Solution by Shin

1. Determine whether the statement is true or false. If the statement is false explain why.

   a) If the f has an absolute maximum value at z, then the $f'(z) > 0$

   (TRUE / FALSE)

   False because absolute value would be at f'(z) = 0 or where f'(z) DNE

   b) The function $f(x) = 2x(x+4)^3$ has inflection points at: (0,0) and (4,0)

   (TRUE / FALSE)

   Take the second derivative of the function to find the inflection point and set f''(x) = 0 to find the inflection point.

   $f'(x) = 2(x+4)^3 + 2x \cdot 3(x+4)^2$

   $(x+4)^2(2(x+4) + 6x)$

   $(x+4)^2(8x+8)$

   $f''(x) = 0 = 2(x+4)(8x+8) + (x+4)^2(8)$

   $0 = (x+4)(16x+16+8x+32)$

   $0 = (x+4)(24x+48)$

   $x = -4, -2$

2. Find two negative numbers that add up to -50 whos product is large as possible.

   Optimization problem

   $x_1 x_2 = \text{max value}$

   $-x_1 + -x_2 = -50$

   $x_1 = 50 - x_2$

   $50x_2 - x_2^2 = \text{max}$

   $\frac{d}{dx}[-50x_2 - x_2^2]$

   $-50 - 2x_2 = 0$

   $\boxed{x_2 = -25 \quad x_1 = -25}$

3. Find the local extrema and the intervals where the following function is increasing or decreasing: $f(x) = x^{\frac{2}{3}}(x-10)$

   Find the derivative and set the f'(x) = 0 and find all the x where f'(x) = 0

   (x

   $f(x) = x^{5/3} - 10x^{2/3}$

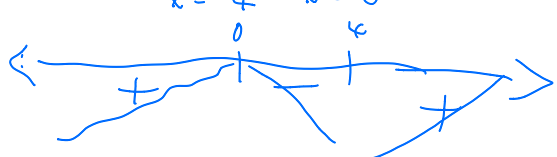
   $f'(x) = \frac{5}{3}x^{2/3} - \frac{20}{3}x^{-1/3}$

   $\frac{5x^{2/3}}{3} - \frac{20}{3x^{1/3}}$

   $f'(x) = \frac{5x^{3/3} - 20}{3x^{1/3}}$

   $f'(x) = 0$ & $f'(x) = \text{DNE}$

   $x = 4 \quad x = 0$



   $f(0) = 0$

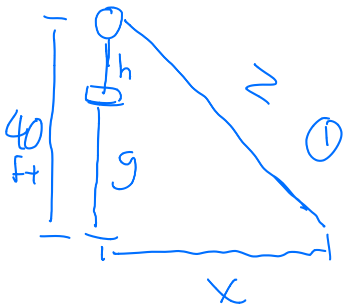
   $f(4) = (4)^{2/3}(-6) \approx -15.11$

   increase @ $(-\infty, 4) \cup (4, \infty)$

   decrease @ $(0, 4)$

   local mx: $(0, 0)$ local min: $(4, -15.11)$

4. A piano is suspended by a 90 ft rope through a pulley system that is vertically 40 ft above a man's arm. The piano is at some height above the ground. At t = 0, the man is 30 ft horizontally from the piano and walks away at 12 ft/s. How fast is the piano being pulled up?



$h + z = 90$

$h + g = 40$

$x^2 + y^2 = z^2$

① Find z

$t = 0$

$x = 30 \quad y = 40$

$30^2 + 40^2 = 50^2$

$z = 50$

② Take the derivative

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dy}{dt} = 0$ since constant

$\cancel{2}x\frac{dx}{dt} = \cancel{2}z\frac{dz}{dt}$

$\frac{x}{z}\frac{dx}{dt} = \frac{dz}{dt}$

$\frac{30}{50}(12)$

$\boxed{\frac{36}{5}\ \frac{ft}{s}}$

5. Determine the critical numbers of the following functions:

a)

$$V(t) = 1 + 80t^3 + 5t^4 - 2t^5$$

$V'(t) = 240t^2 + 20t^3 - 10t^4 = 0$

$-10t^2(t^2 - 2t - 24) = 0$

$-10t^2(t-6)(t+4) = 0$

$\boxed{c = 0,\ 6,\ -4}$

b)

$$Q(t) = (2 - 8x)^4(x^2 - 9)^3$$

$Q'(t) = 32(2-8x)^3(x^2-9)^3 + 6x(x^2-9)^2(2-8x)^4$

$0 = -4(2-8x)^3(x^2-9)^2[20x^2 - 3x - 72]$

$\boxed{x = \frac{1}{4},\ \pm 3,\ \frac{3 \pm \sqrt{5769}}{40}}$

6. Use linear approximation to find the approximate value of sin(122°)
HINT Re-express degrees in terms of radians by using 120°

$\sin(122°)$

$120° \Rightarrow \frac{3\pi}{2}$

$2° \Rightarrow \frac{\pi}{90}$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$

Linear Approx.

$f(x) = f'(a)(x-a) + f(a)$

$f(x) = \cos(\frac{3\pi}{2})(x - \frac{3\pi}{2}) + \sin(\frac{3\pi}{2})$

$\boxed{f(x) \approx \frac{\sqrt{3}}{2} + (\frac{\pi}{90})(-\frac{1}{2})}$

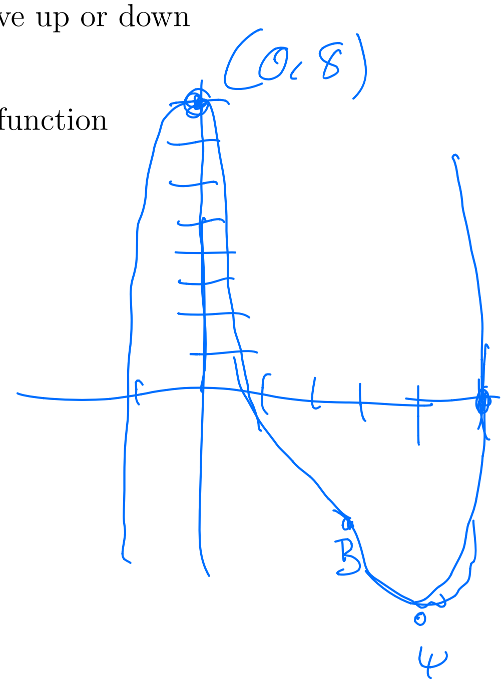
7. For the following functions answer each of the following

- identify the critical points of the function and classify them as local maximum, local minimum, or neither
- identify the intervals on which the function is increasing/decreasing
- determine the interval on which the function is concave up or down
- determine the inflection points of the function
- use the information found to sketch the graph of the function

a)

$$g(t) = t^5 - 5t^4 + 8$$

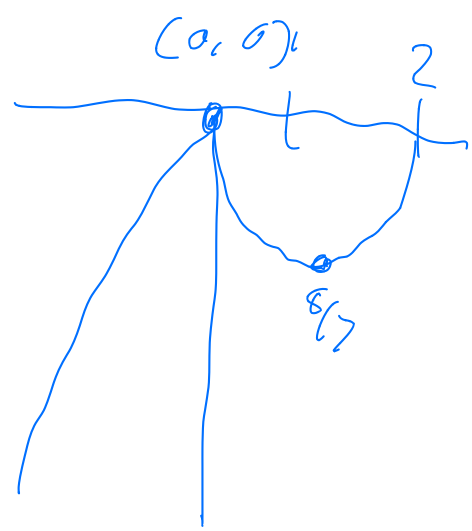
critical # $t=0$ $t=4$
local max @ $t=0$
local min @ $t=4$
Increase $(-\infty, 0) \cup (4, \infty)$
Decrease $(0, 4)$
Inflection: $t=3$
concave up $(3, \infty)$
concave down $(-\infty, 0)$ & $(0, 3)$



b)

$$x^{4/3}(x-2)$$

Critical # : $x = 0$ & $x = 8/7$
Increase $(-\infty, 0) \cup (\frac{8}{7}, \infty)$
Decrease $(0, \frac{8}{7})$
Local max : $x = 0$
Local min : $x = 8$
inflection # : $x = 2/7$
concave up $(\frac{2}{7}, \infty)$
concave down $(0, \infty)$



8. A car is heading "away" from the intersection at 5 m/s. And a bus is heading "towards" the intersection on the other street (at a right angle) at 4 m/s. How fast is the distance changing when the car is is 30 m away from the intersection and the bus is 40 m from the intersection? Additionally, is the distance increasing or decreasing?

$$x^2 + y^2 = z^2$$

$$30^2 + 40^2 = 50^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2(30)(5) + 2(40)(-4) = 2(50)\left(\frac{dz}{dt}\right)$$

$$300 + -320 = -20$$

$$\boxed{\frac{dz}{dt} = -20\ \frac{m}{s}}$$

9. Find the horizontal asymptote(s) of the following functions:

a)

$$\frac{8-4x^2}{9x^2+5x}$$

$$\lim_{x\to\infty} \frac{9-4x^2}{9x^2+5x} = \boxed{\frac{-4}{9}}$$

b)

$$\frac{\sqrt{7+9x^2}}{1-2x} = g(x)$$

$$\lim_{x\to\infty} g(x) = -\frac{3}{2}$$

$$\lim_{x\to-\infty} g(x) = \frac{3}{2}$$

10. Compute the differential dy of the following functions:

a)

$$f(x) = x^2 - \sec(x)$$

$f'(x) = 2x - \sec(x)\tan(x)$

$\boxed{df = 2x - \sec(x)\tan(x)\,dx}$

b) Compute the dy and $\triangle y$ for $y = x^5 - 2x^3 + 7x$ as x changes from 6 to 5.9

$\frac{dy}{dx} = 5x^4 - 6x^2 + 7$

$dy = 5x^4 - 6x^2 + 7\,dx$

$dy = 5(6)^4 - 6(6)^3 + 7\,(5.9 - 6.0)$

$\boxed{dy = -627.1}$

11. Multiple Choice. Read each question and answer choice carefully and choose the ONE best answer.

a) A right cylindrical cone has a radius of 4 cm and a height of 2.0 cm. If the height increases at 0.5 cm/min, but the radius remains constant, then what will be the rate of change of the volume?

A) 8.4 cm^3/min
B) 1.1 cm^3/min
C) 4.2 cm^3/min
D) 2.1 cm^3/min

$V = \frac{1}{3}\pi r^2 h$

$r = 4 \quad h = 2.0 \quad \frac{dh}{dt} = 0.5$

$\frac{dr}{dt} = 0 \leftarrow$ const

$$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt}$$

$$\frac{1}{3}(\pi)(4)^2(0.5)$$

input to calculator

$\boxed{\frac{dV}{dt} \approx 8.4}$

b) Given that the cost function $C(x) = 144 + 0.1x + 0.04x^2$, what is the minimum average cost per unit?

C(x) ← gives cost not cost per unit
so divide x to all sides

A) 20 dollars
B) 40 dollars
C) 60 dollars
D) 80 dollars

$$Z(x) = \frac{C(x)}{x} = \frac{144}{x} + 0.1 + 0.04x$$

$$Z'(x) = -\frac{144}{x^2} + 0.04 = 0$$

$$x^2 = \frac{144}{0.04} = 3600$$

$$\boxed{x = 60}$$

c) Find the approximate value of $(5.2)^3$ using linear approximation

$f(x) = x^3 \quad f'(x) = 3x^2$

$a = 5$

$x = 5.2$

$f(5.2) = f'(a)(x-a) + f(a)$

$f(5.2) = \underset{3(5)^2}{75}(5.2 - 5.0) + 125 = 15 + 125 = 140$

A) 130
B) 140
C) 150
D) 160

12. Determine the number(s) c that satisfies the conclusion of the Mean Value Theorem for the given function and interval.

$$f(z) = 4z^3 - 8z^2 + 7z - 2$$

on the interval [2,5]

$f(5) = 333$

$f(2) = 12$

$f'(z) = 12z^2 - 16z + 7$

$$\frac{333 - 12}{5 - 2} = \frac{321}{3} = 107$$

$f'(c) = 107 = 12z^2 - 16z + 7$

$12z^2 - 16z - 100$

13. Answer the following questions with the given graph:

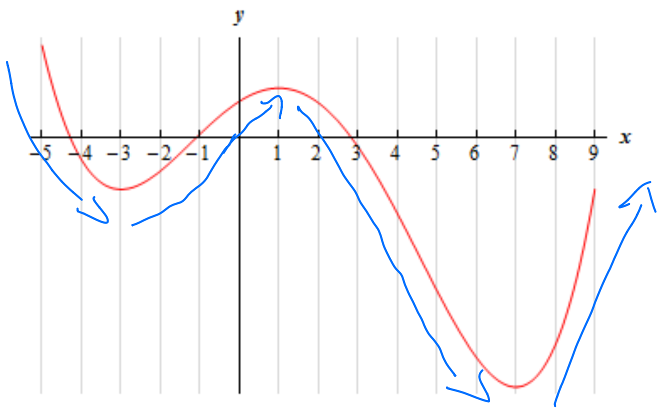


Figure 1: graph of f(x)

a) Write down the interval in which the graph is increasing:

$(-3, 1) \cup (7, \infty)$

b) Write down the interval in which the function is decreasing:

$(-\infty, -3) \cup (1, \infty)$

14. Use Newton's method to determine x_2 for $= x^3 - 7x^2 + 8x - 3$ if $x_0 = 5$

~~**SOLUTION** $3x^2 - 14x + 8$~~

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 5 - \frac{-13}{13} = 6$

$x_2 = 6 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{f(6)}{f'(6)} = 6 - \frac{9}{32} =$

$\frac{192 - 9}{32} = \frac{181}{32}$

$\boxed{\frac{183}{32}}$

15. Sketch a graph with the following information:

$$f(4) = 0, f'(2) = 0, f"(4) = 0 \lim_{x\to\infty} f(x) = 1 \lim_{x\to-\infty} = -\infty$$

Vertical Asymptote at x = 0, $f'(x) > 0$ for $x < 0 \;\; x > 2$
$f'(x) < 0$ for $0 < x < 2$
$f''(x) > 0$ for $x < 0$ and for $0 < x < 4$; $f''(x) < 0$ for $x > 4$.