Problem 1

Given that the buyer can spend 1500 per month then it follows that the buyer can spend $1500 \times 12 = 18000$ dollars per year. Since the mortgage is 30 years then let N(t) be a function such that the buyer returns the money loaned. Then at N(30) = 0.
Given that the rate of the mortgage is 0.06 then we have an initial value problem of

$$N'(t) = 0.06N - 18000$$

Let P(t) = - 0.06 then the integrating factor is $e^{-0.06t}$ and multiplied to the differential equaiton results to

$$\frac{d}{dt}e^{-0.06t}N = -18000e^{-0.06t}$$

integrating both sides results to

$$e^{-0.06t}N = 300000e^{-0.06t} + C$$

or

$$N(t) = 300000 + Ce^{0.06t}$$

Then since at t = 30 then

$$N(30) = 0 = 300000 + Ce^{0.06(30)}$$

algebraically tells that $C \approx -49589.666$.
Then the original amount of the loan at N(0) is

$$N(0) = 300000 - 49589.666e^{0.06(0)} \approx 250410$$

Therefore, the original amount of the loan is 250410 dollars.
Since we are making 1500 per month payment for a 30 year period span then in total

$$1500 \times 12 \times 30 = 540000$$

then total interest = 360000 − 250410 = 289590.

Problem 2

To begin with multiply the differentiable equation with a factor of $\frac{1}{4-t^2}$ such that the first order differential equation may be rewritten as

$$y' + \frac{2t}{4-t^2}y = \frac{3t^2}{4-t^2}$$

Existence Proof
Let f(t,y) = $\frac{3t^2}{4-t^2} - \frac{2ty}{4-t^2}$. This is continuous near any (a,b) where $a \neq 2$ as the denominator will be zero. Therefore, this is continuous "around" (1,-3).
Uniqueness Proof
Take the partial derivative at y of f(t,y) then $f_y(t,y) = \frac{2t}{4-t^2}$. Then this is continuous near any point (a,b) when $a \neq 2$ with similar reasoning mentioned in the existence proof. Therefore, there is a unique solution "around" (1,-3).

Problem 3

The given differential equation $y' = 2y - 1$ is a separable differential equation. Rewrite it as a Lipschitz differential equation such as

$$\frac{dy}{dx} = 2y - 1$$

or

$$\frac{dy}{2y-1} = dx$$

Then integrating both sides result to and equation as

$$\frac{\ln(2y-1)}{2} = x$$

Then isolating y to itself results to

1. $\ln(2y-1) = 2x$
2. $2y - 1 = e^{2x}$
3. $y = \frac{e^{2x}+1}{2}$

Therefore, $y = \frac{e^{2x}+1}{2}$ is a solution for the given differential equation is proved.

Problem 4

Let the following be

- w = 180 (weight of the skydiver)
- h = 5000 ft (height of falling down)
- let v be the velocity
- let r be the air resistance

Since the air resistance is opposite to velocity then

$$F = m\frac{dV}{dt}$$

and let F be

$$F = mg - rv$$

then the following is made

$$mg - rv = m\frac{dV}{dt}$$

or

$$m\frac{dV}{dt} + rv = gm$$

Muliplying with $\frac{1}{m}$ to the entire equation results to

$$\frac{dV}{dt} + \frac{rv}{m} = g$$

let P(t) = $\frac{r}{m}$ then the integrating factor is $e^{\frac{r}{m}t}$ and multiplying it results to

$$\frac{d}{dt} e^{\frac{r}{m}t} V = g e^{\frac{r}{m}t}$$

Integrating both sides results to

$$e^{\frac{r}{m}t} V = \frac{gm}{r} e^{\frac{r}{m}t} + gC$$

or

$$V(t) = \frac{gm}{r} + gCe^{-\frac{r}{m}t}$$

since V(0) = 0, g = 32, m = 180, r = 0.75 then algebraically solving for C results to $C = -240$.
Setting t = 10 then

$$V(10) = \frac{32 \times 180}{0.75} + 32(-240)e^{-\frac{0.75 \times 10}{180}} \approx 313.424$$

Recall that $\int V(t)dt = X(t)$ then integrating V(t) results to

$$X(t) = 7680t + 1843200e^{-\frac{t}{240}} + C$$

Since X(0) = 0 then C = -1843200.
Then at t = 10

$$X(10) = 7680(10) + 1843200e^{-\frac{10}{240}} - 1843200 \approx 1578.007$$

Therefore after 10 seconds the sky diver has fallen 1578 ft.