Section 7/8 HW1

6.1 24, 26, 40, 42

6.2* 26 34 62 70

6.3* 42, 48, 56, 86

6.1

24) $f(x) = \frac{4x-1}{2x+3}$

$x = \frac{4y-1}{2y+3}$

$(2y+3)(x) = 4y-1$

$2yx + 3x = 4y-1$

$2yx - 4y = -1 - 3x$

$y(2x-4) = -1-3x$

$y = \frac{-1-3x}{2x-4}$

26) $y = x^2 - x$

$x = y^2 - y$

$0 = y^2 - y - x$ <- quadratic formula

$\frac{y \pm \sqrt{y^2 - 4(1)(-x)}}{2}$

$y = \frac{1 \pm \sqrt{1^2 + 4x}}{2}$

since $x \geq \frac{1}{2}$

then $f^{-1} = \frac{1}{2} + \frac{\sqrt{1+4x}}{2}$

40) $f(x) = x^3 + 3\sin(x) + 2\cos x \qquad a = 2$

$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$

$f(0) = 2$ so $f^{-1}(2) = 0$

$f'(x) = 3x^2 + 3\cos(x) - 2\sin(x)$

$f'(0) = 3(0) + 3\cos(0) - 2\sin(0) = 3$

$(f^{-1})'(2) = \frac{1}{3}$

42) $f(x) = \sqrt{x^3+4x+4} \qquad a = 3$

$f(1) = 3$ so $f^{-1}(3) = 1$

$f' = \frac{1}{2}(x^3 + 4x + 4)^{-\frac{1}{2}}(3x^2+4)$

$f'(1) = \frac{1}{2}(1+4+4)^{-\frac{1}{2}}(3+4)$

$f' = \frac{7}{6}$

$\frac{1}{\frac{7}{6}} = \frac{6}{7}$

$(f^{-1})'(3) = \frac{6}{7}$

6.2

26 $g(t) = \sqrt{1+\ln t}$

Chain Rule &

$g'(t) = \frac{1}{2}(1+\ln t)^{-\frac{1}{2}} \cdot \left(\frac{1}{t}\right)$

34 $y = \ln \tan^2 x$

Chain Rule, trig deri

$y' = \frac{1}{\tan^2 x} \cdot 2\tan x \cdot \sec^2 x$

62 $y = \frac{(x+1)^4 (x-5)^3}{(x-3)^8}$

$\ln y = 4\ln(x+1) + 3\ln(x-5) - 8\ln(x-3)$

$\frac{1}{y} \cdot y' = 4\left(\frac{1}{x+1}\right) + 3\left(\frac{1}{x-5}\right) - \frac{8}{x-3}$

$y' = y\left(\frac{4}{x+1} + \frac{3}{x-5} - \frac{8}{x-3}\right)$

$y' = \left(\frac{(x+1)^4(x-5)^3}{(x-3)^8}\right)\left(\frac{4}{x+1} + \frac{3}{x-5} - \frac{8}{x-3}\right)$

70 $\int_e^6 \frac{dx}{x \ln x}$

u sub: $u = \ln x \quad du = \frac{dx}{x}$

$\int \frac{du}{u}$

$\ln|u|$

$\ln|\ln(x)| \Big|_e^6$

$\ln|\ln(6)| - \ln|\ln e|$

$\ln|\ln(6)| - \ln|1|$

$\ln(\ln(6)) - 0$

16.3*

(42) $y = x^2 e^{-\frac{1}{x}}$

Product Rule: $uv = u'v + v'u$

$y' = 2xe^{-\frac{1}{x}} + x^2 e^{-\frac{1}{x}} \cdot -\ln|x|$

(48) $y = e^{\sin 2x} + \sin(e^{2x})$

Sum Rule + Chain Rule

$y' = e^{\sin(2x)} \cdot \cos(2x) \cdot 2 + \cos(e^{2x}) \cdot e^{2x} \cdot 2$

$y' = 2(\cos(2x)e^{\sin(2x)} + e^{2x}\cos(e^{2x}))$

(56) Tangent line: $y = f'(a)(x-a) + f(a)$

$a = 0 \quad f(a) = 1$

Implicit differentiation

$(e^y + xe^y \cdot y') + e^x + y'e^x = 0$

$e^1 + 0e^1 \cdot y' + e^0 + y'e^0 = 0$

$e^1 + 0 + 1 + y' = 0$

$y' = -e - 1 = -(e+1)$

$f'(a) = -(e+1)$

$y = -(e+1)(x-0) + 1$

$y = -(e+1)(x) + 1$

(86) $\int x^2 e^{x^3} dx$

u sub: $u = x^3 \quad du = 3x^2 dx \quad \frac{du}{3} = x^2 dx$

$\int e^u \frac{du}{3} = \frac{e^u}{3} + C$

$\frac{e^{x^3}}{3}$