Practice Problems

1. $y' + y^2 \sin(x) = 0$
   Rewrite the equation such that $y' = \frac{dy}{dx}$ and move all the variables to one side such that the new rewritten equation is:

$$\frac{dy}{y^2} = -\sin(x)dx$$

   Integrate the equation to its respective variables such that

$$\int_y \frac{dy}{y^2} = \int_x -\sin(x)dx$$

$$-\frac{1}{y} = \cos(x) + C$$

   such that $C \in \mathbb{R}$

2. $y' = \frac{ty(4-y)}{3}; \quad y(0) = 1$
   Rewrite the equation by having only a single variable on one side such $y' = \frac{dy}{dt}$ that the rewritten equation may look like

$$\frac{dy}{y(4-y)} = \frac{t}{3}dt$$

   Similar to number one we are going to integrate both sides with their respective terms such and to integrate y it requires us to take the partial fraction decomposition of it such that

$$\frac{1}{y(4-y)} = \frac{A}{y} + \frac{B}{4-y}$$

   where A and B are constants and by using algebraic method it follows that A $= \frac{1}{4}$ and B $= \frac{1}{4}$. Thus from partial fraction integration and the sum rule of integration:

$$\int_y \frac{1}{4y} = \frac{1}{4}\ln(y)$$

   +

$$\int_y \frac{1}{4(4-y)} = -\frac{1}{4}\ln(4-4y)$$

   As a result, from the logarithmic properties

$$\frac{1}{4}(\ln(y) - \ln(4-y)) = \frac{1}{4}(\ln(\frac{y}{4-y}))$$

   Integrating the right side results to

$$\frac{t^2}{6} + D$$

such that D is a constant.
Therefore, the general solution:

$$\frac{1}{4}(\ln(\frac{y}{4-y})) = \frac{t^2}{6} + D$$

such that $y \neq 4, 0$
Given that the initial conditions y(0) = 1 then

$$\frac{1}{4}(\ln(\frac{1}{4-1})) = \frac{0^2}{6} + D$$
$$\frac{1}{4}\ln(\frac{1}{3}) = D$$

3. $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$
Divide the numerator and denominator with x^2 such that the equation can be rewritten as

$$\frac{dy}{dx} = \frac{1+3(\frac{y}{x})^2}{2(\frac{y}{x})}$$

Let y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ then the equation rewritten is

$$v + x\frac{dv}{dx} = \frac{1+3v^2}{2v}$$

and we have a separable equation so rewriting it so one variable has one sides then

$$\frac{dx}{x} = \frac{2v}{1+v^2}dv$$

Thus integrating each sides with their corresponding

$$\ln(x) + k = \ln(1+v^2)$$

such that $k \in \mathbb{R}$.
Thus,

$$\ln(x) + k = \ln(1+(\frac{y}{x})^2)$$

4. $\frac{dr}{d\theta} = \frac{r^2}{\theta}; \quad r(1) = 2$
Separable so move individual variables on one sides such that

$$\frac{dr}{r^2} = \frac{d\theta}{\theta}$$

Integrating both to respect of its variable results to

$$-\frac{1}{r} = \ln(\theta) + c$$

Given that r(1) = 2 implies that when $\theta = 1$ then r = 2 therefore,

$$-\frac{1}{2} = \ln(1) + c$$

then c = $-\frac{1}{2}$ since $\ln(1) = 0$.
Therefore, the solution is $-\frac{1}{r} = \ln(\theta) - \frac{1}{2}$