

Answer the questions and show all work clearly. No calculator or notes allowed.

1. Evaluate the following

(a)

$$\log_7^{49}$$

(b)

$$\log_{1/2}^4$$

(c)

$$\ln(\sqrt{e})$$

2. Differentiate the following functions. No need to simplify.

(a) $g(r) = \sec(e^{\sqrt{r}})$

(b) $y = \log_2(x^2 - 4)$

(c) $h(x) = x^{\ln(x)}$

(d) $j(t) = \cos(e^{\sin(t)})$

(e) $y = \ln(2xe^x)$

(f)

$$z = \arcsin(e^{-x})$$

3. Due to environmental changes, the population of a certain species of ant is decreasing at a rate proportional to its size. If the relative decay is 10%, in how many years will the population be half of its current value? Leave your answer unsimplified.

4. Evaluate the following integrals:

(a)

$$\int x^2 \sin(x) dx$$

(b)

$$\int_0^{\ln(2)} \frac{e^x}{1 + e^x}$$

(c)

$$\int x \ln(x) dx$$

(d)

$$\int e^x (1 + e^x)^3 dx$$

(e)

$$\int \tan^4(x) \sec^4(x) dx$$

(f)

$$\int \frac{dx}{(9 - x^2)^{3/2}} dx$$

5. Suppose you invest 500 dollars at a 7% interest. If the interest is compounded continuously, calculate how many years must pass in order for the investment to be value 1500 dollars. Leave your answer unsimplified.
6. Evaluate the following limits. Remember to use proper notation and to indicate if you are using L'Hopital's rule

(a)

$$\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^3}$$

(b)

$$\lim_{x \rightarrow \infty} \arccos\left(\frac{x^3 - 2}{x^3 + 1}\right)$$

(c)

$$\lim_{x \rightarrow \infty} \arctan(e^x)$$

(d)

$$\lim_{x \rightarrow 0} \frac{2 - 2 \cos(x)}{e^x - x - 1}$$

(e)

$$\lim_{x \rightarrow 0} \cos(x)^{1/x^2}$$

7. Let $f(x) = x^2 - 2x - 8, x > 1$. Find the value of $f^{-1}(x)$ at $x = 0 = f(4)$.
8. The half life of Polonium 210 is 140 days. How much sample of 200 mg will be left after 1 year (365 days)?
9. Find the $f^{-1}'(a)$ for the following functions.
- (a) $f(x) = x^3 + 3 \sin(x) + 2 \cos(x)$, $a = 2$
- (b) $f(t) = \sqrt{t^3 + 4t + 4}$, $a = 3$
10. Suppose g is an increasing function such that $g(2) = 8$ and $g'(2) = 5$. calculate $g^{-1}'(2)$