1.2 TRUTH TABLE

2 MAKE A TRUTH TABLES FOR THE FOLLOWING FORMULA

a $\neg[P \wedge (Q \vee \neg P)]$

P	Q	$\neg P$	$Q \vee \neg P$	$P \wedge (Q \vee \neg P)$	$\neg[P \wedge (Q \vee \neg P)]$
T	T	F	T	T	F
T	F	F	F	F	T
F	T	T	T	F	T
F	F	T	T	F	T

b $(P \vee Q) \wedge (\neg P \vee R)$

P	Q	R	$P \vee Q$	$\neg P \vee R$	$P \vee Q) \wedge (\neg P \vee R)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	F	T	F
F	F	F	F	T	F

3a) construct a truth table for P + Q

P	Q	$P + Q$
T	T	F
T	F	T
F	T	T
F	F	F

3b Construct same meaning with only using the logical connective and use a truth table

P	Q	$(P \wedge \neg Q) \vee (\neg P \wedge Q)$
T	T	F
T	F	T
F	T	T
F	F	F

12a Using any laws simplify the given statements

$$\neg(\neg P \vee Q) \vee (P \wedge \neg R)$$

Using DeMorgan's Law and the Double Negation Law we can simplify the statement on the left side into: $P \wedge \neg Q$.
Now the updated statement is:

$$P \wedge \neg Q \vee (P \wedge \neg R)$$

By using the distributive law, we can take out the P and simplify it even further into $P \wedge (\neg Q \vee \neg R)$. Additionally, if we use DeMorgan's Law once more, we can simplify it even to:

$$P \wedge \neg(Q \wedge R)$$

17Find a formula only using the logical connectives

P	Q	$(P \vee Q) \wedge (\neg P \wedge \neg Q)$
T	T	F
T	F	T
F	T	T
F	F	F

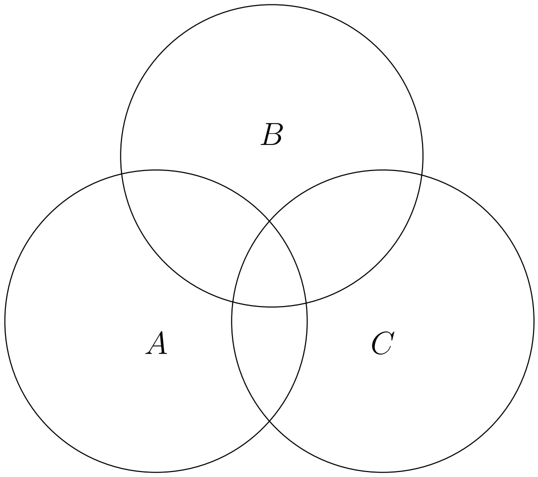
18 Since the given conclusion is a tautology, by the definition of the word we make the assumption that the given conclusion will be true. As long as the premises, regardless of the condition, the argument is valid.
However, if the given conclusion is a contradiction, meaning it is false, by the tautology law, we can assume that there was a negation to the tautology statement.

1.3 VARIABLES AND SETS

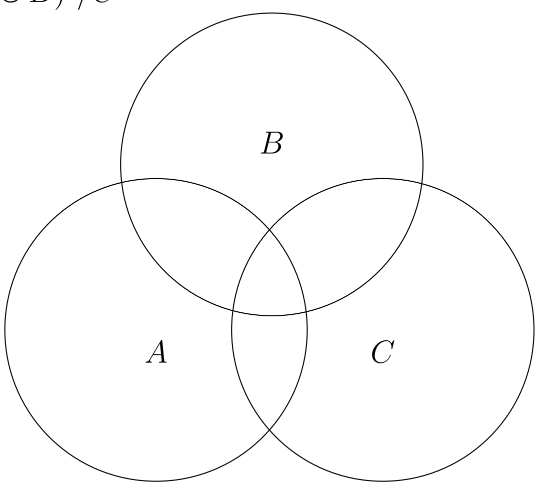
8 What are the truth sets of the following statements?
(a) x is a real number and $x^2 - 4x + 3 = 0$
By factoring $x^2 - 4x + 3 = 0$ we find that $(x-3)(x-1)$, we find that x = 1, 3. Since 1,3 $\in \mathbb{R}$. Therefore the truth sets is $\{1, 3\}$.
(b) x is a real number and $x^2 - 2x + 3 = 0$
By taking the discriminant, we can find if a quadratic equation will have real roots. When the equation $b^2 - 4ac$ is either equal to or greater than 0, there will be real roots.
However, when we take the discriminant, $4 - 4(1)(3) = -8$, it shows that the quadratic equation only has non-real roots, therefore, contradicting the first statement of $x \in \mathbb{R}$.
Thus, the truth value for this statement is $\emptyset$.
c x is a real number and 5 $\in \{y \in \mathbb{R} | x^2 + y^2 < 50\}$
By plugging in y with 5 since 5 is a set in y. As a result, we have $x^2 < 25$.
By doing algebraic process we see that $|x| < 5$.
Therefore, x $\in$ infinite sets since it is all real numbers.

1.4 OPERATION ON SETS

9
Let A be a set containing $\{5, 6\}$ and B be a set $\{6, 7\}$ Therefore $A \cup B = \{5, 6, 7\}$ and $(A \cup B)/B = \{5\}$. Hence, the result does not equal.
11 a)



$(A \cup B) \ /C$



$A \cup (B/C)$

Since the unit Venn diagram of the two are not the same outcome, the conclusion we can make $(A \cup B) \ /C \neq A \cup (B/C)$

11b Give an example where this would contradict the statement in 11a

Let A = $\{3, 4, 5, 6\}$, B = $\{5, 6, 7\}$, and C = $\{6, 7, 8\}$.

Proving the top diagram:

$A \cup B = \{3, 4, 5, 6, 7\}$

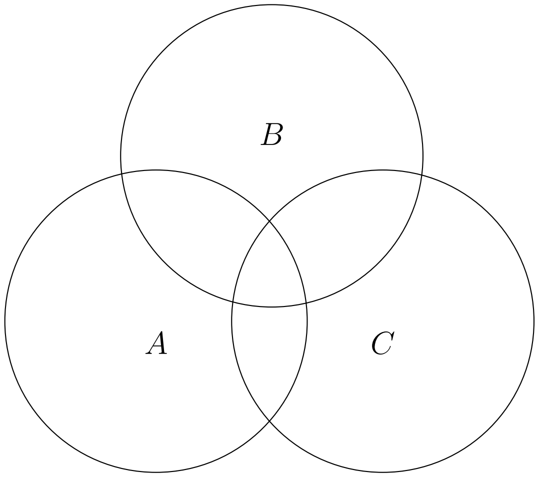
$(A)/C = \{3, 4, 5\}$

Proving the bottom diagram:

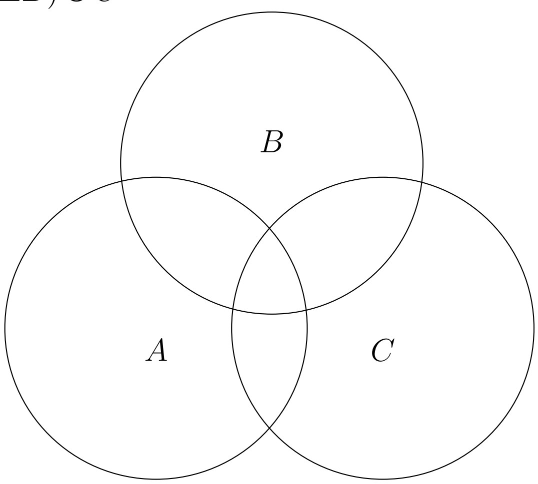
$(B \ /C) = \{5\}$

$A \cup (B/C) = \{3, 4, 5, 6\}$

13a $(A \triangle B) \cup C = (A \cup C) \triangle (B)$.

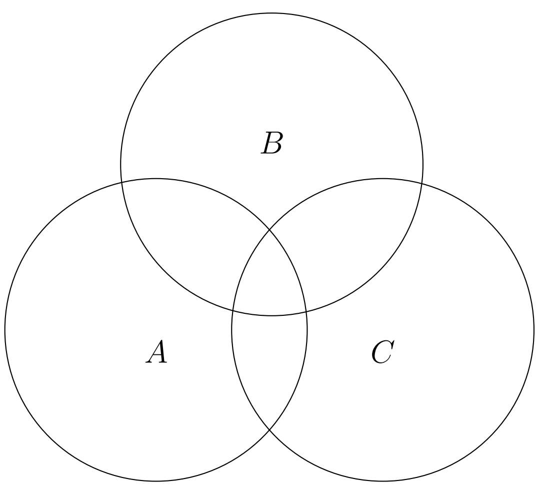


$(A \triangle B) \cup C$

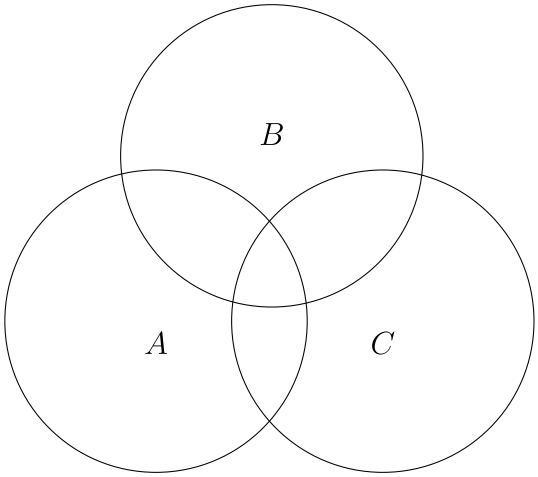


$(A \cup C) \triangle (B)$

13b $(A \triangle B) \cap C = (A \cap C) \triangle (B \cap C)$

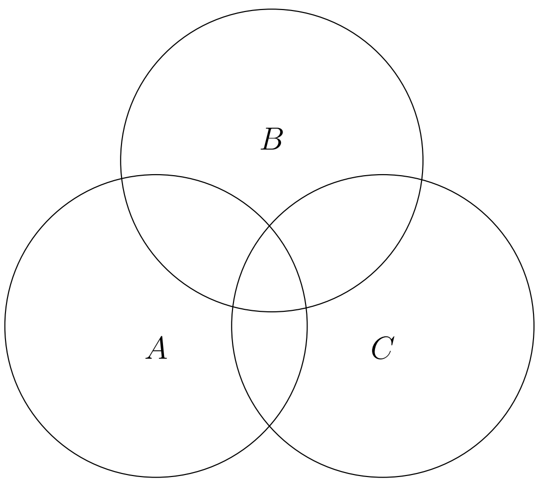


$(A \triangle B) \cap C$

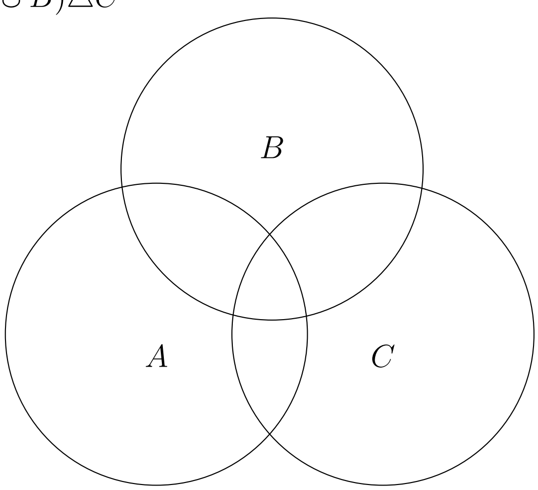


$(A \cap C) \triangle (B \cap C)$

14a $(A \cup B) \triangle C = (A \triangle C) \triangle (B/A)$



$(A \cup B) \triangle C$



$(A \triangle C) \triangle (B/A)$