Direction Read each question and evaluate the answers. Try doing your work on a seperate piece of paper.

1. A spherical balloon is being inflated. What is the volume of the sphere at the instant when the rate of increase of the surface area is four times the rate of increase of the radius of the sphere?
(~~**Solution** 8π cubic units~~ ← sorry typo)

2. A camera on the ground is 100 m away from a model rocket launch pad. When the rocket is launched, it rises at a constant velocity if 20 m/s. How fast must the camera's angle change when the rocket is 100 m high?
Solution 5.7 deg/s

3. You have 500 ft of fence to enclose your children in a rectangular area. What are the dimensions of the fence that will give you the maximum area?
Solution length = 125 ft width = 125 ft

4. Find the most general antiderivatives of the following functions: a)f'(x) = $x^4 - 3x^2 + 1$ b) g'(x) = $5\sec(x)2\tan(x)$ c) h'(x) = $(x)(x-3)^2$ d) z'(x) = $(x)(x-5)(x-2)$

5. If z is a differentiable function and z(5) = 10 and z'(5) = 2, what is the approximate value of z(5.5)?
Solution 11

6. What is the approximate value of $\cos 62°$?
Solution 0.47

7. Find the critical point(s) of the function f(x) = $4x^2 - 3x + 2$.

8. Given the cost function C(x) = $500x + 6x + 0.2x^2$ and the revenue function R(x) = 20x, how many units should be produced to have the maximum profit?
HINT Profit can be found by using R(x) - C(x)
Solution x = 35

9. A plane lifts off from a runway at an angle of 30 degrees. If the speed of the plane is 500 $\frac{mi}{hr}$, then how fast is the plane gaining altitude?
Solution 250 $\frac{mi}{hr}$

10. Find two positive numbers that add up to 30 such that the maximum product is possible.
Solution 15 & 15

11. Using Newton's Method, determine x_2 for f(x) = xcos(x)-x^2 if $x_0 = 1$.

12. Find the local maximum, local minimum, absolute maximum, absolute minimum, & inflection point(s) of the following function f(x) = $\frac{x^2+4}{8x}$

1) $S = 4\pi r^2$

$\frac{ds}{dt} = 8\pi r \frac{dr}{dt}$

Given: $\frac{ds}{dt} = 4\frac{dr}{dt}$

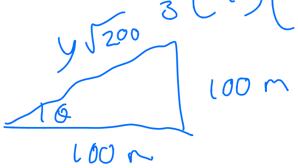
$8\pi r \frac{dr}{dt} = 4\frac{dr}{dt}$

$r = \frac{1}{2\pi}$

$V = \frac{4}{3}\pi r^3$

$dV = \frac{4}{3}(\pi)\left(\frac{1}{2\pi}\right)^3 = \frac{4\pi}{3}\cdot\frac{1}{8\pi^3} = \boxed{\frac{1}{6\pi^2}}$

2)



$\tan\theta = \frac{x}{100}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{100}\frac{dx}{dt}$

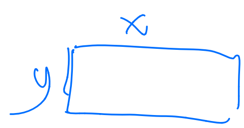
$\frac{dx}{dt} = 20$ m/s

$\frac{d\theta}{dt} = \left(\frac{1}{100}\right)\left(\frac{1}{\sec^2\theta}\right)\left(\frac{20}{1}\right)$

Find $\sec\theta = \frac{h}{a} = \frac{\sqrt{200}}{100}$

$\frac{d\theta}{dt} = \left(\frac{1}{100}\right)\left(\frac{100}{\sqrt{200}}\right)^2\left(\frac{20}{1}\right)$

3) $500 = 2x + 2y$



$xy = \text{Max}_{\text{area}}$

$-2y + 500 = 2x$

$-y + 250 = x$

$-y^2 + 250y = \text{Max Area}$

$-2y + 250 = 0$

$y = 125$

$x = 125$

4) a) $f'(x) = x^4 - 3x^2 + 1$

$f(x) = \frac{x^{4+1}}{4+1} - \frac{3x^{2+1}}{2+1} + x + C$

$f(x) = \frac{x^5}{5} - \frac{3x^3}{3} + x$

b) $g'(x) = 5\sec(x)\ 2\tan(x)$

$g'(x) = 10\sec(x)\tan(x)$

$g(x) = 10\sec(x) + C$

c) $h'(x) = (x)(x-3)^2$

$x(x^2 - 6x + 9)$

$$x^3 - 6x^2 + 9x$$

$$\int \frac{x^{3+1}}{3+1} - \int \frac{6x^{2+1}}{2+1} + \int \frac{9x^{1+1}}{1+1}$$

$$\boxed{\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2} + C}$$

d

5

$$z(x) = z'(z)(a - x) + f(z)$$

$$z(x) = 2(a - 5) + 10$$

$$2(5.5 - 5) + 10$$

$$2(0.5) + 10$$

$$= 1 + 10$$

$$= 11$$

$$\boxed{z(5.5) = 11}$$

6

$$f(x) = \cos(x)$$

$$f'(x) = -\sin(x)$$

$$60^\circ \rightarrow \frac{\pi}{3}$$

$$-\sin\left(\frac{\pi}{3}\right)\left(\frac{\pi}{90}\right) + \cos\left(\frac{\pi}{3}\right)$$

7 $f'(x) = 8x + 3$

$$0 = 8x + 3$$

$$\boxed{x = -\frac{3}{8}}$$

8

$$R(x) - C(x) = \text{Profit}$$

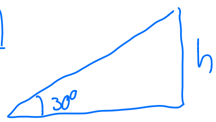
$$20x - (500 + 6x + 0.2x^2) = \text{Profit}$$

$$-500 + 14x - 0.2x^2 = \text{Profit}$$

$$14x - 0.4x = 0$$

$$x = 35 \text{ units}$$

9



$$\sin(30) = \frac{h}{500t}$$

$$h = \sin(30)\,500t$$

$$\frac{dh}{dt} = \sin(30)\,500$$

$$\frac{dh}{dt} = 0.5(500)$$

$$\boxed{\frac{dh}{dt} = 250 \text{ mi/h}}$$

10

$$x + y = 30$$

$$xy = \text{MAX}$$

$$y = 30 - x$$

$$\frac{d}{dx}[30x - x^2 = \text{MAX}]$$

$$30 - 2x = 0$$

$$2x = 30$$

$$x = 15$$

$$\boxed{x = 15 \quad y = 15}$$

11

$$f(x) = x\cos(x) - x^2$$

$$f'(x) = \cos(x) - x\sin(x) - 2x$$

$$x_0 = 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.808$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.744$$

$$\boxed{x_2 = 0.744}$$