

1. Determine whether the statement is true or false. If the statement is false explain why.

a) If the  $f$  has an absolute maximum value at  $z$ , then the  $f'(z) > 0$

(TRUE / FALSE)

**SOLUTION** FALSE because at either maximum or minima then  $f'(z) = 0$  or be undefined or its on the interior of the domain.

b) The function  $f(x) = 2x(x + 4)^3$  has inflection points at:  $(0,0)$  and  $(4,0)$

(TRUE / FALSE)

**SOLUTION** FALSE because the inflection point is at  $(-4,0)$   $(-2, -32)$

2. Find two negative numbers that add up to -50 whos product is large as possible.

**SOLUTION**  $x_1 = -25$  &  $x_2 = -25$

3. Find the local extrema and the intervals where the following function is increasing or decreasing:  $f(x) = x^{\frac{2}{3}}(x - 10)$

**SOLUTION** Local max at  $x = 0$ , local min at  $x = 4$ , increasing  $(-\infty, 0) \cup (4, \infty)$ , decreasing  $(0,4)$

4. A piano is suspended by a 90 ft rope through a pulley system that is vertically 40 ft above a man's arm. The piano is at some height above the ground. At  $t = 0$ , the man is 30 ft horizontally from the piano and walks away at 12 ft/s. How fast is the piano being pulled up?

**SOLUTION**  $36/5$  ft/s

5. Determine the critical numbers of the following functions:

a)

$$V(t) = 1 + 80t^3 + 5t^4 - 2t^5$$

**SOLUTION**  $t = 0, -4, 6$

b)

$$Q(t) = (2 - 8x)^4(x^2 - 9)^3$$

**SOLUTION**  $t = \frac{1}{4}, t = 3, t = -3, t =$

6. Use linear approximation to find the approximate value of  $\sin(122^\circ)$

**HINT** Re-express degrees in terms of radians by using  $120^\circ$

**SOLUTION**

$$\frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)\left(\frac{\pi}{90}\right)$$

7. For the following functions answer each of the following

- identify the critical points of the function and classify them as local maximum, local minimum, or neither
- identify the intervals on which the function is increasing/decreasing
- determine the interval on which the function is concave up or down
- determine the inflection points of the function
- use the information found to sketch the graph of the function

a)

$$g(t) = t^5 - 5t^4 + 8$$

**SOLUTION** CRITICAL POINT:  $t = 0$  and  $t = 4$

CLASSIFICATION OF CRITICAL POINTS:

RELATIVE MINIMUM: 4

RELATIVE MAXIMUM: 0

INCREASING:  $(-\infty, 0) \cup (4, \infty)$

DECREASING:  $(0, 4)$

CONCAVE UP:  $(3, \infty)$

CONCAVE DOWN:  $(-\infty, 0)$  &  $(0, 3)$

INFLECTION POINT:  $t = 3$

b)

$$x^{4/3}(x - 2)$$

8. A car is heading "away" from the intersection at 5 m/s. And a bus is heading "towards" the intersection on the other street (at a right angle) at 4 m/s. How fast is the distance changing when the car is 30 m away from the intersection and the bus is 40 m from the intersection? Additionally, is the distance increasing or decreasing?

**SOLUTION**

distance is changing at  $-\frac{2}{10}$  m/s

The distance is decreasing.

9. Find the horizontal asymptote(s) of the following functions:

a)

$$\frac{8 - 4x^2}{9x^2 + 5x}$$

**SOLUTION**  $-\frac{4}{9}$

b)

$$\frac{\sqrt{7 + 9x^2}}{1 - 2x}$$

**SOLUTION**  $x \rightarrow -\infty : y = -\frac{3}{2}$   
 $x \rightarrow \infty : y = \frac{3}{2}$

10. Compute the differential  $dy$  of the following functions:

a)

$$f(x) = x^2 - \sec(x)$$

**SOLUTION**  $df = (2x - \sec(x) \tan(x)) dx$

b) Compute the  $dy$  and  $\Delta y$  for  $y = x^5 - 2x^3 + 7x$  as  $x$  changes from 6 to 5.9

**SOLUTION**  $dy = (5x^4 - 6x^2 + 7) dx$  and  $-627.1$

11. Multiple Choice. Read each question and answer choice carefully and choose the ONE best answer.

a) A right cylindrical cone has a radius of 4 cm and a height of 2.0 cm. If the height increases at 0.5 cm/min, but the radius remains constant, then what will be the rate of change of the volume?

A) 8.4 cm<sup>3</sup>/min

B) 1.1 cm<sup>3</sup>/min

C) 4.2 cm<sup>3</sup>/min

D) 2.1 cm<sup>3</sup>/min

**SOLUTION** A) 8.4

- b) Given that the cost function  $C(x) = 144 + 0.1x + 0.04x^2$ , what is the minimum average cost per unit?

- A) 20 dollars
- B) 40 dollars
- C) 60 dollars
- D) 80 dollars

**SOLUTION** C) 60 dollars

- c) Find the approximate value of  $(5.2)^3$  using linear approximation using  $x = 5$

- A) 130
- B) 140
- C) 150
- D) 160

**SOLUTION** [B] 140

12. Determine the number(s)  $c$  that satisfies the conclusion of the Mean Value Theorem for the given function and interval.

$$f(z) = 4z^3 - 8z^2 + 7z - 2$$

on the interval  $[2, 5]$

**SOLUTION**  $c = \frac{2+\sqrt{79}}{3}$

13. Answer the following questions with the given graph:

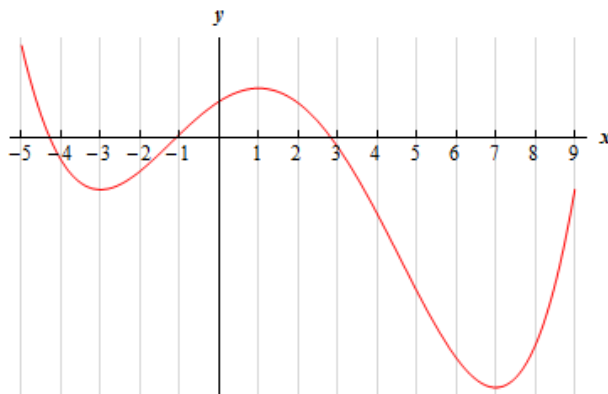


Figure 1: graph of  $f(x)$

a) Write down the interval in which the graph is increasing:

b) Write down the interval in which the function is decreasing:

**SOLUTION** Increasing from  $(-3, 1)$   $(7, \infty)$  Decreasing from  $(-\infty, -3)$   $(1, 7)$

14. Use Newton's method to determine  $x_2$  for  $f(x) = x^3 - 7x^2 + 8x - 3$  if  $x_0 = 5$

**SOLUTION**  $3x^2 - 14x + 8$

15. Sketch a graph with the following information:

$$f(4) = 0, f'(2) = 0, f''(4) = 0 \quad \lim_{x \rightarrow \infty} f(x) = 1 \quad \lim_{x \rightarrow -\infty} f(x) = -\infty$$

Vertical Asymptote at  $x = 0$ ,  $f'(x) > 0$  for  $x < 0$   $x > 2$

$f'(x) < 0$  for  $0 < x < 2$

$f''(x) > 0$  for  $x < 0$  and for  $0 < x < 4$ ;  $f''(x) < 0$  for  $x > 4$ .