Practice problems to be added. Please report any typos found.

Legend:
$\bullet$ = need to know
$\circ$ = need to know, sub-bullet of $\bullet$
$\diamond$ = useful, but not tested
$*$ = should know, but will be provided on exam formula sheet.

Inverse Functions

- Know how to solve for the inverse function of a given function.
- Know how to sketch a graph of the inverse function given the graph of the original function.
- Know how to find the derivative of the inverse function given the original function, namely

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Exponential and Logarithmic Functions

- Definition: $\log_a(x) = y$ is the number satisfying the equation $x = a^y$.
- Derivatives:

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}a^x = a^x \ln a \qquad \frac{d}{dx}\ln x = \frac{1}{x} \qquad \frac{d}{dx}\log_a x = \frac{1}{x \ln a}$$

- Know how and when to apply the technique of Logarithmic Differentiation.
- Know the graphs, domain and range of exponential and logarithmic functions.
- Limits:

$$\lim_{x\to\infty} e^x = \infty \qquad \lim_{x\to-\infty} e^x = 0 \qquad \lim_{x\to\infty} \ln x = \infty \qquad \lim_{x\to 0^+} \ln x = -\infty$$

- Integrals:

$$\int e^x \, dx = e^x + C \qquad \int \frac{1}{x} \, dx = \ln|x| + C$$

Separable Differential Equations

- Attempt to separate the two variables to opposite sides of the equation, and integrate, and remember the $+C$ *immediately* when you integrate, not the end of the problem.
- Solve explicitly for y (or whatever the dependent variable is) when possible.
- If an initial value problem, use the initial value to solve for the constant C.

Exponential Growth & Decay (i.e. modeling the diff. eq. $\frac{dy}{dx} = ky$)

- This model applies to where the rate of change of some quantity is proportional to its size.
- The solution to this diff. eq. is $y = y_0 e^{kt}$, where $y_0 = y(0)$ is the value of function at $t = 0$.
- The method is usually to solve for the two unknown constants y_0, k using two data points. Though in many problems, y_0 is never known.
- If given a half-life λ, then a data point is $\frac{1}{2}y_0 = y_0 e^{k\lambda}$ This lets you solve for k, and y_0 is irrelevant.

Inverse Trigonometric Functions

- Domains and Ranges:
  - $\sin^{-1} x = \arcsin x$ Domain: [-1,1] Range: $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$
  - $\cos^{-1} x = \arccos x$ Domain: [-1,1] Range: $0 \leq \cos^{-1} x \leq \pi$
  - $\tan^{-1} x = \arctan x$ Domain: $(-\infty, \infty)$ Range: $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

◇ Useful: $\arcsin(-x) = -\arcsin(x), \arccos(-x) = -\pi - \arccos(x), \arctan(-x) = -\arctan(x)$.

- Limits: $\lim_{x\to\infty} \arctan x = \pi/2$ $\lim_{x\to-\infty} \arctan x = -\pi/2$

∗ Derivatives / Integrals:

$$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}} \qquad \frac{d}{dx}\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) = \frac{1}{a^2 + x^2} \qquad \frac{d}{dx}\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) = \frac{1}{|x|\sqrt{x^2 - a^2}}$$

∗ $\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}(\frac{x}{a}) + C$ where $x^2 < a^2$

∗ $\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$

∗ $\int \frac{1}{x\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}|\frac{x}{a}| + C$ where $|x| > a > 0$

L'Hôpital's Rule

- Type notation
- L'Hôpital's Rule applies *only* when the type of the limit is $0/0$ or ∞/∞. Always check type first.
- For type $\infty \cdot 0$, rewrite the product as a quotient by taking one of the factors and inverting it. That is $AB = \frac{A}{1/B} = \frac{B}{1/A}$. One of these "new" limits will be type $0/0$, and the other will be type ∞/∞.
- For types 1^∞, 0^0, and ∞^0, set the limit equal to L, and take the natural logarithm of both sides, yielding $\ln L$ as being a limit whose type is $0/0$, ∞/∞, or ∞/∞, and then can be solved using L'H rule. Don't forget: $\ln L$ is not your answer! $L = e^{\ln L}$ is!
- For the type $\infty - \infty$, try to combine the terms by factoring, common denominator, logarithm identities, etc.
- Remember to simplify as much as possible before resorting to L'Hôpitals Rule again.
- Not all limits are best done, *or even doable*, by L'Hôpitals Rule. If L'H rule does not apply, it is because the type is not appropriate, or that a Calc I method solves the problem.

Integration By Parts ($\int u\, dv = uv - \int v\, du$)

- Choose dv to be something easily integrated, such as e^{ax} or $\sin(ax)$.
- Choose u to be something which simplifies when differentiated, such as $\ln x$ or $\sin^{-1} x$.
- If there is a x^n in your integral, apply one of the above two rules, then chose x^n to be the remaining u or dv.

◇ The acronym LIPTE (**L**ogarithmic, **I**nverse Trig, **P**olynomial, **T**rig, **E**xponential) can be used to decide which function to let be u (L being first, E being last.) This rule is not guaranteed to work.

- In general, the integral $\int v\, du$ should be less complicated than $\int u\, dv$. If it is more complicated, try switching your choice for u and dv.
- Typical IBP problems are:
  - $\int x^n f(x)\, dx$, where $f(x)$ is easily integrated, such as e^{ax}, $\sin(ax)$, etc. If $n > 1$, then IBP will need to be used more than once.
  - $\int x^n g(x)\, dx$, where $g(x)$ simplifies when differentiated, such as $\ln x$ or an inverse trig function.
  - $\int g(x)\, dx$, where $g(x)$ is easily differentiated (special case of above, let $u = 1$.)

- ◦ $\int f(x)g(x)\ dx$, where both $f(x)$ and $f(x)$ are from the group e^{ax}, $\sin(ax)$, and $\cos(ax)$. The choice for u or dv doesn't matter, and doing IBP twice will get back an expression with the original integral, which can be solved by the "back to self" method. Note, once you choose u and dv, be "consistent" with your choice the second time you perform IBP, or else the problem "starts over."

Trigonometric Integrals (Integrals like $\int \sin^n x \cos^m x\ dx$, $\int \sec^n x \tan^m x\ dx$, or other combinations.)

- The basic strategy is to use Pythagorean Identities to rewrite the integral as powers of different trig functions, then do a u-substitution with one of the present trig functions. You may have to use a trig identity, then split the integral in several smaller ones, and repeat the process on those integrals.
- $\int \sin^n x \cos^m x\ dx$
  - ◦ n is odd. Let $w = \cos x$, so $dw = -\sin x\ dx$, and $\sin^2 x = 1 - \cos^2 x = 1 - w^2$. This makes the problem into an integral of a polynomial (in terms of w.)
  - ◦ m is odd. Let $w = \sin x$. The process is similar to the above case.
  - ◦ both n and m are even. Use the half angle formulas: $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ to reduce the exponents. Expand and treat each term as a new separate integral. Each integral will be of the same form as the integral in the above black bullet (except $2x$ is the argument, not x). Apply the appropriate technique to each integral (which may be again be the technique discussed here.)
- $\int \sec^n x \tan^m x\ dx$
  - ◦ m is odd. Let $w = \sec x$, so $dw = \sec x \tan x\ dx$, and $\tan^2 x = \sec^2 x - 1 = w^2 - 1$. This makes the problem into an integral of a polynomial (in terms of w.)
  - ◦ n is even. Let $w = \tan x$, so $dw = \sec^2 x\ dx$, and $\sec^2 x = \tan^2 x + 1 = w^2 + 1$. This makes the problem into a polynomial (in terms of w.) One exceptional case is $\int \tan^2 x\ dx = \int (\sec^2 x - 1)\ dx = \tan x - x + C$.
  - ◦ m is even and n is odd. Convert all instances of tangent into secant by using $\tan^2 x = \sec^2 x - 1$ Expand so the you have several separate integrals, each that are of powers of $\sec x$. For each integral, use integration by parts with $dv = \sec^2 x$, and u being whatever the remaining power of secant is. This process eventually terminates, or you will use the "back to self" method. (You may have to recall $\int \sec x\ dx = \ln|\sec x + \tan x| + C$.)
- $\int \csc^n x \cot^m x\ dx$. This is handled in a similar way to combinations of secant and tangent, except that negatives appear when taking derivatives.
- For any other combination, try rewriting all trig functions in terms of only $\sin x$ and $\cos x$, or only in terms of $\sec x$ and $\tan x$.

- For quotients of trig functions, the same principle applies: m and n above can be negative numbers.
- Integrals:

$$\int \sin x\, dx = \cos x + C \qquad * \int \tan x\, dx = \ln|\sec x| + C \qquad * \int \sec x\, dx = \ln|\sec x + \tan x| + C$$

$$\int \cos x\, dx = -\sin x + C \qquad \int \cot x\, dx = -\ln|\csc x| + C \qquad \int \csc x\, dx = \ln|\csc x + \cot x| + C$$

<u>**Trigonometric Substitution**</u> (Pattern $\pm a^2 \pm [\ ]^2$)

- If the pattern is $a^2 - [\ ]^2$, let $[\ ] = a\sin\theta$, so $\sqrt{a^2 - [\ ]^2} = a\cos\theta$.
- If the pattern is $a^2 + [\ ]^2$, let $[\ ] = a\tan\theta$, so $\sqrt{a^2 + [\ ]^2} = a\sec\theta$.
- If the pattern is $[\ ]^2 - a^2$, let $[\ ] = a\sec\theta$, so that $\sqrt{[\ ]^2 - a^2} = a\tan\theta$
- BUT: <u>memorizing is bad.</u> Always make sure to draw a reference triangle to jog your memory of the substitutions, and to make sure your substitution makes sense!
- Some integrals may have a quadratic which is not of the above forms, and you will need to complete the square to make it of these forms.
- After the trigonometric substitution, the resulting integral should be a product or quotient of trigonometric functions, and so you use techniques for solving trigonometric integrals (see above).
- Don't forget take your solved integral and rewrite in terms of your original variable.

Practice Problems

1. Evaluate the following: (a) $\log_7 49$ (b) $\log_{\sqrt{3}} 9$ (c) $\log_2 \frac{1}{\sqrt{2}}$ (d) $\log_{1/2} 4$ (e) $\ln \sqrt{e}$

2. Differentiate the following functions: (a) $y = (e^{-x^2} - x)^\pi$ (b) $r = \frac{1}{\ln(\cos\theta)}$
(c) $f(x) = (x^2-1)^{-x}$ (d) $y = \ln(x^2+1) - e^{\sin x}$

3. Evaluate the following integrals: (a) $\int_0^1 e^x\sqrt{2e^x-1}\,dx$ (b) $\int_0^1 \frac{x}{x^2+9}dx$ (c) $\int_1^2 \frac{\ln x}{x}dx$
(d) $\int e^{7x}dx$ (e) $\int \frac{\sin(\ln x)}{x}dx$ (f) $\int \frac{1}{1-x}dx$

4. Solve the following differential equations. Express y as a function of x.

$$\text{(a) } y' = x + xy^2 \quad \text{(b) } e^y + y'\cos x = 0 \quad \text{(c) } y' = \frac{1+x^2}{y}, y(0) = -1$$

5. Bacteria grow at a rate proportional to its size. The count in a bacteria colony that started at 1000 was 2500 after 3 hours. How long will it take for the population to reach 10000?

6. The half-life of Polonium-210 is 140 days. How much of a sample of 200 mg will be left after 1 year (365 days)?

7. Evaluate the following to an exact value: (a) $\arcsin(-1/2)$ (b) $\tan^{-1}(\sqrt{3})$ (c) $\sin^{-1}(\cos(\pi/3))$

8. Differentiate the following functions: (a) $y = \arcsin(e^{-x})$ (b) $y = e^{\arctan x}$

9. Evaluate the following limits:

$$\text{(a) } \lim_{x\to 0+} \frac{\arcsin x}{x-1} \quad \text{(b) } \lim_{x\to\infty} e^{\arctan x} \quad \text{(c) } \lim_{x\to 0+} e^{\arctan(1/x)} \quad \text{(d) } \lim_{x\to -\infty} \arcsin\left(\frac{e^{-x}+\sqrt{3}}{e^{-x}+3}\right)$$

$$\text{(e) } \lim_{x\to 0} \frac{e^{-x^2}-1}{x^2} \quad \text{(f) } \lim_{x\to 0+} |\ln x|^x \quad \text{(g) } \lim_{x\to 0} \frac{\cos x}{x^2} \quad \text{(h) } \lim_{x\to\pi} \frac{\sin x}{\pi - x} \quad \text{(i) } \lim_{x\to 0+} \frac{x - \ln(x+1)}{x^2}$$

$$\text{(j) } \lim_{x\to 1+} x^{\frac{1}{x-1}} \quad \text{(k) } \lim_{x\to 2} \frac{x^3-8}{x^2-2} \quad \text{(l) } \lim_{x\to 0+} e^x \ln x \quad \text{(m) } \lim_{x\to 0}(1+3x^2)^{1/x^2} \quad \text{(n) } \lim_{x\to\infty} \frac{\arctan x}{x}$$

$$\text{(o) } \lim_{x\to\infty} x^{1/\ln x} \quad \text{(p) } \lim_{x\to 1} \frac{1+\cos\pi x}{(x-1)^2} \quad \text{(q) } \lim_{x\to 0+} (\sin x)^{\sqrt{x}} \quad \text{(r) } \lim_{x\to\infty} (\ln(2x^2+1) - 2\ln x)$$

10. Evaluate the following integrals:

$$\text{(a) } \int (x+\cos 4x)dx \qquad \text{(b) } \int e^{3x}dx \qquad \text{(c) } \int \frac{dx}{\cos^2(3x)} \qquad \text{(d) } \int \sec x \tan x \, dx$$

$$\text{(e) } \int \frac{1}{x^2+9}\,dx \qquad \text{(f) } \int \frac{x}{x^2+9}\,dx \qquad \text{(g) } \int \frac{1}{\sqrt{9-x}}\,dx \qquad \text{(h) } \int \frac{1}{\sqrt{9-x^2}}$$

(i) $\int x \ln x \, dx$ (j) $\int \sqrt{x} \ln x \, dx$ (k) $\int \frac{\ln x}{x} \, dx$ (l) $\int \frac{\ln x}{\sqrt{x}} \, dx$

(m) $\int xe^x \, dx$ (n) $\int xe^{3x} \, dx$ (o) $\int x \sin x \, dx$ (p) $\int x \sin(3x) \, dx$

(q) $\int \sin(x) \cos(x) e^{\sin(x)} \, dx$ (r) $\int \sin(x) e^{3x} \, dx$ (s) $\int \sqrt{9 - x^2}$

(t) $\int \frac{\sqrt{9 - x^2}}{x} \, dx$ (u) $\int \frac{\sqrt{9 - x^2}}{x^2} \, dx$ (v) $\int \frac{\sqrt{x^2 - 9}}{x} \, dx$ (w) $\int \frac{\sqrt{x^2 + 9}}{x^4} \, dx$

(x) $\int \frac{x^2}{x^2 + 9} \, dx$ (y) $\int \cos^5 x \, dx$ (z) $\int \sec^6 x \, dx$ (aa) $\int \sin^3 x \cos^2 x \, dx$

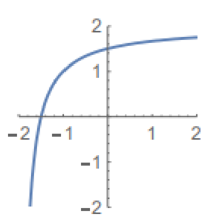
(ab) $\int \cos^2 x \, dx$ (ac) $\int \tan^3 x \sec^3 x \, dx$ (ad) $\int \tan^4 x \sec^4 x \, dx$

11. Find a formula for the inverse function of $f(x) = e^{3x+5}$

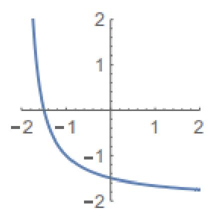
12. For the function $f(x) = e^{3x} + 4x + 1$, note that $f(0) = 2$. Find $(f^{-1})'(2)$.

13. For the following two graphs, choose the correct graph of its inverse function from the four choices of (I), (II), (III), (IV).

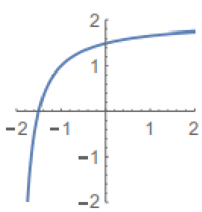
(a)



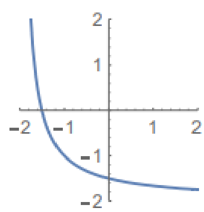
(b)



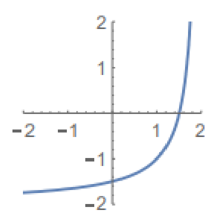
(I)



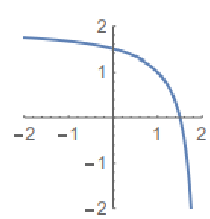
(II)



(III)



(IV)



Practice Problem Solutions

1. (a) 2 (b) 4 (c) $-\frac{1}{2}$ (d) -2 (e) $\frac{1}{2}$

2.
$$\text{(a) } \frac{dy}{dx} = \pi(e^{x^2} - x)^{\pi-1}(2xe^{x^2} - 1) \qquad \text{(b) } r' = \frac{\sin\theta}{\ln(\cos\theta)^2 \cos\theta}$$

$$\text{(c) } f'(x) = (x^2-1)^{-x}\left(\ln(x^2-1) - \frac{2x^2}{x^2-1}\right) \qquad \text{(d) } y' = \frac{2x}{x^2+1} - e^{\sin x}\cos x$$

3. (a) u-sub. Let $u = 2e^x + 1$. Get $\frac{1}{3}((2e-1)^{3/2} - 1)$.

   (b) u-sub. Let $u = x^2 + 9$. Get $\frac{1}{2}(\ln 10 - \ln 9)$.

   (c) u-sub. Let $u = \ln x$. Get $\frac{1}{2}(\ln 2)^2$.

   (d) u-sub. Let $u = 7x$. Get $\frac{1}{7}e^{7x} + C$.

   (e) u-sub. Let $u = \ln x$. Get $-\cos(\ln x)) + C$.

   (f) u-sub. Let $u = 1 - x$. Get $-\ln|1-x| + C$.

4.
$$\text{(a) } y = \tan\left(\frac{1}{2}x^2 + C\right) \qquad \text{(b) } y = -\ln(\ln|\sec x + \tan x| + C) \qquad \text{(c) } y = -\sqrt{2x + \frac{2}{3}x^3 + 1}$$

5. $\dfrac{3\ln 10}{\ln(5/2)}$ hours. (Note, there are several ways of writing this number. It's about 7.54.) In this problem, $C = 1000$, $k = \frac{\ln(5/2)}{3}$.

6. $200e^{365\ln(1/2)/140}$ mg. (Note, there are several ways of writing this number. It's about 32.8.) For this problem $C = 200$, $k = \frac{\ln(1/2)}{140}$.

7. (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$.

8.
$$\text{(a) } y' = \frac{-e^{-x}}{\sqrt{1-e^{-2x}}} \qquad \text{(b) } y' = e^{\arctan x}\frac{1}{1+x^2}$$

9. (a) 0 (b) $e^{\pi/2}$ (c) $e^{\pi/2}$ (d) $\pi/4$ (e) -1 (f) 1 (g) ∞ (h) 1 (i) 1/2 (j) e (k) 0 (l) $-\infty$ (m) e^3 (n) 0 (o) e (p) $\pi^2/2$ (q) 1 (r) $\ln 2$.

10. (a) $\frac{1}{2}x^2 + \frac{1}{4}\sin(4x) + C$.

(b) $\frac{1}{3}e^{3x} + C$

(c) $\frac{1}{3}\tan(3x) + C$ $(1/\cos^2 x = \sec^2 x = d/dx \tan x)$.

(d) $\sec x + C$

(e) $\frac{1}{3}\arctan\frac{x}{3} + C$. Can use trig-sub, or remember this common integral.

(f) $\frac{1}{2}\ln(x^2+9) + C$. Use u-sub, $u = x^2 + 9$. Also, note that there are no absolute-values here since $x^2 + 9$ is always positive.

(g) $-2\sqrt{9-x} + C$. Use u-sub, with $u = 9 - x$.

(h) $\arcsin(x/3) + C$. Can use trig-sub, or remember this common integral.

(i) $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$. Use integration by parts, $\ln x = u$, $xdx = dv$.

(j) $\frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$. Use integration by parts, $u = \ln x$, $\sqrt{x}dx = dv$.

(k) $\frac{1}{2}(\ln x)^2 + C$. Use u-sub, $u = \ln x$.

(l) $2\sqrt{x}\ln x - 4\sqrt{x} + C$. Use integration by parts, $u = \ln x$, $x^{-1/2}dx = dv$.

(m) $xe^x - e^x + C$. Use integration by parts, $u = x$, $e^x dx = dv$.

(n) $\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$. Use integration by parts, $u = x$, $e^{3x}dx = dv$.

(o) $-x\cos x + \sin x + C$. Use integration by parts, $u = x$, $\sin x dx = dv$.

(p) $-\frac{1}{3}x\cos(3x) + \frac{1}{9}\sin(3x) + C$. Use integration by parts, $u = x$, $\sin(3x)dx = dv$.

(q) $e^{\sin x}\sin x - e^{\sin x} + C$. Use w-sub, then integration by parts. Let $w = \sin x$, giving $\int we^w\, dw$. Then use IBP with $u = w$ and $e^w dw = dv$.

(r) $-\frac{1}{10}e^{3x}\cos x + \frac{3}{10}e^{3x}\sin x + C$. "Back to self" method of integration by parts. First let $u = \sin x$ and $e^{3x}dx = dv$. In the second IBP, let $u = \cos x$ and $e^{3x}dx = dv$ (note, this is the "consistent" choice), then solve your equation for $\int \sin(x)e^{3x}\, dx$.

(s) $\frac{1}{2}x\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{x}{3} + C$. Trig substitution with $x = \sin\theta$, get $\int 9\cos^2\theta\, d\theta$. Use the half-angle formula to reduce the power of cosine and solve, then use the half-angle formula for sine in reverse when substituting back in the x's.

(t) $-3\ln|\frac{3}{x}+\frac{\sqrt{9-x^2}}{x}|+\sqrt{9-x^2}+C$. Trig-substitution with $x=3\sin\theta$. Get $\int\frac{3\cos^2\theta}{\sin\theta}d\theta$. Rewrite numerator as $1-\sin^2\theta$, split integral into $3\int\csc\theta-\sin\theta\ d\theta$.

(u) $-\frac{\sqrt{9-x^2}}{x}-\arcsin\frac{x}{3}+C$. Trig-sub, with $x=3\sin\theta$. Get $\int\frac{\cos^2\theta}{\sin^2\theta}d\theta$. Rewrite numerator as $1-\sin^2\theta$, split integral into $\int csc^2\theta-1d\theta$.

(v) $\sqrt{x^2-9}-3\tan^{-1}\left(\frac{\sqrt{x^2-9}}{3}\right)+C$, or $\sqrt{x^2-9}-3\sec^{-1}\frac{x}{3}+C$. Trig-sub with $x=3\sec\theta$, get $\int\tan^2\theta d\theta=3\int(\sec^2\theta-1)d\theta$.

(w) $=\frac{1}{27}\left(\frac{\sqrt{x^2+9}}{x}\right)^3+C$. Trig-sub with $x=3\sec\theta$, get $\int\frac{\sec^3\theta}{\tan^4\theta}d\theta$. No obvious substitution seems present, so rewrite integral in terms of sine and cosine to get $\int\frac{\cos\theta}{\sin^4\theta}d\theta$. which is quickly solved with u-sub, letting $u=\sin\theta$.

(x) $x-3\arctan(\frac{x}{3})+C$. Trig-sub with $x=\tan\theta$. Get $3\int\tan^2\theta d\theta=3\int(sec^2-1)d\theta$.

(y) $\sin x-\frac{2}{3}\sin^3x+\frac{1}{5}\sin^5x+C$. Integral is trigonometric with odd power of $\cos x$: $\int\cos^5x\ dx=\int(\cos^2x)^2\cos x\ dx=\int(1-\sin^2x)^2\cos x\ dx=\int(1-u^2)^2\ du$, when $u=\sin x$.

(z) $\tan x+\frac{2}{3}\tan^3x+\frac{1}{5}\tan^5x+C$. Integral is trigonometric with an even power of $\sec x$: $\int\sec^6x\ dx=\int(\sec^2x)(\sec^2x)^2\ dx=\int(\sec^2x)(1+\tan^2x)^2\ dx=\int(1+u^2)^2\ du$ when $u=\tan x$.

(aa) $-\frac{1}{3}\cos^3x+\frac{1}{5}\cos^5x+C$. Integral is trigonometric with an odd power of $\sin x$. $\int\sin^3x\cos^2x\ dx=\int\sin^2x\sin x\cos^2x\ dx=\int(1-\cos^2x)\sin x\cos^2x\ dx=$
$-\int(1-u^2)u^2\ du$, letting $u=\cos x$.

(ab) $\frac{1}{2}x+\frac{1}{4}\sin(2x)+C$. Use half angle / power reduction formula.

(ac) $-\frac{1}{3}\sec^3x+\frac{1}{5}\sec^5x+C$. Odd power of $\tan x$: $\int\tan^2x\tan x\sec^3x\ dx$
$=\int(sec^2x-1)sec^2x\sec x\tan x\ dx=\int(u^2-1)u^2\ du$

(ad) $\frac{1}{5}\tan^5x+\frac{1}{7}\tan^7x+C$. Even power of secant: $\int\tan^4x\sec^4x\ dx=\int\tan^4x\sec^2x\sec^2x\ dx$
$=\int\tan^4x(\tan^2x+1)\sec^2x\ dx=\int u^4(u^2+1)\ du$, for $u=\tan x$.

11. $f^{-1}(x)=\frac{(\ln x)-5}{3}$

12. $(f^{-1})'(2)=\frac{1}{f'(0)}=\frac{1}{7}$.

13. (a) III (b) II