

1. **10 pts** Determine whether the statement is true or false. If the statement is false explain why.

a) If the f has an absolute maximum value at z , then the $f'(z) > 0$

(TRUE / FALSE)

SOLUTION FALSE because at either maximum or minima then $f'(z) = 0$

b) The function $f(x) = 2x(x + 4)^3$ has inflection points at: $(0,0)$ and $(4,0)$

(TRUE / FALSE)

SOLUTION FALSE because the inflection point is at $(-4,0)$

2. **5 pts** Find two negative numbers that add up to -50 such that the maximum product is possible.

SOLUTION $x_1 = -25$ & $x_2 = -25$

3. **5 pts** Find the local extrema and the increasing/decreasing intervals of the following function: $f(x) = x^{\frac{2}{3}}(x - 10)$

4. **10 pts** A piano is suspended by a 90 ft rope through a pulley system that is 40 ft above a man's arm. The piano is at some height above the ground. At $t = 0$, the man is 30 ft horizontally from the piano and walks away at 12 ft/s. How fast is the piano being pulled up?
SOLUTION $36/5 = 5.2$ ft/s

5. **10 pts** Determine the critical points of the following functions:

a)

$$V(t) = 1 + 80t^3 + 5t^4 - 2t^5$$

SOLUTION $t = 0, -4, 6$

b)

$$Q(t) = (2 - 8x)^4(x^2 - 9)^3$$

SOLUTION $t = \frac{1}{4}, t = 3, t = -3, t =$

6. **10 pts** Use linear approximation to find the approximate value of $\sin(122^\circ)$

SOLUTION

$$\frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)\left(\frac{\pi}{90}\right)$$

7. **20 pts** For the following functions answer each of the following

- identify the critical points of the function and classify them as relative maximum, relative minimum, or neither
- identify the intervals on which the function is increasing/decreasing
- determine the interval on which the function is concave up or down
- determine the inflection points of the function
- use the information found to sketch the graph of the function

a)

$$g(t) = t^5 - 5t^4 + 8$$

SOLUTION CRITICAL POINT: $t = 0$ and $t = 4$

CLASSIFICATION OF CRITICAL POINTS:

RELATIVE MINIMUM: 4

RELATIVE MAXIMUM: 0

INCREASING: $(-\infty, 0) \cup (4, \infty)$

DECREASING: $(0, 4)$

CONCAVE UP: $(3, \infty)$

CONCAVE DOWN: $(-\infty, 0)(0, 3)$

INFLECTION POINT: $t = 3$

b)

$$x^{4/3}(x - 2)$$

8. **10 pts** A car is heading "away" from the intersection at 5 m/s. And a bus is heading "towards" the intersection on the other street (at a right angle) at 4 m/s. How fast is the distance changing when the car is 30 m away from the intersection and the bus is 40 m from the intersection? Additionally, is the distance increasing or decreasing?

SOLUTION

distance is changing at $-\frac{2}{10}$ m/s

The distance is decreasing.

9. **10 pts** Evaluate the horizontal asymptote of the following functions:

a)

$$\frac{8 - 4x^2}{9x^2 + 5x}$$

SOLUTION $-\frac{4}{9}$

b)

$$\frac{\sqrt{7 + 9x^2}}{1 - 2x}$$

SOLUTION $x \rightarrow -\infty : y = -\frac{3}{2}$
 $x \rightarrow \infty : y = \frac{3}{2}$

10. **15 pts** Compute the differentials of the following functions:

a)

$$f(x) = x^2 - \sec(x)$$

SOLUTION $df = (2x - \sec(x) \tan(x)) dx$

b) Compute the dy and Δy for $y = x^5 - 2x^3 + 7x$ as x changes from 6 to 5.9

SOLUTION $dy = (5x^4 - 6x^2 + 7) dx$ and -627.1