Additional Problem Sets

The previous common final exams are excellent source of practice. Previous exams and quizzes are also great source. Here is the link where the previous common finals can be found.

https://math.hawaii.edu/ yuen/241finalinfo.htm

1. Find the integrals of the following problems

   a)
   $$\int_0^4 x(x+3)dx$$

   b)
   $$\int \sin(x)\cos(x)dx$$

   c)
   $$\int 7x^3\cos(2+x^4) + \frac{4\sin(8x)}{1+9\cos(8z)}dx$$

   d)
   $$\int \frac{\sin(\sqrt{x})}{\sqrt{x}}dx$$

2. Let x and y be two positive numbers such that $x + 2y = 50$ and (x+1)(y+2) is a maximum.

3. Using the **Definition**, find the derivatives of the function

   a)
   $$Q(t) = 10 + 5t - t^2$$

   b)
   $$f(x) = 2x^3 - 1$$

   c)
   $$z(x) = \frac{5}{x+2}$$

   d)
   $$m(w) = \sqrt{1-9w}$$

4. What is the approximate value of $\cos(62°)$ using linear approximation?

5. If $f'(x) = 2x^2 - 5$, find the interval where f is decreasing and increasing.

6. What is the rate of change of the function f(x) = $\sqrt{9-x^3}$ at x = 3?

7. Two cars leave the same intersection and drive away. Car drives due east at 100 km/hr while char B drives south at 50 km/hr. After 1 hour from the intersection, how fast is the distance between them increasing?

8. Find the value of x at which the graph $x^2 = y$ and $y = 4x$ have parallel tangent.

9. A ball is dropped from the roof of a building and hits the ground 15 seconds later. The position of the ball is given as $s(t) = -16t^2 - v_0 t + s_0$ where s_0 is the initial position measured in meters and v_0 is the initial velocity. Find the height of the building.

10. Find the limit of the following functions:

    a)
    $$\lim_{x \to \frac{\pi}{4}} \tan(x)$$

    b)
    $$\lim_{z \to -3} \frac{z+3}{z^2 - 9}$$

    c)
    $$\lim_{y \to 0} \frac{\tan(y)\cos(y)}{y}$$

    d)
    $$\lim_{x \to 0} \frac{4x^2}{1 - \cos(2x)}$$

11. Find $\frac{dy}{dx}$ of the following functions:

    a)
    $$\sec(y) = (y - x)^3$$

    b)
    $$y = \frac{2x + 7}{5 - 2x}$$

    c)
    $$x^2 y^2 - 3x = 5$$

12. Find the exact area under the curve f(x) = sin(x) and g(x) = cos(x), and the lines x = 0 to x = $\frac{\pi}{2}$

13. The marginal profit of manufacturing and selling a flu vaccine is given by P'(x) = $1000 - 0.04x$, where x is the number of units vaccine sold. How much profit should the company expect if it sells 20,000 units of this vaccine?

14. The cost of producing a brand of computer is given by the function C(x) = $200 + 16x + 0.1x^2$. If the computer sells for 500 dollars each and 1000 are produced and sold, then what is the marginal unit?

15. Evaluate the following definite integrals:

 a)
$$\int_0^4 (x^3 - 3x + 1)dx$$

 b)
$$\int_0^{2\pi} (x - \cos(x))dx$$

 c) If $\int_0^k (5 - x)dx = -12$ and $k > 0$, find k

16. The velocity of the particle moving on a line given by the equation v(t) = $2t^2 - 14t - 5$. Find the average velocity from t = 1 to t = 3.

17. Determine the second and fourth derivatives of the following functions,

 a)
$$g(t) = 3t^7 - 6t^4 + 8t^3 - 12t + 18$$

 b)
$$f(x) = 7\sin(x) - 6\cos(3x - 1)$$

18. Find the absolute extrema for the function $f(x) = 2x^3 - 3x^2$ in the interval [0,2].

19. Suppose you are given f'(x) = $\frac{x^2-4}{x^2+4}$. Find where f(x) has a local extrema, the intervals of increase/decrease of f(x), inflection points, and the concavity of f(x).

20. Determine a value c which satisfies the conclusion of the Mean Value Theorem:

 a)
$$f(x) = x^3 - 4x^2 + 3$$

 on [0,4]

 b)

21. For the following functions determine the following things:
critical points, interval of increase/decrease, classify the local maxima/minima, interval of concavity, and inflection points

 a)
$$f(x) = 5 - 8x^3 - x^4$$

 b)
$$z^4 - 2z^3 - 12z^2$$

22. Let S be a solid having as base the region in the first quadrant enclosed by the curve xy = 3 and the line y = 4 - x. Suppose further that parallel cross sections of S perpendicular to the x axis are rectangles having as base the vertical line connecting the graphs, having height twice the base. Find the volume of S.

23. Evaluate the following limits:

    a)
    $$\lim_{x\to\infty} \frac{2\sqrt{x} + 7\sin(x)}{\sqrt{x}}$$

    b)
    $$\lim_{x\to 0} \frac{\sqrt{9+x} - 3}{x}$$

    c)
    $$\lim_{x\to 0} x^3 \cos\frac{1}{x^2}$$

    d)
    $$\lim_{x\to 0} \frac{5}{\sin(8x)}$$

24. Show that there are three solutions to $x^3 - 7x + 1$ in the interval of [-3,3].

25. Find the $\frac{dy}{dx}$, tangent line, and the normal line of the following functions:

    a) $y = \frac{1}{x^2}$ at the point (1,1)
    b) $y^2 = \frac{x^2-4}{x^2+4}$ at the point (2,0)
    c) $(x+y)^3 = x^3 + y^3$ at the point (-1,1)

26. Coffee is draining from a conical filter into a cylindrical coffee pot at a rate of 10 cm^3/min.

    a) How fast is the coffee rising in the pot when the coffee in the pot is 5 in deep.
    b) How fast is the level in the cone falling then?

SOLUTIONS

1.
2. x $= \frac{53}{2} \& y = \frac{47}{4}$
3. $\frac{1}{2} + \frac{-\sqrt{3}}{2}\frac{\pi}{90}$
4.
5.
6.
7. 112 km/hr
8. x = 2
9. 3600 ft
10. a) 1 b) -1/6 c)1 d) 2
11. a) $\frac{3x^2-6xy+3y^2}{\sec(y)+3x-3y}$
12. $2(\sqrt{2}-1)$
13. 12,000,000
14. 284 dollars
15. a) 44 b) $2\pi^2$ c) k = 12
16. -73/3
17.
18. Min at x = 1 and Max at x = 2
19.
20.
21.
22. 0.6
23.
24.
25.
26. a) $10/9\pi$ b) $-8/5\pi$