \to \mathbb{R}$ is continuous and $E \subset \mathbb{R}$ is open, then $f(E)$ is open. (F)
(4) If $f : \mathbb{R} \to \mathbb{R}$ is differentiable and $f'(0) = 0$, then f is not one-to-one. (F)
(5) If $f : (2, 3) \to \mathbb{R}$ is uniformly continuous, then $\lim_{x\to 2} f(x)$ exists. (T)
(6) A union of any collection of closed sets of real numbers is a closed set. (F)
(7) Let $f : [a, b] \to \mathbb{R}$ be continuous. Then the image of f is a closed interval. (T)
(8) If a set is not open, it is closed. (F)

Addendum from Quiz 2. Prove that the curves $f(x) = 2x^3$ and $g(x) = 3x^2 - 2$ intersect on the interval $[-1, 1]$. Justify your answer.