Solution

Math 241 Quiz 02

This quiz contains 07 questions. Answer at least 04 questions.

1. Suppose $x^2 + y^2 + z^2 = 9$, $\frac{dx}{dt} = 5$, and $\frac{dy}{dt} = 4$. Find $\frac{dz}{dt}$ when $x = 2, y = 2$ and $z = 1$.

$(x^2+y^2+z^2)' = (9)'$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} = 0$

$2z\frac{dz}{dt} = -2x\frac{dx}{dt} - 2y\frac{dy}{dt}$

$2(1)\frac{dz}{dt} = -2(2)(5) - 2(2)(4)$

$2\frac{dz}{dt} = -20 - 16$

$2\frac{dz}{dt} = -36$

$\boxed{\frac{dz}{dt} = -18}$

2. Find the linearization of the function $f(x) = x^3 - x^2 + 3$ at $a = -2$.

$f'(x) = 3x^2 - 2x$

$f(a) = f(-2) = -8 - 4 + 3 = -9$

$f'(a) = f'(-2) = 3(-2)^2 - 2(-2) = 12 + 4 = 16$

$f(x) = 16(x+2) + -9)$

$f(x) = 16x + 23$

3. The height in meters of a projectile shot vertically upward from a point 2 m above ground level with an initial velocity of 24.5 m/s is $h = 2 + 24.5t - 4.9t^2$ after t seconds.

(a) Find the velocity after 2 s and 4 s.

$h \leftarrow$ position

$h' \leftarrow$ velocity

$h' = 24.5 - 9.8t$

$h'(2) = 24.5 - 9.8(2) = \boxed{4.9 m/s}$

$h'(4) = 24.5 - 9.8(4) = \boxed{-14.7 \ m/s}$

(b) When does the projectile reach its maximum height?

$f'(t) = 0$ give max

$24.5 - 9.8t = 0$

$\boxed{t = 2.5\text{s}}$

(c) What is the maximum height?

$h = 2 + 24.5(2.5) - 4.9(2.5)^2$

$= \boxed{32.625}$ m

(d) When does it hit the ground?

$h = 0$ cause ground

$2 + 24.5t - 4.9t^2 = 0$

use quadratic formula

$\boxed{t = 5.08}$ because $t > 2.5$

(e) With what velocity does it hit the ground?

$24.5 - 9.8(5.08) = -25.3$

$\boxed{-25.3 \text{ m/s}}$

4. Find the absolute maximum of the function $f(x) = 5 + 54x - 2x^3$ in the interval $[0, 4]$.

End points

$f(0) = 5$

$f(4) = 93$

critical points

$f'(x) = 0$

$f'(x) = 54 - 6x^2$

$x = \pm 3$, but -3 is not in $[0, 4]$

so

$f(3) = 113$

$\therefore$ absolute max: $f(3) = 113$ or $(3, 113)$

5. Suppose f is differentiable everywhere and $3 \leq f'(x) \leq 5$ for all x. Show that $18 \leq f(8) - f(2) \leq 30$. *Hint: Use Mean Value Theorem.*

Because f(x) is differentiable everywhere its continuous everywhere. Therefore, it is continuous on [2,8], and differentiable on (2,8). Thus by MVT there is an arbitrary element d within [2,8] such that

$$f'(d) = \frac{f(8) - f(2)}{8 - 2}$$

so $3 \leq f'(c) \leq 5$ implies $3 \leq \frac{f(8) - f(2)}{6} \leq 5$.

6. Consider the function $f(x) = x^3 - 12x + 2$. Find the interval(s) in which the function is increasing. Also, find the interval(s) in which the function is decreasing.

$f'(x) = 3x^2 - 12$

$0 = 3x^2 - 12 = 3(x^2 - 4) = 3(x+2)(x-2)$

$\boxed{\text{Decreasing } (-2, 2) \quad \text{Increasing } (-\infty, -2) \cup (2, \infty)}$

7. Find each of the following limits if it exists.

1.

$$\lim_{x \to -\infty} \frac{x - 2}{x^2 + 1}$$

$\frac{x^1}{x^2}$ $2 > 1$

when the exponent of the denominator is greater $\lim_{x\to\infty} = 0$

$\boxed{0}$

2.

$$\lim_{x \to \infty} x \sin(\frac{1}{x})$$

Assuming you do not know what L'Hopital Rules is.

The limit interval can be rewritten as:

$$\lim_{x \to \infty} \frac{\sin(\frac{1}{x})}{\frac{1}{x}}$$

We know that as x approaches infinity to 1/x we get 0.

So we can rewrite or manipulate the limit as:

$$\lim_{\frac{1}{x} \to 0} \frac{\sin(\frac{1}{x})}{\frac{1}{x}}$$

For every 1/x replace that with z.

$$\lim_{z \to 0} \frac{\sin(z)}{z} = 1$$

Final Answer is 1.

$\sim END \sim$