1.5 The Conditional and BiConditional Connectives

2 Analyze the logical form of the following statements:
a Mary will sell her house only if she can get a good price and find a nice apartment.
Let S represent the phrase: Mary will sell her apartment.
Let G represent the phrase : "Mary can get a good price."
Let A represent the phrase: " Mary finds a nice apartment."
Therefore the logical form of the statement is going to be:
$S \rightarrow (G \wedge A)$
b Having both good credit history and an adequate down payment is a necessary condition for getting a mortage.
Let G represent the phrase: "Having a good credit history."
Let D represent the phrase: "Having an adequate down payment."
Let M represent the phrase: "Getting a mortage".
Therefore the logical form of the statement is going to be:
$(G \wedge D) \rightarrow M$
3 Analyze the logical form of the following statements:
a It is raining, then it is windy and the sun is not shining.
Let R represent the phrase: "It is raining."
Let W represent the phrase: "It is windy."
Let S represent the phrase:"The sun is shining."
When it is raining the sun is not shining, therefore, the logical form of this is:
$R \rightarrow (W \wedge \neg S)$
The converse of the logical connective of a is
$(W \wedge \neg S) \rightarrow R$.
Analyze the following statements and determine whether it is equivalent or converse from 3a
b It is windy and not sunny only if it is raining.
Using the same letters to represent the statements from 3a.
The logcial form of this is going to be:
$(W \wedge \neg S) \rightarrow R$ which is the converse of 3a.
c Rain is a sufficient condition for wind with no sunshine.
In logical form of the statement, using the same letters from 3a we get
$R \rightarrow (W \wedge \neg S)$
This phrase is equivalently identical to the statement from 3a.
d Rain is a ncessary condition for wind with no sunshine.
Using the same letters from 3a, the logical form of the statement is:
$(W \wedge \neg S) \rightarrow R$, which is the converse from 3a.
e It's not raining, if either the sun is shining or it's not windy.
Using the same letters from 3a the logical form of the statement is:
$(S \wedge \neg W) \rightarrow \neg R$
Rewriting this in a different form of a logical connective using DeMorgan's Law, conditional law, and double negation law gives us $R \rightarrow (\neg S \wedge W)$. This statement is equivalent to 3a.
f Wind is a necessary condition for it to be rainy, and so is a lack of sunshine.
$(R \rightarrow W) \wedge (R \rightarrow \neg S)$

Rewriting the statement using conditional law and distributive law gives us:
$R \to (\neg S \wedge W)$

g Either it is windy only if it is raining, or it is not sunny only if it is raining.
$(W \to R) \vee (\neg S \to R)$
Rewriting this using conditional law, double negation law, and conditional law, the rewritten form is
(W $\wedge \neg S) \to R$

5a Show that $P \leftrightarrow Q$ is equivalent to $(P \wedge Q) \vee (\neg P \wedge \neg Q)$
$Webeginbyconstructingatruthtableandiftheconclusionsofthetwostatementareidenticalwitheachotherthe$

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Next we write the truth table for: $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.

P	Q	$P \wedge Q$	$\neg P \wedge \neg Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
T	T	T	F	T
T	F	F	F	F
F	T	F	F	F
F	F	F	T	T

Based off the truth table, since the two have the same conclusions, the two phrases are equivalent.

5b Show that $(P \to Q) \wedge (P \to R)$ is equivalent to $P \to (Q \vee R)$
Similar to 5a, we will construct a truth table, and if the conclusion comes out identical, then we have proven that the two statements are equivalent.

P	Q	R	$P \to Q$	$P \to R$	$(P \to Q) \wedge (P \to R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

We write a truth table for $P \to (Q \vee R)$

P	Q	R	$Q \vee R$	$P \to (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

Since we get the same conclusions, we can conclude that the two statements are equivalent from each other.

2.1 Quantifiers

2 Analyze the logical form of the following statements:
a Anyone who has bought a Rolls Royce chocolate with cash must have a rich uncle.
Let R(x) represent the phrase: ”x bought Rollys Royce chocolate”.
Let C(x) represent the phrase: ”x purchased with cash.”
Let U(x, y) represent the phrase: ”x has an uncle y”.
Let L(y) represent the phrase: ”y is a rich.”
Therefore, the logical form of the statement is:
$\forall x(R(x) \wedge C(x)) \rightarrow (\exists y(U(x,y) \wedge L(y)$
b If anyone in the dorm has measles, then everyone who has a friend in the dorm will have to be quarantined.
Let M(x) represent the phrase: ”x has measles.”
Let D(x) represent the phrase: ”x is in the dorms.”
Let F(x,y) represent the phrase: ”x is friends with y.”
Let Q(y) represent the phrase: ”y will have to be quarantined.”
Therefore, the logical form of the statement is:
$\forall x[(D(x) \wedge M(x)) \rightarrow \forall y(D(y)) \wedge \exists z(F(y,z) \rightarrow Q(z))]$
c If nobody failed the test, then everybody who got an A will tutor someone who got a D.
Let P(x) represent the phrase: ”x failed the test.”
Let A(x) represent the phrase: ”x got an A on the test.”
Let D(x) represent the phrase: ”x got a D on the test.” Let T(x,y) represent the phrase: ”x will tutor y.”
Therefore the logical form of the statement is:
$\forall x[(\neg P(x) \rightarrow \forall y(A(y) \rightarrow \exists z D(z) \wedge T(y,z))]$
3 Analyze the logical forms of the following statements:
a Every number that is larger than x is larger than y.
x and y are the free variables.
$\forall L(z)((L(z) > x) \rightarrow ((L(z) > y))$
b For every number a, the equation $ax^2 + 4x - 2$ has at least one solution iff $a \geq -2$
no free variables.
$\forall a \exists x((ax^2 + 4x - 2) \iff (a \geq -2))$
c All solutions of the inequality $x^3 - 3x < 3$ are smaller than 10.
There is no free variables.
$\forall x((x^3 - 3x < 3) \rightarrow (x < 10))$
d If there is a number x such that $x^2 + 5x = w$ and there is a number y such that $4 - y^2$, then w is between -10 and 10.
W is the free variables.
$\exists x((x^2 + 5x = w) \wedge \exists y(4 - y^2 = w)) \rightarrow (-10 < w < 10)$
8 Same as exercise 7 but with R as its environment.
a) $\forall x \exists y(2x - y = 0)$
$2x = y$, true for all real numbers.

b $\exists y \forall x (2x - y = 0)$
There exists one y where it produces for all x in 2x is
false.
c $\forall x \exists y (x - 2y = 0)$
True because $y = \frac{x}{2}$ is where all x gives one y.
d $\forall x (x < 10 \rightarrow \forall y (y < x \rightarrow y < 9)$
Since we are dealing with a universe discourse of $\mathbb{R}$, this statement is false. For example, while x may be 9.4 and y may be 9.2. While x is less than 10 and y < x but it fails to meet y < 9 since 9.2 is greater than 9.
e $\exists y \exists z (y + z = 100)$
There exists at least one y + z that the sum is 100. This is true because y = 42 and z = 38 that brings the sum into 100, therefore, there is one to one of real numbers that make the sum of y and z to 100.
f $\forall x \exists y (y > x \wedge \exists z (y + z = 100))$
This is true in the discourse of $\mathbb{R}$.