Problem 1: (8 pts) Suppose $f : [a, b] \to \mathbb{R}$ is a bounded function.
(a) (2 pts) Define what it means for P to be a partition of $[a, b]$. What is a marked partition?
(b) (2 pts) Define a lower sum $L(P, f)$.
(c) (2 pts) Define a Riemann sum $S(P, f)$.
(d) (2 pts) Define the lower integral of f.

Problem 2: (4 pts) State one of the Theorems that gives a necessary and sufficient condition for f to be Riemann integrable on the interval $[a, b]$. (One of Theorems 5.2, 5.5, 5.6, or 5.7.)

Problem 3: (4 pts) The following statement is false. Explain why.
There is a function $f \in R(x)$ on $[-1, 1]$ and a partition P of $[-1, 1]$ such that $L(P, f) = 1$, $U(P, f) = 2$, and $\int_{-1}^{1} f(x)dx = 3$.

Problem 4: (10 pts) Prove that if $f : [a, b] \to \mathbb{R}$ is continuous, then f is Riemann integrable on $[a, b]$. (This is Theorem 5.4 from the book, prove it).

Problem 5: (5 pts) A set $A \subset [0, 1]$ is dense in $[0, 1]$ iff every open interval that intersects $[0, 1]$ contains a point of A. Suppose $f : [0, 1] \to \mathbb{R}$ is integrable and $f(x) = 0$ for all $x \in A$ with A dense in $[0, 1]$. Show that $\int_{0}^{1} f(x)dx = 0$.

Problem 6: (6 pts) Either give an example, or explain why there is no such example of ...
(a) (3 pts) ... a continuous function $f : [0, 1] \to \mathbb{R}$ that is not Riemann integrable.
(b) (3 pts) ... a Riemann integrable function $g : [0, 1] \to \mathbb{R}$ that is not continuous.