Answer the questions and show all work clearly. No calculator or notes allowed.

1. Evaluate the following

   (a)
   $$\log_7^{49}$$

   (b)
   $$\log_{1/2}^{4}$$

   (c)
   $$\ln(\sqrt{e})$$

   (d)
   $$\arcsin(-1/2)$$

   (e)
   $$\sin^{-1}(\cos(\pi/3))$$

2. Compute the following limits. Justify the solution using algebraic manipulation or L'Hopitals rule.

   (a)
   $$\lim_{x\to\infty} \frac{\ln(x)}{e^x}$$

   (b)
   $$\lim_{x\to-\infty} x^2 e^x$$

   (c)
   $$\lim_{x\to 0} \frac{x - \tan(x)}{x - \sin(x)}$$

   (d)
   $$\lim_{x\to-\infty} \arcsin \frac{e^{-x} + \sqrt{3}}{e^{-x} + 3}$$

3. Differentiate the following functions. No need to simplify.

   (a) $g(r) = \sec(e^{\sqrt{r}})$

   (b) $y = \log_2^(x^2 - 4)$

   (c) $h(x) = x^{\ln(x)}$

   (d) $j(t) = \cos(e^{sin(t)})$

   (e) $y = \ln(2xe^x)$

   (f)
   $$z = \arcsin(e^{-x})$$

4. Due to environmental changes, the population of a certain species of ant is decreasing at a rate proportional to its size. If the relative decay is 10%, in how many years will the population be half of it's current value? Leave your answer unsimplified.

5. Evaluate the following integrals:

   (a)
   $$\int x^2 sin(x)dx$$

   (b)
   $$\int_0^{\ln(2)} \frac{e^x}{1+e^x}$$

   (c)
   $$\int x\ln(x)dx$$

   (d)
   $$\int e^x(1+e^x)^3dx$$

   (e)
   $$\int \tan^4(x)\sec^4(x)dx$$

   (f)
   $$\int \frac{dx}{(9-x^2)^{3/2}}dx$$

6. Suppose you invest 500 dollars at a 7% interest. If the interest is compounded continuously, calculate how many years must pass in order for the investment to be value 1500 dollars. Leave your answer unsimplified.

7. Evaluate the following limits. Remember to use proper notation and to indicate if you are using L'Hopital's rule

   (a)
   $$\lim_{x\to 0}\frac{\sin(x)-x}{x^3}$$

   (b)
   $$\lim_{x\to\infty}\arccos(\frac{x^3-2}{x^3+1})$$

   (c)
   $$\lim_{x\to\infty}\arctan(e^x)$$

   (d)
   $$\lim_{x\to 0}\frac{2-2\cos(x)}{e^x-x-1}$$

(e)
$$\lim_{x\to 0} \cos(x)^{1/x^2}$$

8. Let f(x) = $x^2 - 2x - 8, x > 1$. Find the value of $f^{-1}(x)$ at x = 0 = f(4).

9. The half life of Polonium 210 is 140 days. How much sample of 200 mg will be left after 1 year (365 days)?

10. Find the $f^{-1\prime}(a)$ for the following functions.

    (a) $f(x) = x^3 + 3\sin(x) + 2\cos(x)$, a = 2

    (b) $f(t) = \sqrt{t^3 + 4t + 4}$, a = 3

11. Suppose g is an increasing function such that g(2) = 8 and g'(2) = 5. calculate $g^{-1\prime}(2)$

12. A bacteria culture grows with constant relative growth rate. The bacteria count was 260 after 1 hour and 20,000 after 5 hours.

    (a) What is the relative growth rate?

    (b) What was the inital size of the culture?

    (c) Find an expression for the number of bacteria after t hours?

13. Evaluate the following integrals

    (a)
    $$\int \cot(x)dx$$

    (b)
    $$\int 2e^{2x}(\tan^2(1+e^{2x})+1)dx$$

    (c)
    $$\int 5^{\tan(x)}(\tan^2(x)+1)dx$$

    (d)
    $$\int_0^{\pi/3} \sin(x)\ln(\cos^3(x))dx$$

    (e)
    $$\int \theta^3\cos(\theta^2)d\theta$$

    (f)
    $$\int x\sec^{-1}(x)dx$$

    (g)
    $$\ln(x)xdx$$

14. The half life of a radioactive isotope is 32 days.

    (a) A sample has a mass of 35 mg initially. Find a formula for the mass remaining after t days.

    (b) Find the mass remaining after 12 days.

15. Bacteria grow at a rate proportional to its size. The count in a bacteria colony that started at 1000 was 2500 after 3 hours. How long will it take for the population to reach 10000?

16. Differentiate the following functions

    (a)
    $$f(x) = e^x \ln(arcsec(x))$$

    (b)
    $$y = (\tan^{-1}(x))^2$$

    (c)
    $$f(x) = \cos(x)^{\arctan(x)}$$

    (d)
    $$y = \arcsin(e^{-x})$$