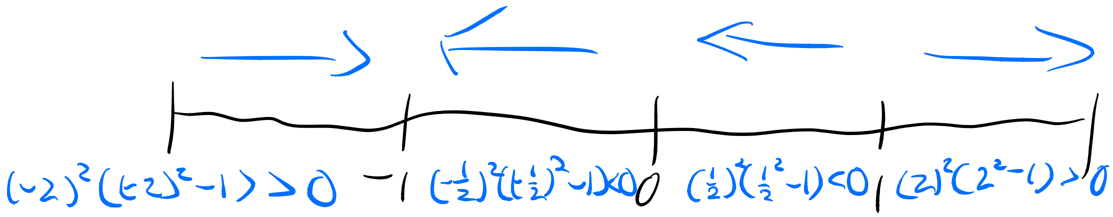
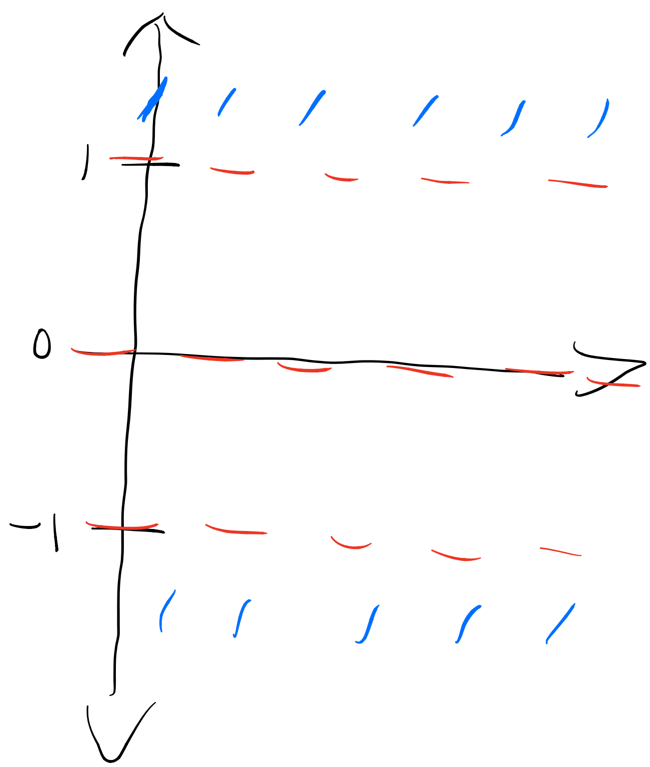
Problem 1

Critical Points To find the critical points set $y' = 0$. Then algebraically solving for y we find that that y $= 0, \pm 1$.
To check the stability at each point, create a number line such that: Therefore, x = -1 is



stable, x = 0 is semistable, and x = 1 is unstable.
Thus the directional field of it is will look like



Problem 2

Rewrite the differential equation as

$$bxe^{2xy}dy + ye^{2xy} + xdx = 0$$

Then by definition of exactness, let M = bxe^{2xy} and let N = $ye^{2xy} + xdx$ then a differential equation is exact if and only if $M_x = N_y$.
Taking the partial derivative at x of M results $M_x = be^{2xy} + 2bxye^{2xy}$ and taking the partial derivative of y of N results to $N_y = e^{2xy} + 2xye^{2xy}$ then algebraically solving for b by setting $N_y = M_x$ results to b = 1.
Let f(x,y) be a solution of the given such that $f_x = ye^{2xy} + xdx$ and $f_y = xe^{2xy}$. Hence

$$\int f_x dx = \int ye^{2xy} + xdx = \frac{e^{2xy}}{2} + \frac{x^2}{2} + \phi(y)$$

such that $\phi(y)$is a function with a respect of y.
Differentiating the previous equation to respect of y results to

$$M = f_y dy = xe^{2xy} + \phi'(y)$$

Since from $M = xe^{2xy}$ then

$$\phi'(y) = 0$$

or

$$\phi(y) = K \in \mathbb{R}$$

Therefore, the general solution for the given differential equation is

$$f(x,y) = \frac{e^{2xy}}{2} + \frac{x^2}{2} + K$$

Problem 3

Rewrite the given differential equation of

$$y(1-x^2)dy - (xy^2 - \sin(x)\cos(x)dx = 0$$

Let M = $-xy^2 + \sin(x)\cos(x)$ and let N = $y(1-x^2)$ then the differential equation is considered exact iff $M_y = N_x$. Then taking the partial derivatives

$$M_y = -2xy + 0$$

and

$$N_x = 0 - 2xy$$

Therefore, the equation is exact.
Taking the integration of N results to

$$f(x,y) = \int N dy = \int y - yx^2 dy = \frac{y^2}{2} - \frac{x^2y^2}{2} + \theta(x)$$

where $\theta(x)$ is a function of x and since the equation is exact then using M = f_x then

$$-xy^2 + \theta'(x) = -xy^2 - \sin(x)\cos(x)$$

Then solving for $\theta(x)$ results to

$$\theta(x) = \frac{\sin^2(x)}{2} + K$$

where $K \in \mathbb{R}$. Therefore,

$$f(x, y) = \frac{y^2}{2} - \frac{x^2y^2}{2} + \frac{\sin^2(x)}{2} + K$$

Problem 4

Given

$$xydx + (2x^2 + 3y^2 - 20)dy = 0$$

Let M = xy and let N = $2x^2 + 3y^2 - 20$, then taking the partial derivatives of each then it arrives that

$$M_y = x \neq N_x = 4x$$

Thus the differential equation is not exact.
~~Thus~~ But, finding the integrating factor

$$e^{\frac{-M_y+N_x}{M}} = e^{\int \frac{3x}{xy}dy} = e^{\ln(y)^3} = y^3$$

Then multiplying the integrating factor to the differential equation results to

$$xy^4dx + (2x^2y^3 + 3y^5 - 20y^3)dy = 0$$

and by setting M = xy^4 and N = $2x^2y^3 + 3y^5 - 20y^3$ shows that the differential equation is exact as

$$M_y = 4xy^3 = N_x$$

Integrating M dx results to

$$f(x, y) = \int Mdx = \int xy^4dx = \frac{x^2y^4}{2} + \theta(y)$$

where $\theta(y)$ is a function of y. By taking the partial derivative in terms of y and setting it equal to N it results to

$$2x^2y^3 + \theta'(y) = 2x^2y^3 + 3y^5 - 20y^3$$

or

$$\theta'(y) = 3y^5 - 20y^3$$

and taking the integration of it results

$$\theta(y) = 0.5y^6 - 5y^4 + K$$

where K $\in \mathbb{R}$.
Therefore, f(x,y) = $\frac{x^2y^4}{2} + 0.5y^6 - 5y^4 + K$.

Problem 5

From the given let $x_0 = 2, y_0 = 4, h = 0.1$ then using Euler's method.

$$f(2.1) = 4 + (0.1)(0.1\sqrt{4} + 0.4(2)^2) \approx 4.18$$

Then $(2.1, 4.18)$

$$f(2.2) = 4.18 + 0.1(0.1\sqrt{4.18} + 0.4(2.1)^2) \approx 4.3768$$

Then $(2.2, 4.3768)$

$$f(2.3) = 4.3768 + 0.1(0.1\sqrt{4.3768} + 0.4(2.2)^2) \approx 4.5913$$
$$f(2.4) = 4.5913 + 0.1(0.1\sqrt{4.5913} + 0.4(2.3)^2) \approx 4.8243$$
$$f(2.5) = 4.8243 + 0.1(0.1\sqrt{4.8243}^2 + 0.4(2.4)^2) \approx 5.07675$$

From Euler's approximation then $(2.5, 5.077)$