

Practice Exam 1

Math 241 FALL 2019

September 30, 2019

Name:

Section Number:

Read all of the following information before starting the exam:

- **CALCULATORS ARE NOT ALLOWED.**
- Show all work, clearly and in order using proper notations, if you want to get full credit. We reserve the right to take off points if we cannot see how you arrived at your answer (even if your final answer is correct).
- Box or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. We will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 pages total including this cover sheet and the scrap page and is worth 145 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

Question 1	20	
Question 2	5	
Question 3	5	
Question 4	5	
Question 5	10	
Question 6	5	
Question 7	5	
Question 8	25	
Question 9	5	
Question 10	15	
Question 11	20	
Question 12	25	
Total	145	

1. 20 pts Compute the following limits. IF the limits do not exist write "DNE". DO NOT USE L'HOPITAL'S RULE specify +/- for ∞ .

a)

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 4x - 12}$$

b)

$$\lim_{x \rightarrow 0} \frac{x}{|x|}$$

c)

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x^2 + 2}$$

d)

$$\lim_{x \rightarrow 0^+} \frac{7}{x^2 - 3x}$$

2. 5 pts Show that $\cos(x) = x$ has at least one solution:

3. 5 pts Prove that $\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$

4. 5pts Find the line tangent to the graph of $f(x) = x^4 + 5x^3$ at $x = 5$.

5. **10 pts** Use the definition of derivatives to differentiate the following:

a) $f(x) = \frac{3}{5x+1}$

b) $f(x) = x^3 + x$

6. **5pts** Consider the equation: $1 + x = x^3$ Stating the theorems you use, prove that there is at least one solution in the interval $[1,2]$.

7. **5 pts** Suppose the amount of the air in a balloon in after t is given by $V(t) = t^3 - 6t^2$, estimate the instantaneous rate of change after 6 hours:

8. **25 pts** Differentiate the following equations using any method

a)

$$f(x) = \frac{4x}{5x + 10x}$$

b)

$$m(x) = \sin(\cos(x^4)) + e^4 + \frac{5x}{10x^2 + 5x}$$

c)

$$h(x) = \frac{\sin(x)}{x} + 4x$$

d)

$$g(x) = 4x^4 + 5x \sin(2x)$$

e)

$$z(x) = \frac{(x^4 + 2)(x^3)}{4 - x^4}$$

9. **5 pts** Find the limits:

$$T(x) = \begin{cases} 2x^3 & \text{if } x < 2 \\ x^2 & \text{if } x > 2 \end{cases}$$

a)

$$\lim_{x \rightarrow 2^+} T(x)$$

b)

$$\lim_{x \rightarrow 2} T(x)$$

10. **15 pts** Evaluate the following limits state DNE if the limit does not exist. **DO NOT USE L'HOPITALS RULES**

a)

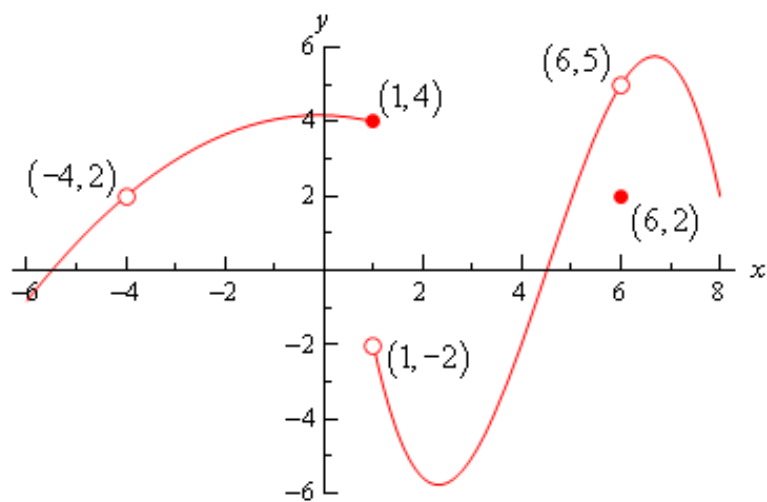
$$\lim_{x \rightarrow 0} \frac{1 + 2 \sin(x)}{x}$$

b)

$$\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$$

c)

$$\lim_{x \rightarrow -3} \frac{\sqrt{2x + 22} - 4}{x + 3}$$



11. 20pts Evaluate the following limits using the graph above

a)

$$\lim_{x \rightarrow -4} f(x)$$

b)

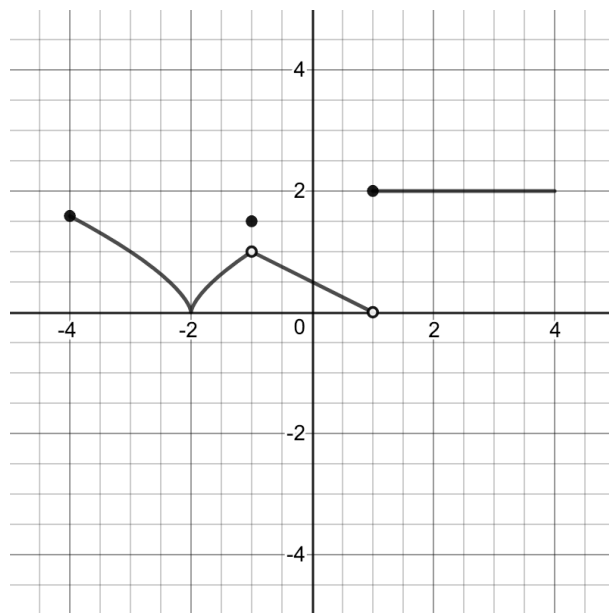
$$\lim_{x \rightarrow 1^-} f(x)$$

c)

$$\lim_{x \rightarrow 1} f(x)$$

d)

$$\lim_{x \rightarrow 6} f(x)$$



12. Solve the following questions using the given graph:

- a Does the function appear differentiable at $x=-2$, why or why not?
- b Evaluate $f'(2)$.
- c Evaluate $f'(0)$
- d True or False: $f'(-3) < 0$
- e True or False: the functions is differentiable at $x=0$.