Answer the questions and show all work clearly. No calculator or notes allowed.

1. Given g(x) = $(2x+5)^{1/3}$, find a formula for the inverse function g^{-1}.
2. Let $f(x) = \ln(x) + x^5 + 1$ and f(1)= 2. Find $f^{-1\prime}(2)$
3. Find the Taylor series expansion for f(x) = $\frac{1}{x^2}$ centered at a = 1
4. Fill in the following
   (a) arcsin(0) =
   (b) $\cos^{-1}(0) =$
   (c) $\log_2^{1/8}$
   (d) $\arctan(\sqrt{3})$
   (e) $\arcsin(\cos(\frac{\pi}{3}))$
5. Differentiate the following functions. No need to simplify.
   (a) $f(x) = 3^{\sin(x)}$
   (b) $y = (\sin(x)^{5x}$
   (c) $f(x) = \ln(x^2 - 3x + 2) + \sin(\ln(x))$
   (d) $g(x) = e^{3x-2} + 5^{\sin(x)}$
   (e) $h(t) = \sin(t)^{\cos(t)}$
   (f) $f(x) = \ln(\arctan x) + \sin^{-1}(e^{3x})$
   (g) $g(x) = \arctan(\sqrt{x})$
   (h) $f(x) = \ln(e^{3x} + 7x) + 2^{\sqrt{x}}$
   (i) $y = \arctan(x)^x$
   (j) $y = \arcsin(e^{-x})$
6. Evaluate the integrals of the following
   (a) $\int \frac{5x^2}{x^3+7} dx$
   (b) $\int \sin(x) e^{\cos(x)} dx$
   (c) $\int x e^{x^2} dx$
   (d) $\int \frac{dx}{\sqrt{7-4x^2}}$ Hint: Let u = 2x
   (e) $\int \frac{x dx}{5+x^4}$
   (f) $\int \frac{dx}{7+x^2}$
   (g) $\int x e^{2x} dx$
   (h) $\int \sec^6(x) dx$

(i) $\int \frac{dx}{x\sqrt{4-x^2}}$

(j) $\int \frac{3x+2}{(x+1)(x+3)}dx$

(h) $\int \frac{x+6}{x^2+4x+7}dx$

(j) $\int_3^\infty \frac{1}{x^2-4}dx$

(k) $\int xe^{3x}dx$

(l) $\int x\ln(3x)dx$

(m) $\int \sin^5(x)dx$

(n) $\int \tan^6(x)\sec^6(x)dx$

(o) $\int \frac{dx}{x^2\sqrt{x^2+4}}$

(p) $\int \frac{dx}{x\sqrt{4-x^2}}$

(q) $\int \frac{x+2}{x^2+6x+5}$ dx

(r) $\int_0^\infty \frac{4}{5x^2+6x+1}$ dx

(s) $\int_1^2 \frac{1}{\sqrt{4-x^2}}$dx

(t) $\int_0^\infty \frac{dx}{2x^2+3x+1}$

(u) $\int \frac{dx}{(9-x^2)^{3/2}}$

(v) $\int_0^1 \frac{dx}{x^2}$

(w) $\int \frac{x+5}{x^2+2x+4}$ dx

(x) $\int x\ln(3x)$ dx

7. Evaluate the following limits. Remember to show the algebraic process or indicate L'Hopital's Rule.

(a)
$$\lim_{x\to 0}(2\sin(x)+\cos(x))^{3/x}$$

(b)
$$\lim_{x\to 0}\frac{x^4-x^2}{1-\cos(x)}$$

(c)
$$\lim_{x\to 0^+} x\ln(\sin(x))$$

(d)
$$\lim_{x\to 0}(1+\sin(x))^{1/x}$$

(e)
$$\lim_{x\to 0}\frac{1-e^{2x}}{1-e^{3x}}$$

(f)

$$\lim_{x\to 0}(\cos(x))^{1/x^2}$$

8. Solve the following differential equations

   (a) $x^3y' = y^2$

   (b) $e^y(1+x^2)y' = 1$ with the initial value $y(0) = 1$

   (c) $\frac{dy}{dx} = x^2\sqrt{1-y^2}$

   (d) $\frac{dy}{dx} - 5y = 10x$

   (e) $xy' + 3y = \frac{\sin x}{x^2}$ $x > 0$

   (f) $t\frac{dy}{dt} + 2y = t^3$ where y(2) = 1 $t > 0$

   (g) $\theta y' - 2y = \theta^3 \sec(\theta)\tan(\theta)$

   (h) $3y'' + 4y' + y = 0$ y(0) = 0, y'(0) = 2

   (i) $y'' + 4y' + 7y = 0$

   (j) $xy' = 3y + x^3 + 3$

9. Consider the integral $\int_{-2}^{2} x^4$ dx. Use Simpson's Rule to find S_4.

10. Find n such that Simpson's Rule is within 10^{-8} of $\int_2^3 \frac{1}{x}dx$.

11. Consider the integral, $\int_1^5 x^2dx$. Use the n = 4 with trapezoidal rule and simpsons rule

12. Find teh 3rd Taylor Polynomial for $\sqrt{4+x}$ centered at a = 0

13. For each of the following series, using the any of the tests state whether they converge absolutely, converge conditionally, or diverge.

    (a)

    $$\sum_{n=1}^{\infty} \frac{-4}{n^{3/2}}$$

    (b)

    $$\sum_{n=2}^{\infty} (-1)^n n^2$$

    (c)

    $$\sum_{n=1}^{\infty} \frac{|\sin(n^2)|}{n^2+2}$$

    (d)

    $$\sum_{n=1}^{\infty} \sin(1/n)$$

(e)
$$\sum_{n=2}^{\infty} \frac{1}{n(\ln(n)^2}$$

(f)
$$\sum_{n=1}^{\infty} \frac{3^n}{n!}$$

(g)
$$\sum_{n=1}^{\infty} \frac{\arctan n^n}{3}$$

(h)
$$\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$$

(i)
$$\sum_{n=1}^{\infty} (\ln(n) - \ln(n+1))$$

(j)
$$\sum_{n=1}^{\infty} \frac{2^n}{3^n}$$

(k)
$$\sum_{n=2}^{\infty} \frac{1}{3+2^{-n}}$$

(l)
$$\sum_{n=1}^{\infty} (\sqrt{2})^n$$

(m)
$$\sum_{n=1}^{\infty} \frac{-2}{n\sqrt{n}}$$

14. For the given power series determine the values of x which the series absolutely converges, conditionally converges, and divergent. State the interval of convergence, and the radius of convergence.

(a)
$$\sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}$$

(b)
$$\sum_{n=1}^{\infty} \frac{3^n x^n}{n!}$$

(c)
$$\sum_{n=1}^{\infty} \frac{n!}{1000x^n}$$

(d)
$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{7^n n}$$

(e)
$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{5^n \sqrt{n}}$$

15. Find the third order Taylor polynomial for each of the following function at the specified base point

(a) $f(x) = \sqrt{4+x}$ at a = 0

(b) $f(x) = \cos(2x)$ at a = $\frac{\pi}{6}$