

1. Determine whether the statement is true or false. If the statement is false explain why.

a) If the f has an absolute maximum value at z , then the $f'(z) > 0$

(TRUE / FALSE)

SOLUTION FALSE because at either maximum or minima then $f'(z) = 0$ or be undefined or its on the interior of the domain.

b) The function $f(x) = 2x(x + 4)^3$ has inflection points at: $(0,0)$ and $(4,0)$

(TRUE / FALSE)

SOLUTION FALSE because the inflection point is at $(-4,0)$ $(-2, -32)$

2. Find two negative numbers that add up to -50 whos product is large as possible.

SOLUTION $x_1 = -25$ & $x_2 = -25$

3. Find the local extrema and the intervals where the following function is increasing or decreasing: $f(x) = x^{\frac{2}{3}}(x - 10)$

SOLUTION Local max at $x = 0$, local min at $x = 4$, increasing $(-\infty, 0) \cup (4, \infty)$, decreasing $(0,4)$

4. A piano is suspended by a 90 ft rope through a pulley system that is vertically 40 ft above a man's arm. The piano is at some height above the ground. At $t = 0$, the man is 30 ft horizontally from the piano and walks away at 12 ft/s. How fast is the piano being pulled up?

SOLUTION $36/5 = 5.2$ ft/s

5. Determine the critical numbers of the following functions:

a)

$$V(t) = 1 + 80t^3 + 5t^4 - 2t^5$$

SOLUTION $t = 0, -4, 6$

b)

$$Q(t) = (2 - 8x)^4(x^2 - 9)^3$$

SOLUTION $t = \frac{1}{4}, t = 3, t = -3, t =$

6. Use linear approximation to find the approximate value of $\sin(122^\circ)$

HINT Re-express degrees in terms of radians by using 120°

SOLUTION

$$\frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)\left(\frac{\pi}{90}\right)$$

7. For the following functions answer each of the following

- identify the critical points of the function and classify them as local maximum, local minimum, or neither
- identify the intervals on which the function is increasing/decreasing
- determine the interval on which the function is concave up or down
- determine the inflection points of the function
- use the information found to sketch the graph of the function

a)

$$g(t) = t^5 - 5t^4 + 8$$

SOLUTION CRITICAL POINT: $t = 0$ and $t = 4$

CLASSIFICATION OF CRITICAL POINTS:

RELATIVE MINIMUM: 4

RELATIVE MAXIMUM: 0

INCREASING: $(-\infty, 0) \cup (4, \infty)$

DECREASING: $(0, 4)$

CONCAVE UP: $(3, \infty)$

CONCAVE DOWN: $(-\infty, 0)$ & $(0, 3)$

INFLECTION POINT: $t = 3$

b)

$$x^{4/3}(x - 2)$$

8. A car is heading "away" from the intersection at 5 m/s. And a bus is heading "towards" the intersection on the other street (at a right angle) at 4 m/s. How fast is the distance changing when the car is 30 m away from the intersection and the bus is 40 m from the intersection? Additionally, is the distance increasing or decreasing?

SOLUTION

distance is changing at $-\frac{2}{10}$ m/s

The distance is decreasing.

9. Find the horizontal asymptote(s) of the following functions:

a)

$$\frac{8 - 4x^2}{9x^2 + 5x}$$

SOLUTION $-\frac{4}{9}$

b)

$$\frac{\sqrt{7 + 9x^2}}{1 - 2x}$$

SOLUTION $x \rightarrow -\infty : y = -\frac{3}{2}$
 $x \rightarrow \infty : y = \frac{3}{2}$

10. Compute the differential dy of the following functions:

a)

$$f(x) = x^2 - \sec(x)$$

SOLUTION $df = (2x - \sec(x) \tan(x)) dx$

b) Compute the dy and Δy for $y = x^5 - 2x^3 + 7x$ as x changes from 6 to 5.9

SOLUTION $dy = (5x^4 - 6x^2 + 7) dx$ and -627.1

11. Multiple Choice. Read each question and answer choice carefully and choose the ONE best answer.

a) A right cylindrical cone has a radius of 4 cm and a height of 2.0 cm. If the height increases at 0.5 cm/min, but the radius remains constant, then what will be the rate of change of the volume?

A) 8.4 cm³/min

B) 1.1 cm³/min

C) 4.2 cm³/min

D) 2.1 cm³/min

SOLUTION A) 8.4

- b) Given that the cost function $C(x) = 144 + 0.1x + 0.04x^2$, what is the minimum average cost per unit?

- A) 20 dollars
- B) 40 dollars
- C) 60 dollars
- D) 80 dollars

SOLUTION C) 60 dollars

- c) Find the approximate value of $(5.2)^3$ using linear approximation using $x = 5$

- A) 130
- B) 140
- C) 150
- D) 160

SOLUTION [B] 140

12. Determine the number(s) c that satisfies the conclusion of the Mean Value Theorem for the given function and interval.

$$f(z) = 4z^3 - 8z^2 + 7z - 2$$

on the interval $[2, 5]$

SOLUTION $c = \frac{2+\sqrt{79}}{3}$

13. Answer the following questions with the given graph:

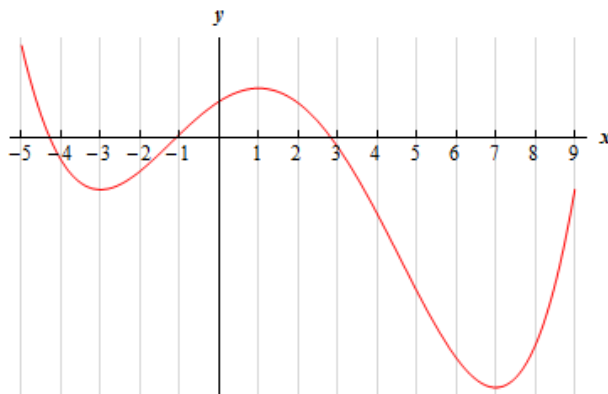


Figure 1: graph of $f(x)$

a) Write down the interval in which the graph is increasing:

b) Write down the interval in which the function is decreasing:

SOLUTION Increasing from $(-3, 1)$ $(7, \infty)$ Decreasing from $(-\infty, -3)$ $(1, 7)$

14. Use Newton's method to determine x_2 for $= x^3 - 7x^2 + 8x - 3$ if $x_0 = 5$

SOLUTION $3x^2 - 14x + 8$

15. Sketch a graph with the following information:

$$f(4) = 0, f'(2) = 0, f''(4) = 0 \lim_{x \rightarrow \infty} f(x) = 1 \lim_{x \rightarrow -\infty} = -\infty$$

Vertical Asymptote at $x = 0$, $f'(x) > 0$ for $x < 0$ $x > 2$

$f'(x) < 0$ for $0 < x < 2$

$f''(x) > 0$ for $x < 0$ and for $0 < x < 4$; $f''(x) < 0$ for $x > 4$.