Some solutions to the worksheet for week 15

1. $\int x^{1.4} + 9x^{3.5}$ dx

   1) Apply the sum and power rule to the indefinite integral

$$\frac{x^{1.4+1}}{1.4+1.0} + 9\frac{x^{3.5+1.0}}{3.5+1.0}$$

   2) Simplify the answer and plus C (for an arbitrary constant) since its indefinite

$$\frac{x^{2.4}}{2.4} + 9\frac{x^{4.5}}{4.5} + C$$

2. $\int \sqrt[8]{x^9}$ dx

   1) Rewrite the root into fractional exponents

$$\int x^{\frac{8}{9}} dx$$

   2) Apply the power rule to find the anti-derivative

$$\int \frac{x^{\frac{8}{9}+1}}{8/9+1}$$

   3) Simplify the answer and plus C to it

$$\frac{9x^{\frac{17}{9}}}{17} + C$$

3. $\int_0^2 (2x-9)(8x^2+7)dx = -226$

   1) Expand the given by foiling

$$\int_0^2 16x^3 - 72x^2 + 14x - 63dx$$

   2) Then apply the sum and power rule to find the antiderivatives

$$4x^4 - 24x^3 + 7x^2 - 63x\Big|_0^2$$

   3) Then find the difference between the given boundaries

$$64 - 192 + 28 - 126 - 0 = -226$$

4. $\int_{-1}^2 (x - 6|x|)dx = -\frac{27}{2}$

1) Apply the sum rule to find the antiderivatives
$\int_{-1}^{2} x = \frac{3}{2}$

2) -6 $\int_{-1}^{2} |x|$
Given that we want positive values based from the absolute value sign we will rewrite the integral as:
-6 [(-$\int_{0}^{-1}$ x dx) + ($\int_{0}^{2} xdx$)] = -6 [$\frac{5}{2}$] = -15

3) Combine the two:
$\frac{3}{2} - 15 = \frac{-27}{2}$

5. $\int x^2\sqrt{x^3+11}dx = \frac{2}{9}(x^3+11)^{3/2} + C$

1) take a u sub with : $u = (x^3+11)$; $du = 3x^2dx$; Rewrite du to match dx $\frac{du}{3} = x^2dx$

2) Find the antiderivative at the u sub $\int \sqrt{u}du$

3) Evaluate the integral and rewrite the outcome in terms of x

6. $\int(\sin^2(\theta)\cos(\theta))d\theta = \frac{sin^3(\theta)}{3} + C$

1) take a u sub at u = $\sin(\theta)$; d$\theta = \cos(\theta)$

2) rewrite the integral: $\int u^2du$ and evaluate it

7. $\int \sec^2(\theta)\tan^8(\theta)d\theta = \frac{1}{9}\tan^9(\theta) + C$

1) take u sub at u = $\tan(\theta)$; du = $\sec^2(\theta)d\theta$

2) evaluate the integral as: $\int u^8du$

3) once evaluate rewrite the antiderivative in terms of θ

8. $\int x^3\sqrt{x^2+27}dx = \frac{1}{5}(x^2+27)^{5/2} - 9(x^2+27)^{3/2} + C$

1) expand x^3 as x^2x to $\int xx^2\sqrt{x^2+27}dx$

2) take a u sub at u = x^2+27; $\frac{du}{2} = x$

3) rewrite the integral: $\int \frac{1}{2}(u-27)\sqrt{u}du$

4) evaluate the integral: $\frac{1}{2}\int u^{3/2} - 27u^{1/2}du$

5) rewrite the outcome in terms of x instead of u's