1. Finding the Laplace transform of $L(20e^{-3t}) = 20\int_0^\infty e^{(-s-3)t}dt$ therefore solving the integral
$$\lim_{A\to\infty}\int_0^\infty e^{(-s-3)t}dt = \lim_{A\to 0} -\frac{e^{(-s-3)t}}{(s+3)}\Big|_0^A = \frac{20}{s}$$
Next, finding the laplace transformation of $L(7t) = \int_0^\infty \int 7te^{-st}dt$ and by using method of improper integrals and integration by parts of letting u = t and dv = e^{-st} then
$$7\lim_{a\to\infty}\int_0^a te^{-st}dt = 7\lim_{a\to 0}(\frac{te^{-ts}}{s} - \frac{e^{-st}}{s^2}\Big|_0^a = \frac{7}{s^2}$$
Finally taking the Laplace Transform of L(9) = $\int_0^\infty 9e^{-st}dt$ then using direct integration and improper integral method results to
$$9\lim_{a\to\infty}\int e^{-st}dt = 9(-\frac{e^{-st}}{s}\Big|_0^a) = \frac{9}{s}$$
Therefore, using the sum rule of Laplace transform then $L(20e^{-3t}+7t-9) = L(20e^{-3t}) + L(7t) - L(9) =$
$$\frac{20}{s} + \frac{7}{s^2} - \frac{9}{s}$$

2. To begin with take the partial fraction decomposition such that
$$\frac{A}{s-1} + \frac{B}{s-2} + \frac{Cs+D}{s^2+4} = \frac{(s+3)^2}{(s-1)(s-2)(s^2+4)}$$
where A, B, C, and D are undetermined constants. Then algebraically solving for A, B, C, and D results to
   - $A = -\frac{16}{5}$
   - $B = \frac{25}{8}$
   - $C = \frac{3}{40}$
   - $D = -\frac{82}{40}$

   Then it results to
$$-\frac{16}{5(s-1)} + \frac{25}{8(s-2)} + \frac{\frac{3}{40}s - \frac{82}{40}}{(s^2+4)}$$
Then taking the inverse Laplace transformation
   - $-\frac{16}{5}\mathbf{L^{-1}}[\frac{1}{s-1}] = -\frac{16}{5}e^t$
   - $\frac{25}{8}\mathbf{L^{-1}}[\frac{1}{(s-2)} = \frac{25}{8}e^{2t}$
   - $\frac{3}{40}\mathbf{L^{-1}}[\frac{s}{(s^2+4)}] = \frac{3}{40}\cos(2t)$
   - -$\frac{41}{40}\mathbf{L^{-1}}[\frac{2}{s^2+4}] = -\frac{41}{40}\sin(2t)$

Therefore, the function f(t) is

$$f(t) = -\frac{16}{5}e^t + \frac{25}{8}e^{2t} + \frac{3}{40}\cos(2t) - \frac{41}{40}\sin(2t)$$

3. Take the Laplace transformation of y"' then

$$\mathbf{L}[y'''] = s^3F(s) - s^2f(0) - sf'(0) - f''(0) = s^3F(s) - s$$

Then taking the Laplace transformation of y"

$$\mathbf{L}[y^{''}] = s^2F(s) - sf(0) - f'(0) = s^2F(s) - 1$$

Then the left hand side of the differential transforms into

$$(s^3 - s^2)F(s) - s + 1$$

Taking the right side transformation becomes

$$\frac{2}{s^2} + \frac{1}{s} + \frac{3}{s^2+1}$$

Then

$$(s^3 - s^2)F(s) - s + 1 = \frac{2}{s^2} + \frac{1}{s} + \frac{3}{s^2+1}$$

or

$$F(s) = \frac{1}{(s^3 - s^2}[\frac{2}{s^2} + \frac{1}{s} + \frac{3}{s^2+1}]$$

Then from simplifications

$$Y(s) = -6\frac{1}{s} - 5\frac{1}{s^2} - \frac{3}{s^3} - \frac{2}{s^4} + 4.5\frac{1}{s-1} + 1.5\frac{s}{s^2+1} + 1.5\frac{1}{s^2+1}$$

Then taking the inverse Laplace results to

$$y(t) = -6 - 5t - 1.5t^2 - \frac{1}{3}t^3 + 4.5e^t + 1.5\cos(t) + 1.5\sin(t)$$

4. Taking the Laplace transformation of the given the right hand side becomes

$$L(1 + e^{-t}) = \frac{1}{s} + \frac{1}{s+1}$$

and the left hand side becomes

$$L(y" + 4y' + 6y) = y(s)[s^2 + 4s + 6]$$

therefore,

$$Y(s) = \frac{1}{6s} + \frac{1}{3(s+1)} - \frac{s/2 + 1/3}{s^2 + s + 6}$$

Therefore, taking the inverse Laplace transformation results to

$$Y(t) = \frac{1}{6} + \frac{1}{3}e^{-t} - \frac{1}{2}e^{-2t}\cos(\sqrt{2}t) - 3e^{-2t}\sin(\sqrt{2}t)$$

5. From the right hand side using the laplace transform

   - $$\int_0^2 2e^{-st}dt = 2[-\frac{e^{-2s}}{s} + \frac{1}{s}]$$
   - $$\int_2^3 -e^{-st}dt = \frac{e^{-3s} - e^{-2s}}{s}$$

   And taking the Laplace transformation of the left side

   - $\mathbf{L}[y'] = sf(s) - 5$
   - $\mathbf{L}[y] = f(s)$

   Then the overall Laplace transformation is

   $$f(s) = \frac{-3e^{-2s}}{s(s+1)} + \frac{e^{-3s}}{s(s+1)} + \frac{2}{s(s+1)}$$

   Then taking the inverse transformation

   $$y(t) = 3u_2(t)e^{-(t-2)} - 3u_2(t) + u_3(t) - u_3(t)e^{(-(t-3)} + 2 - 2e^{-t}$$