MATH 241 RECITATION JANUARY 15, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Evaluate the following limits.

   (a) $\lim_{x \to -2} \dfrac{x^3 + 6x^2 - 2}{x^2 - 1}$

   (b) $\lim_{x \to 4} \dfrac{x^2 + 6x - 40}{x^3 - 64}$

   (c) $\lim_{x \to 9} \dfrac{\sqrt{x+7} - 4}{3x - 27}$

   (d) $\lim_{x \to 4} \dfrac{4 - x}{5 - \sqrt{x^2 + 9}}$

   (e) $\lim_{x \to 3} \dfrac{x^4 - 81}{x - 3}$

2. Using the Squeeze Theorem, evaluate $\lim_{x\to 0} x^3 \cos\left(\frac{1}{x^2}\right)$

3. Given $f(x) = \begin{cases} \frac{x}{x-2} & x \neq 4 \\ 4 & x = 4 \end{cases}$, evaluate the following limits.

   (a) $\lim_{x\to 2^-} f(x)$

   (b) $\lim_{x\to 2^+} f(x)$

   (c) $\lim_{x\to 2} f(x)$

   (d) $\lim_{x\to 4^-} f(x)$

   (e) $\lim_{x\to 4^+} f(x)$

   (f) $\lim_{x\to 4} f(x)$

The End.

MATH 241 Recitation January 22, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Given the function

$$f(x) = \begin{cases} x^2 + 2x + 1 & x < 1 \\ -\dfrac{x}{2} & 1 \leq x < 2 \\ \sqrt{x-2} - 1 & x \geq 2 \end{cases}$$

   (a) Is this function continuous at $x = 1$? Explain why or why not.

   (b) Is this function continuous at $x = 2$? Explain why or why not.

2. Given the function

$$f(x) = \begin{cases} -x & x < -1 \\ (x+1)^2 + 1 & x > -1 \end{cases}$$

   Is the function continuous at $x = -1$? Explain why or why not.

3. Show that there are three solutions to $x^3 - 7x + 1 = 0$ in the interval $[-3, 3]$.

4. Using the limit definition of the derivative, find the derivative of $f(x) = \sqrt{x}$.

MATH 241 RECITATION JANUARY 29, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Given the function $f(x) = \dfrac{1}{x^2}$, find the equation of the tangent line at the point $(1, 1)$.

2. Let $f(x) = 2x^4 + 3x^3 - 3x$. Find the equation of the tangent line at the point $(1, -2)$.

3. Find the derivatives of the following:

   (a) $f(x) = (3x^3 - 2x^{-2} + 5x) \cdot (4x^3 - 5x + 7)$

   (b) $f(x) = \dfrac{x^2 + 4x}{x - 4}$

   (c) $f(x) = \dfrac{12 - 6x^{3/2}}{x^3 - 2x}$

   (d) $f(x) = (x^7 + 3x^5 - 5x^3 + x)\sqrt{x}$

   (e) $f(x) = \dfrac{4x^3 - 7x + 5}{2x^2}$

   (f) $f(x) = (4x^3 + 7x^2 - 12x) \cdot (3x^5 - 4x - 2)$

4. An object is dropped from the top of the Empire State Building. Its height after t seconds is given by $s(t) = 1250 - 4.9t^2$ meters.

   (a) What is the object's position after 2 seconds?

   (b) How fast is the object falling after 2 seconds?

   (c) What is its acceleration after 2 seconds?

The End.

MATH 241 RECITATION FEBRUARY 5, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Evaluate the following limits

   (a) $\lim_{x\to 0} \dfrac{\sin(3x)}{4x}$

   (b) $\lim_{x\to 0} \dfrac{5x}{\sin(8x)}$

   (c) $\lim_{x\to 0} \dfrac{\sin(2x)}{\sin(4x)}$

   (d) $\lim_{x\to 0} \dfrac{1-\cos^2(x)}{x}$

   (e) $\lim_{x\to 0} \dfrac{\frac{1}{\cos(x)} - 1}{x}$

2. Find the derivative of the following

   (a) $f(x) = x^3\sqrt{1+\sqrt{x}}$

   (b) $g(x) = \sqrt[3]{\sin(3x^3+4x^2-7x)}$

(c) $f(x) = \dfrac{\cos^2(x^3)}{\sin(x^2+1)}$

(d) $h(x) = (\sqrt{x^3 - 2x^2})(\tan(x^3))$

3. Use implicit differentiation to find $\dfrac{dy}{dx}$

(a) $x^{3/5} + y^{4/5} = 3xy$

(b) $xy^2 + \tan(y^3) = x^2y$

(c) $\cos(y^2+1) + \sin(x^2-1) = xy$

(d) $y^3 = \dfrac{3x-1}{3y+1}$

The End.

MATH 241 RECITATION FEBRUARY 12, 2019

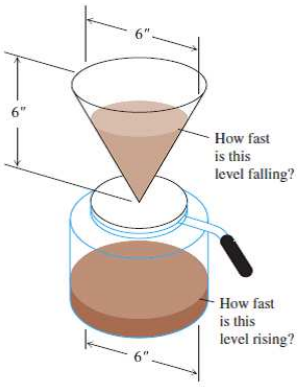
Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. A snowball is placed under the sun. Its radius decreases at a rate of $\frac{1}{2}$ inch an hour. Find the rate of change of its volume at the instant the radius is 3 inches. Volume of sphere: $V = \frac{4}{3}\pi r^3$.

2. A person flies a kite at a height of 300 feet. The wind carries the height, horizontally, at a rate of 25 feet per second. What is the rate of change of the length of the kite string (the distance between the person and kite), when the kite is 500 feet away from the person?

3. Coffee is draining from a conical filter into a cylindrical coffeepot at a rate of 10 in^3/min. Volume of a cone: $V = \frac{1}{3}\pi r^2 h$ Volume of a cylinder: $V = \pi r^2 h$



(a) How fast is the coffee rising in the pot when the coffee in the cone is 5 in deep?

(b) How fast is the level in the cone falling then?

The End.

MATH 241 RECITATION FEBRUARY 19, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Use a linear approximation to estimate the value of the following.

   (a) $\sqrt[3]{7.99}$

   (b) $\dfrac{2}{3.002}$

   (c) $\sin(\dfrac{\pi}{7})$

2. Find the critical points of $f(x) = \cos(x), \quad [0, 2\pi]$

3. Find the critical points of $g(x) = 1 + 80t^3 + 5t^4 - 2t^5$

4. Find the global maximum and global minimum of the following in the given interal.

(a) $f(x) = \frac{1}{3}x^3 + x^2 - 8x + 3, \ [0, 3]$

(b) $g(x) = 2\sin(x) - \sqrt{3}x, \quad [0, \frac{\pi}{2}]$

(c) $h(x) = -x^3 + 3x^2 - 2, \quad [-2, 2]$

(d) $f(x) = x^{1/3}, \quad [1, 8]$

The End.

MATH 241 RECITATION FEBRUARY 26, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Let $f(x) = \dfrac{x^2+4}{8x}$

   (a) Find the local maxima and local minima of the given function.

   (b) Using intervals, describe the concavity of $f(x)$.

2. Find the inflection points of $h(x) = 2\sin(x) - \sqrt{3}x$ in the interval $[0, 2\pi]$

3. Find the inflection points of $f(x) = x^4 - 2x^3$ and describe the concavity of $f(x)$.

4. Let $g(x) = x^5 - 5x^4$.

   (a) Find all local maxima and local minima.

   (b) Describe the intervals of increase and decrease of $g(x)$

   (c) Find the inflection points and describe the concavity of $g(x)$.

5. Show that $f(x) = x^5 + 10x^3 - 1$ has *exactly* one root in the interval $[0, 1]$

The End.

MATH 241 RECITATION MARCH 5, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Evaluate the following limits

   (a) $\lim_{x\to\infty} \dfrac{\sqrt{1+4x^6}}{x^3}$

   (b) $\lim_{x\to\infty} \dfrac{2x^3-7x^2+9x-12}{3x^3+4}$

   (c) $\lim_{x\to-\infty} \dfrac{x^4+4x^3-7x^2+1}{3x-4}$

   (d) $\lim_{x\to\infty} \sqrt{9x^2+4x}-3x$

2. Sketch the following functions. Label local maxima and local minima where applicable.

   (a) $f(x) = \dfrac{1}{x^2 - 4}$

   (b) $f(x) = \dfrac{x^2 - 4}{2x^2}$

The End.

MATH 241 RECITATION MARCH 12, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. A rectangular garden is to be constructed using a rock wall on one side of the garden and some fencing on the other three sides. I have 200 feet of fencing to work with. What are the dimensions I should use to maximize area?

2. A rectangle is inscribed in a semicircle of radius 2. What is the largest area the rectangle can have and what are its dimensions? Recall the equation of a semicircle, $\sqrt{4-x^2}$.

3. Find the point (x, y) on the graph $y = \sqrt{x}$ that will minimizes the distance to the point $(4, 0)$. Recall the distance formula: $\sqrt{(x-x_0)^2 + (y-y_0)^2}$

4. A cylindrical can holds 20π m^3. The material for the top and bottom of the can costs \$10 per m^2 and the material for the side of the can costs \$8 per m^2. Find the radius and height that minimizes the cost. Recall the volume of a cylinder: $V = \pi r^2 h$.

5. We want to build a box whose base length is 6 times the base width and the box will enclose 20 in^3. The cost of the material of the sides is \$3 per in^2 and the cost of the top and bottom is \$15 per in^2. Determine the dimensions of the box that will minimize the cost.

The End.

MATH 241 RECITATION APRIL 2, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Let $f(x) = x^2+1$. Using Riemann Sums (partitions using rectangles) where $n = 4$, find the approximate area under the curve on the interval $[0, 4]$ using

   (a) Left Endpoints

   (b) Right Endpoints

   (c) Midpoints

2. Using the Fundamental Theorem of Calculus, find the derivatives of the following

   (a) $g(x) = \int_2^x (2t^3 - 9\sqrt{t} + \cos(t))^2 dt$

(b) $h(x) = \int_x^5 3t\cos(t)dt$

(c) $f(x) = \int_0^{x^2} \cos(t)dt$

3. Calculate the following integrals

(a) $\int_0^4 (x^2+1)dx$

(b) $\int_1^2 (4x^3 - 3x^2 + x - 7)dx$

(c) $\int_0^1 \frac{2}{3}x^2 - 8x - \pi \; dx$

(d) $\int_1^5 2 \; dx$

(e) $\int \cos(x) - \sin(x)dx$ *Hint: Be careful. What type of integral do you have? Definite or indefinite?*

The End.

MATH 241 RECITATION APRIL 9, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Use an appropriate u-substitution to calculate the following

   (a) $\int_{-2}^{2} (3x^2 - 4x)(x^3 - 2x^2)^3 \, dx$

   (b) $\int \frac{5x - 3}{\left(\frac{15}{2}x^2 - 9x\right)^{5/3}} \, dx$

   (c) $\int \frac{\sin(\sqrt{x})}{\sqrt{x}} \, dx$

   (d) $\int_{1}^{5} \frac{2x^3 + x}{(x^4 + x^2 + 1)^2} \, dx$

2. Find the area of the region enclosed by the following curves.

   (a) $y = 2 - x^2$ and $y = -x$

   (b) $y = x^4 - 4x^2 + 4$, $y = x^2$, and the line $x = 0$. *Hint: You will have to split this into two integrals*

3. Using the *disk* method, find the volume of region bounded by $y = \dfrac{1}{x}$, $y = 0$, $x = 1$, and $x = 4$, rotated about the x-axis.

The End.

MATH 241 RECITATION APRIL 16, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Find the resulting volumes of the region bounded by the given curves

   (a) $y = \sqrt{x}$ bounded by the line $x = 4$ and the x-axis, rotated about the y-axis

   (b) $y = x^3$ bounded by the lines $x = 1$, $x = 2$, and $y = 0$, rotated about the y-axis

   (c) $y = \sqrt{x}$ and $y = x$, rotated about the y-axis

(d) $y = x^2 + 2$, $y = -x + 2$ and the line $x = 1$, rotated about the y-axis

2. Repeat (c) except now, rotate about the line $x = -2$.

3. Repeat (d) except now, rotate about the line $x = 3$.

4. Find the resulting volume when $x = (y - 2)^2$ and $y = x$ is rotated about the x-axis.

The End.

MATH 241 RECITATION APRIL 23, 2019

Name: _____ **Section (circle one): 3 4**

Complete the following problems in small groups. Please show all your work! Julian and I will come around if you need any assistance.

1. Evaluate the following limits

   (a) $\lim_{x \to 2^-} \dfrac{|x-2|}{x-2}$

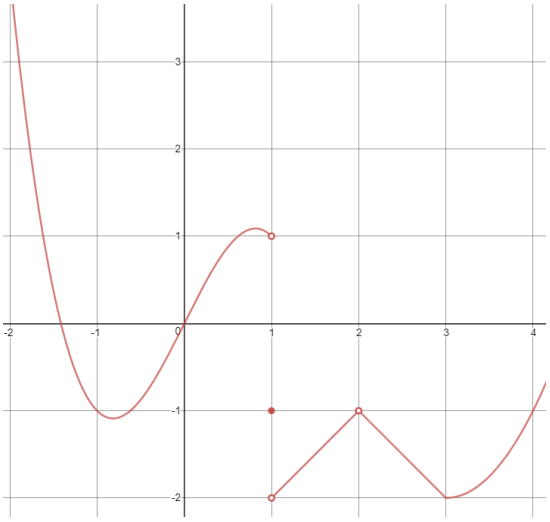
   (b) $\lim_{x \to 1} \dfrac{\sqrt{x^2+1} - \sqrt{2}}{x-1}$

   (c) $\lim_{x \to 0} \dfrac{\sin(2x)}{3x}$

   (d) $\lim_{x \to \infty} \dfrac{\cos(3x)}{x^2} + 2$ *Hint: Squeeze Theorem*

2. Is $f(x)$ continuous at $x = 2$ if $f(x) = \begin{cases} 8x - 10 & x \leq 2 \\ 4x^2 - 5x & x > 2 \end{cases}$?

3. Is $f(x)$ continuous at $x = 1$? What about $x = 2$? Is $f(x)$ continuous at $x = 3$? Is $f(x)$ differentiable at these points?



4. Given $2x^2y + xy^2 = 3x - y$, find dy/dx.

5. Approximate 21^2 using linear approximation. *Hint: A linear approximation should look like* $f'(c)(x - c) + f(c)$.

6. Find all local max and inflection points of $f(x) = 2x^3 - 3x^2$. Describe the behavior of the graph (increase, decrease, and concavity).

The End.